Four-nucleon reactions

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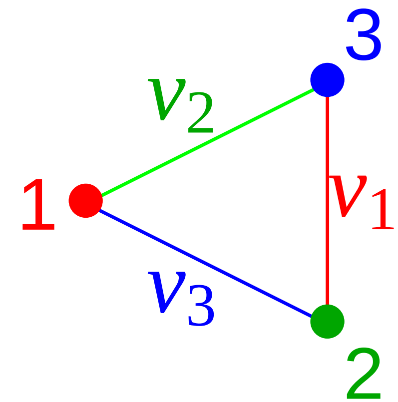
Four-nucleon scattering

- Three-particle scattering equations
- 3N reactions

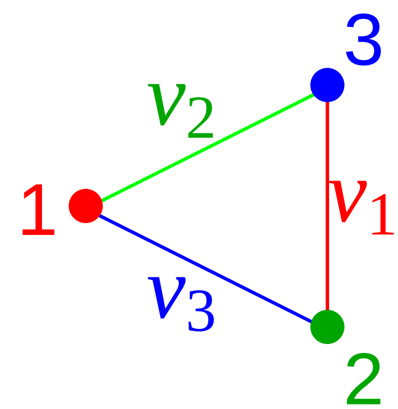
- Four-nucleon scattering equations
- 4N reactions below 3-cluster breakup
- 4N reactions above 3- and 4-cluster breakup

Three-particle system



Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$

Three-particle system



Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$

- Faddeev equations

$$(E - H_0 - v_{\alpha})|\psi_{\alpha}\rangle = v_{\alpha} \sum_{\gamma} \bar{\delta}_{\alpha\gamma} |\psi_{\sigma}\rangle$$

$$|\Psi\rangle = \sum_{\alpha} |\psi_{\alpha}\rangle$$

Alt, Grassberger, and Sandhas equations

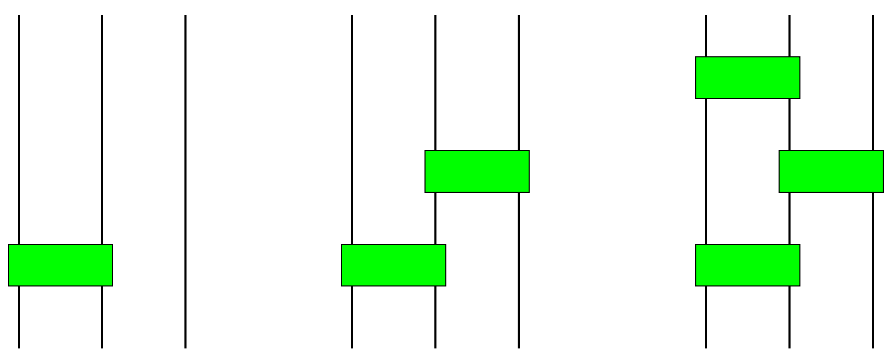
$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} T_{\gamma} G_0 U_{\gamma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\gamma} T_{\gamma} G_0 U_{\gamma\alpha}$$

$$T_{\gamma} = v_{\gamma} + v_{\gamma} G_0 T_{\gamma}$$

$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $(E - H_0 - v_{\alpha})|\phi_{\alpha}\rangle = 0$



...

AGS equations with 3BF

$$V_{3BF} = \sum_{\alpha=1}^{3} w_\alpha$$

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\gamma \bar{\delta}_{\beta\gamma} T_\gamma G_0 U_{\gamma\alpha}$$

$$+ w_\alpha + \sum_\gamma w_\gamma G_0 (1 + T_\gamma G_0) U_{\gamma\alpha}$$

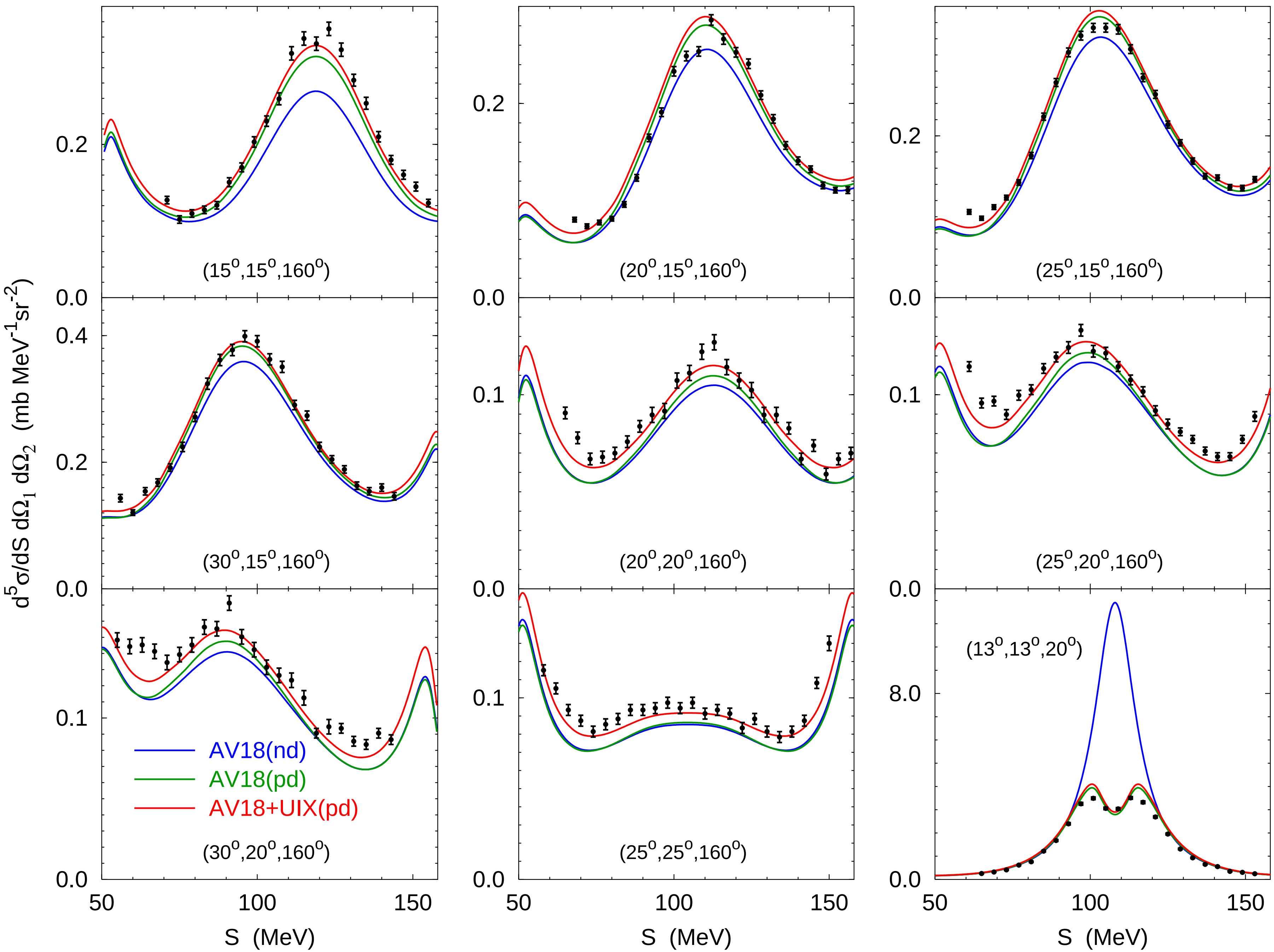
AGS equations: numerical solution

$$U = PG_0^{-1} + PTG_0U$$
$$+ (1+P)w + (1+P)wG_0(1+TG_0)U$$

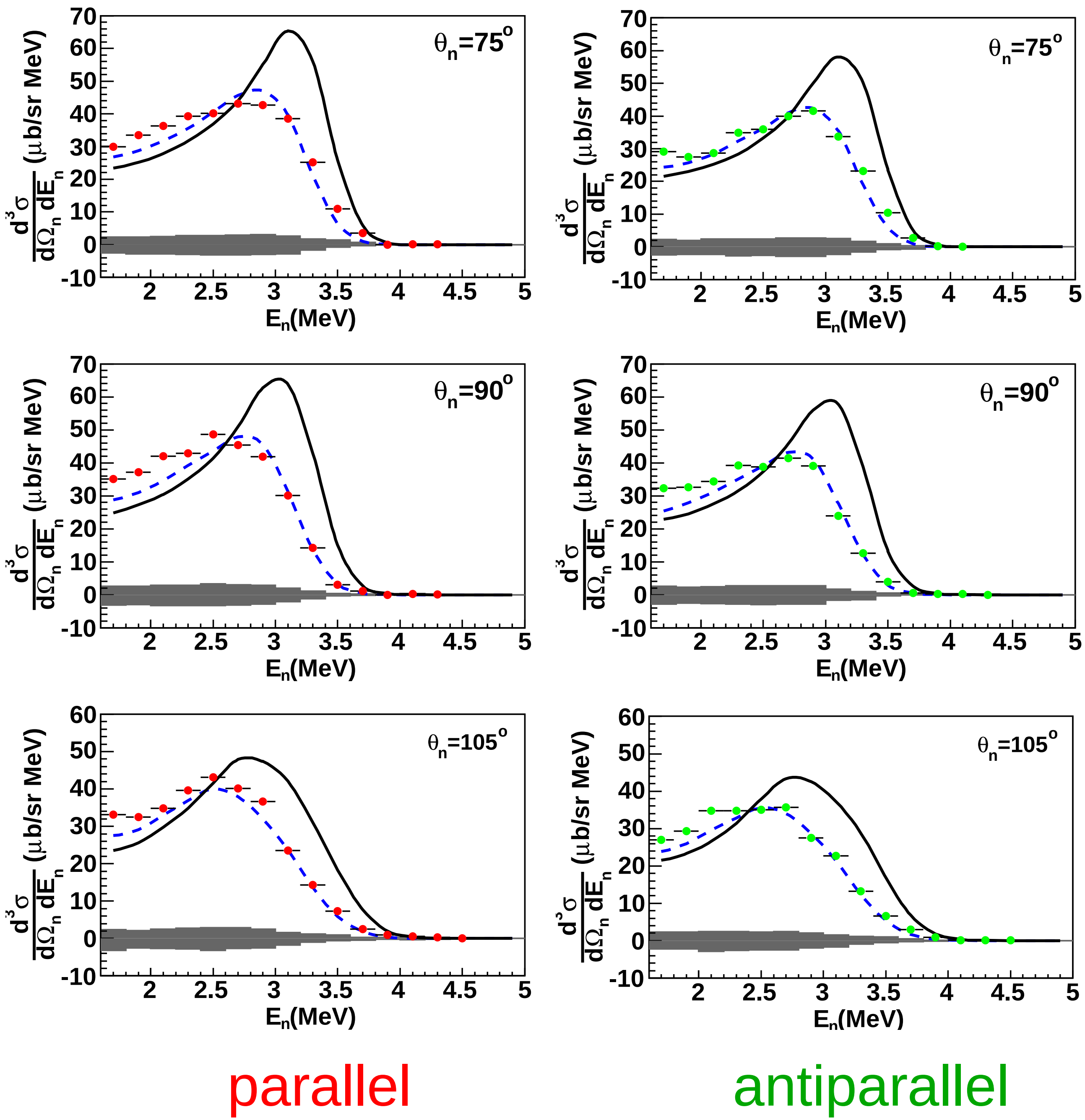
- symmetrized for 3N: $P = P_{12}P_{23} + P_{13}P_{23}$
- momentum-space partial wave basis
- set of coupled 2-variable integral equations
- integrable singularities in kernel
- Coulomb interaction: screening and renormalization

[PRC 71, 054005; PRC 72, 054004; PRC 80, 064002]

Coulomb vs 3NF: ^{1}H(d,pp)n at $E_d = 130$ MeV



$^3\vec{\mathrm{He}}(\vec{\gamma},n)pp$ at $E_\gamma = 12.8$ MeV

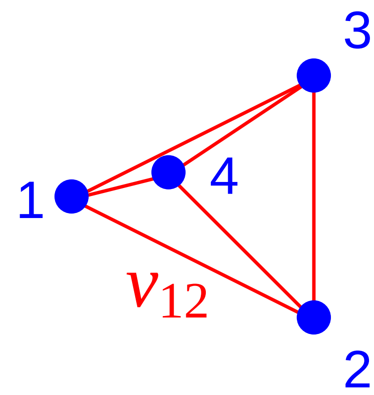


TUNL data [PRL 110, 202501]

CD Bonn + Δ (with Coulomb) (Lisbon)

AV18 + UIX (no Coulomb) (Cracow)

Four-particle scattering



Hamiltonian $H_0 + \sum_{i>j} v_{ij}$

- Wave function:
  Schrödinger equation
- Wave function components:
  Faddeev-Yakubovsky equations
- Transition operators:
  Alt-Grassberger-Sandhas equations

