Study of Regulator Dependence of Chiral Potentials in Calculations for Infinite Nuclear Matter

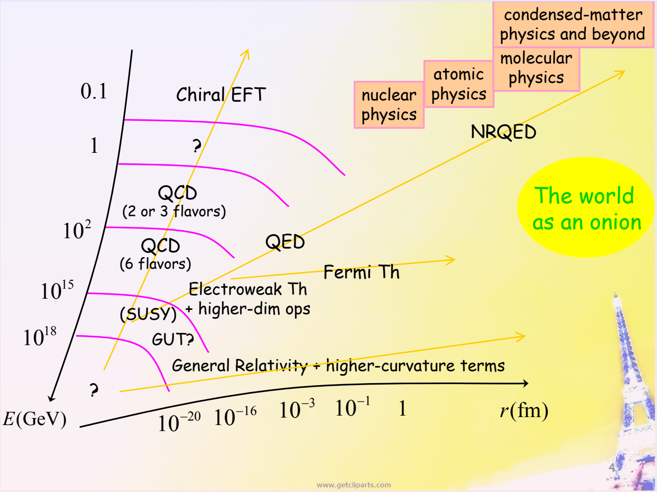
Luigi Coraggio

Istituto Nazionale di Fisica Nucleare - Sezione di Napoli

June, 13th 2013

Acknowledgements

- J. W. Holt (University of Washington)
- N. Itaco (University of Naples and INFN)
- R. Machleidt (University of Idaho)
- L. E. Marcucci (University of Pisa and INFN)
- F. Sammarruca (University of Idaho)
- L. C. (INFN)



Courtesy of U. van Kolck

lnQ

perturbative QCD

~ 1 GeV $M_{QCD} \sim m_N,\ m_\rho,\ 4\pi f_\pi,\ \ldots$

hadron theory with chiral simmetry

~ 100 MeV $M_{nuc} \sim f_\pi,\ 1/r_{NN},\ m_\pi,\ \ldots$

~ 30 MeV $\xi \sim 1/a_{NN}$

Nuclear physics exhibits a separation of scales

$\Downarrow$

To resort to EFT could be a valuable way to describe the physics of nuclei, since the underlying theory QCD is not solvable

Weinberg's "theorem"

"If one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates S-matrix elements with this Lagrangian to any order in perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles" [1]

[1] *S. Weinberg, Physica A* **96** *327 (1979)*

Chiral EFT for nuclear theory

- Identify the relevant degrees of freedom (nucleons,pions, deltas) and symmetries of the problem (chiral symmetry) .

Chiral EFT for nuclear theory

- Identify the relevant degrees of freedom (nucleons,pions, deltas) and symmetries of the problem (chiral symmetry) .
- Build up the most general Lagrangian consistent within these constraints.

$$\mathcal{L}_{\pi\pi} = \frac{f_\pi^2}{4}\,\mathrm{tr}\left[\,\partial_\mu U \partial^\mu U^\dagger + m_\pi^2 (U + U^\dagger)\,\right] + \ldots$$

$$\mathcal{L}_{\pi N} = \bar{\Psi}\left(i\gamma^\mu D_\mu - M_N + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu + \ldots\right)\Psi$$

$$\mathcal{L}_{NN} = -\frac{1}{2}C_S \bar{N}N\bar{N}N - \frac{1}{2}C_T(\bar{N}\vec{\sigma}N)\cdot(\bar{N}\vec{\sigma}N) + \ldots$$

Chiral EFT for nuclear theory

- Identify the relevant degrees of freedom (nucleons,pions, deltas) and symmetries of the problem (chiral symmetry) .
- Build up the most general Lagrangian consistent within these constraints.

