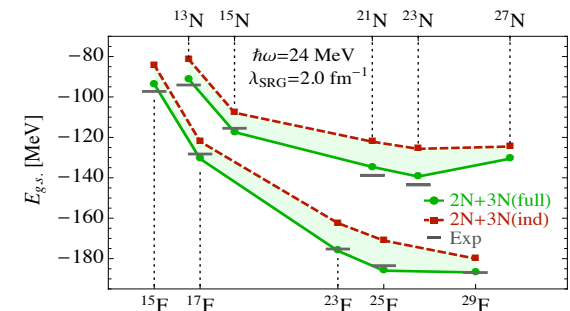# Three-Nucleon Forces in Neutron Rich and Open-Shell Isotopes

C. Barbieri and A. Cipollone *(U. of Surrey, UK)*

V. Somà   *(TU Darmstadt & EMMI, Germany)*
P. Navrátil *(TRIUMF, Canada)*
T. Duguet   *(CEA Saclay, France)*

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TRIUMF

TECHNISCHE UNIVERSITÄT DARMSTADT

EMMI

cea

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Outilne:

- Concept of spectral functions

- Dyson formalism for finite (closed-shell) nuclei
- Inclusion of 3NF and fluorine/nitrogen driplines
- Gorkov-GF for open shells

A. Cipollone, CB, P. Navrátil, arXiv:1303.4900 [nucl-th]

V. Somà, CB, and T. Duguet, Phys. Rev. C 87, 011303 (2013)
V. Somà, T. Duguet, and CB, Phys. Rev. C 84, 064317 (2011)
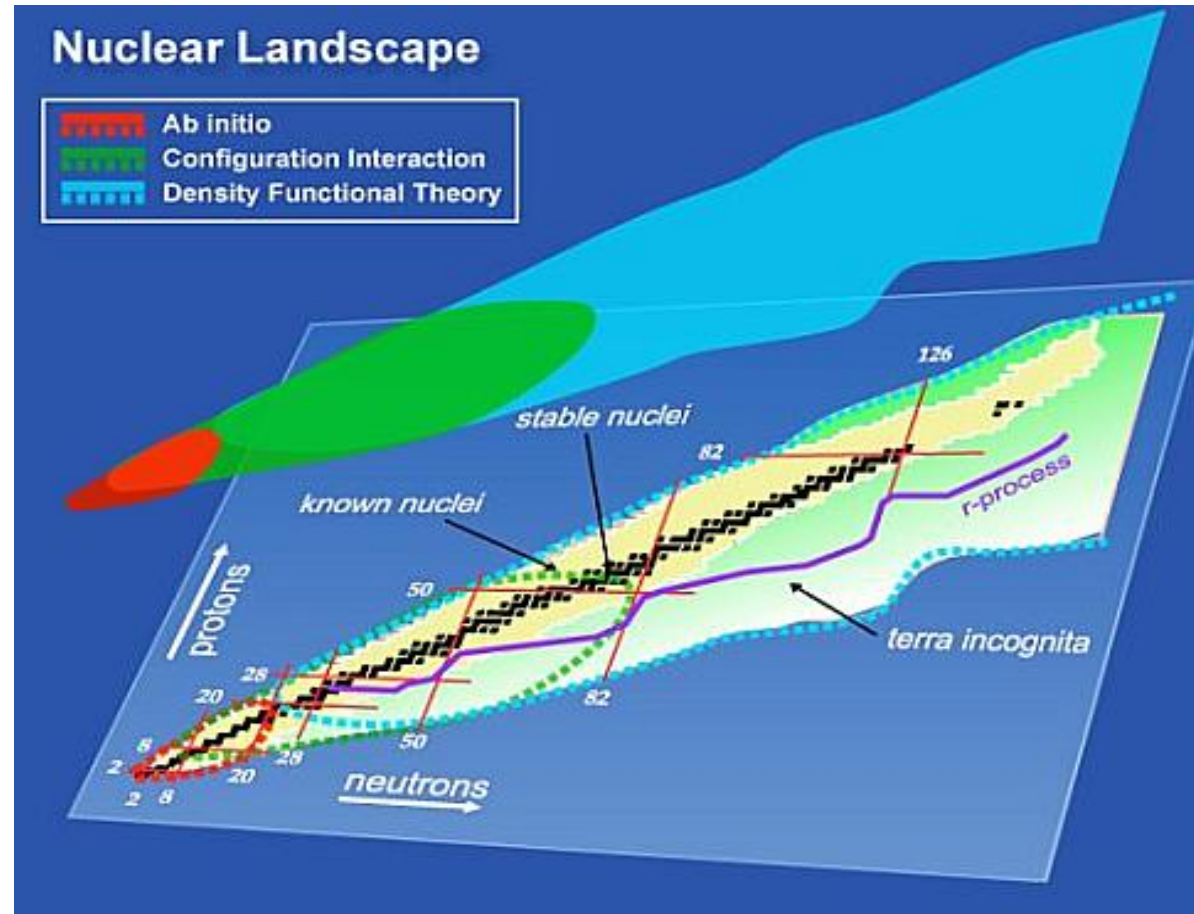
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Towards a unified description of nuclei

Open issues @ mid masses are:

→ Need of good nuclear Hamiltonians (3N forces mostly!)

→ Structure calculations are limited to closed-shells or A±1, A±2

→ Ab-Initio link between structure and reactions. (BUT calculations are GOOD!!!)

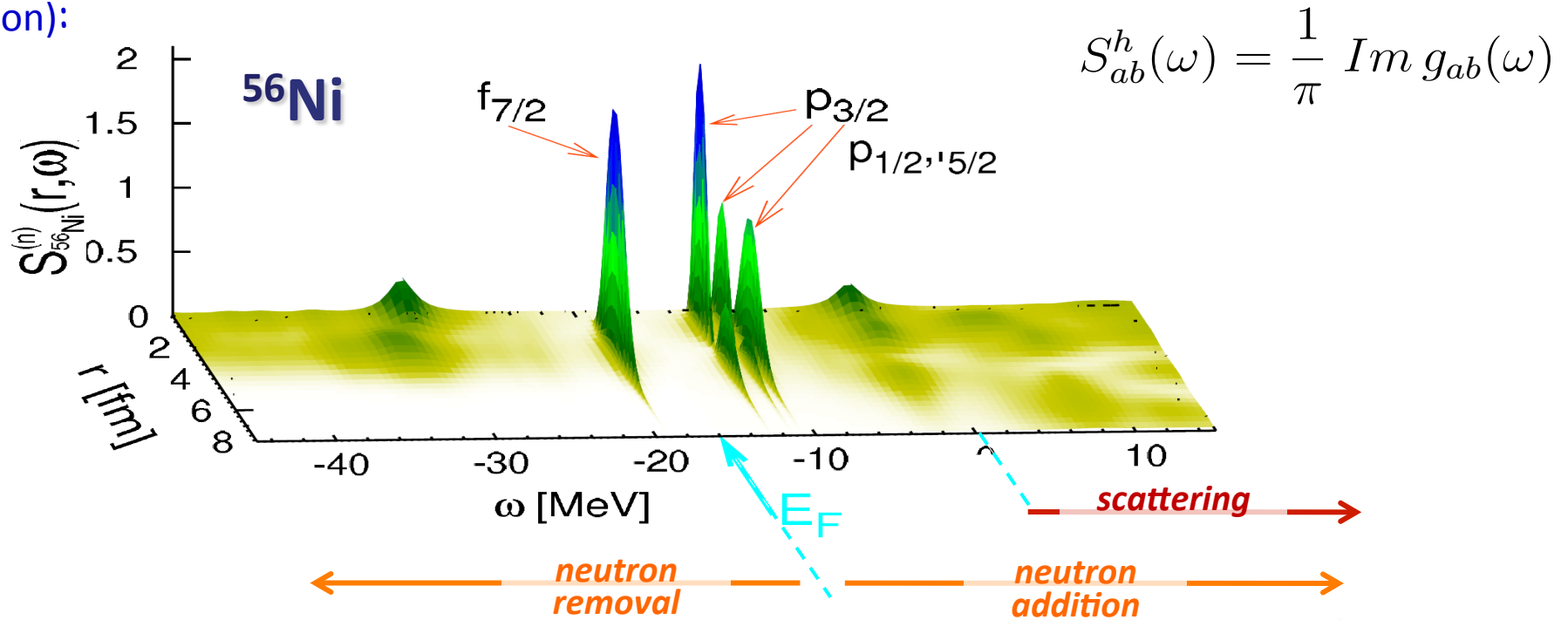**Nuclear Landscape**

- Ab initio
- Configuration Interaction
- Density Functional Theory

stable nuclei

known nuclei

r-process

terra incognita

protons

neutrons

126

82

50

28

20

8

2

→ Green's functions can be naturally extended to: Scattering observable
Open shell nuclei

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Green's functions in many-body theory

One-body Green's function (or propagator) describes the motion of quasi- particles and holes:

$$g_{\alpha\beta}(E) = \sum_n \frac{\langle \Psi_0^A | c_\alpha | \Psi_n^{A+1} \rangle \langle \Psi_n^{A+1} | c_\beta^\dagger | \Psi_0^A \rangle}{E - (E_n^{A+1} - E_0^A) + i\eta} + \sum_k \frac{\langle \Psi_0^A | c_\beta^\dagger | \Psi_k^{A-1} \rangle \langle \Psi_k^{A-1} | c_\alpha | \Psi_0^A \rangle}{E - (E_0^A - E_k^{A-1}) - i\eta}$$

...this contains <u>all the structure information</u> probed by nucleon transfer (spectral function):
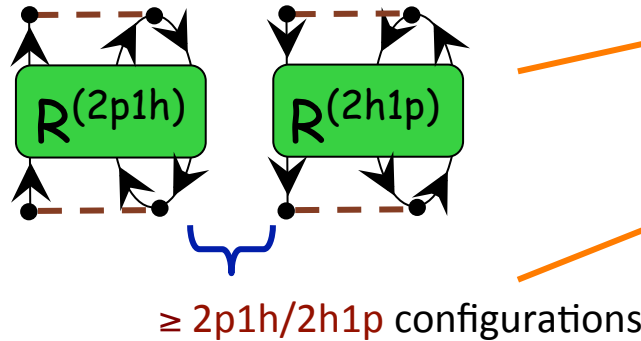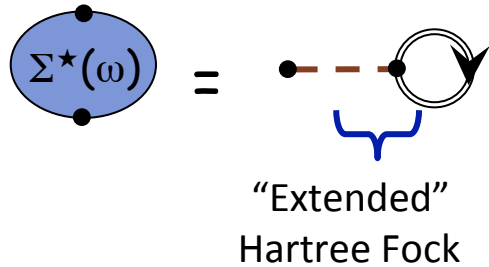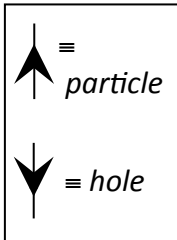
$$S_{ab}^h(\omega) = \frac{1}{\pi} \, Im \, g_{ab}(\omega)$$

[CB, M.Hjorth-Jensen, Pys. Rev. **C79**, 064313 (2009); CB, Phys. Rev. Lett. **103**, 202502 (2009)]

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Calculating the spectral function:

FRPA, ADC(3), and the like...

Faddeev-RPA in two words...