Symmetrized AGS equations

$$t = v + vG_0 t$$

$$G_0 = (E + i\varepsilon - H_0)^{-1}$$

$$u_j = P_j G_0^{-1} + P_j t G_0 u_j$$

$$3 + 1: \quad P_1 = P_{12} P_{23} + P_{13} P_{23}$$

$$2 + 2: \quad P_2 = P_{13} P_{24}$$

$$U_{11} = (G_0 t G_0)^{-1} \zeta P_{34} + \zeta P_{34} u_1 G_0 t G_0 U_{11} + u_2 G_0 t G_0 U_{21}$$

$$U_{21} = (G_0 t G_0)^{-1} (1 + \zeta P_{34}) + (1 + \zeta P_{34}) u_1 G_0 t G_0 U_{11}$$

$$U_{12} = (G_0 t G_0)^{-1} + \zeta P_{34} u_1 G_0 t G_0 U_{12} + u_2 G_0 t G_0 U_{22}$$

$$U_{22} = (1 + \zeta P_{34}) u_1 G_0 t G_0 U_{12}$$

$\zeta = -1\ (+1)$ for fermions (bosons)

basis states partially symmetrized

Scattering amplitudes: $E + i\varepsilon \to E + i0$

2-cluster reactions:

$$T_{fi} = s_{fi} \langle \phi_f | U_{fi} | \phi_i \rangle$$

$$|\phi_j\rangle = G_0 t P_j |\phi_j\rangle$$

$$|\Phi_j\rangle = (1 + P_j) |\phi_j\rangle$$

3-cluster breakup/recombination:

$$T_{3i} = s_{3i} \langle \phi_3 | [(1 + \zeta P_{34}) u_1 G_0 t G_0 U_{1i} + u_2 G_0 t G_0 U_{2i}] | \phi_i \rangle$$

4-cluster breakup/recombination:

$$T_{4i} = s_{4i} \{ \langle \phi_4 | [1 + (1 + P_1) \zeta P_{34}] (1 + P_1) t G_0 u_1 G_0 t G_0 U_{1i} | \phi_i \rangle + \langle \phi_4 | (1 + P_1)(1 + P_2) t G_0 u_2 G_0 t G_0 U_{2i} | \phi_i \rangle \}$$

Wave function

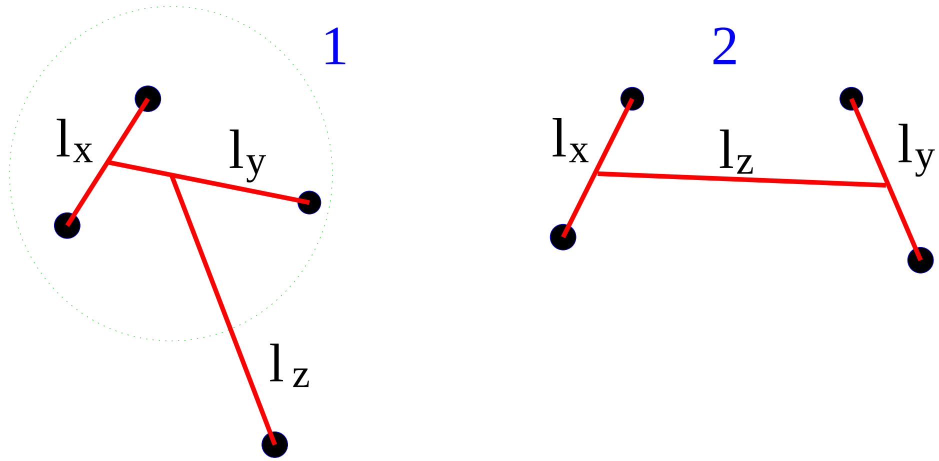
$$|\Psi_i\rangle = s_i\{[1+(1+P_1)\zeta P_{34}](1+P_1)|\psi_{1,i}\rangle + (1+P_1)(1+P_2)|\psi_{2,i}\rangle\}$$

with Faddeev-Yakubovsky components

$$|\psi_{j,i}\rangle = \delta_{ji}|\phi_i\rangle + G_0 t G_0 u_j G_0 t G_0 U_{ji}|\phi_i\rangle$$

Solution of 4N AGS equations

$$U_{11}|\phi_1\rangle = -G_0^{-1}P_{34}P_1|\phi_1\rangle - P_{34}u_1G_0\,t\,G_0\,U_{11}|\phi_1\rangle + u_2G_0\,t\,G_0\,U_{21}|\phi_1\rangle$$



- momentum-space partial-wave basis
  $|k_xk_yk_z[l_z(\{l_y[(l_xS_x)j_xs_y]S_y\}J_ys_z)S_z]JM,[(T_xt_y)T_yt_z]TM_T\rangle_1$
  $|k_xk_yk_z[l_z\{(l_xS_x)j_x[l_y(s_ys_z)S_y]j_y\}S_z]JM,[T_x(t_yt_z)T_y]TM_T\rangle_2$
- large system (up to 30000) of coupled 3-variable integral equations with integrable singularities
- Coulomb interaction: screening and renormalization [PRC 75, 014005, PRL 98, 162502]

Singularities of 4N AGS equations

^{3}H, ^{3}He, or d+d bound state poles

$$G_0 u_j G_0 \to \frac{P_j |\phi_j\rangle s_{jj} \langle\phi_j| P_j}{E + i\varepsilon - E_j^b - k_z^2/2\mu_j}$$

treated by subtraction below 3-cluster threshold

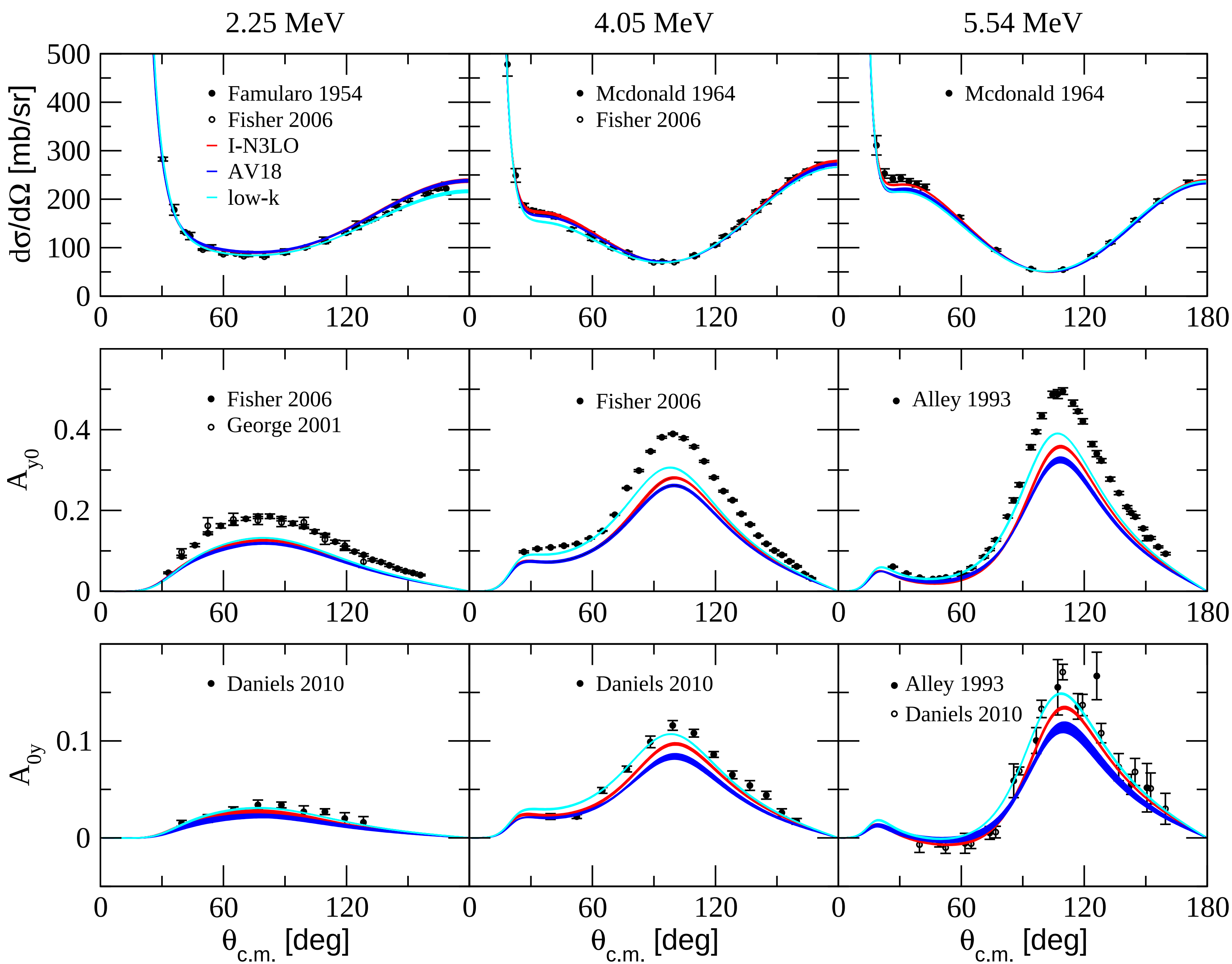
$$\int_p^q k_z^2\, dk_z \frac{F(k_z)}{k_0^2 - k_z^2 + i0}$$

$$= \mathcal{P} \int_p^q k_z^2\, dk_z \frac{F(k_z)}{k_0^2 - k_z^2} - \frac{1}{2} i\pi k_0 F(k_0)$$

$$= \int_p^q dk_z \frac{k_z^2 F(k_z) - k_0^2 F(k_0)}{k_0^2 - k_z^2}$$

$$- \frac{1}{2} k_0 F(k_0) \left[i\pi + \ln \frac{(k_0 + p)(q - k_0)}{(k_0 - p)(k_0 + q)} \right]$$

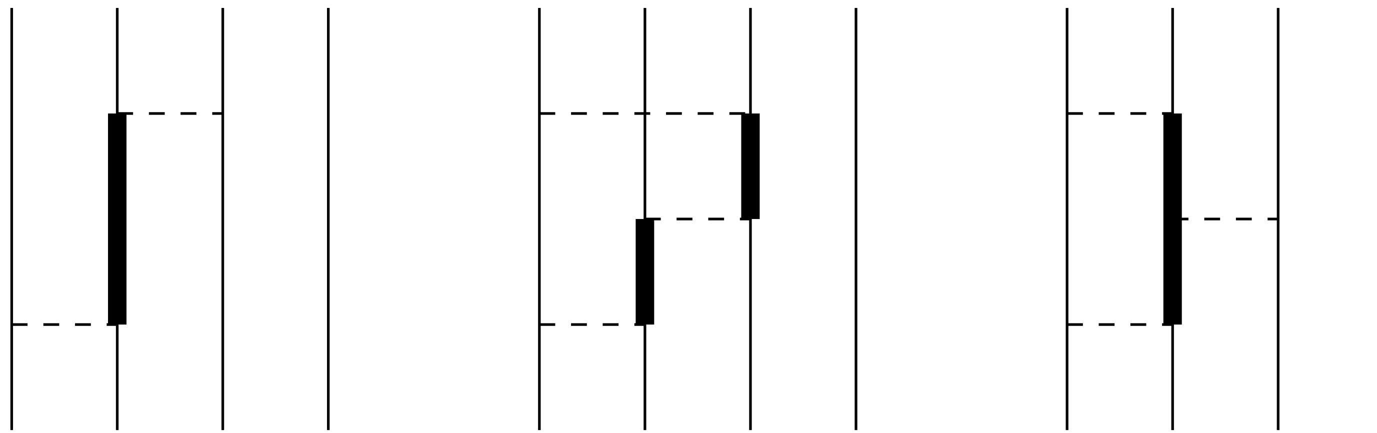
p-^{3}He scattering



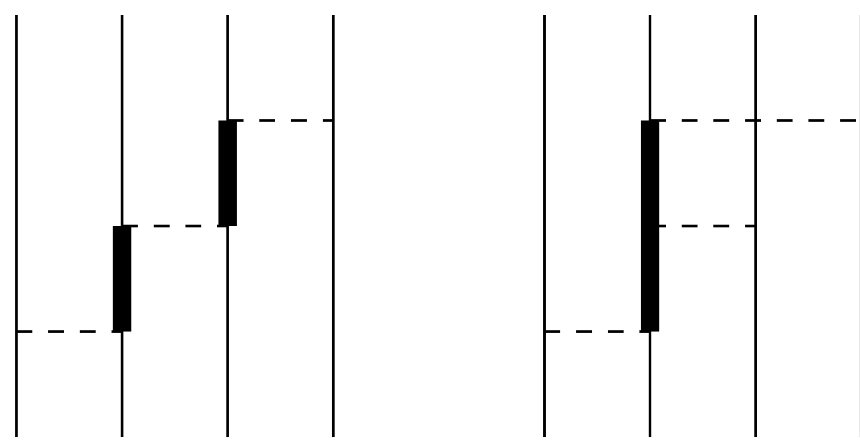
AGS/HH/FY (Lisbon/Pisa/Strasbourg, PRC 84, 054010)

