Infinite Cut-off Regularization of Chiral Nucleon-Nucleon Forces

From Few-Nucleon Forces to Many-Nucleon Structure

ECT June 2013*

D. R. Entem

Thanks to my Collaborators R. Machleidt, Ch. Zeoli, E. Ruiz-Arriola, M. Pavón-Valderrama

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Outline

- EFT for the NN system
- The perturbative amplitude and the definition of the potential
- Singular potentials
- Renormalization with $\Lambda \gg \Lambda_\chi$
  - The $^1S_0$ partial wave
  - Renormalization of the N^3LO potential.

  'Infinite-cutoff renormalization of the chiral nucleon-nucleon interaction at N^3LO
  Ch. Zeoli, R. Machleidt, D.R. Entem, Few Body Systems (2012) 1-15
  (arXiv:1208.2657)
- Renormalization with $\Lambda < \Lambda_\chi$
  'Recent Progress in the Theory of Nuclear Forces'
  R. Machleidt, Q. MacPherson, E. Marji, R. Winzer, Ch. Zeoli, D.R. Entem,
  arXiv:1210.0992

Chiral EFT

- Effective degrees of freedom
  - nucleons
  - Chiral symmetry breaking $\to$ pions
- Simetries: non linear realization of chiral symmetry
- Lagrangian:
  $\mathcal{L} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \ldots =$ all terms consistent with the symmetries
- Non renormalizable theory
- Power counting

$$\nu = 4 - N + 2(L - C) + \sum_i V_i \Delta_i$$

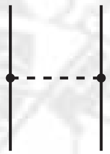
$$\Delta_i = d_i + \frac{n_i}{2} - 2 \geq 0$$

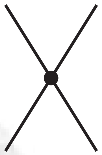
- Low energy expansion $(Q/\Lambda)^\nu$
- Contact terms.

$$L \leq \frac{\nu}{2}$$

Chiral EFT

Order $\nu = 0$





$$V_{1\pi}(\vec{p}\,', \vec{p}) = -\frac{g_A^2}{4 f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2}$$

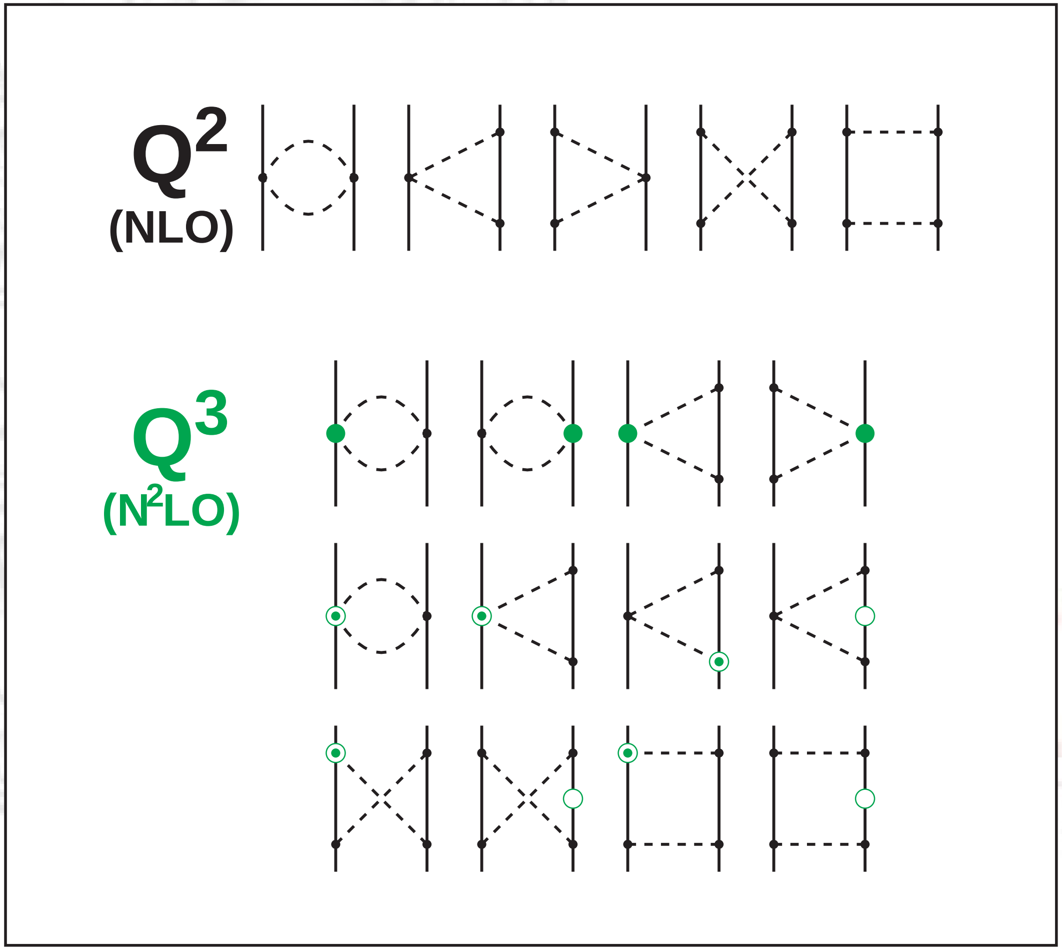
$$V_{ct}^{(0)}(\vec{p}\,', \vec{p}) = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