Self-energy (optical potential):

"Extended" Hartree Fock

$\geq$ 2p1h/2h1p configurations

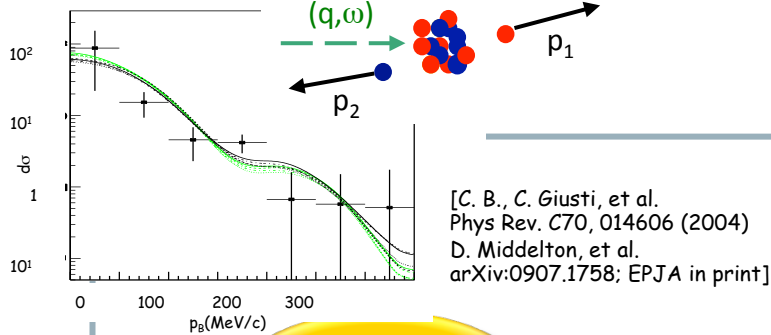## Faddeev-RPA:

$\equiv$ particle

$\equiv$ hole
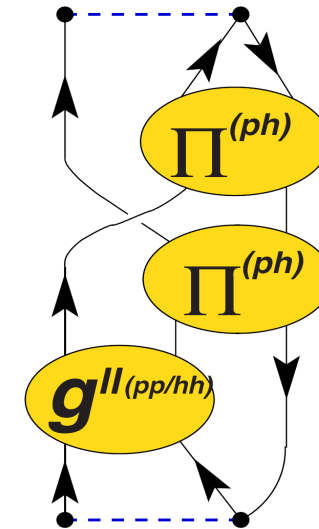
Phys.Rev.C63, 034313 (2001)
Phys.Rev.C65, 064313 (2002)
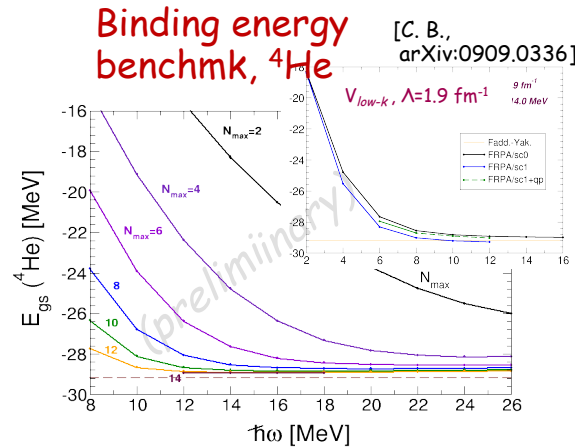Phys.Rev.A76, 052503 (2007)

- A complete expansion requires *all types* of particle-vibration coupling:
  - ✓ $g^{II}(\omega)$ → pairing effects, two-nucleon transfer
  - ✓ $\Pi^{(ph)}(\omega)$ → collective motion, using RPA or beyond
  - ✓ Pauli exchange effects
- The Self-energy $\Sigma^{\star}(\omega)$ yields *both* single-particle states and scattering
- Finite nuclei:→ require high-performance computing

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Self-Consistent Green's Function Approach

$^{16}O(e,e'pn)^{14}N$ @ MAINZ

(q,ω)

p1

p2

[C. B., C. Giusti, et al.
Phys Rev. C70, 014606 (2004)
D. Middelton, et al.
arXiv:0907.1758; EPJA in print]

Binding energy benchmk, ^{4}He

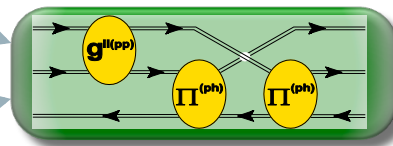[C. B., arXiv:0909.0336]

V_{low-k}, Λ=1.9 fm^{-1}

Ionization energies/ affinities, in atoms

[CB, D. Van Neck,
AIP Conf.Proc.1120,104 ('09) & in prep]

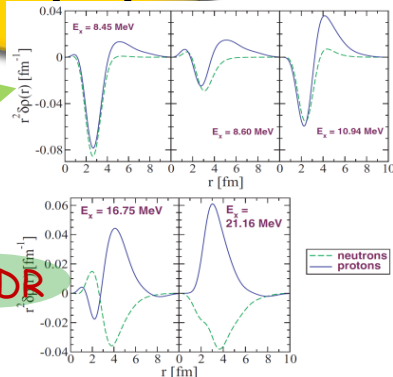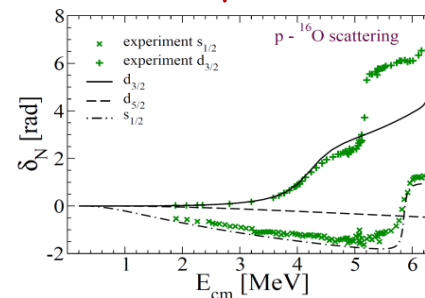|		Hartree-Fock	FRPAc	Experiment [16, 17]
He:	1s	0.918 (+14)	0.9008 (-2.9)	0.9037
Be^{2+}:	1s	5.6672 (+116)	5.6551 (-0.5)	5.6556
Be:	2s	0.3093 (-34)	0.3224 (-20.2)	0.3426
	1s	4.733 (+200)	4.5405 (+8)	4.533
Ne:	2p	0.852 (+57)	0.8037 (+11)	0.793
	1s	1.931 (+149)	1.7967 (+15)	1.782
Mg^{2+}:	2p	3.0068 (+56.9)	2.9537 (+3.8)	2.9499
	1s	4.4827	4.3589	
Mg:	3s	0.253 (-28)	0.280 (-1)	0.281
	2p	2.282 (+162)	2.137 (+17)	2.12
Ar:	3p	0.591 (+12)	0.579 (≈0)	0.579
	3s	1.277 (+202)	1.065 (-10)	1.075
	3s		1.544	
	2p	9.571 (+411)	9.219 (+59)	9.160

$g^{II}(\omega)$

$\Pi^{(ph)}(\omega)$

$g^{II(pp)}$

$\Pi^{(ph)}$ $\Pi^{(ph)}$

Dyson Eq.

Isovector response for ^{32}Ar, ^{34}Ar

Proton Pygmy

IVGDR

b_{HO}=1.8 fm, N$_{max}$=15

full space
truncated

[C. B., K. Langanke, et al., Phys Rev. C77, 024304 (2008)]

$^{16}O(p,\gamma)$

p - ^{16}O scattering

experiment s$_{1/2}$
experiment d$_{3/2}$

d$_{3/2}$
d$_{5/2}$
s$_{1/2}$

$^{16}O(p,\gamma)^{17}F$

total

s$_{1/2}$

d$_{5/2}$
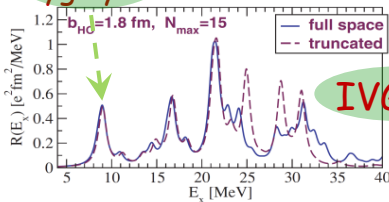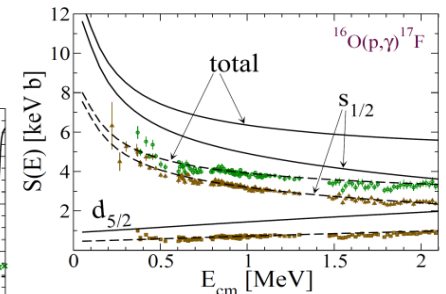
[C. B., B. K. Jennings
Nucl. Phys A758, 395c (2005)
Phys Rev. C72, 014613 (2005)]

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Accuracy of FRPA – *simple atoms/molecules*

binding, eq. bond distances, →
ionization energies (molecules)

98-99% of correlation energy
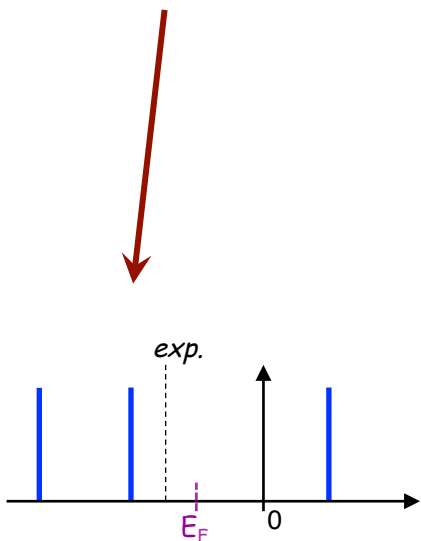is recovered

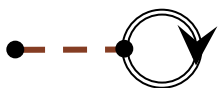< 1% of tot. binding energy

		FTDAc	FRPAc	CCSD(T)	FCI	Expt.
H_2						
	E_0	−1.161	−1.161	−1.164	−1.164	−1.175
	r_{H-H}	0.757	0.757	0.761		0.741
	I	16.03	16.03	16.12		16.08
BeH_2						
	E_0	−15.831	−15.832	−15.835	−15.836	-
	r_{Be-H}	1.337	1.337	1.339		1.340
	I	11.78	11.76	11.89		-
HCl						
	E_0	−460.256	−460.255	−460.254		-
	r_{H-Cl}	1.297	1.293	1.290		1.275
	I	12.24	12.24	12.26		-
HF						
	E_0	−100.224	−100.228	−100.228	−100.231	-
	r_{H-F}	0.916	0.913	0.920		0.917
	I	15.70	15.54	15.42		16.12
H_2O						
	E_0	−76.240	−76.236	−76.241		-
	r_{H-O}	0.964	0.962	0.967		0.958
	Λ_{O-H-O}	102	102	102		104
	I	12.15	12.21	11.94		12.61

binding
energies (atoms)

NB: energies in Hartree
errors in mHartree

	Hartree-Fock	FTDA	FRPA	CCSD	Experiment
He	−2.8617(+42.0)	−2.9028(+0.9)	−2.9029(+0.8)	−2.9039(−0.2)	−2.9037
Be^{2+}	−13.6117(+43.9)	−13.6559(−0.3)	−13.6559(−0.3)	−13.6561(−0.5)	−13.6556
Be	−14.5731(+94.3)	−14.6438(+23.6)	−14.6436(+23.8)	−14.6522(+15.2)	−14.6674
Ne	−128.5505(+387.8)	−128.9343(+4.0)	−128.9381(+0.2)	−128.9353(+3.0)	−128.9383
Mg^{2+}	−198.837(+444)	−199.226(−5)	−199.228(−7)	−199.225(−4)	−199.221
Mg	−199.616(+438)	−200.048(+6)	−200.052(+2)	−200.050(+4)	−200.054
Ar	−526.820(+724)	−527.543(+1)	−527.548(−4)	−527.536(+8)	−527.544
σ_{rms} [mH]	392	9.5(3.6)	9.5(3.4)	6.9(4.2)	

[M. Degroote, D. van Neck, C. B. Phys. Rev. A **83**, 042517 (2011); **85**, 012501 (2012)]

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Accuracy of FRPA – *simple atoms/molecules*

ADC(1) ≡ HF

ADC(2) ≡ 2nd ord.

ADC(3) ≡ FTDA
FRPA

$R^{(2p1h)}$
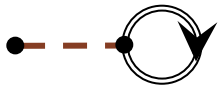
exp.

E_F 0

exp.

E_F 0

exp.