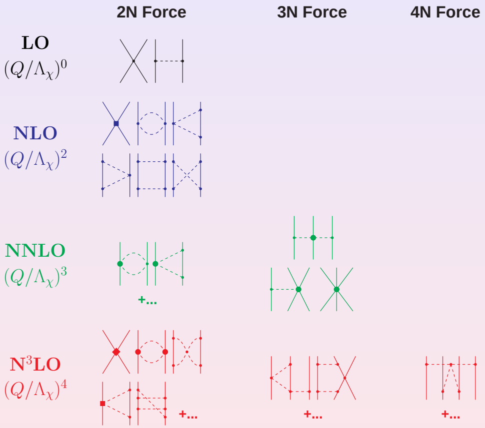
$$\mathcal{L}_{\pi\pi} = \frac{f_\pi^2}{4} \mathrm{tr} \left[\partial_\mu U \partial^\mu U^\dagger + m_\pi^2 (U + U^\dagger) \right] + \ldots$$

$$\mathcal{L}_{\pi N} = \bar{\Psi} \left(i\gamma^\mu D_\mu - M_N + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu + \ldots \right) \Psi$$

$$\mathcal{L}_{NN} = -\frac{1}{2} C_S \bar{N} N \bar{N} N - \frac{1}{2} C_T (\bar{N} \vec{\sigma} N) \cdot (\bar{N} \vec{\sigma} N) + \ldots$$

- Perform a perturbative expansion of this Lagrangian for momenta $q < \Lambda$, and adjust the coefficients to the physical observables (renormalization).

The chiral perturbative expansion



An important observation: ChPT allows the contruction of nuclear two- and many-body forces on an equal footing.

An important observation: ChPT allows the contruction of nuclear two- and many-body forces on an equal footing.

More precisely: most interaction vertices in the 3NF, as well as in the 4NF, occur in the 2NF too.

An important observation: ChPT allows the contruction of nuclear two- and many-body forces on an equal footing.

More precisely: most interaction vertices in the 3NF, as well as in the 4NF, occur in the 2NF too.

Consequently, the corresponding parameters LECs are consistently the same in the 2NF and 3NF.

A main issue: ChPT introduces a cutoff Λ and the calculated observables depends on its choice

A main issue: ChPT introduces a cutoff Λ and the calculated observables depends on its choice

Necessarily, the chiral hamiltonian has to be renormalized for each chosen cutoff via fixing the chosen LECs to fit the available experimental data (*NN* scattering data, deuteron and triton binding energies, ...).

A main issue: ChPT introduces a cutoff Λ and the calculated observables depends on its choice

Necessarily, the chiral hamiltonian has to be renormalized for each chosen cutoff via fixing the chosen LECs to fit the available experimental data (*NN* scattering data, deuteron and triton binding energies, ...).

Cutoff invariance can be then guaranteed, at least for the two- and three-body systems.

What about the many-body systems?

Infinite nuclear matter: this is an interesting environment to study the dependence on Λ of results of a many-mody calculation

Infinite nuclear matter: this is an interesting environment to study the dependence on Λ of results of a many-mody calculation

We calculate infinite nuclear matter EOS starting from chiral 2NF and 3NF defined within different cutoffs

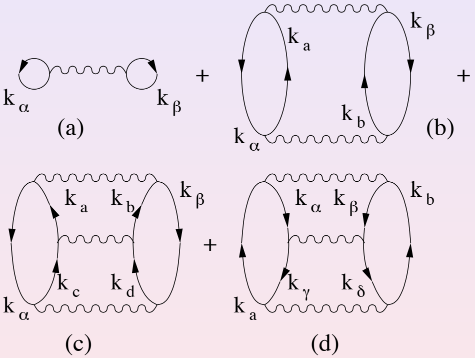
Infinite nuclear matter: this is an interesting environment to study the dependence on Λ of results of a many-mody calculation

We calculate infinite nuclear matter EOS starting from chiral 2NF and 3NF defined within different cutoffs

Perturbative approach: we perform a Goldstone expansion of the binding energy per nucleon E/A up to third order in the energy.

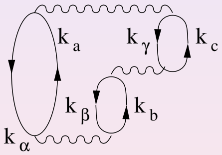
L. C., J. W. Holt, N. Itaco, R. Machleidt, Phys. Rev. C **87**, *014322 (2013)*

The perturbative expansion



What has been left out ...