Charge dependent OPE

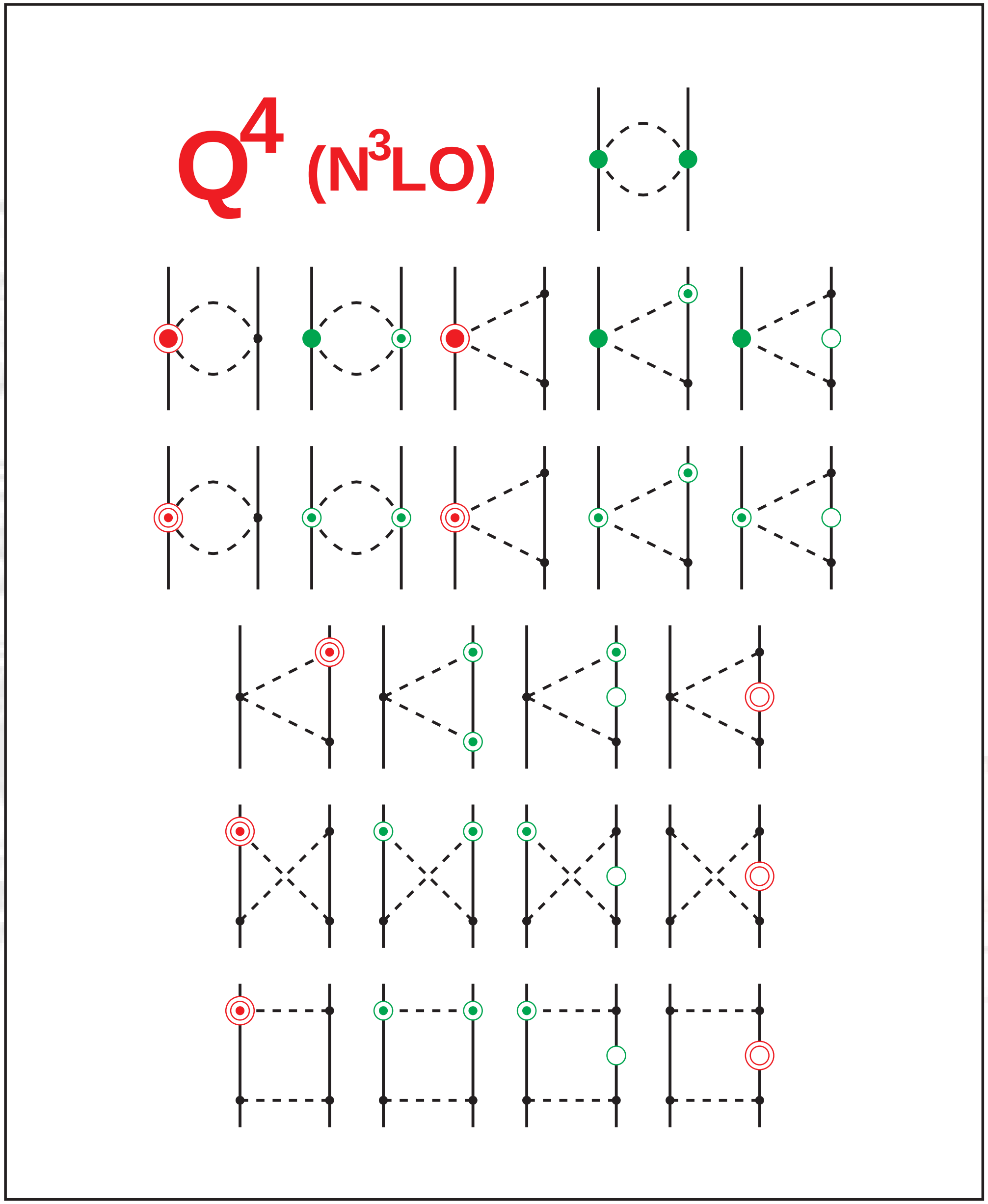
$$V_{1\pi}^{(np)}(\vec{p}\,', \vec{p}) = -V_{1\pi}(m_{\pi^0}) + (-1)^{T+1} 2 V_{1\pi}(m_{\pi^\pm})$$