E_F 0
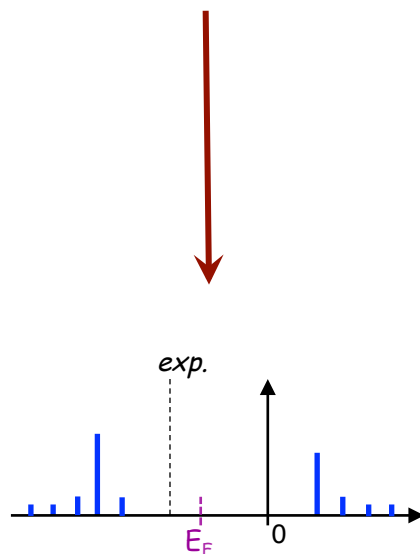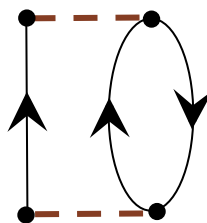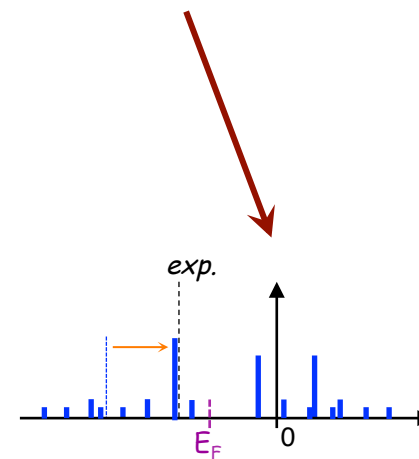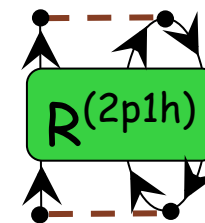
98-99% of correlation energy is recovered
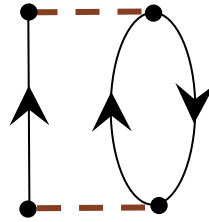
binding, eq. bond distances, →
ionization energies (molecules)

Accuracy of FRPA – *simple atoms/molecules*

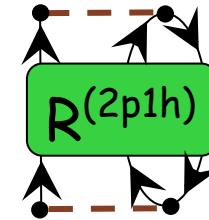ADC(1) ≡ HF

ADC(2) ≡ 2nd ord.

ADC(3) ≡ FTDA
FRPA

$R^{(2p1h)}$

← ionization energies (atoms)

		Hartree-Fock	Second order	FTDA	FRPA	Experiment [63,64]
He	$1s$	0.918(+14)	0.9012(−2.5)	0.9025(−1.2)	0.9008(−2.9)	0.9037
Be^{2+}	$1s$	5.6672(+116)	5.6542(−1.4)	5.6554(−0.2)	5.6551(−0.5)	5.6556
Be	$2s$	0.3093(−34)	0.3187(−23.9)	0.3237(−18.9)	0.3224(−20.2)	0.3426
	$1s$	4.733(+200)	4.5892(+56)	4.5439(+11)	4.5405(+8)	4.533
Ne	$2p$	0.852(+57)	0.752(−41)	0.8101(+17)	0.8037(+11)	0.793
	$2s$	1.931(+149)	1.750(−39)	1.8057(+24)	1.7967(+15)	1.782
Mg^{2+}	$2p$	3.0068(+56.9)	2.9217(−28.2)	2.9572(+7.3)	2.9537(+3.8)	2.9499
	$2s$	4.4827	4.3283	4.3632	4.3589	
Mg	$3s$	0.253(−28)	0.267(−14)	0.272(−9)	0.280(−1)	0.281
	$2p$	2.282(+162)	2.117(−3)	2.141(+21)	2.137(+17)	2.12
Ar	$3p$	0.591(+12)	0.563(−16)	0.581(+2)	0.579(≈ 0)	0.579
	$3s$	1.277(+202)	1.111(+36)	1.087(+12)	1.065(−10)	1.075
	$3s$		1.840	1.578	1.544	
σ_{rms} [mH]		81.4	29.3	13.7	10.6	

[M. Degroote, D. van Neck, C. B. Phys. Rev. A **83**, 042517 (2011); **85**, 012501 (2012)]

NB: energies in Hartree
errors in mHartree

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Scaling – *intermediate state representation methods*

TABLE I. Characteristics of nD-ADC and CC methods (explicit configuration space, perturbation-theoretical consistency for ionization energies (Ω), and ground-state (E_0) energies scaling).

Method	Configuration space	Ω		E_0	Scaling[a]
		$1h$	$2h\text{-}1p$		
ADC(2)	$1h, 2h\text{-}1p$	2	0	2	n^4
ADC(2)-E	$1h, 2h\text{-}1p$	2	1	2	n^5
CCSD	$1h, 2h\text{-}1p$	2	1	3	$n^6 \approx n_u^4 n_o^2$
ADC(3)	$1h, 2h\text{-}1p$	3	1	3	n^5
CCSDT	$1h, 2h\text{-}1p, 3h\text{-}2p$	3	2	4	n^8

2nd ord., (ADC(2))

FRPA, FTDA, (ADC(3))

[A. B. Trofimov and J. Schirmer, J. Chem. Phys. **123**, 144115 (2005)
F. Mertins and J. Schirmer, Phys. Rev. A**53**, 2140 (1996)]

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Three-nucleon interactions

→ application to nuclei
→ need new formalism?

A. Cipollone, P. Navratil, CB
arXiv:1303.4900 [nucl-th]
arXiv:1211.3315 [nucl-th]

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Modern realistic nuclear forces

Chiral EFT for nuclear forces:

	2N forces	3N forces	4N forces
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$		—	—
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$		—	—
N^{2}LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			—
N^{3}LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			
	+ ...	+ ...	+ ...
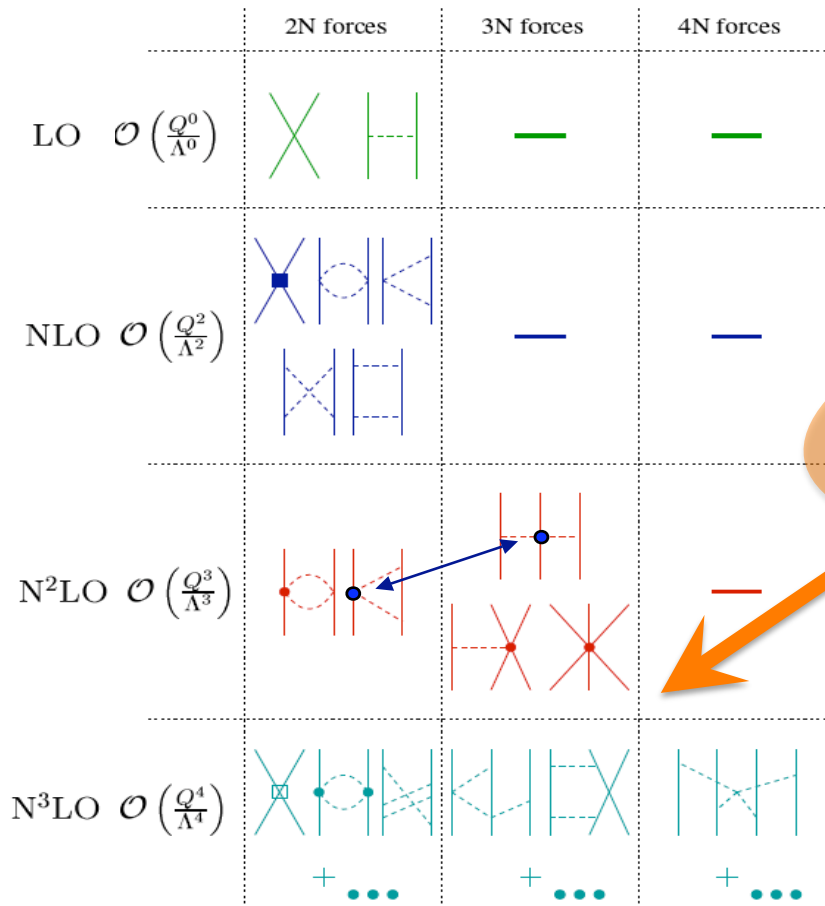
(3NF arise naturally at N2LO)

Single particle spectrum at E$_{fermi}$:
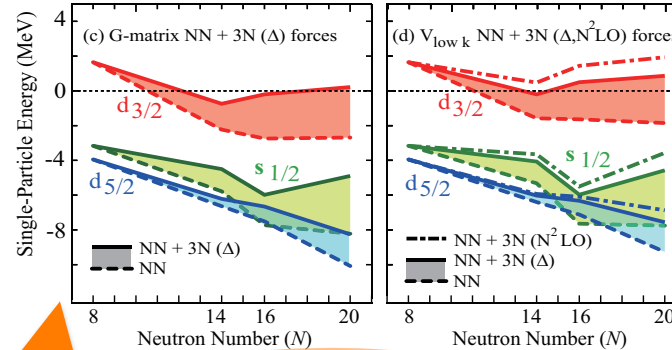
[T. Otsuka et al., Phys Rev. Lett **105**, 32501 (2010)]

Need at LEAST 3NF!!!
("cannot" do RNB physics without...)

Saturation of nuclear matter:
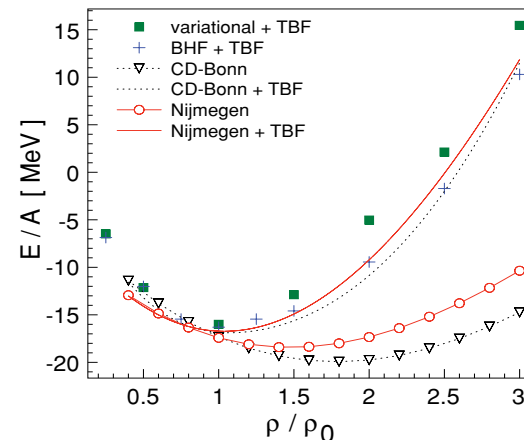
[V. Somà, Phys Rev. C **78**, 054003 (2008))]

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Dyson equation

✳ Propagators solves the Dyson equations

$$g_{ab}(\omega) = g^0_{ab}(\omega) + \sum_{cd} g^0_{ac}(\omega)\, \Sigma_{cd}(\omega)\, g_{db}(\omega)$$
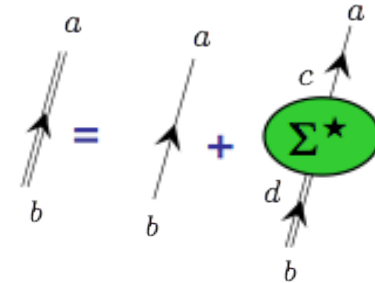
✳ (Hole) single particle spectral function

$$S^h_{ab}(\omega) = \frac{1}{\pi}\, Im\, g_{ab}(\omega) = \sum_k \langle \Psi^{A-1}_k | c_b | \Psi^A_0 \rangle \langle \Psi^A_0 | c^\dagger_a | \Psi^{A-1}_k \rangle\, \delta(\omega - (E^A_0 - E^{A-1}_k))$$

✳ Koltun sum rule (with NNN interactions):

$$\frac{1}{2}\sum_{ab}\int_{-\infty}^{E_F}(t_{ab} + \delta_{ab}\omega)S^h_{ab}(\omega)\, d\omega = \langle T \rangle + \langle V^{NN} \rangle + \frac{3}{2}\langle V^{NNN} \rangle$$

$$\langle V^{NNN} \rangle \approx \frac{1}{6}$$

Inclusion of NNN forces

A. Carbone, A. Cipollone, CB, A. Rios, A Polls

✴ NNN forces can enter diagrams in three different ways:

$$\frac{1}{4}$$ $g^{II}(\omega)$
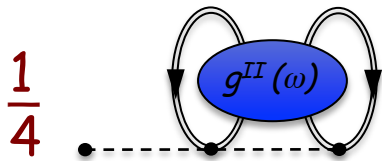
Correction to external
1-Body interaction
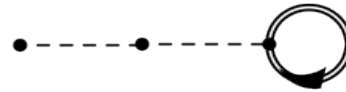
Correction to
non-contracted
2-Body interaction

pure 3-Body
contribution

- Contractions are with *fully correlated density
 matrices* (BEYOND a normal ordering…)