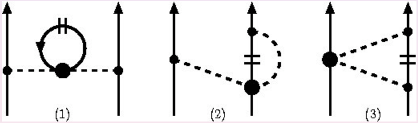
One third-order topology is missing:

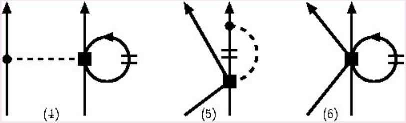


$$3p - 3h$$

The density-dependent effective *NN* potential

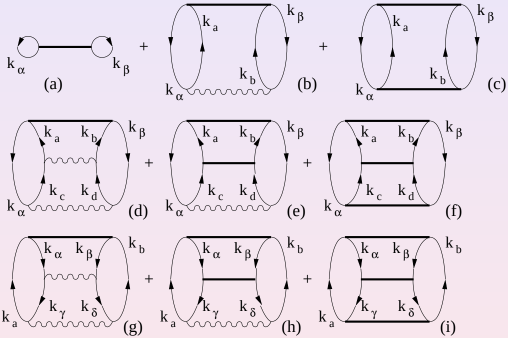
The effect of the chiral N^{2}LO 3NF has been taken into account adding a density-dependent two-body potential $\overline{V}_{NNN}$[1] to the N^{3}LO two-body one (see also Jeremy Holt's talk):





[1] *J. W. Holt, N. Kaiser, and W. Weise, Phys. Rev. C* **81**, *024002 (2010)*

The perturbative expansion



Some details

The first-order HF contribution is explicitly given by

$$E_1 = \frac{8}{\pi}\int_0^{k_F} k^2 dk \left[1 - \frac{3}{2}\frac{k}{k_F} + \frac{1}{2}\left(\frac{k}{k_F}\right)^3\right] \sum_{JLS}(2J+1)[V_{NN}^{JLLS}(k,k) + \frac{1}{3}\overline{V}_{NNN}^{JLLS}(k,k)]$$

The energy denominators of the perturbative expansion are expressed in terms of the self-consistent single-particle potential:

$$E(k,k',K) = \frac{\hbar^2 k'^2}{M} + 2U\left(\sqrt{\frac{K^2}{4} + k'^2}\right) - \frac{\hbar^2 k^2}{M} - 2U\left(\sqrt{\frac{K^2}{4} + k^2}\right)$$

$$U(\tilde{k}) = \quad 8\sum_{JLLS}(2J+1)^2 \left\{\left[\int_0^{\frac{1}{2}(k_F-\tilde{k})} \tilde{k}'^2 d\tilde{k}' + \frac{1}{2\tilde{k}}\int_{\frac{1}{2}(k_F-\tilde{k})}^{\frac{1}{2}(k_F+\tilde{k})} \tilde{k}' d\tilde{k}' (\tfrac{1}{4}(k_F^2 - \tilde{k}^2) - \right.\right.$$
$$\left.\left. - \tilde{k}'(\tilde{k}' - \tilde{k}))\right]\left[V_{NN}^{JLLS}(\tilde{k}',\tilde{k}') + \frac{1}{2}\overline{V}_{NNN}^{JLLS}(\tilde{k}',\tilde{k}')\right]\right\}$$

The second-order diagrams are computed using the so-called angle-average (AA) approximation:

$$E_2 = -\frac{6}{\pi^2 k_F^3} \int_0^{2k_F} K^2 dK \int_0^{\infty} k'^2 dk' \int_0^{\infty} k^2 dk P(k',K) Q(k,K)$$

$$\sum_{JL\bar{L}S} (2J+1) \frac{[V_{NN}^{JL\bar{L}S}(k,k') + \overline{V}_{NNN}^{JL\bar{L}S}(k,k')]^2}{E(k,k',K)}$$

The operators P and Q are defined as:

$$\begin{aligned}
Q(k,K) &= 0 \; , & 0 \le k \le (k_F^2 - \frac{K^2}{4})^{1/2} \\
&= -\frac{k_F^2 - k^2 - K^2/4}{kK} \; , & (k_F^2 - \frac{K^2}{4})^{1/2} \le k \le (k_F + \frac{K}{2}) \\
&= 1 \; , & k \ge (k_F + \frac{K}{2}) \\
P(k,K) &= 1 \; , & 0 \le k \le (k_F - \frac{K}{2}) \\
&= \frac{k_F^2 - k^2 - K^2/4}{kK} \; , & (k_F - \frac{K}{2}) \le k \le (k_F^2 - \frac{K^2}{4})^{1/2} \\
&= 0 \; , & k \ge (k_F^2 - \frac{K^2}{4})^{1/2}
\end{aligned}$$