2π contributions

One loop contributions $\nu = 2L + \sum_i \Delta_i$

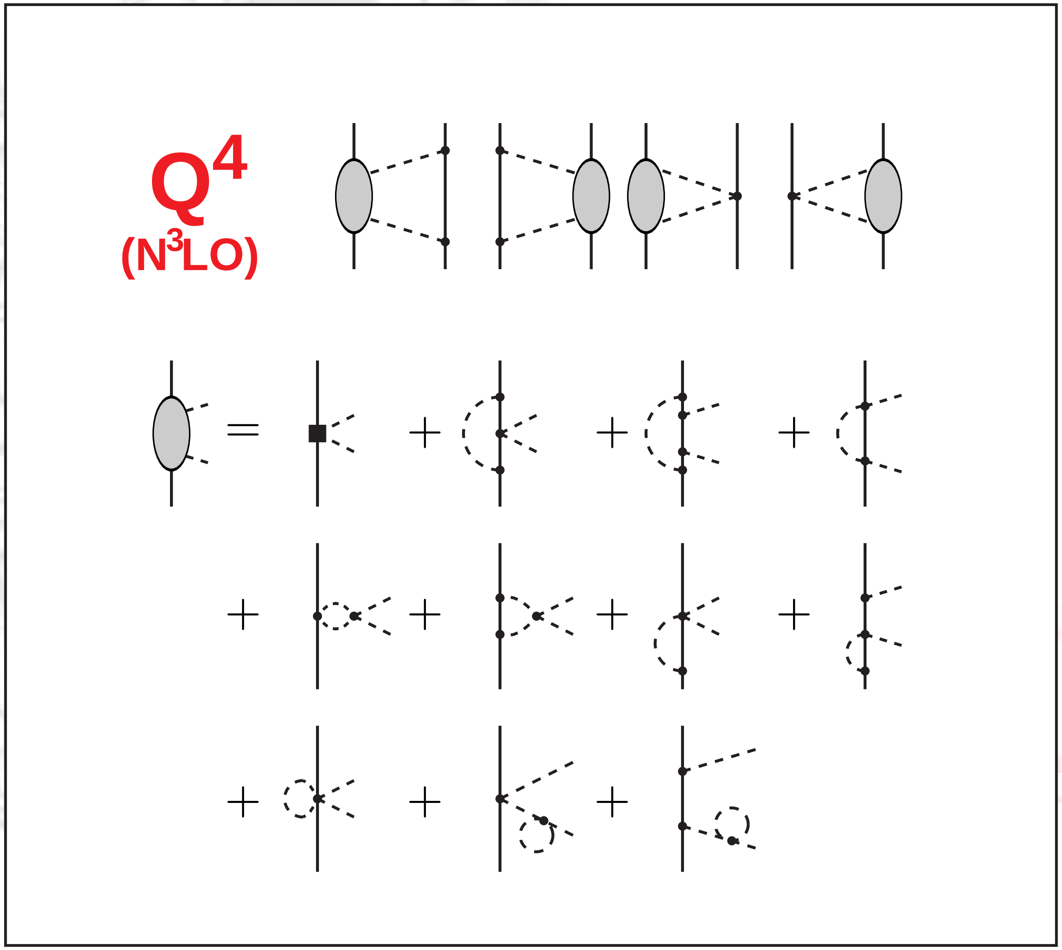


2π exchange contributions



2π exchange contributions

Two loop contributions



The perturbative amplitude

- All amplitudes are evaluated using dimensional regularization and $\overline{MS}$.
- The ampitude is organize as

$$
\begin{aligned}
\mathcal{V}_{LO} &= \mathcal{V}_{ct}^{(0)} + \mathcal{V}_{1\pi}^{(0)} \\
\mathcal{V}_{NLO} &= \mathcal{V}_{LO} + \mathcal{V}_{ct}^{(2)} + \mathcal{V}_{1\pi}^{(2)} + \mathcal{V}_{2\pi}^{(2)} \\
\mathcal{V}_{NNLO} &= \mathcal{V}_{NLO} + \mathcal{V}_{1\pi}^{(3)} + \mathcal{V}_{2\pi}^{(3)} \\
\mathcal{V}_{N^3LO} &= \mathcal{V}_{NNLO} + \mathcal{V}_{ct}^{(4)} + \mathcal{V}_{1\pi}^{(4)} + \mathcal{V}_{2\pi}^{(4)} + \mathcal{V}_{3\pi}^{(4)}
\end{aligned}
$$