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Inclusion of NNN forces

A. Carbone, A. Cipollone, CB, A. Rios, A Polls

✳ NNN forces can enter diagrams in three different ways:

$\frac{1}{2}$

Correction to external
1-Body interaction
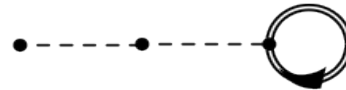
Correction to
<u>non-contracted</u>
2-Body interaction
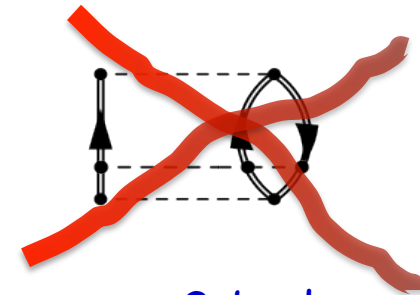
pure 3-body
contribution (small)

- Contractions are with <u>*fully correlated density matrices*</u> (BEYOND a normal ordering…)
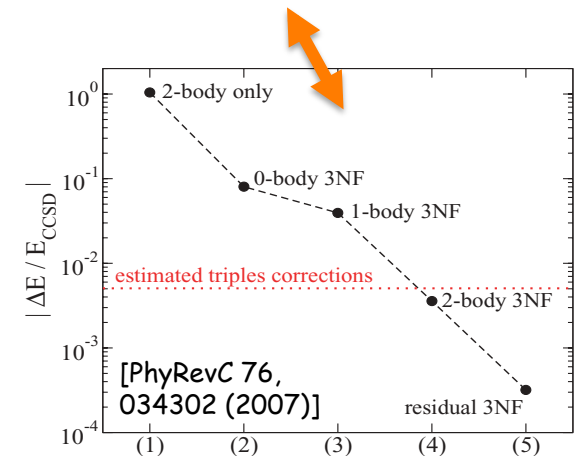
2-body only

0-body 3NF

1-body 3NF

estimated triples corrections

2-body 3NF

10^0

10^{-1}

10^{-2}

10^{-3}

10^{-4}

$|\Delta E / E_{CCSD}|$

[PhyRevC 76, 034302 (2007)]

residual 3NF

(1)   (2)   (3)   (4)   (5)

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Inclusion of NNN forces

A. Carbone, A. Cipollone, CB, A. Rios, A Polls

✳ NNN forces can enter diagrams in three different ways:

→ Define new 1- and 2-body interactions and
use <u>only</u> interaction-irreducible diagrams
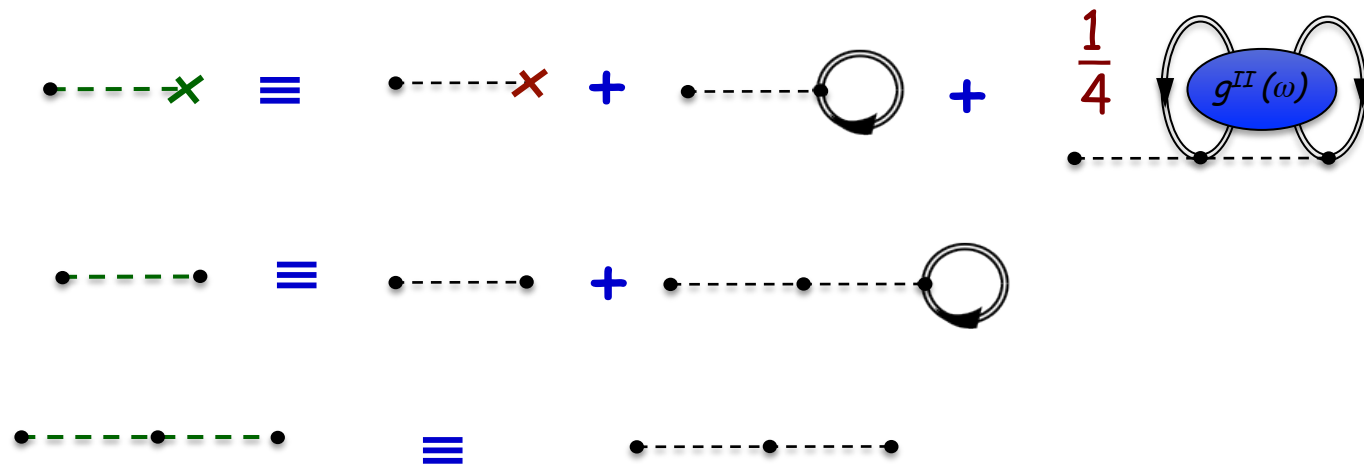
- Contractions are with <u>*fully correlated density matrices*</u>
(BEYOND a normal ordering…)

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NNN forces in FRPA/FTDA formalism

A. Cipollone, CB, P. Navrátil

Use: $\tilde{V}$ = V + W as 2-body potential in all V-irred.
RPA/TDA summations

$$\Pi^{(ph)} = \; \Big| \; \Big| \; + \; \tilde{V} \; \Pi^{(ph)}$$

$$g^{II (pp/hh)} = \; \Big| \; \Big| \; - \; \times \; + \; \tilde{V} \; g^{II (pp/hh)}$$

Then:
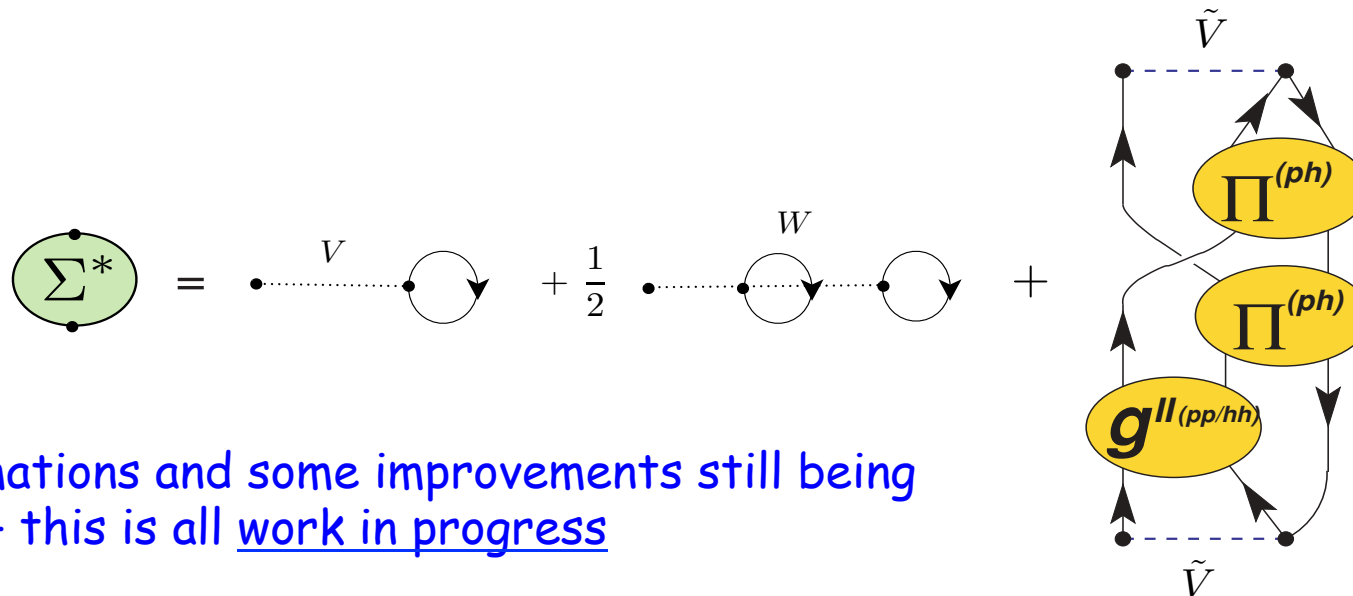
...approximations and some improvements still being assessed – this is all <u>work in progress</u>

NNN forces in FRPA/FTDA formalism

A. Cipollone, CB, P. Navrátil

→ Ladder contributions to static self-energy are negligible (in oxygen)

$\hbar\omega = 24$ MeV
$\lambda_{SRG} = 2.0$ fm^{-1}

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Error estimates for the oxygen chain

- Odd-even isitopes from nucleon addition (A+1) and removal (A-1):

$$E^{A\pm1} = \pm\varepsilon_0^{A\pm1}[\widetilde{H}(A\pm1)] + E_0^A[\widetilde{H}(A\pm1)]$$

- C.O.M. correction important ^{13}O: 1.65 → 0.19 MeV, ^{15}O 1.03→0.02MeV

- discrepancy up to 400 keV (< 1% BE)

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Error estimates for the oxygen chain

A. Cipollone, CB, P. Navrátil, arXiv:1303.4900 [nucl-th]
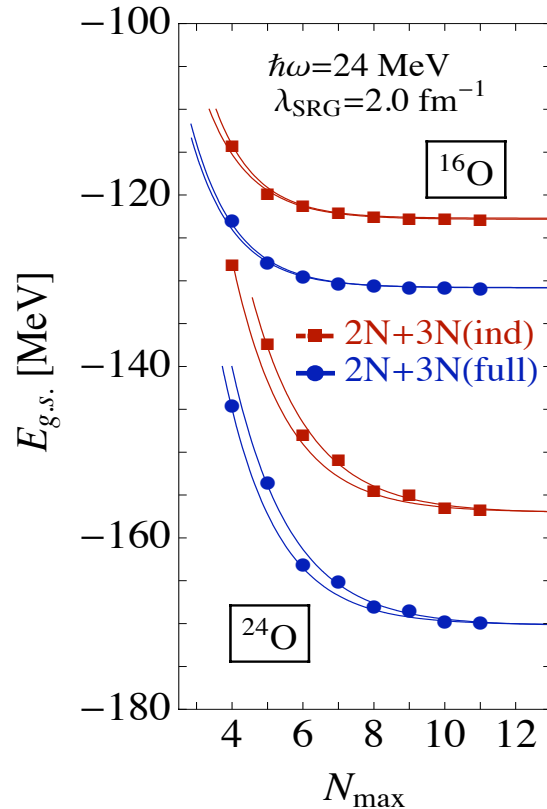
- Convergence: 100-600 keV ≤ 1%
- Many-body truncations ≤ 1%
- λ_{SRG}-dep. (1.88-2.5fm^{-1}) ≈ 3-5%