The particle-particle (pp) and hole-hole (hh) third-order diagrams are also computed in the AA approximation, and their explicit expressions are:

$$E_3(pp) = \frac{12}{(\pi k_F)^3} \int_0^{2k_F} K^2 dK \int_0^{\infty} k^2 dk \int_0^{\infty} k'^2 dk' \int_0^{\infty} k''^2 dk'' P(k,K) Q(k',K) Q(k'',K)$$

$$\sum_{JL\bar{L}\bar{L}'S} (2J+1) [V_{NN}^{JL\bar{L}S}(k,k') + \overline{V}_{NNN}^{JL\bar{L}S}(k,k')][V_{NN}^{J\bar{L}\bar{L}'S}(k',k'') + \overline{V}_{NNN}^{J\bar{L}\bar{L}'S}(k',k'')]$$

$$[V_{NN}^{J\bar{L}'LS}(k'',k) + \overline{V}_{NNN}^{J\bar{L}'LS}(k'',k)]/[E(k'',k) \cdot E(k',k)]$$

$$E_3(hh) = \frac{2}{(\pi k_F)^3} \int_0^{2k_F} K^2 dK \int_0^{\infty} k^2 dk \int_0^{\infty} k'^2 dk' \int_0^{\infty} k''^2 dk'' P(k,K) Q(k',K) P(k'',K)$$

$$\sum_{JL\bar{L}\bar{L}'S} (2J+1) [V_{NN}^{JL\bar{L}S}(k,k') + \overline{V}_{NNN}^{JL\bar{L}S}(k,k')][V_{NN}^{J\bar{L}\bar{L}'S}(k',k'') + \overline{V}_{NNN}^{J\bar{L}\bar{L}'S}(k',k'')]$$

$$[V_{NN}^{J\bar{L}'LS}(k'',k) + \overline{V}_{NNN}^{J\bar{L}'LS}(k'',k)]/[E(k',k'') \cdot E(k',k)]$$

The N^{3}LO two-body potential

We consider for our study three N^{3}LO two-body potentials with different cutoffs and different regulator functions $f(p', p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}]$:

The N^{3}LO two-body potential

We consider for our study three N^{3}LO two-body potentials with different cutoffs and different regulator functions $f(p', p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}]$:

- $\Lambda = 500$ MeV using $n = 2$ in the regulator function[1]

The N^3LO two-body potential

We consider for our study three N^3LO two-body potentials with different cutoffs and different regulator functions $f(p', p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}]$:

- $\Lambda = 500$ MeV using $n = 2$ in the regulator function[1]
- $\Lambda = 450$ MeV using $n = 3$ in the regulator function

The N^{3}LO two-body potential

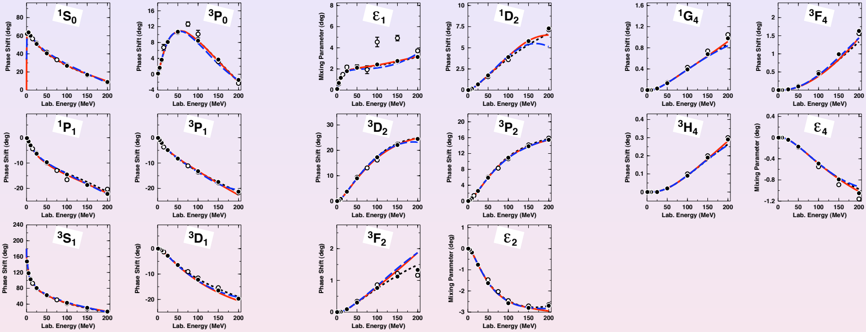
We consider for our study three N^{3}LO two-body potentials with different cutoffs and different regulator functions $f(p',p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}]$:

- $\Lambda = 500$ MeV using $n = 2$ in the regulator function[1]
- $\Lambda = 450$ MeV using $n = 3$ in the regulator function
- $\Lambda = 414$ MeV using $n = 10$ in the regulator function (sharp cutoff)[2]

[1] *R. Machleidt and D.R. Entem, Phys. Rep.* **503** *1 (2011)*

[2] *L. C., A. Covello, and A. Gargano, N. Itaco, D. R. Entem, T. T. S. Kuo, and R. Machleidt Phys. Rev. C* **75**, *024311 (2007)*