Δ-isobar excitation: effective 3N and 4N forces

Fujita-Miyazawa higher order 3N force

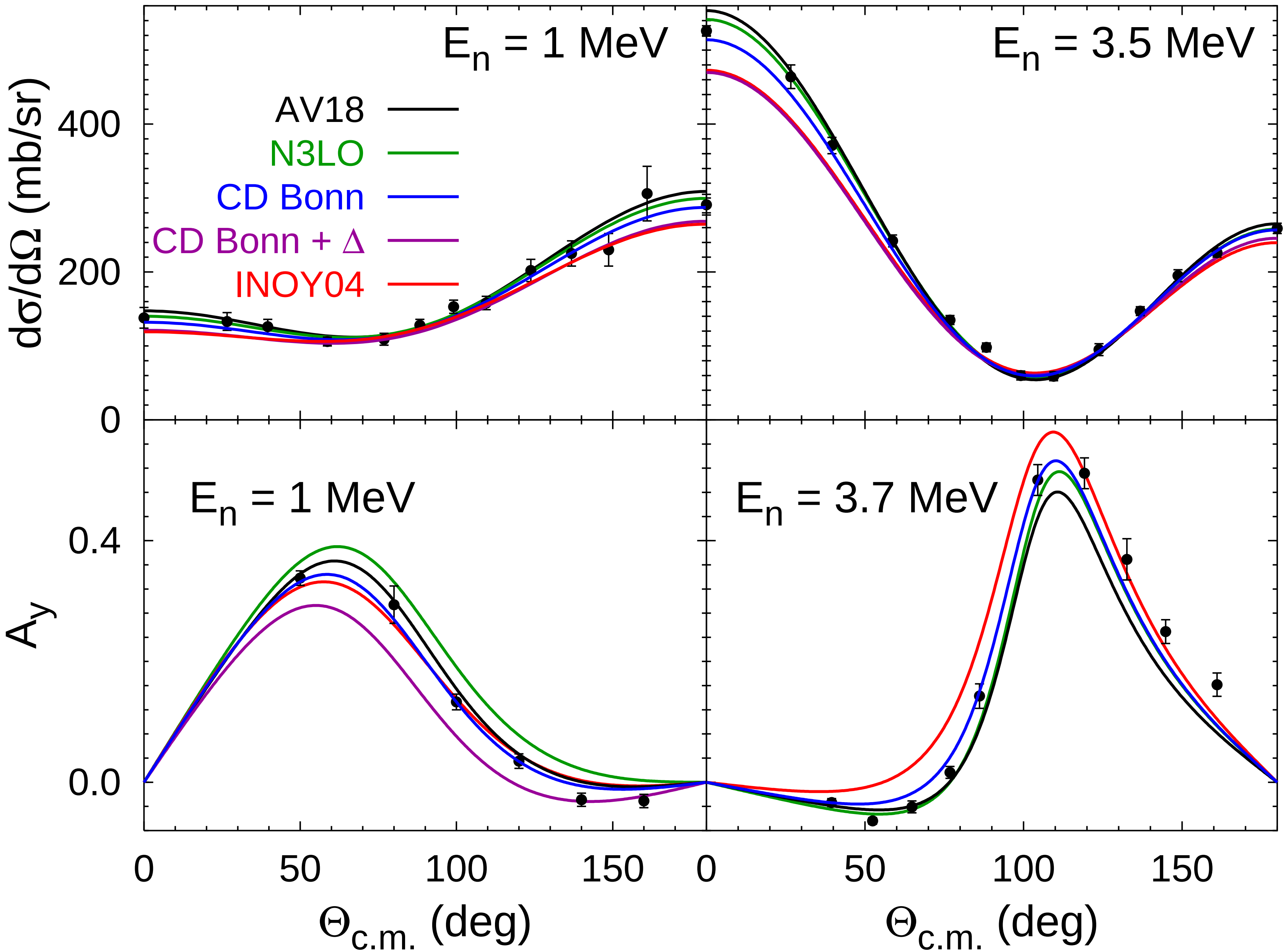


4N force



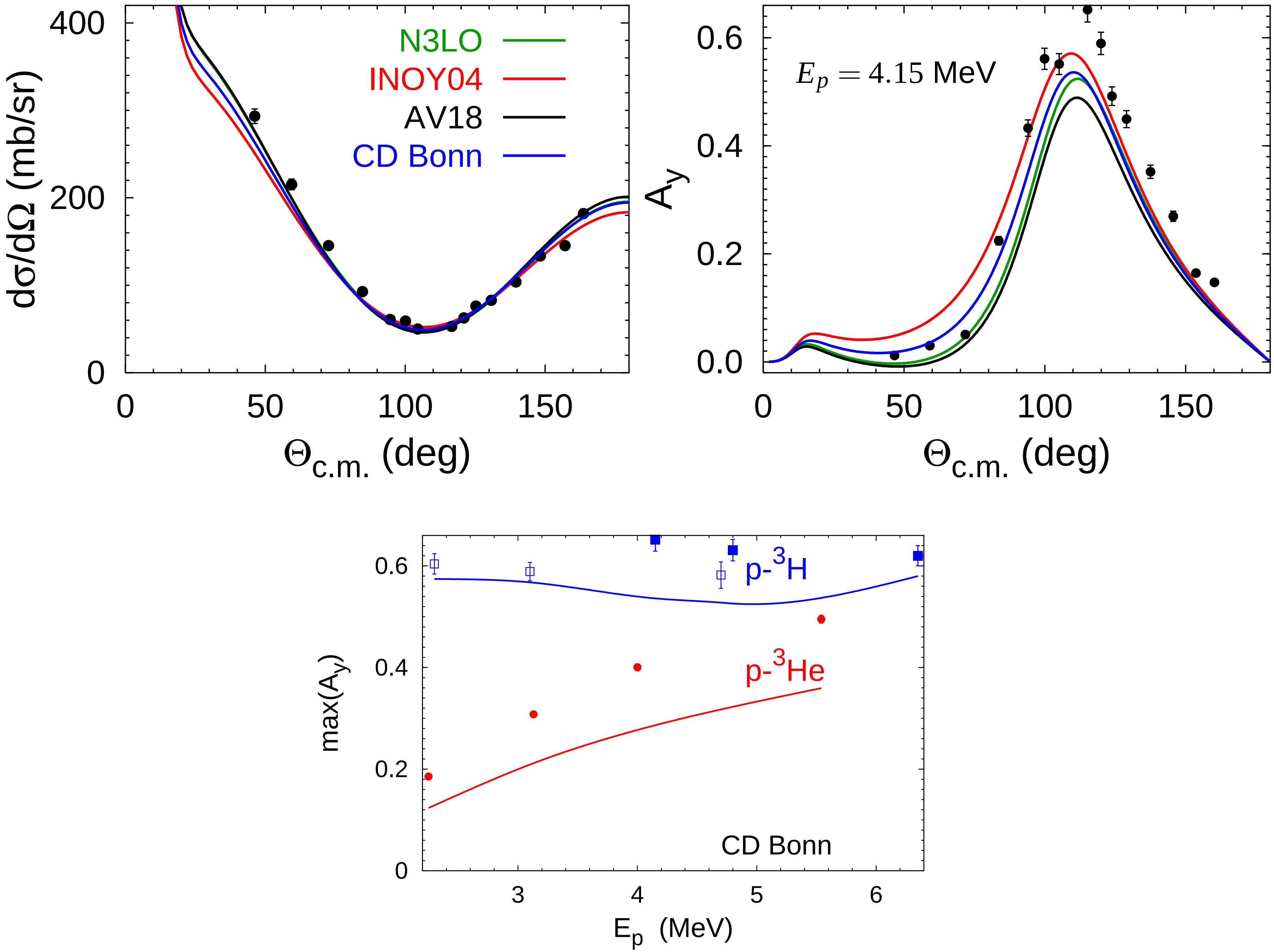
[PLB 660, 471]

n-^{3}He elastic scattering

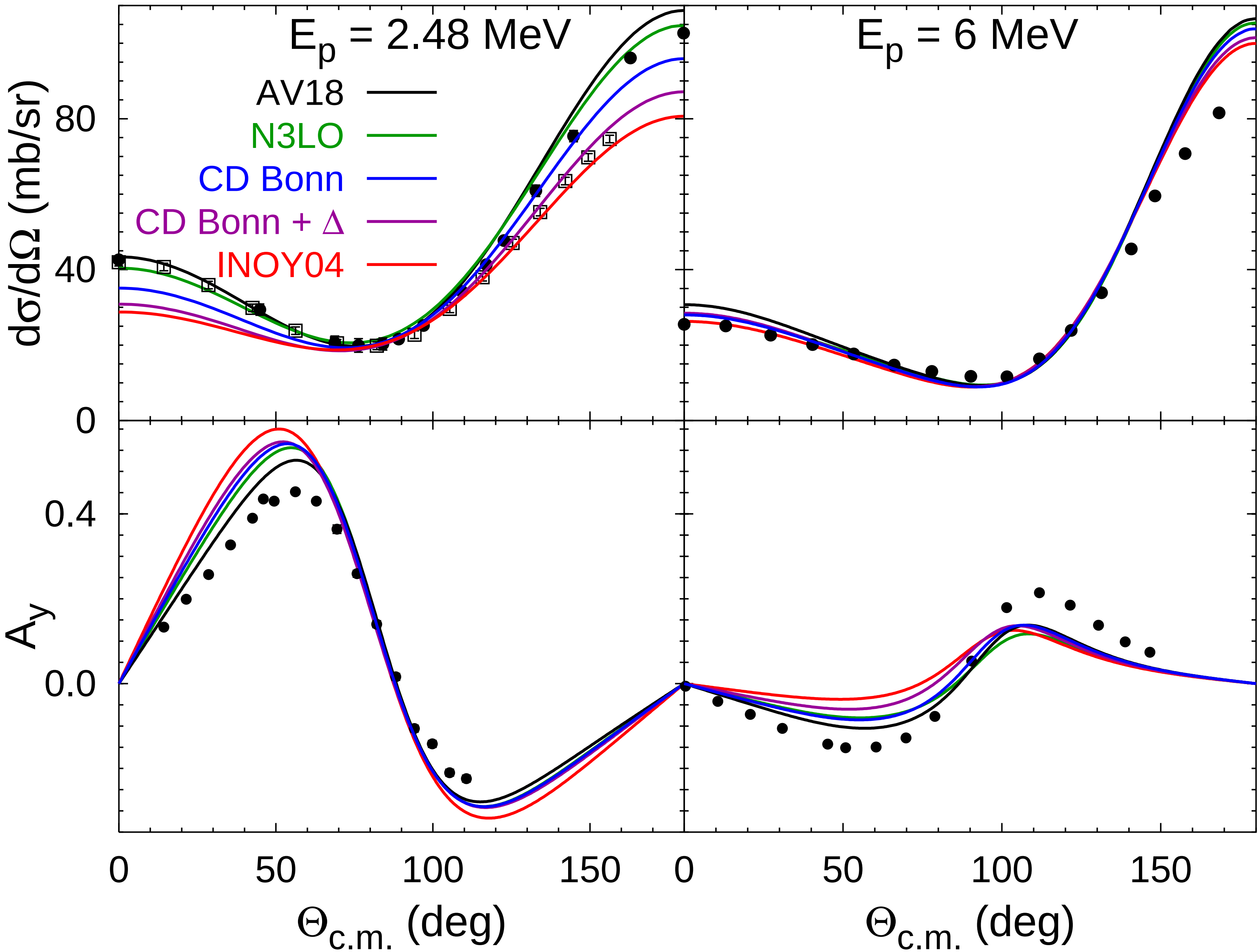


[PRC 76, 021001]