[see S. Binder, Phys. Rev. C, 87, 021303 (2013).]
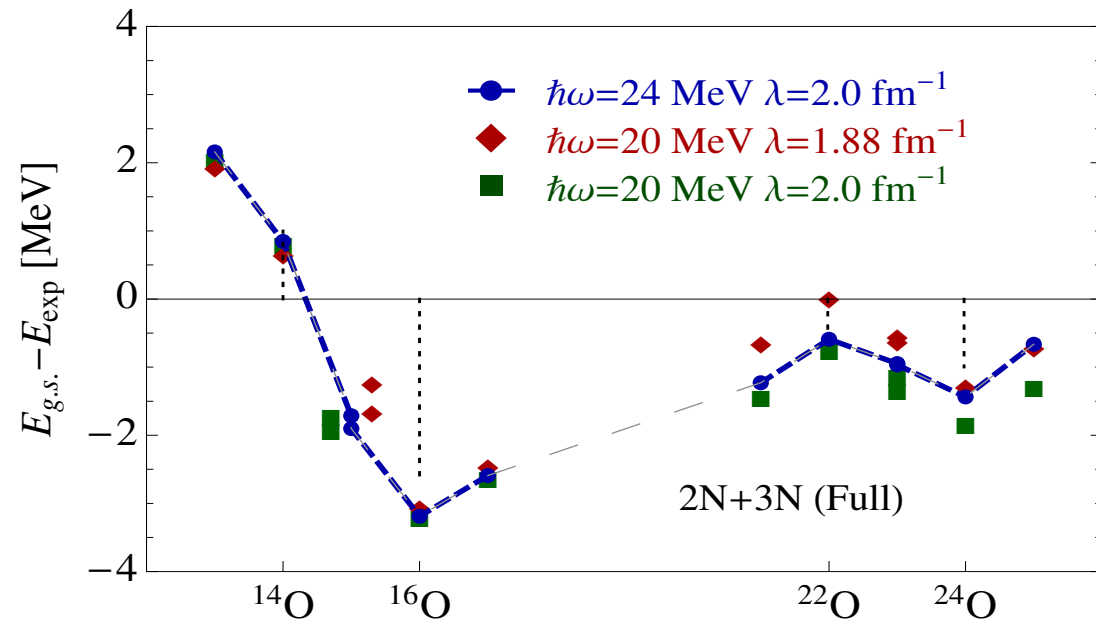
- Total error on predicted binding energies of ≈ 5%

- Max. deviation from experiment ≈ 3%

Results for the N-O-F chains

→ 3NF crucial for reproducing binding energies and driplines around oxygen

→ *d3/2 raised* by genuine 3NF

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

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N3LO (Λ = 500Mev/c) chiral NN interaction evolved to 2N + 3N forces (2.0fm^{-1})
N2LO (Λ = 400Mev/c) chiral 3N interaction evolved (2.0fm^{-1})

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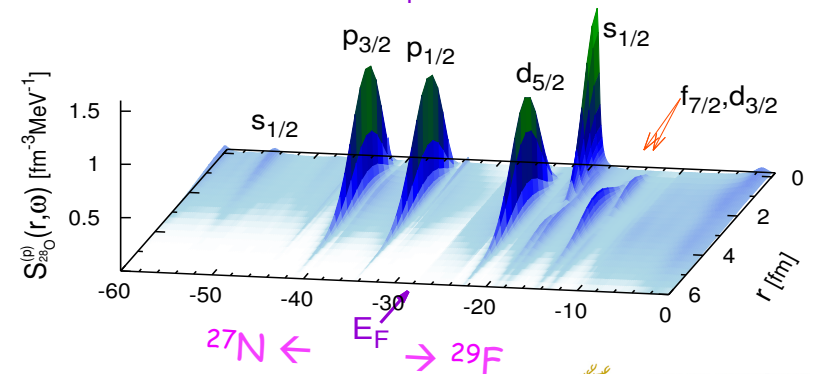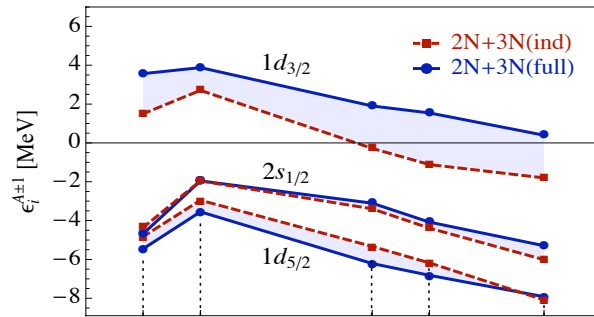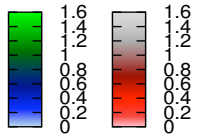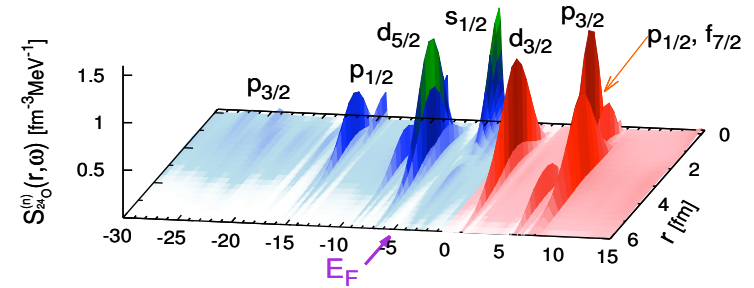# Proton spectral function of Oxygens
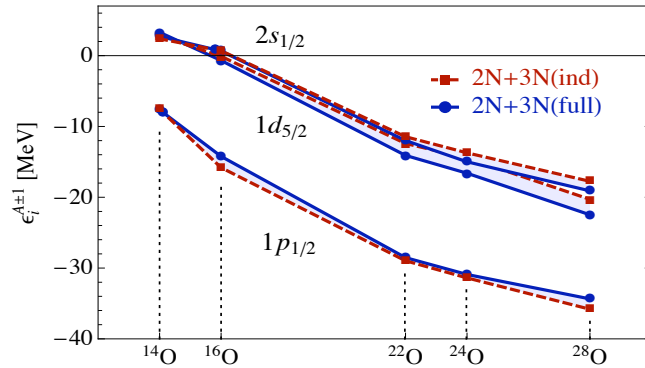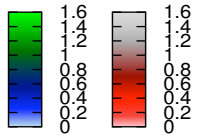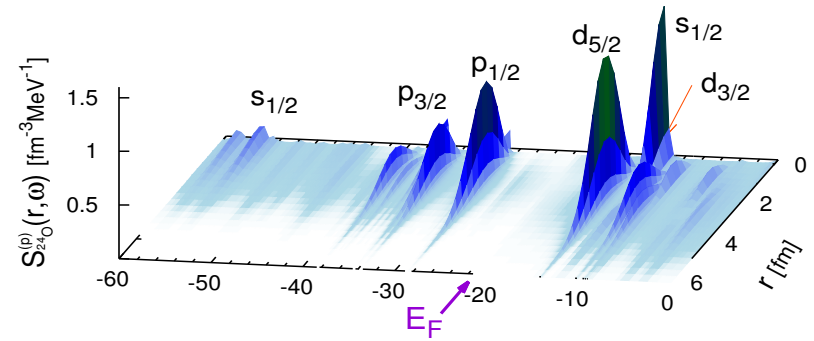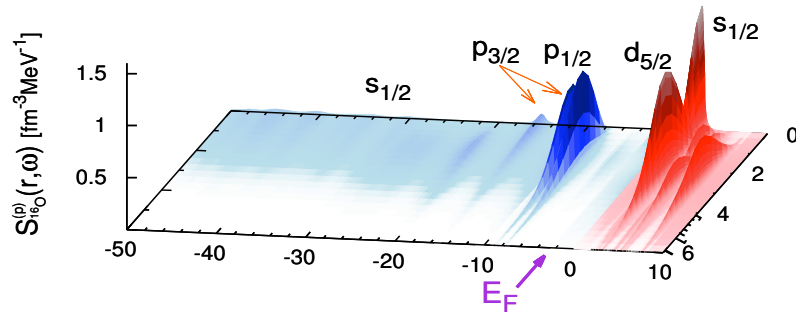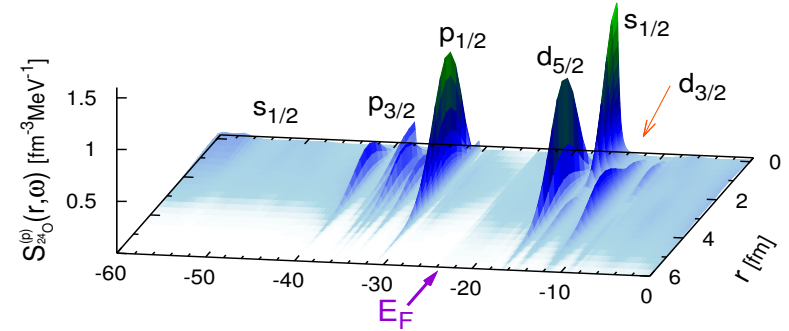
A. Cipollone, CB, P. Navrátil
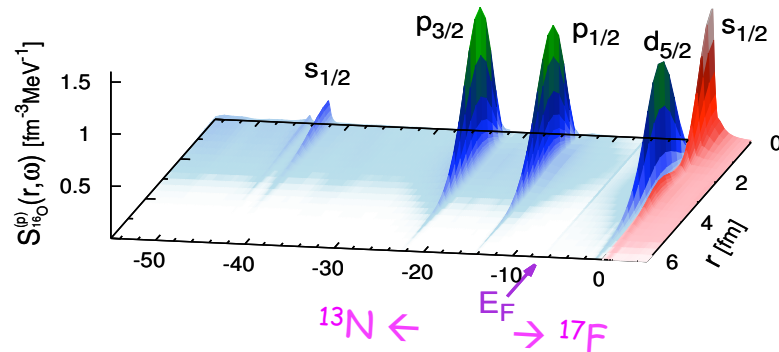
Neutron spectral function of Oxygens

A. Cipollone, CB, P. Navrátil
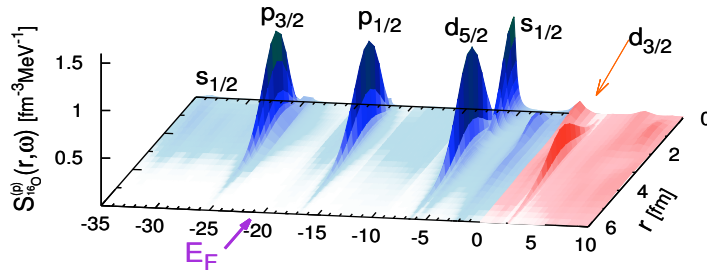
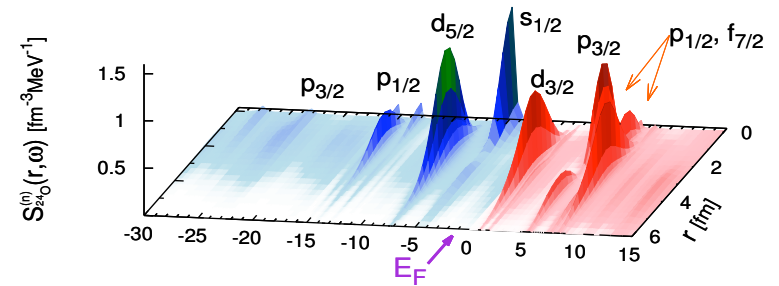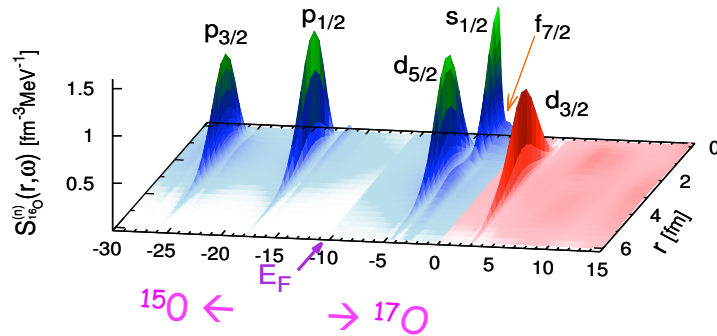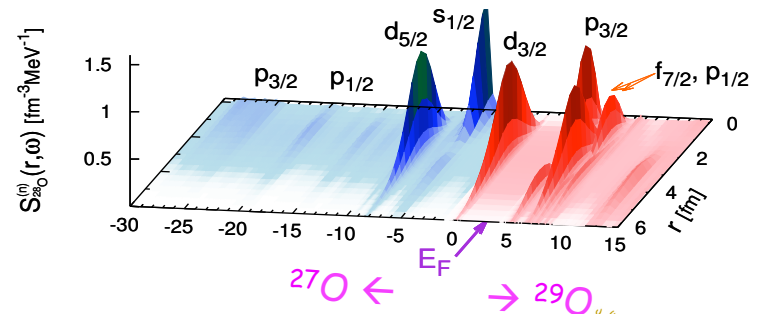# Single nucleon transfer in the oxygen chain