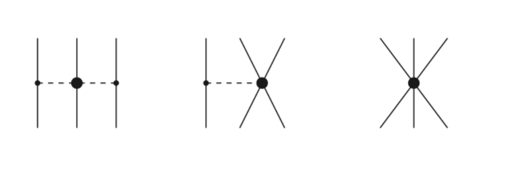
Phase shifts



- Dotted black curve: $\Lambda = 500$ MeV
- Dashed blue curve: $\Lambda = 450$ MeV
- Solid red curve: $\Lambda = 414$ MeV

The N^{2}LO three-body potential

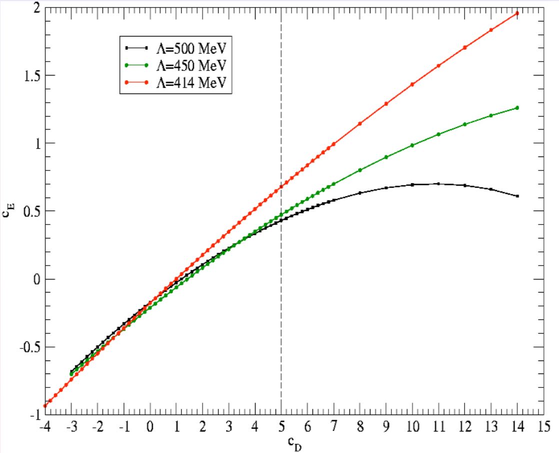
Aside the N^{3}LO two-body potentials we consider also the contribution from their corresponding three-body potentials, calculated at N^{2}LO:



They bring two more coefficients (c_D, c_E) corresponding to the contact and the 2π-exchange terms, to be adjusted to the physics of the three-nucleon system.

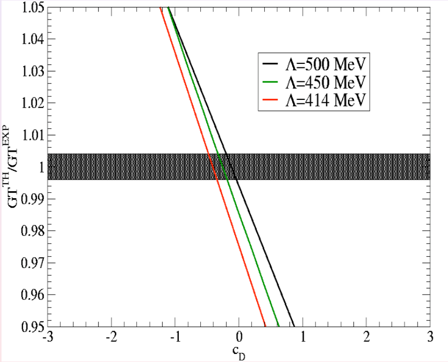
The fit of c_D, c_E parameters

The binding energy of the triton:



The fit of c_D, c_E parameters

The triton Gamow-Teller matrix elements of the triton via the μ capture in $^2\mathrm{H}(\mu^-, \nu_\mu)nn$ and $^3\mathrm{He}(\mu^-, \nu_\mu)^3\mathrm{H}$[1] (see also Michele Viviani's talk):

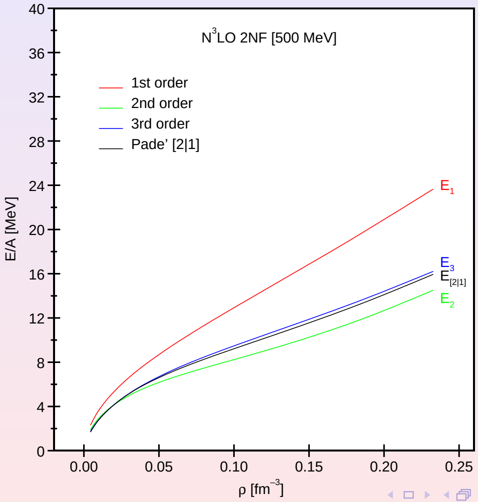


[1] *L. E. Marcucci, A. Kievsky, S. Rosati, R. Schiavilla, and M. Viviani, Phys. Rev. Lett.*

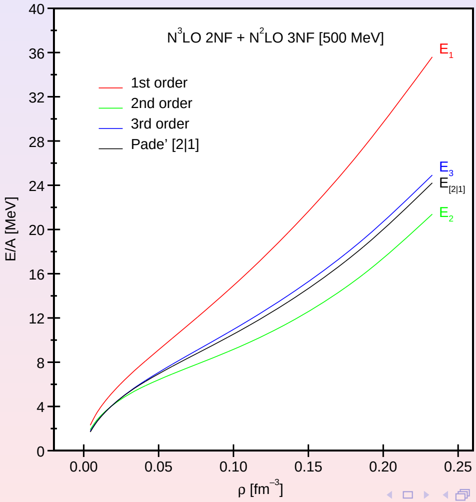
The equation of state for infinite neutron matter