p-^{3}H elastic scattering

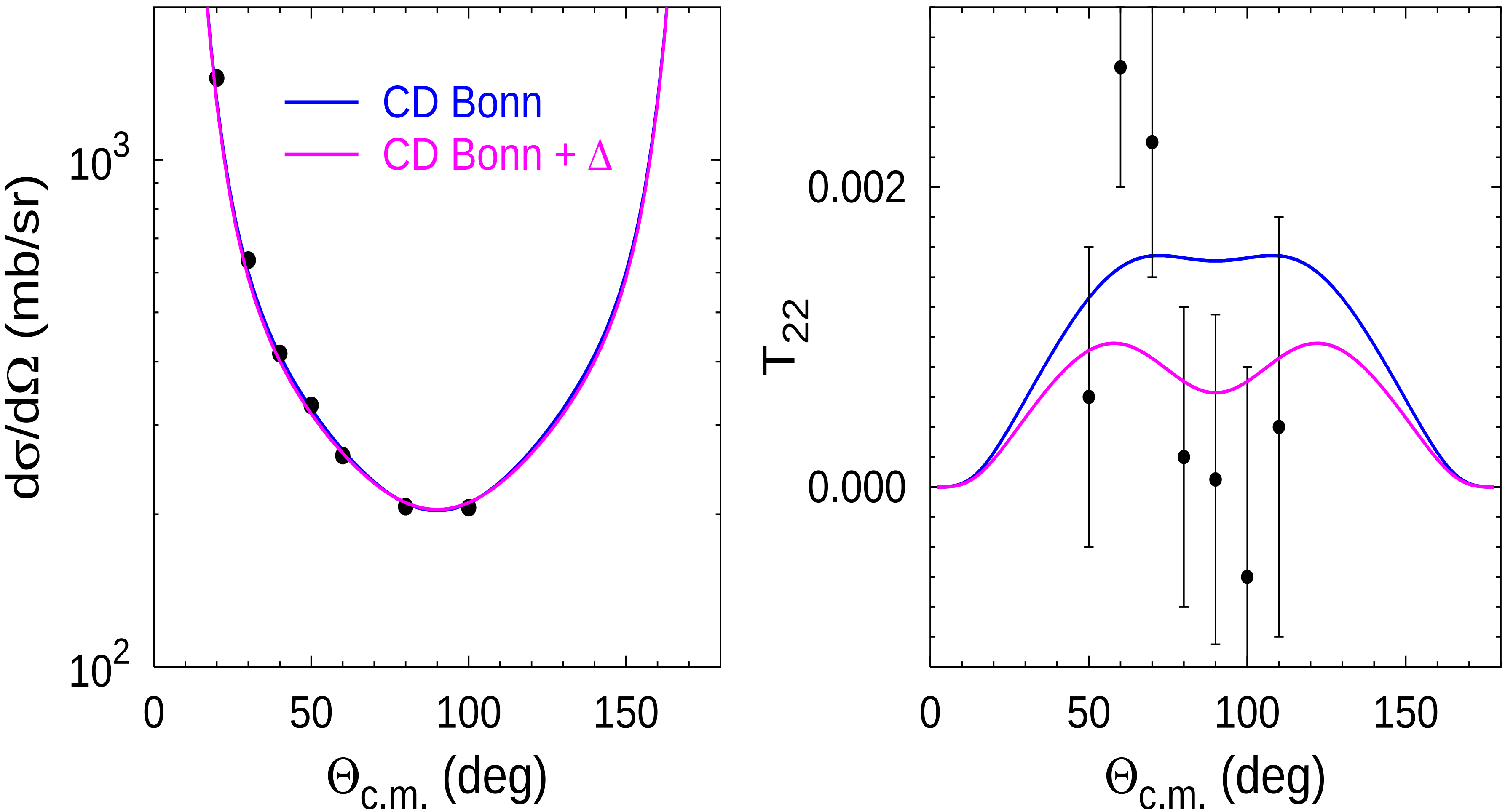


Charge exchange reaction $^3\mathrm{H}(p,n)^3\mathrm{He}$



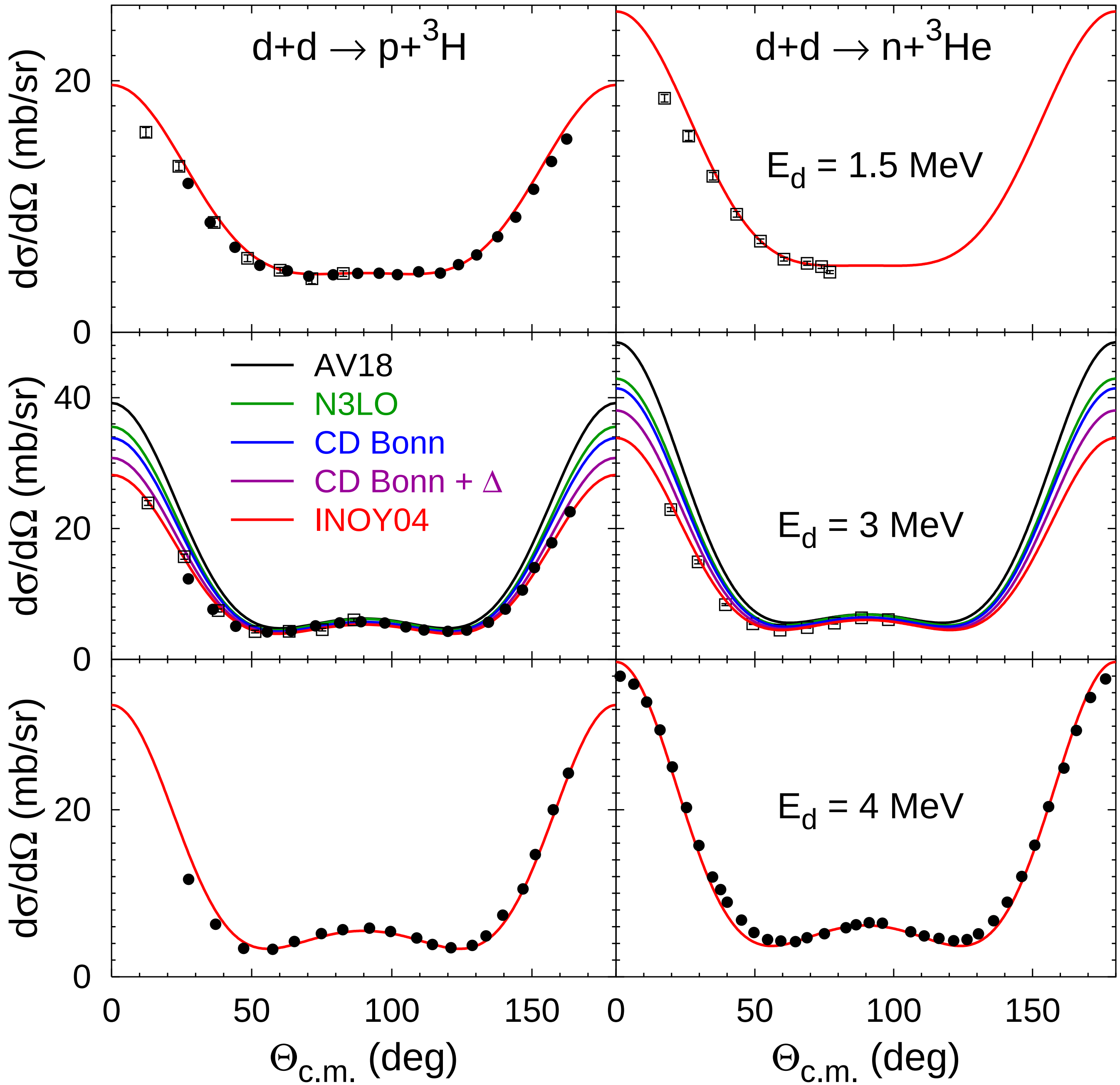
[PRC 76, 021001]

d-d elastic scattering at $E_d = 3$ MeV



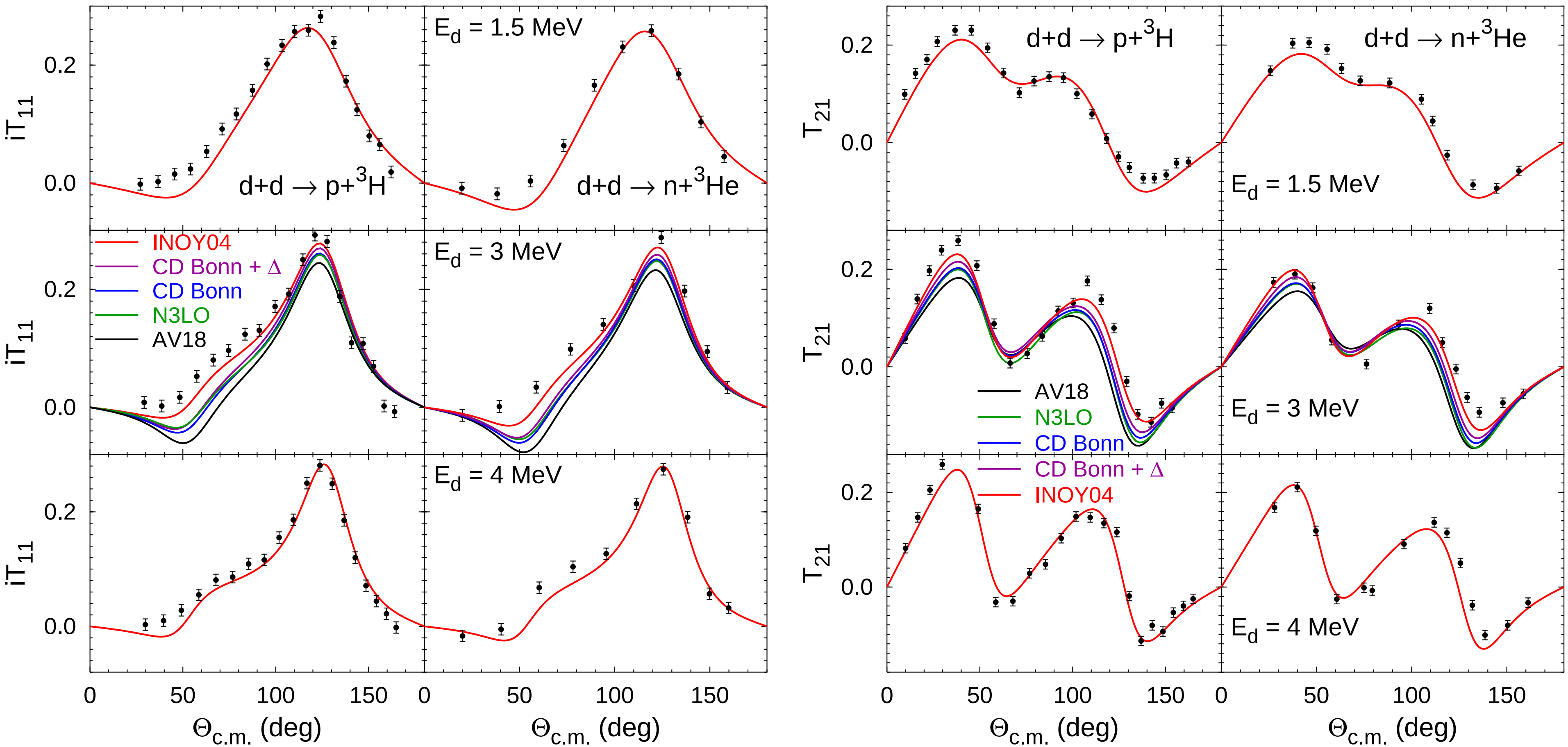
[PLB 660, 471]

^{2}H(d,p)^{3}H and ^{2}H(d,n)^{3}He



[PRC 81, 054002]

$^{2}H(d,p)^{3}H$ and $^{2}H(d,n)^{3}He$



Above breakup: additional singularities in AGS equations

deuteron bound state poles

$$t \longrightarrow \frac{v|\phi_d\rangle\langle\phi_d|v}{E + i\varepsilon - e_d - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

free resolvent

$$G_0 \longrightarrow \frac{1}{E + i\varepsilon - k_x^2/2\mu_j^x - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

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$$G_0 \rightarrow \frac{1}{E + i\varepsilon - k_x^2/2\mu_j^x - k_y^2/2\mu_j^y - k_z^2/2\mu_j}$$

treated by complex-energy method:

1. solve for $U_{fi}(E + i\varepsilon)$ with finite $\varepsilon = \varepsilon_1, \ldots, \varepsilon_n$
2. extrapolate to $\varepsilon \rightarrow 0$ for physical amplitudes $U_{fi}(E + i0)$

[H. Kamada *et al*, Prog. Theor. Phys. 109, 869L (2003)]

Integration with special weights

accuracy & efficiency of the complex-energy method is greatly improved by a special integration

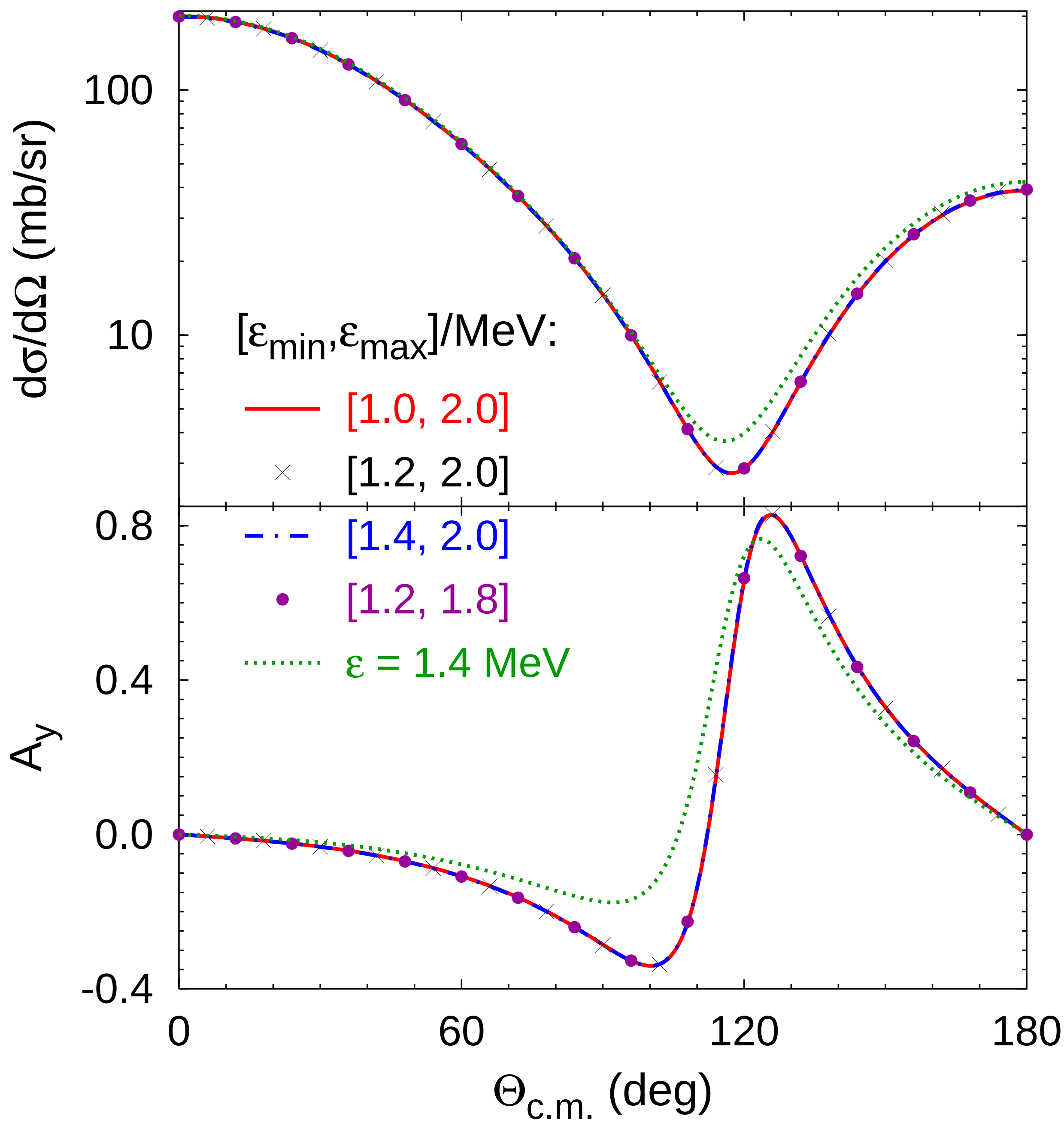
$$\int_a^b \frac{f(x)}{x_0^n + iy_0 - x^n} dx \approx \sum_{j=1}^{N} f(x_j) w_j(n, x_0, y_0, a, b)$$

where the quasi-singular factor is absorbed into special weights

$$w_j(n, x_0, y_0, a, b) = \int_a^b \frac{S_j(x)}{x_0^n + iy_0 - x^n} dx$$

that may be calculated using spline functions $\{S_j(x)\}$ for standard Gaussian grid $\{x_j\}$ [PRC 86, 011001]