[F. Flavigny et al, PRL**110**, 122503 (2013)]

→ Analysis of $^{14}O(d,t)^{13}O$ and $^{14}O(d,^3He)^{13}N$ transfer reactions @ SPIRAL

Reaction	E^* (MeV)	J^π	R_{rms}^{HFB} (fm)	r_0 (fm)	C^2S_{exp} (WS)	C^2S_{th} $0p + 2\hbar\omega$	R_s (WS)	C^2S_{exp} (SCGF)	C^2S_{th} (SCGF)	R_s (SCGF)
$^{14}O\,(d,t)\,^{13}O$	0.00	$3/2^-$	2.69	1.40	1.69 (17)(20)	3.15	0.54(5)(6)	1.89(19)(22)	3.17	0.60(6)(7)
$^{14}O\,(d,^3He)\,^{13}N$	0.00	$1/2^-$	3.03	1.23	1.14(16)(15)	1.55	0.73(10)(10)	1.58(22)(2)	1.58	1.00(14)(1)
	3.50	$3/2^-$	2.77	1.12	0.94(19)(7)	1.90	0.49(10)(4)	1.00(20)(1)	1.90	0.53(10)(1)
$^{16}O\,(d,t)\,^{15}O$	0.00	$1/2^-$	2.91	1.46	0.91(9)(8)	1.54	0.59(6)(5)	0.96(10)(7)	1.73	0.55(6)(4)
$^{16}O\,(d,^3He)\,^{15}N$ [19,20]	0.00	$1/2^-$	2.95	1.46	0.93(9)(9)	1.54	0.60(6)(6)	1.25(12)(5)	1.74	0.72(7)(3)
	6.32	$3/2^-$	2.80	1.31	1.83(18)(24)	3.07	0.60(6)(8)	2.24(22)(10)	3.45	0.65(6)(3)
$^{18}O\,(d,^3He)\,^{17}N$ [21]	0.00	$1/2^-$	2.91	1.46	0.92(9)(12)	1.58	0.58(6)(10)			
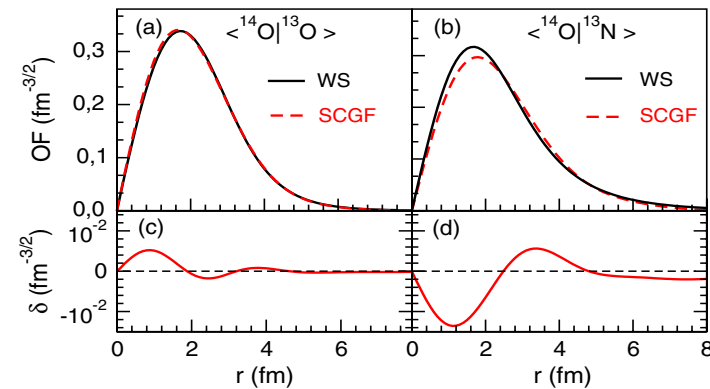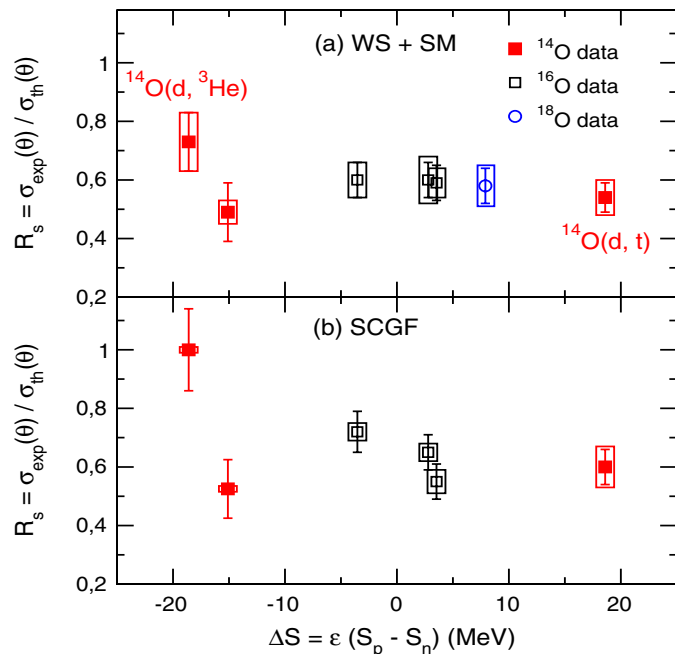
- Overlap functions and strengths from GF

- Rs independent of asymetry

Approaching open-shells in the mid-mass region:

➔ Gorkov theory
➔ proof-of-principle results
   at 2nd order

V. Somà, CB, and T. Duguet, Phys. Rev. C **87**, 011303 (2013)
V. Somà, T. Duguet, and CB, Phys. Rev. C **84**, 064317 (2011)

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Going to open-shells: Gorkov ansatz

✳ Ansatz

$$\ldots \approx E_0^{N+2} - E_0^N \approx E_0^N - E_0^{N-2} \approx \ldots \approx 2\mu$$

✳ Auxiliary many-body state

$$|\Psi_0\rangle \equiv \sum_N^{\text{even}} c_N |\psi_0^N\rangle$$

→ Mixes various particle numbers

→ Introduce a "grand-canonical" potential $\quad \Omega = H - \mu N$

⟹ $|\Psi_0\rangle$ minimizes $\quad \Omega_0 = \langle\Psi_0|\Omega|\Psi_0\rangle$

under the constraint $\quad N = \langle\Psi_0|N|\Psi_0\rangle$

⟹ $\quad \Omega_0 = \sum_{N'} |c_{N'}|^2 \Omega_0^{N'} \approx E_0^N - \mu N$

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Gorkov Green's functions and equations

[V. Somà, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]

✳ Set of 4 Green's functions

$$i\,G^{11}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ a_a(t) a^\dagger_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$

$$i\,G^{21}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ \bar{a}^\dagger_a(t) a^\dagger_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$

$$i\,G^{12}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ a_a(t) \bar{a}_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$

$$i\,G^{22}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ \bar{a}^\dagger_a(t) \bar{a}_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$

[Gorkov 1958]

$$\mathbf{G}_{ab}(\omega) = \mathbf{G}^{(0)}_{ab}(\omega) + \sum_{cd} \mathbf{G}^{(0)}_{ac}(\omega)\, \mathbf{\Sigma}^\star_{cd}(\omega)\, \mathbf{G}_{db}(\omega)$$

Gorkov equations

$$\mathbf{\Sigma}^\star_{ab}(\omega) \equiv \begin{pmatrix} \Sigma^{\star\,11}_{ab}(\omega) & \Sigma^{\star\,12}_{ab}(\omega) \\ \Sigma^{\star\,21}_{ab}(\omega) & \Sigma^{\star\,22}_{ab}(\omega) \end{pmatrix}$$

$$\mathbf{\Sigma}^\star_{ab}(\omega) \equiv \mathbf{\Sigma}_{ab}(\omega) - \mathbf{U}_{ab}$$

1st & 2nd order diagrams

[V. Somà, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]

✳ 1st order ➠ energy-independent self-energy

✳ 2nd order ➠ energy-dependent self-energy

✳ Gorkov equations ⟶ eigenvalue problem

$$\sum_b \left(\begin{array}{cc} t_{ab} - \mu_{ab} + \Sigma_{ab}^{11}(\omega) & \Sigma_{ab}^{12}(\omega) \\ \Sigma_{ab}^{21}(\omega) & -t_{ab} + \mu_{ab} + \Sigma_{ab}^{22}(\omega) \end{array} \right) \Bigg|_{\omega_k} \left(\begin{array}{c} \mathcal{U}_b^k \\ \mathcal{V}_b^k \end{array} \right) = \omega_k \left(\begin{array}{c} \mathcal{U}_a^k \\ \mathcal{V}_a^k \end{array} \right)$$

$$\mathcal{U}_a^{k*} \equiv \langle \Psi_k | \bar{a}_a^\dagger | \Psi_0 \rangle$$
$$\mathcal{V}_a^{k*} \equiv \langle \Psi_k | a_a | \Psi_0 \rangle$$

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Gorkov equations

[V. Somà, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]

$$\sum_b \begin{pmatrix} t_{ab} - \mu_{ab} + \Sigma_{ab}^{11}(\omega) & \Sigma_{ab}^{12}(\omega) \\ \Sigma_{ab}^{21}(\omega) & -t_{ab} + \mu_{ab} + \Sigma_{ab}^{22}(\omega) \end{pmatrix}\Bigg|_{\omega_k} \begin{pmatrix} \mathcal{U}_b^k \\ \mathcal{V}_b^k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}_a^k \\ \mathcal{V}_a^k \end{pmatrix}$$

$$\begin{pmatrix} T - \mu + \Lambda & \tilde{h} & \mathcal{C} & -\mathcal{D}^\dagger \\ \tilde{h}^\dagger & -T + \mu - \Lambda & -\mathcal{D}^\dagger & \mathcal{C} \\ \mathcal{C}^\dagger & -\mathcal{D} & E & 0 \\ -\mathcal{D} & \mathcal{C}^\dagger & 0 & -E \end{pmatrix} \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix}$$
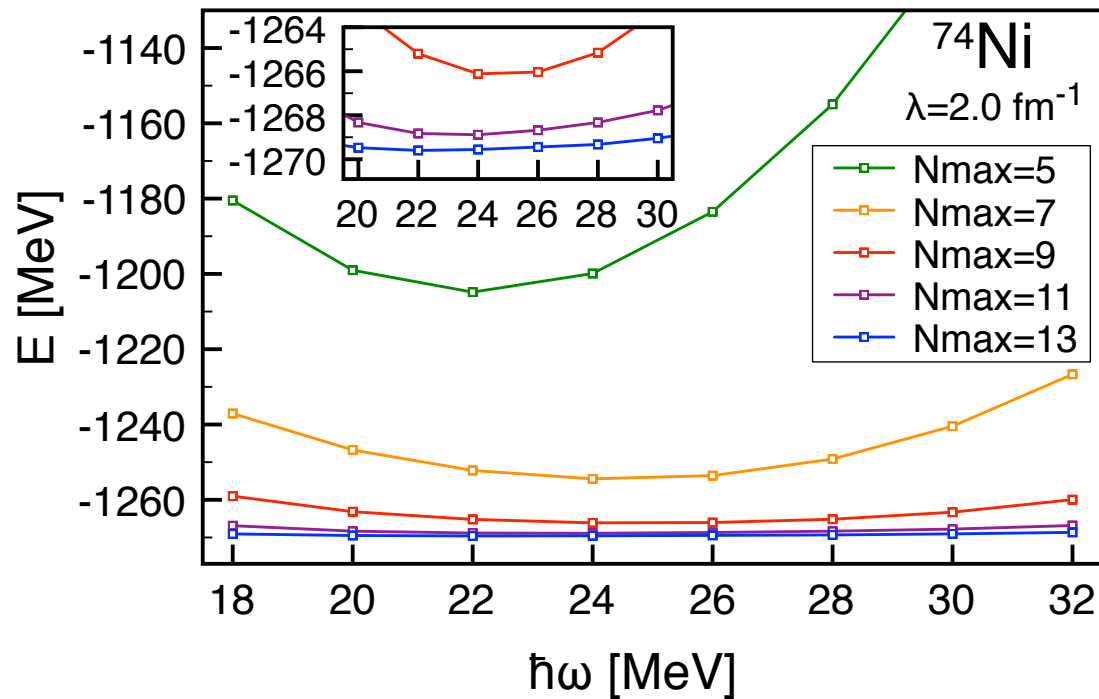
Energy *independent* eigenvalue problem

with the normalization condition $\sum_a \left[|\mathcal{U}_a^k|^2 + |\mathcal{V}_a^k|^2 \right] + \sum_{k_1 k_2 k_3} \left[|\mathcal{W}_k^{k_1 k_2 k_3}|^2 + |\mathcal{Z}_k^{k_1 k_2 k_3}|^2 \right] = 1$