L. C., J. W. Holt, N. Itaco, R. Machleidt, Phys. Rev. C **87**, *014322 (2013)*

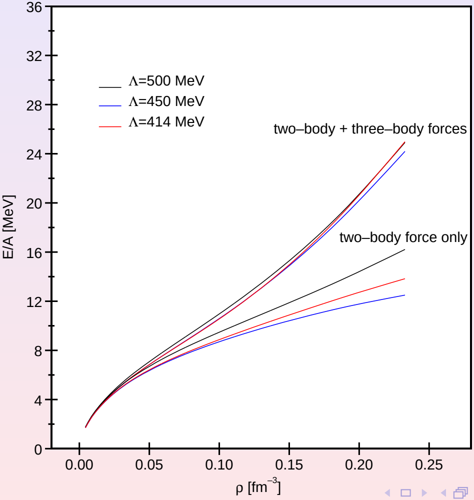
The infinite neutron matter EOS



The infinite neutron matter EOS

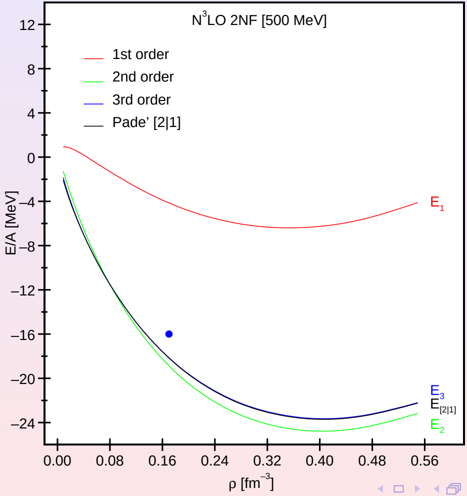


The infinite neutron matter EOS

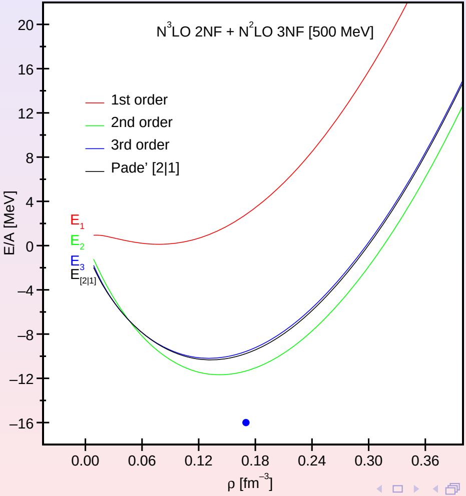


The equation of state for infinite symmetric nuclear matter

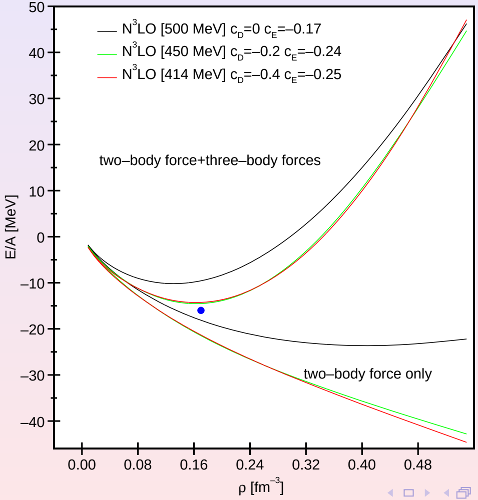
The infinite nuclear matter EOS



The infinite nuclear matter EOS



The infinite nuclear matter EOS



Concluding remarks

- Chiral potentials with a cutoff $\Lambda \leq 500$ MeV exhibit a perturbative behavior both for infinite neutron and nuclear matter calculations
- The EOS for infinite neutron matter shows substantial regulator independence when including 3NF contributions
- The EOS for infinite nuclear matter shows moderate regulator independence

Concluding remarks

- Chiral potentials with a cutoff $\Lambda \leq 500$ MeV exhibit a perturbative behavior both for infinite neutron and nuclear matter calculations
- The EOS for infinite neutron matter shows substantial regulator independence when including 3NF contributions
- The EOS for infinite nuclear matter shows moderate regulator independence

Perspectives

- Improve the calculation of the perturbative expansion
- Need of a N^3LO three-body force?
- Need to include four-body force effects?