Extrapolation $\varepsilon \to 0$: $n+^3H$ at 22.1 MeV

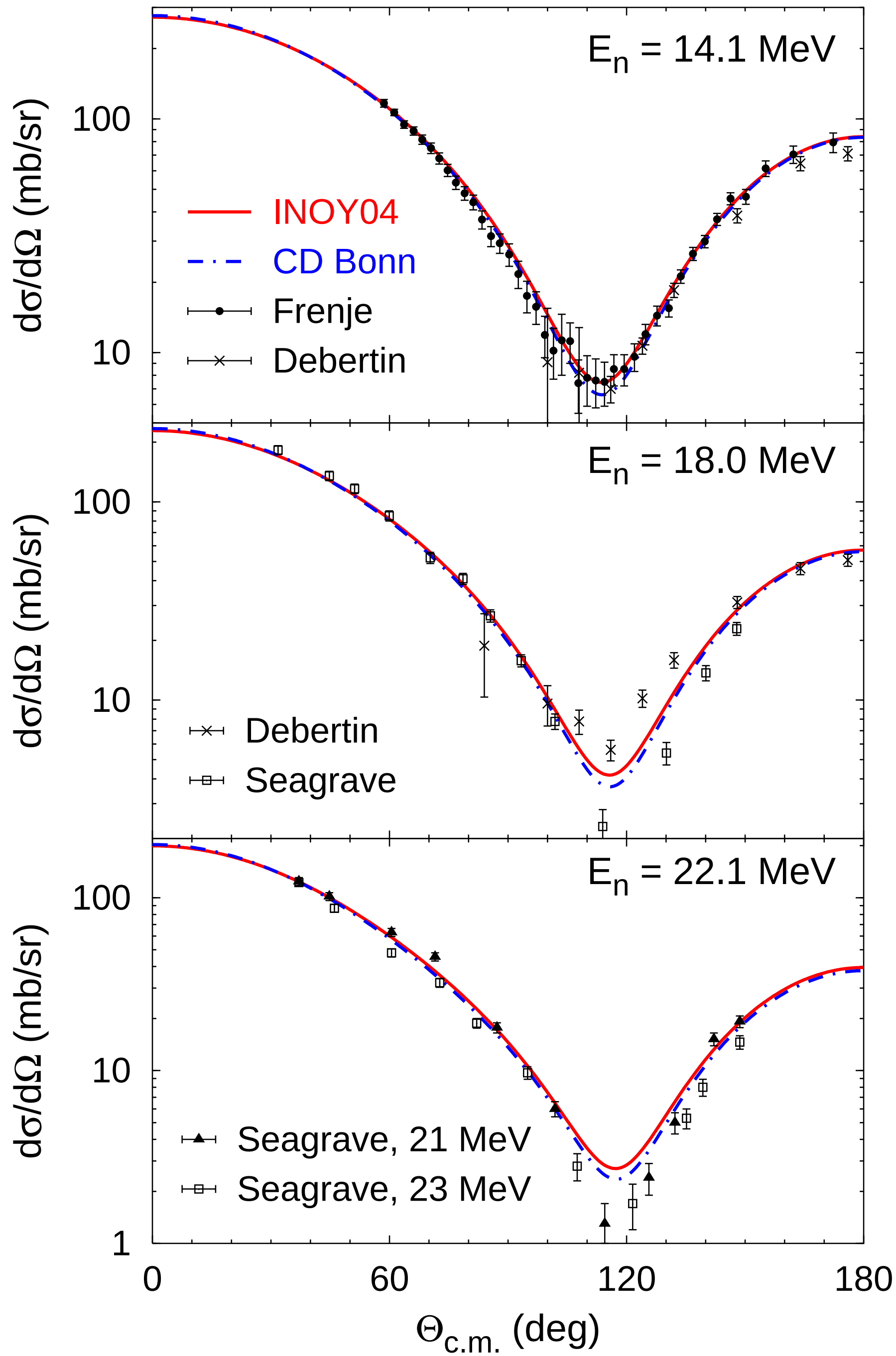


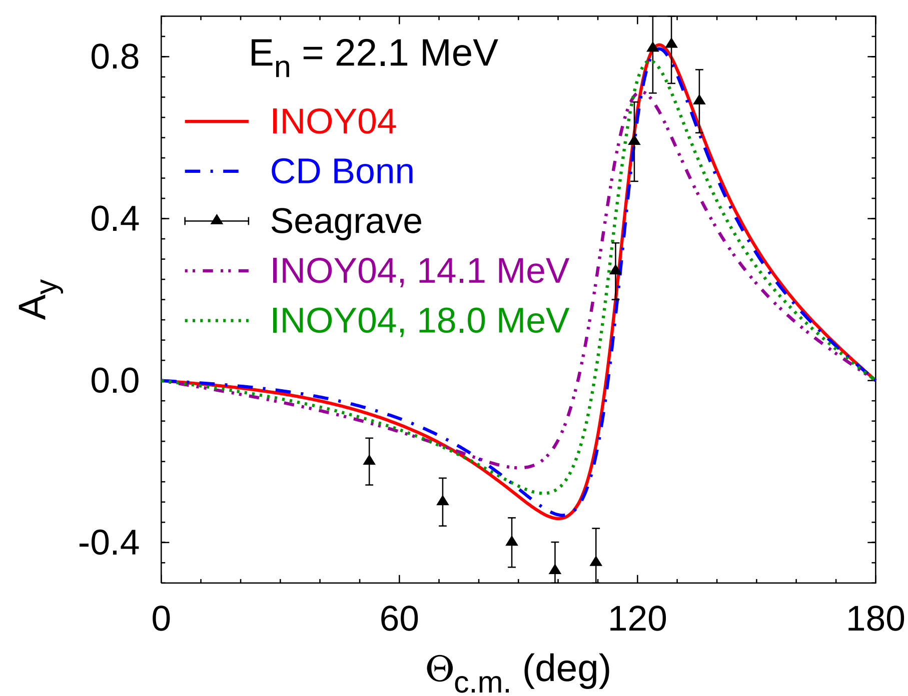
Extrapolation $\varepsilon \to 0$: n+^{3}H at 22.1 MeV

$[\varepsilon_{\min}, \varepsilon_{\max}]$	$\delta(^1S_0)$	$\eta(^1S_0)$	$\delta(^3P_0)$	$\eta(^3P_0)$	$\delta(^3P_2)$	$\eta(^3P_2)$
$[1.0, 2.0]$	62.63	0.990	43.03	0.959	65.27	0.950
$[1.2, 2.0]$	62.60	0.991	43.04	0.959	65.29	0.951
$[1.4, 2.0]$	62.67	0.991	43.03	0.958	65.27	0.950
$[1.2, 1.8]$	62.65	0.992	43.03	0.959	65.28	0.950
1.4	73.37	0.916	44.77	0.840	67.38	0.933

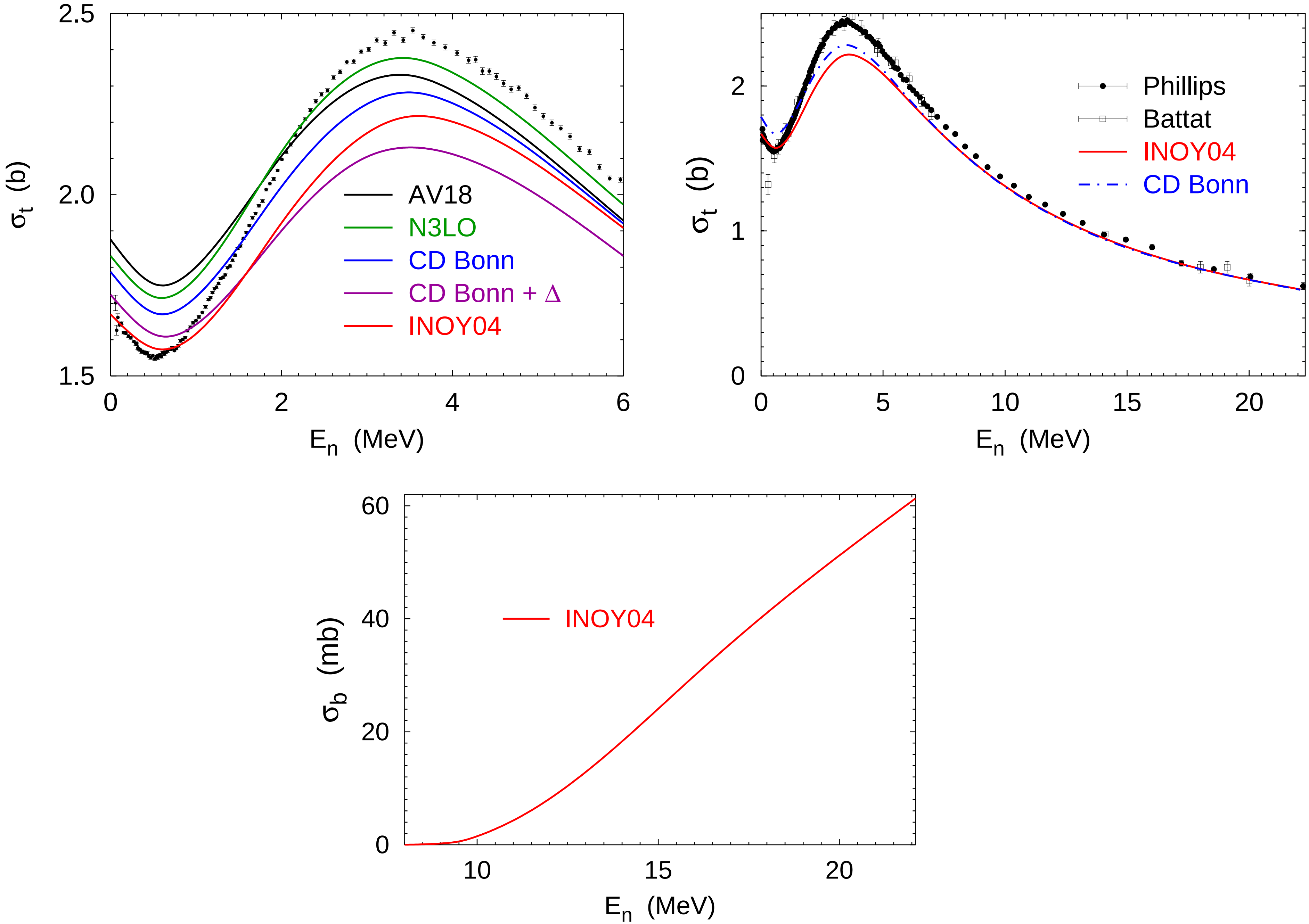
[PRC 86, 011001]