Binding energies

Soma, CB, Duguet, Phys. Rev. C **87**, 011303 (2013)

$\lambda=2.0$ fm^{-1}

^{44}Ca

^{74}Ni

$\lambda=2.0$ fm^{-1}

Legend:
- Nmax=5
- Nmax=7
- Nmax=9
- Nmax=11
- Nmax=13

1st order

2nd order

⟹ NN interaction:
chiral N^{3}LO SRG-evolved to 2.0 fm^{-1}

[Entem and Machleidt 2003]

^{43}Ca

⟹ Very good convergence

^{74}Ni ⟹ From N=13 to N=11 → 200 keV
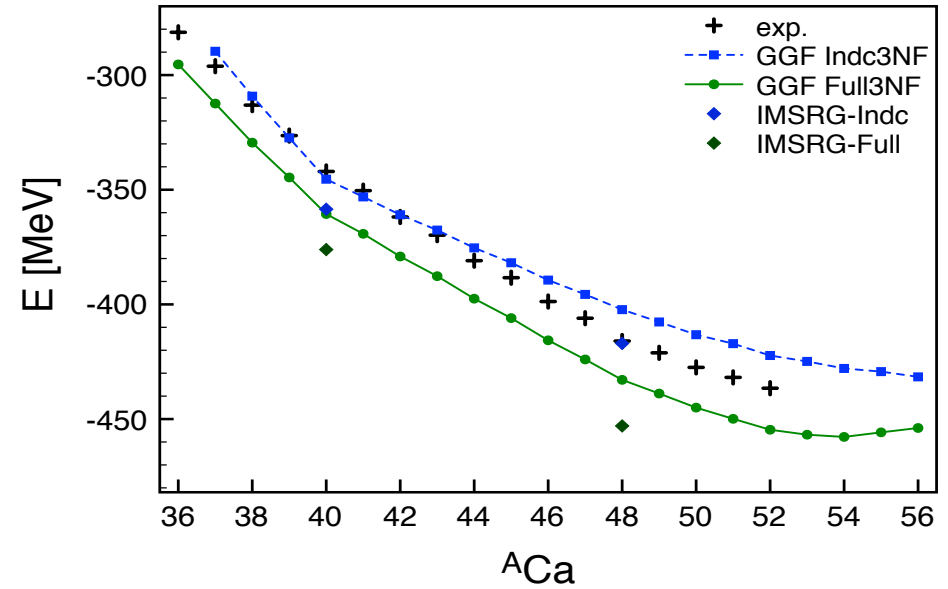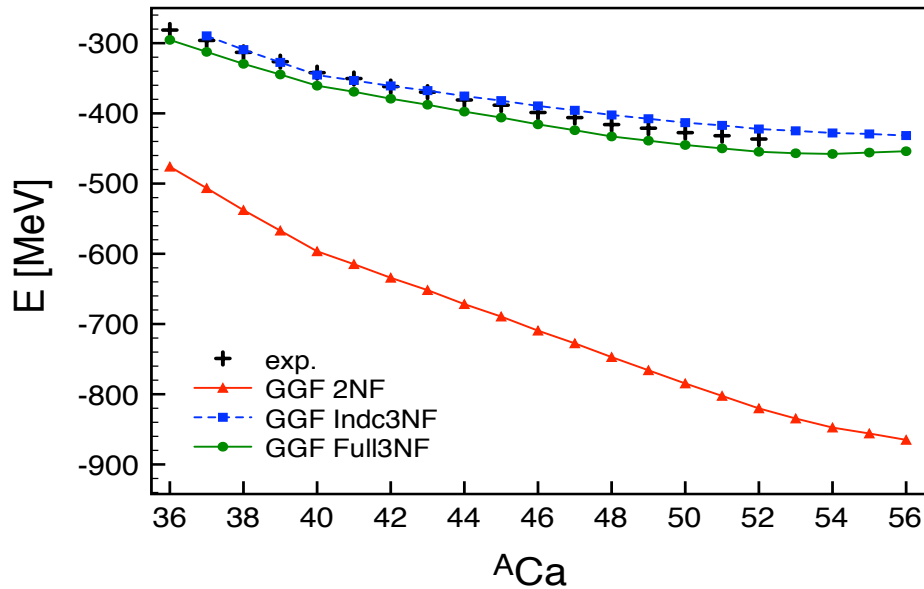
E (N=13) = -1269.6 MeV
E (N=∞) = -1269.7(2) MeV

(Extrapolation to infinite model space from
[Furnstahl, Hagen, Papenbrock 2012] and [Coon et al. 2012])

Calcium isotopic chain

✪ *Ab initio* calculation of the whole Ca chain with NN + 3N forces

➡ 3NF bring energies close to experiment

➡ Induced 3NF and full 3NF investigated

➡ Original 3NF correct the energy curvature

[Somà *et al.* in preparation]

N3LO (Λ = 500Mev/c) chiral NN interaction evolved to 2N + 3N forces (2.0fm^{-1})
N2LO (Λ = 400Mev/c) chiral 3N interaction evolved (2.0fm^{-1})

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Two-neutron separation energies

✪ Neutron-rich extremes of the nuclear chart

➠ Good agreement with measured S2n

➠ Towards a quantitative *ab initio* description of the medium-mass region
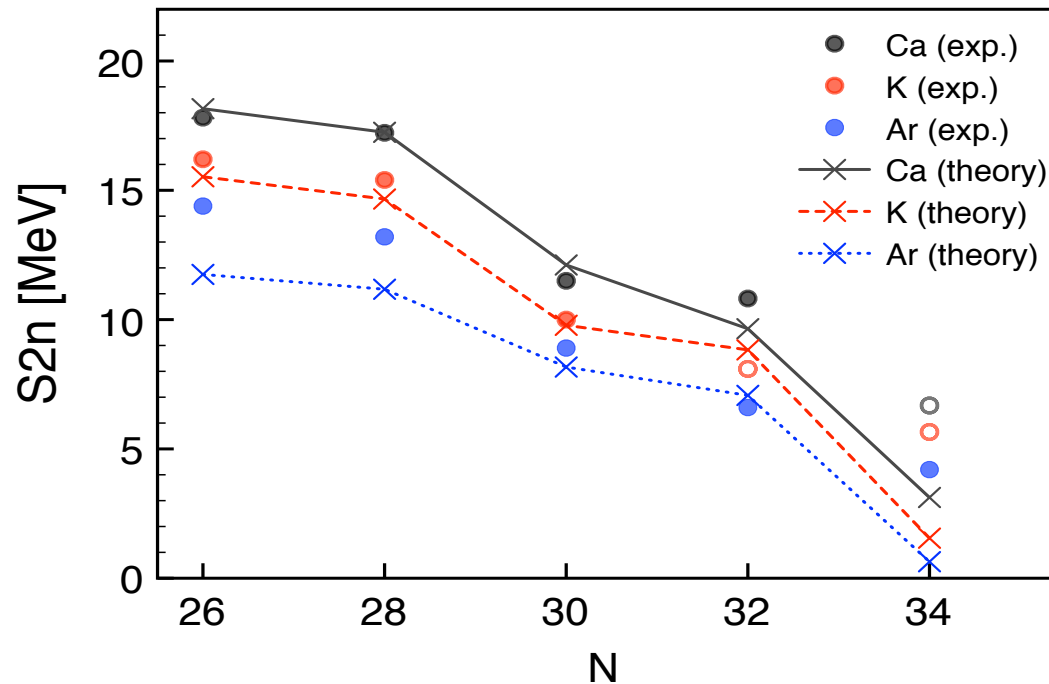
Knockout & transfer experiments

✪ Neutron removal from proton- and neutron-rich Ar isotopes @ NSCL

Isotopes	lj^π	Sn(MeV)	ΔS (MeV)	(theo.) SF(LB-SM)	(expt.) SF(JLM + HF)	Rs(JLM + HF)	(expt.) SF(CH89)	Rs(CH89)
^{34}Ar	$s1/2^+$	17.07	12.41	1.31	0.85 ± 0.09	0.65 ± 0.07	1.10 ± 0.11	0.84 ± 0.08
^{36}Ar	$d3/2^+$	15.25	6.75	2.10	1.60 ± 0.16	0.76 ± 0.08	2.29 ± 0.23	1.09 ± 0.11
^{46}Ar	$f7/2^-$	8.07	-10.03	5.16	3.93 ± 0.39	0.76 ± 0.08	5.29 ± 0.53	1.02 ± 0.10

[Lee *et al.* 2010]

$$\Delta S = Sn - Sp$$

	Sn (MeV)	ΔS (MeV)	SF
^{34}Ar	33.0	18.6	1.46
^{36}Ar	27.7	7.5	1.46
^{46}Ar	16.0	-22.3	5.88

Gorkov GF NN

	Sn (MeV)	ΔS (MeV)	SF
^{34}Ar	22.4	15.5	1.56
^{36}Ar	15.3	7.2	1.54
^{46}Ar	6.5	-15.7	6.64