n+^{3}H elastic scattering

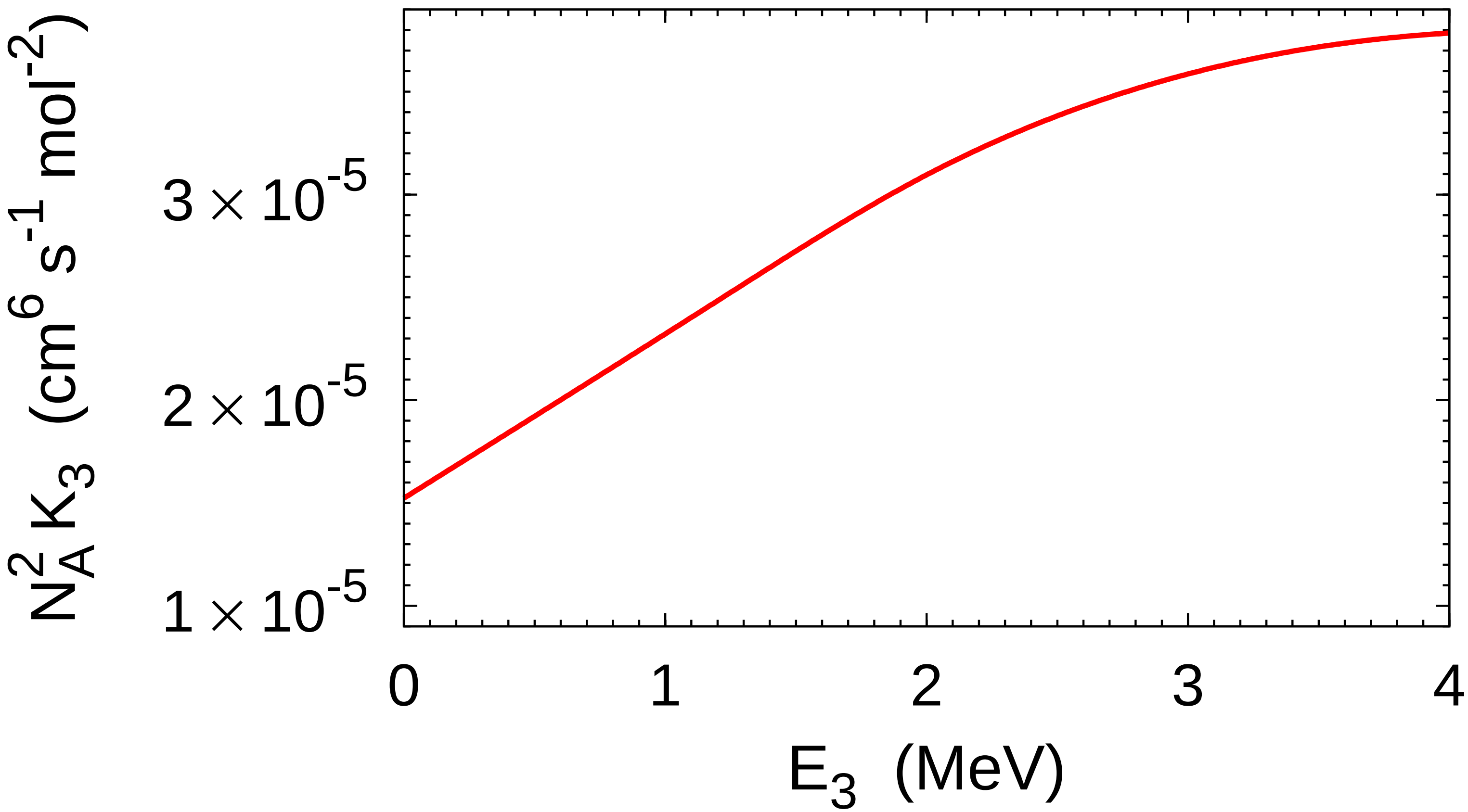




n+^{3}H total and breakup cross sections



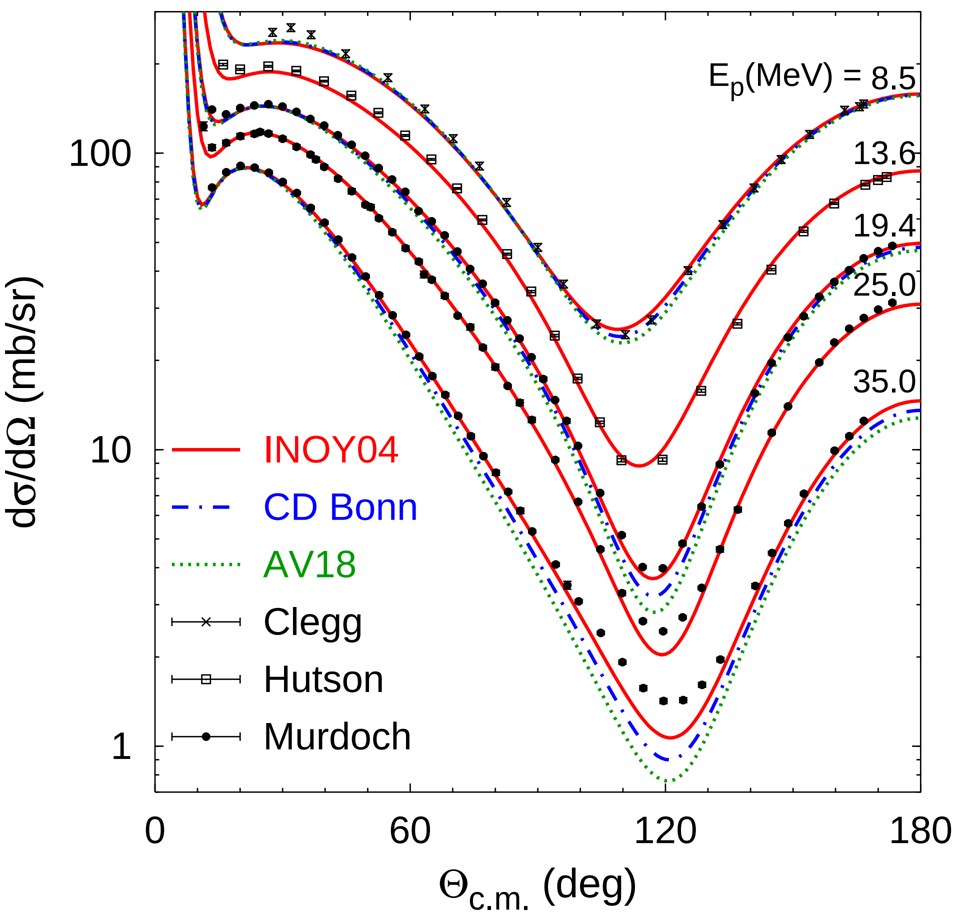
Recombination reaction $^2H+n+n \to n+^3H$

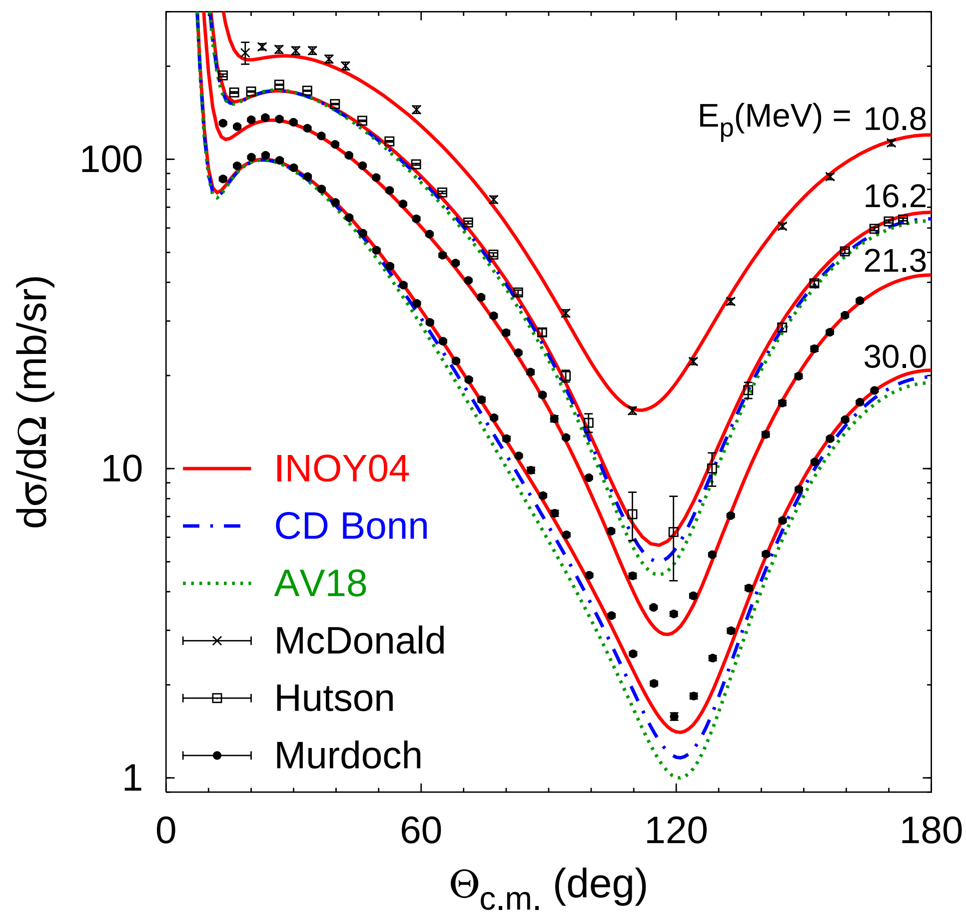


$$\frac{d\rho_t}{dt} = K_2^\gamma \rho_d \rho_n + K_3 \rho_d \rho_n^2 + \ldots$$

[PRC 87, 014002]

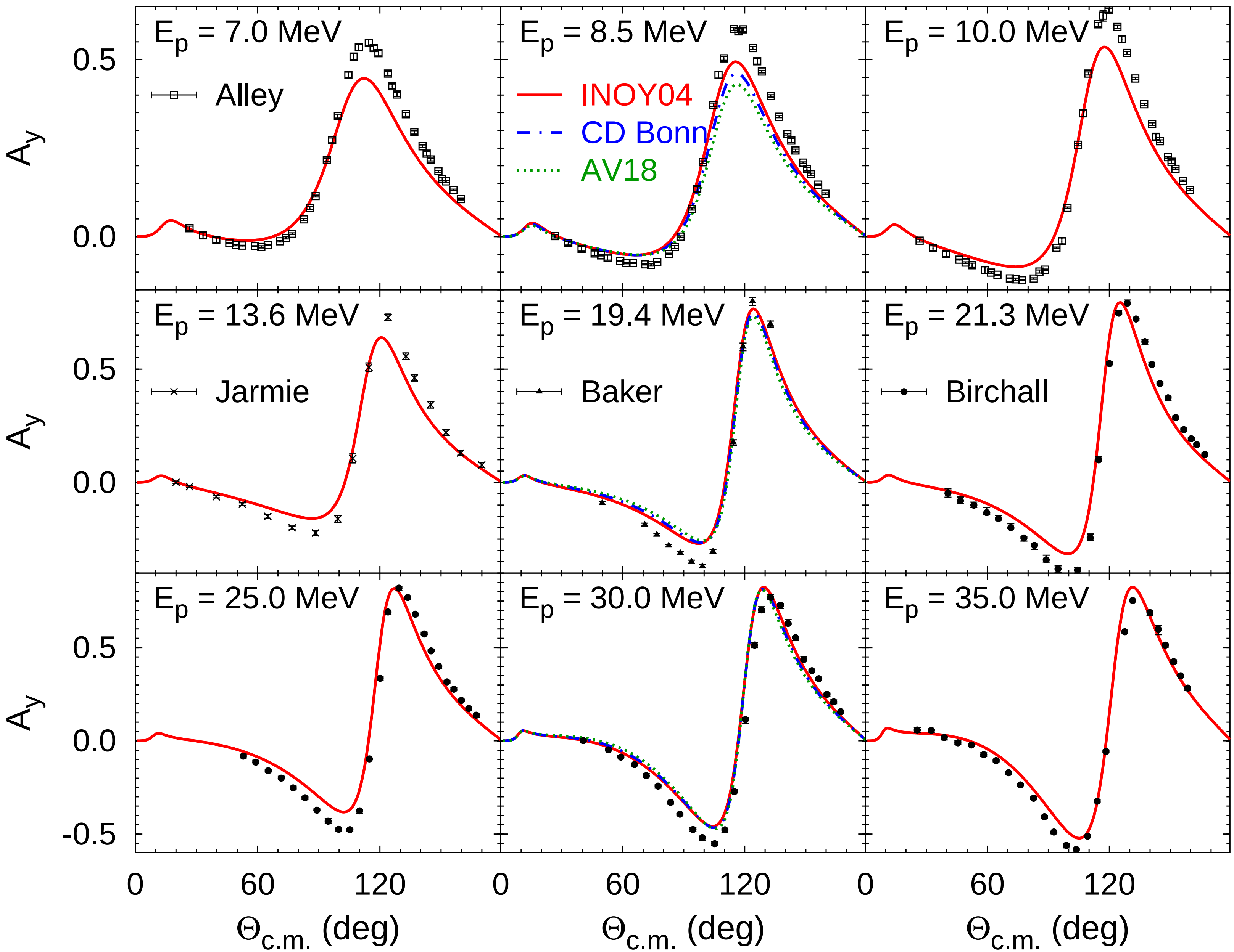
p+^{3}He elastic scattering



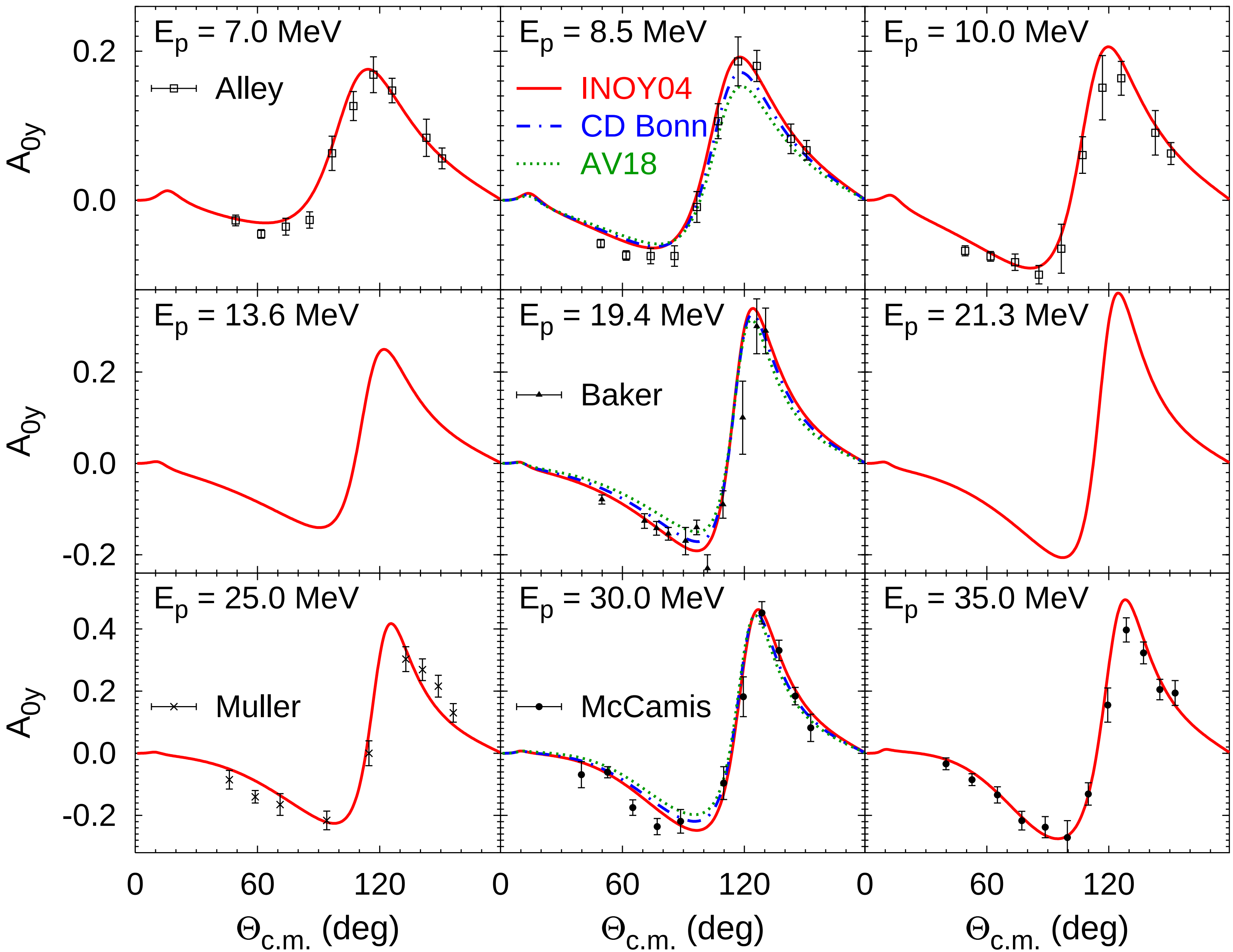


[PRC 87, 054002]