Gorkov GF NN + 3N

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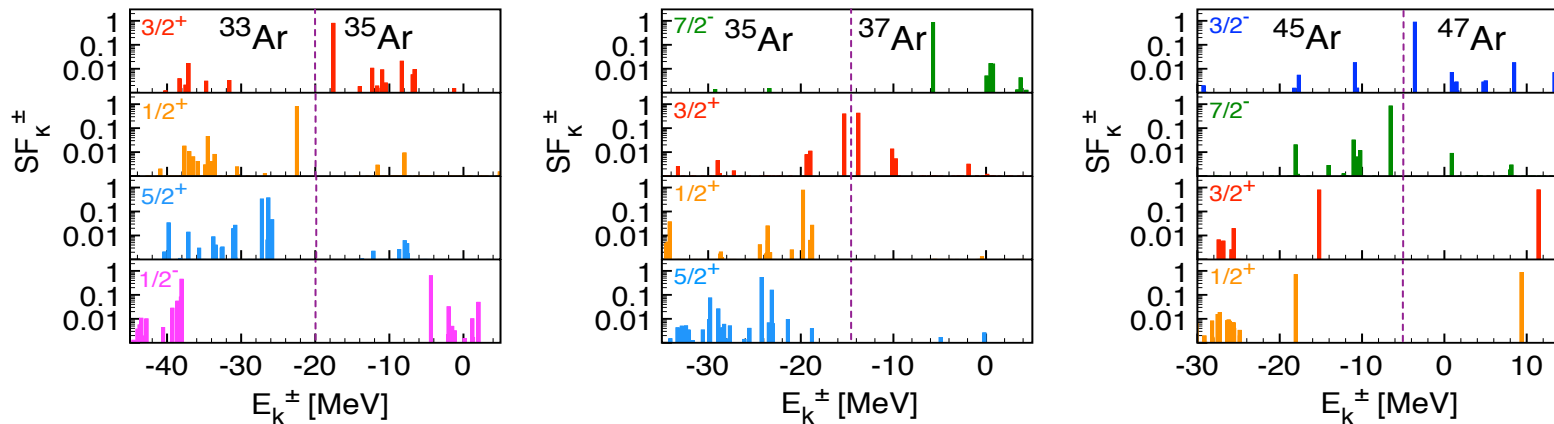
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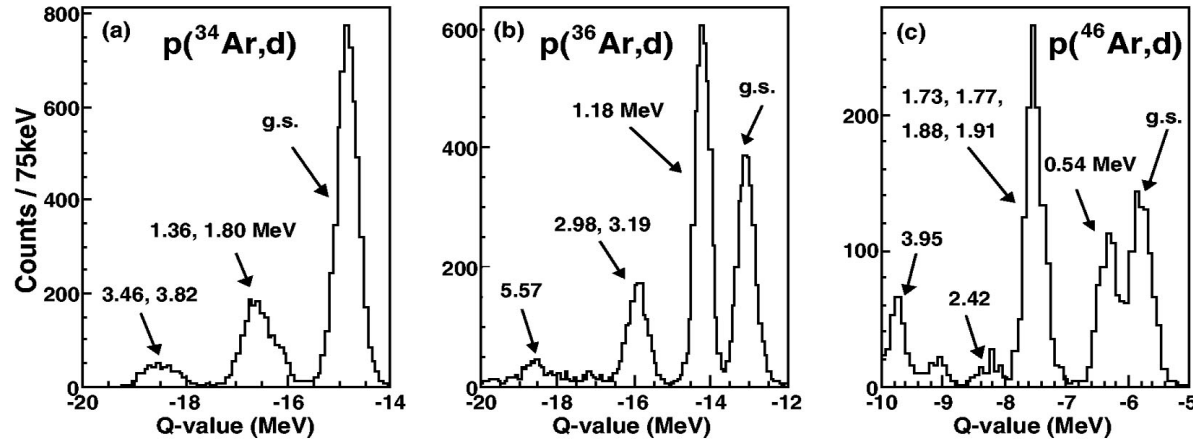
[Lee *et al.* 2010]

Conclusions

• Self-Consistent Green's Functions (SCGF), is a microscopic *ab-initio* method applicable to medium mass nuclei. *Greatest advantage* is the link to several (experimentally accessible) information.

• *Addition of* three nucleon forces (3NF):

→ Effective N+NN hamiltonian with int. irred. diagrams
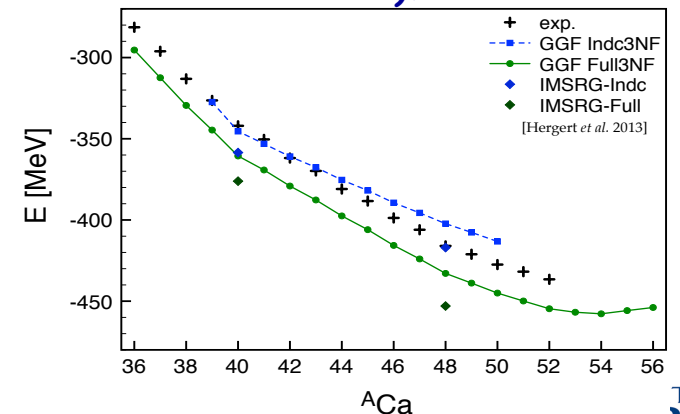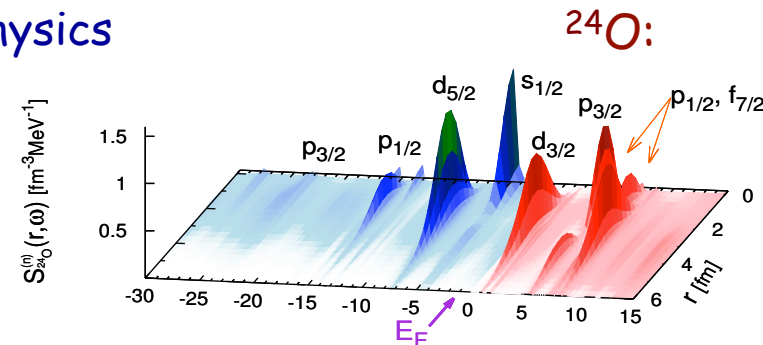
→ Needed to properly predict BEs and dripline physics

→ *First ab-initio study of ^AN and ^AF chains*

^{24}O:

• *Proof of principle calculations Gorgov theory are successful at 2nd order.* This de facto show opens a whole new path:

→ Open-shell nuclei (*many*, *not* previously approachable otherwise!).

→ Reactions at driplines.
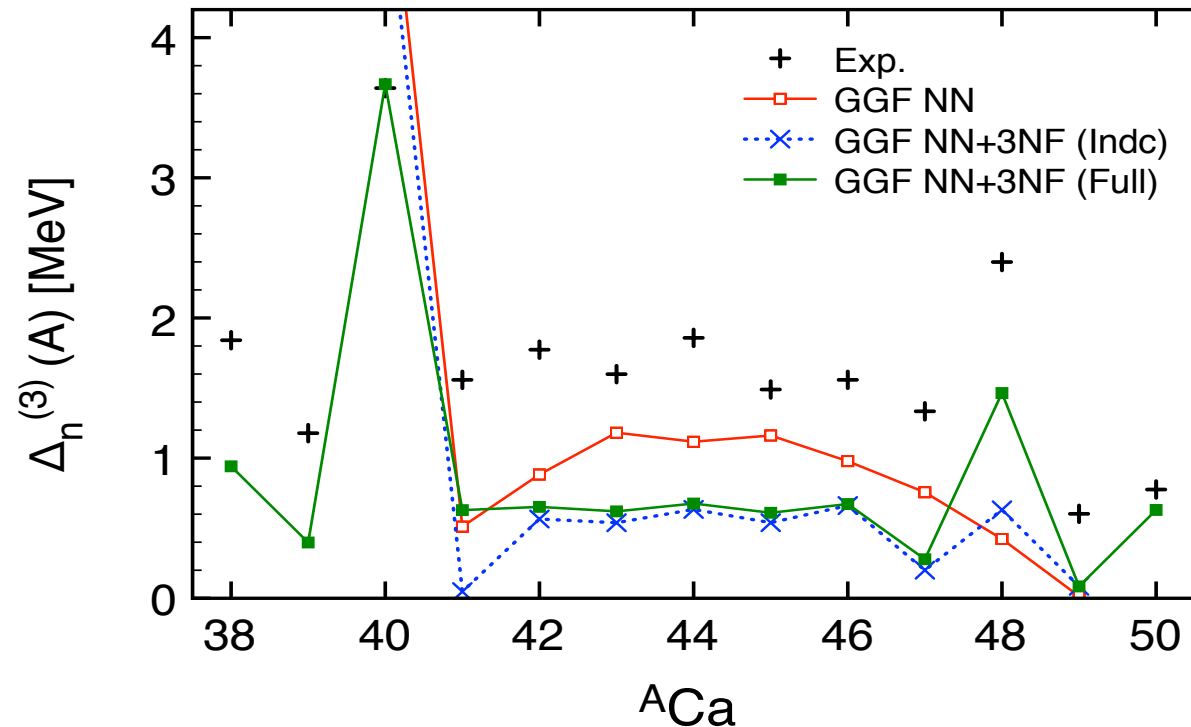
→ Structure of next generation EDF.

Thank you for your attention!!!

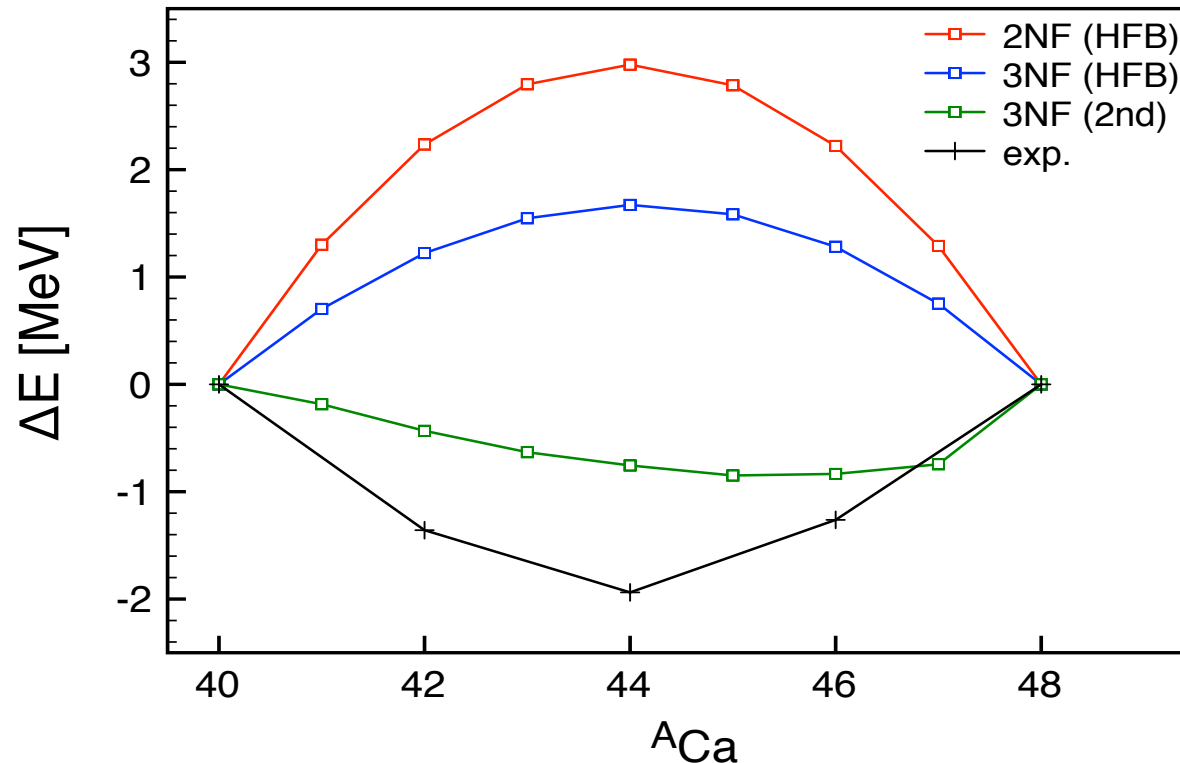[Hergert *et al.* 2013]

exp.
GGF Indc3NF
GGF Full3NF
IMSRG-Indc
IMSRG-Full

Pairing gaps

✪ Three-point mass differences

$$\Delta_n^{(3)}(A) = \frac{(-1)^A}{2} [E_0^{A+1} - 2\,E_0^A + E_0^{A-1}]$$

University of Surrey

Pairing gaps

✪ Inversion of odd-even staggering

➠ Second order _and_ 3NF necessary to invert the staggering