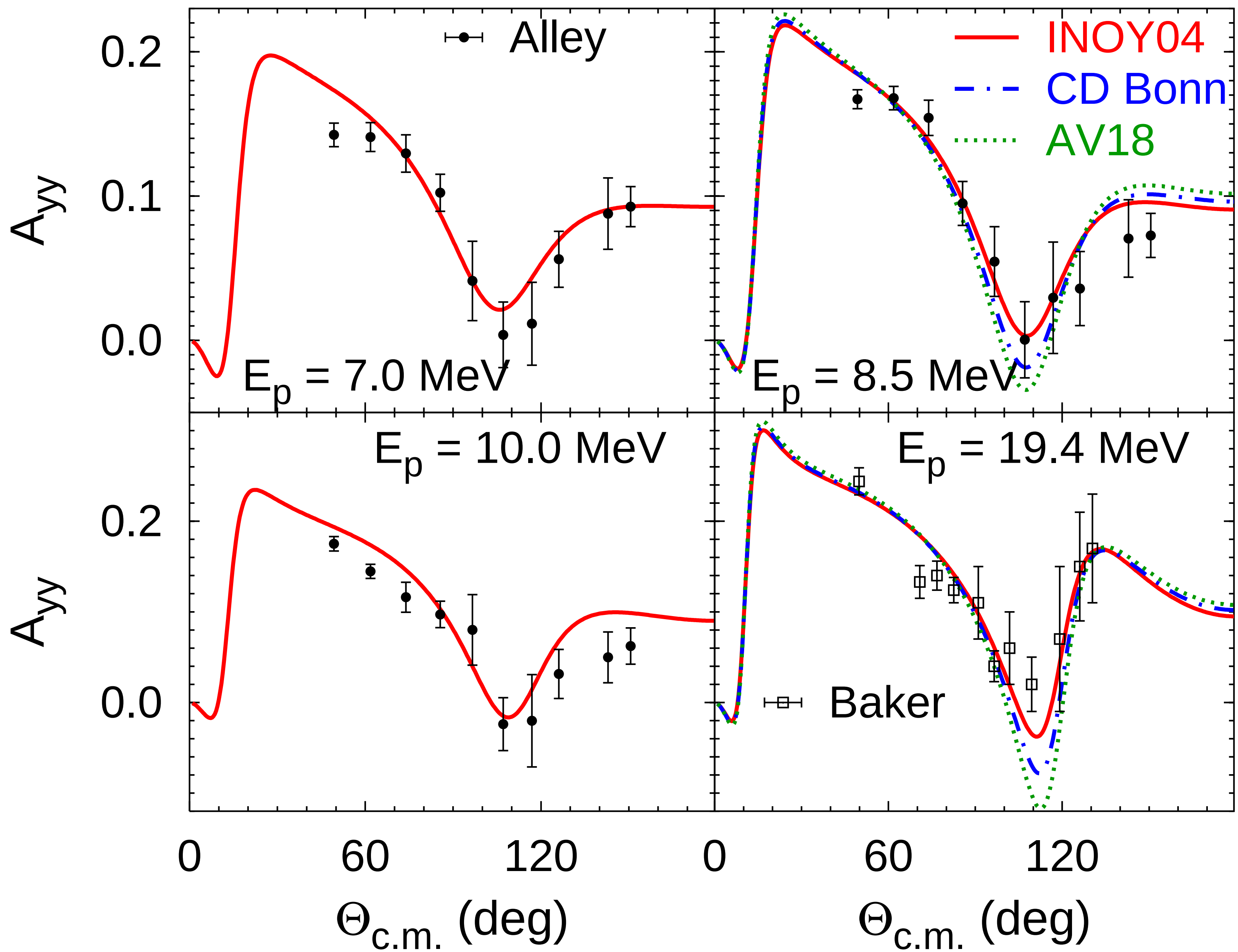
$p+^3He$ elastic scattering



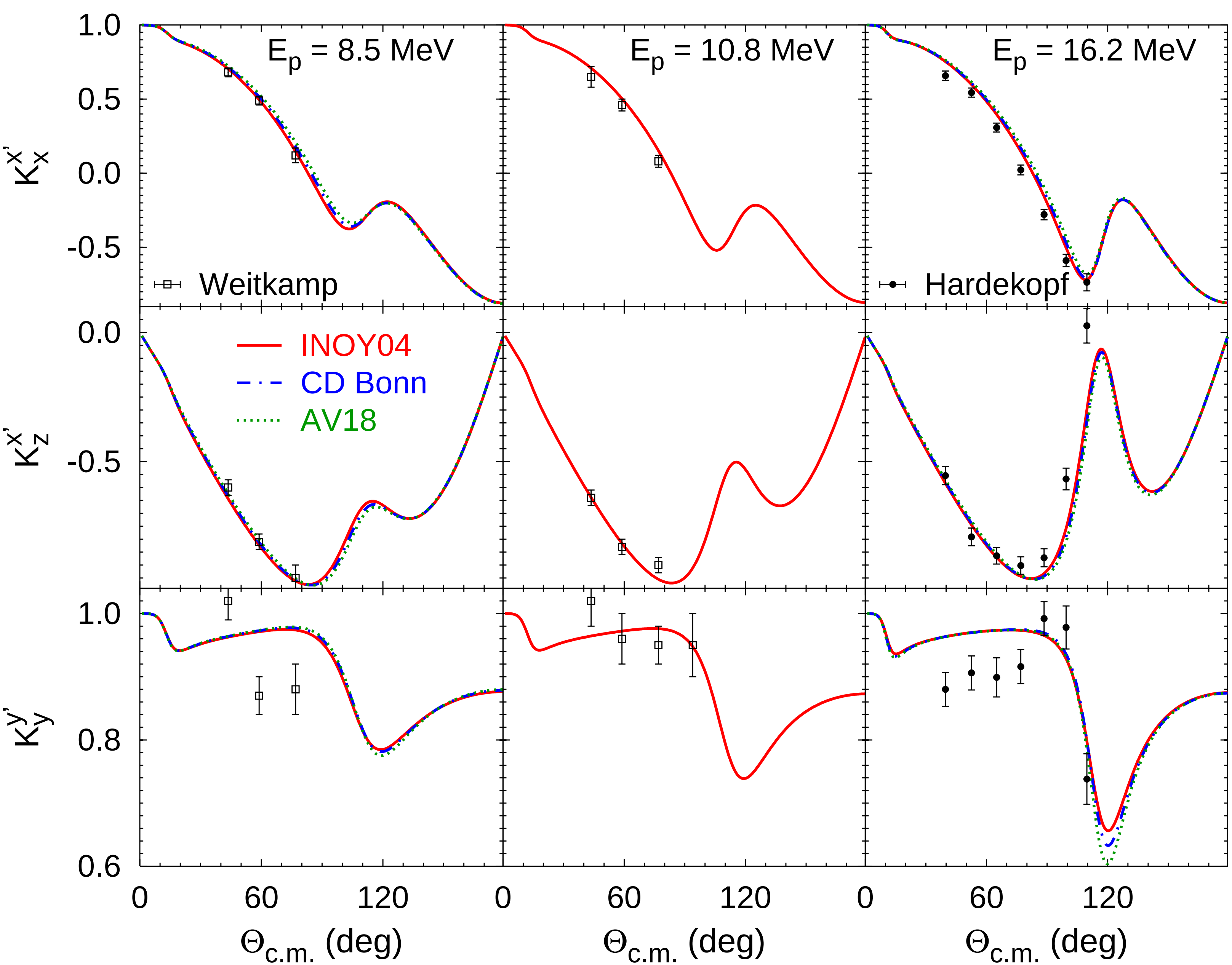
p+^{3}He elastic scattering



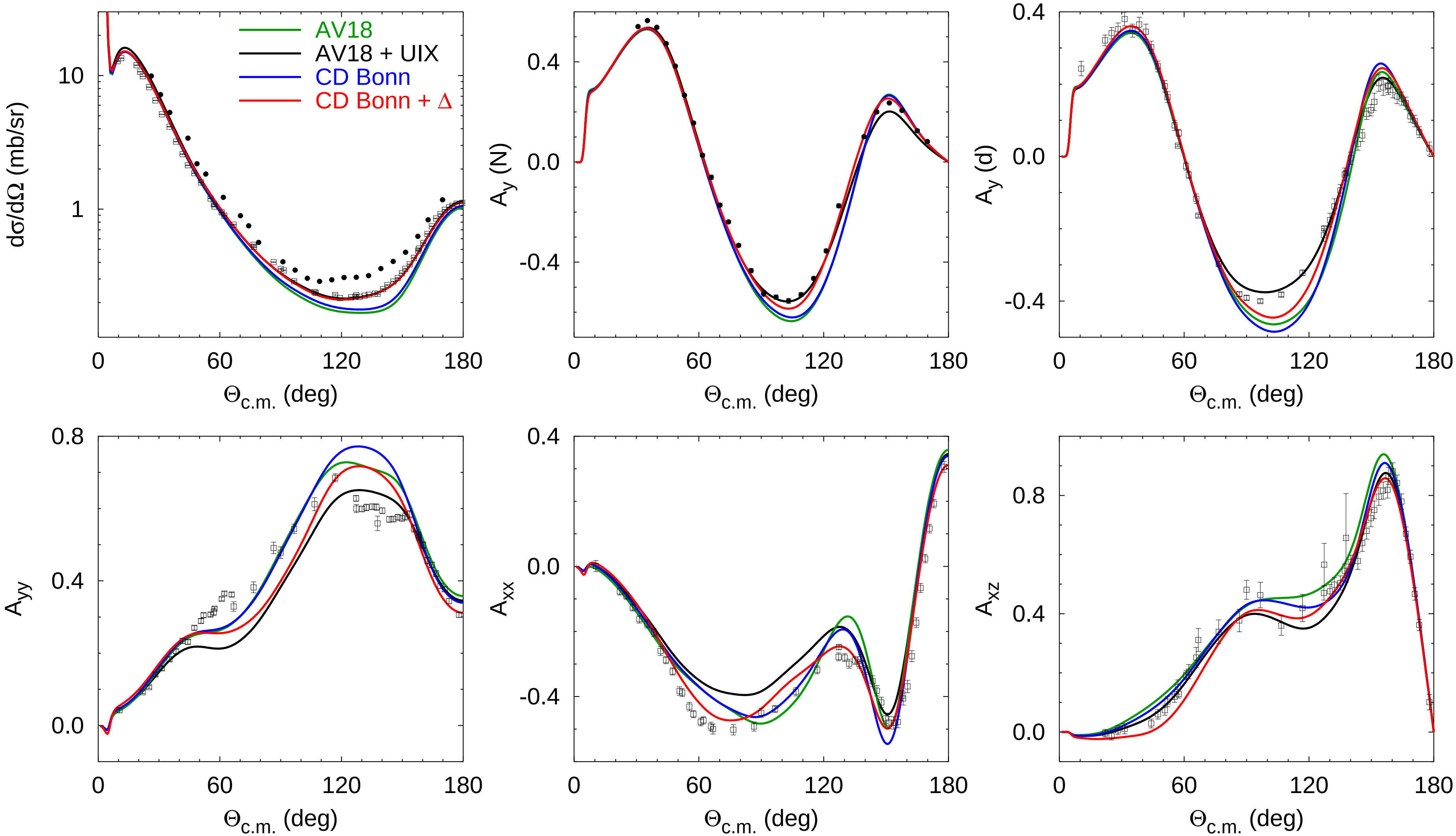
p+^{3}He elastic scattering



p+^{3}He elastic scattering

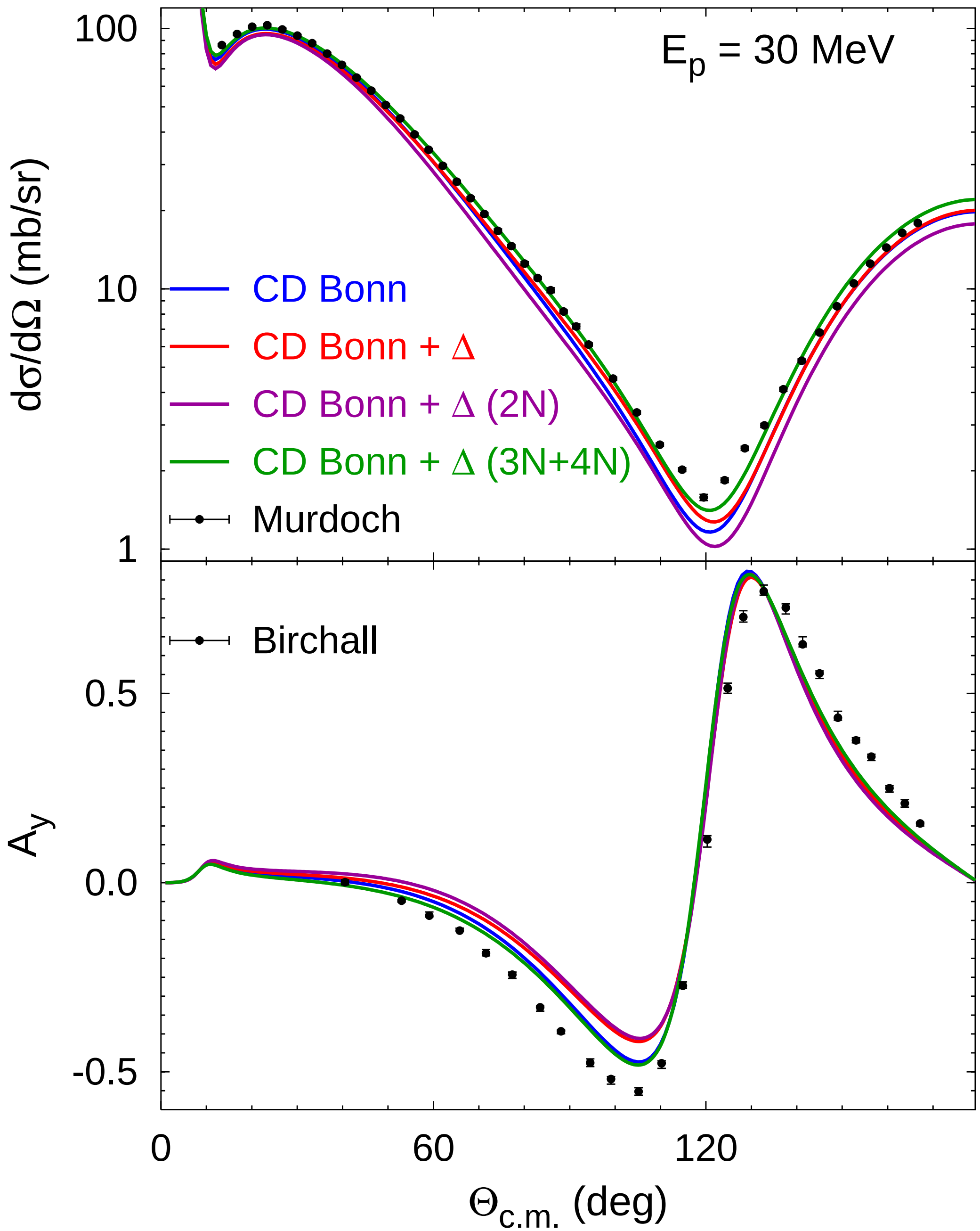


p+d elastic scattering at $E_p = 135$ MeV

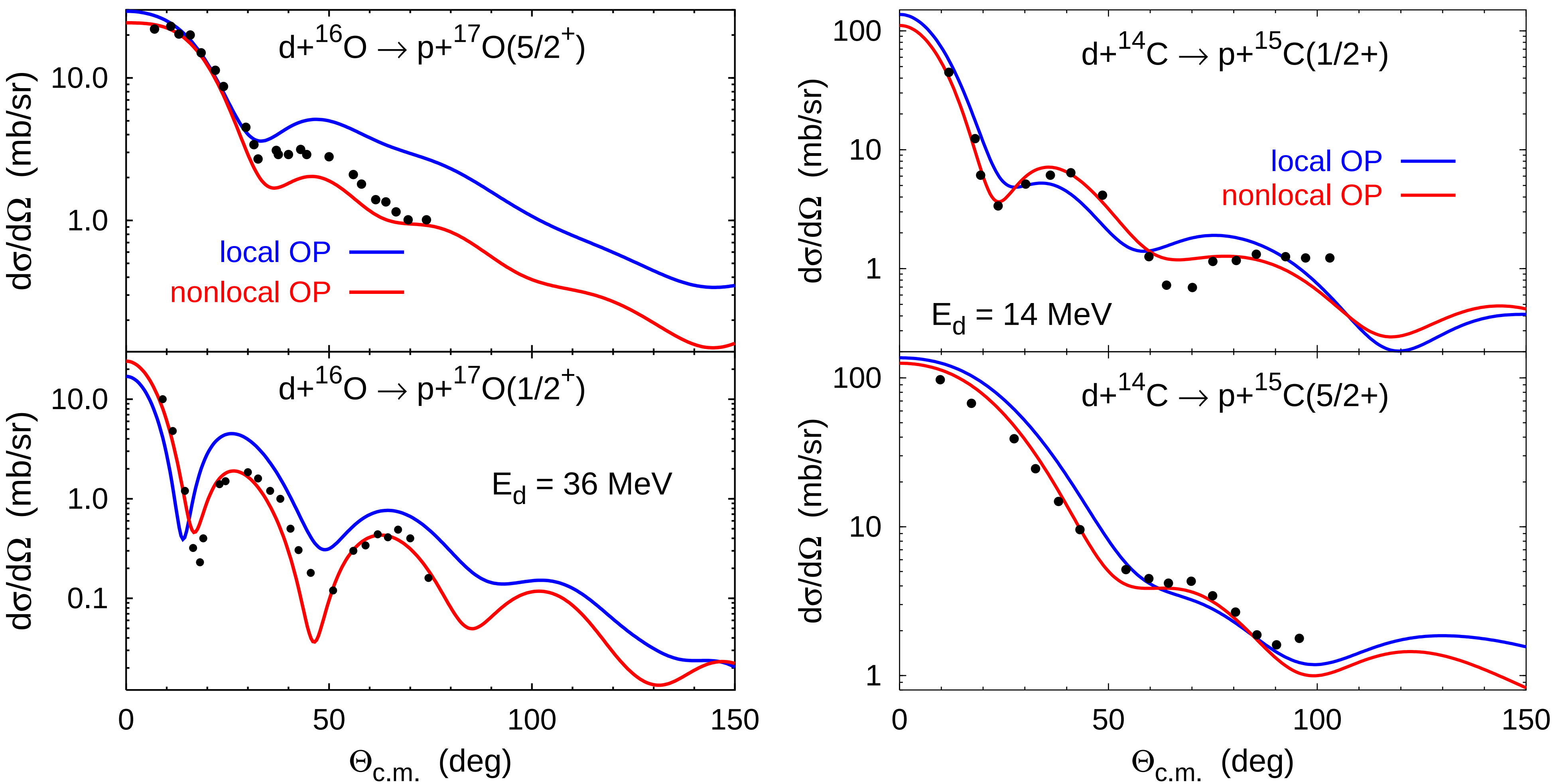


[PRC 80, 064002]

p+^{3}He elastic scattering: Δ effects

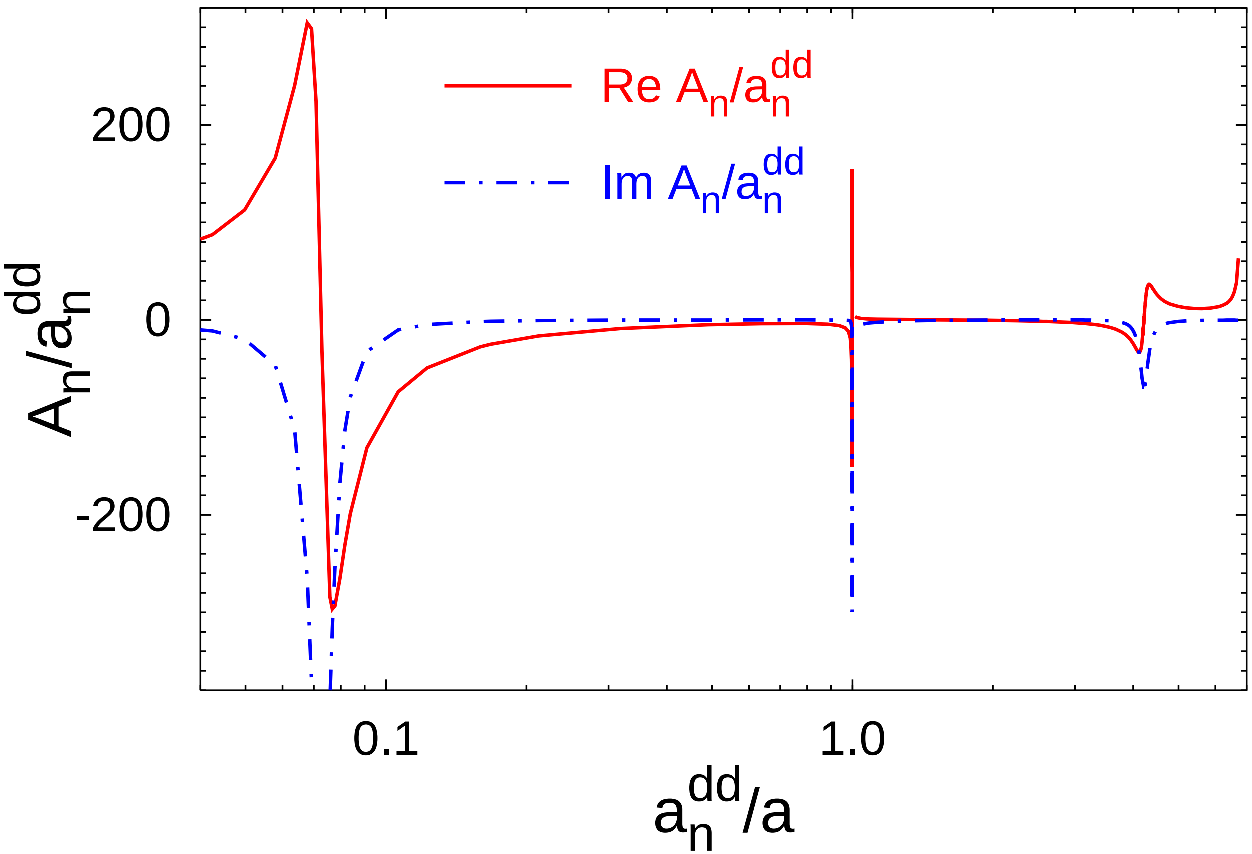


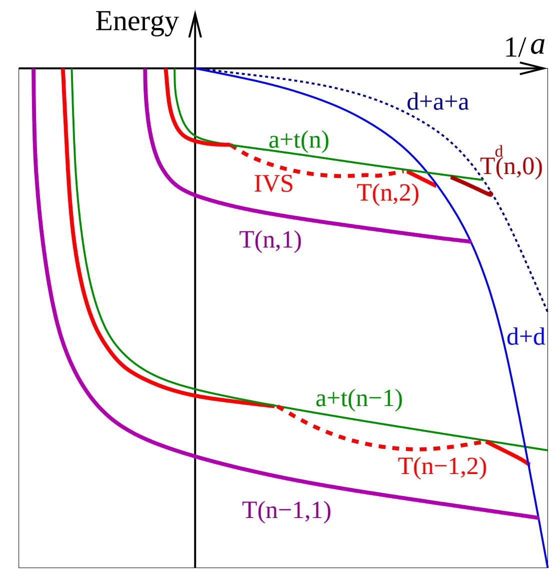
Extension: 3-body nuclear reactions



[PRC 79, 021602, PRC 79, 054603]

Extension: 4-boson universal physics





$$a_n^{dd} : b_n = 2b_d$$

[EPL 95, 43002, PRA 85, 042705]

Summary

- 3/4-body Faddev/AGS equations in momentum space
- complex-energy method
  with special integration weights

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- 3/4-body Faddev/AGS equations in momentum space
- complex-energy method
  with special integration weights

- 3N hadronic and e.m. reactions
- 4N scattering

- 3-body nuclear reactions
- universal 4-boson physics