- Contacts exactly absorb the infinities due to loop diagrams

Weinberg's proposal

- **Evidences of non-perturbative nature: large scattering lengths and a bound state in NN**
- **Iterative diagrams breaks the Chiral expansion (non-perturbative)**

$$\int \frac{d^3q}{(2\pi)^3} V(p',q) \frac{m_N}{k^2 - q^2 + i\epsilon} V(q,p)$$

If $V(p',p) = C_0$

$$\int \frac{d^3q}{(2\pi)^3} C_0 \frac{m_N}{k^2 - q^2 + i\epsilon} C_0 \quad = \quad -iC_0^2 \frac{m_N k}{4\pi}$$

- **Compute the potential using Chiral EFT and include it in a Lippmann-Schwinger Equation to account for the non-perturbative contribution**
- **Can the same counter terms renormalize the final result?**

Definition of the potential

We use the Blankenbecler Sugar reduction of the Bethe-Salpeter equation

$$\mathcal{M} = \mathcal{V} + \mathcal{V}\mathcal{G}\mathcal{M}$$

or

$$\mathcal{M} = \mathcal{W} + \mathcal{W} g \mathcal{M} \quad \textbf{with} \quad \mathcal{W} = \mathcal{V} + \mathcal{V}(\mathcal{G} - g)\mathcal{W}$$

So

$$\begin{aligned} \mathcal{M}(q';q|P) &= \mathcal{V}(q';q|P) + \int d^4k \mathcal{V}(q';k|P)\mathcal{G}(k|P)\mathcal{M}(k;q|P) \\ \mathcal{G}(k|P) &= \frac{i}{(2\pi)^4}\left[\frac{\frac{1}{2}\not{P} + \not{k} + M_N}{(\frac{1}{2}P + k)^2 - M_N^2 + i\epsilon}\right]^{(1)} \left[\frac{\frac{1}{2}\not{P} - \not{k} + M_N}{(\frac{1}{2}P - k)^2 - M_N^2 + i\epsilon}\right]^{(2)} \end{aligned}$$

In the center of mass frame $P = (\sqrt{s}, \vec{0})$ **with** $\sqrt{s}$ the total energy

Definition of the potential

The Blankenbecler Sugar propagator is

$$
\begin{aligned}
g_{BbS}(k|P) &= -\frac{1}{(2\pi)^3}\int_{4M_N^2}^{\infty}\frac{ds'}{s'-s-i\epsilon}\delta^+((\frac{1}{2}P'+k)^2-M_N^2)\delta^+((\frac{1}{2}P'-k)^2-M_N^2) \\
&\quad \left[\frac{1}{2}\not{P}'+\not{k}+M_N\right]^{(1)}\left[\frac{1}{2}\not{P}'-\not{k}+M_N\right]^{(2)} \\
&= \delta(k_0)\bar{g}_{BbS}(\vec{k}|P) = \delta(k_0)\frac{1}{(2\pi)^3}\frac{M_N^2}{E_k}\frac{\Lambda_+^{(1)}(\vec{k})\Lambda_+^{(2)}(-\vec{k})}{\frac{1}{4}s-E_k^2+i\epsilon}
\end{aligned}
$$

with

$$
\Lambda_+^{(i)}(\vec{k}) = \sum_{\lambda_i} u(\vec{k},\lambda_i)\bar{u}(\vec{k},\lambda_i)
$$

- **Propagate only the positive energy solutions**
- **The $\delta(k_0)$ implies that both nucleons are equally off-shell**
- **Initial nucleons on-shell $q_0 = 0$**
- **Integration over k_0**

Definition of the potential

So

$$\mathcal{M}(0,\vec{q}';0,\vec{q}|P) = \mathcal{W}(0,\vec{q}';0,\vec{q}|P) + \int d^3k \mathcal{W}(0,\vec{q}';0,\vec{k}|P)\bar{g}_{BbS}(\vec{k}|P)\mathcal{M}(0,\vec{k};0,\vec{q}|P)$$

with

$$\mathcal{W} = \mathcal{V} + \mathcal{V}(\mathcal{G} - g_{BbS})\mathcal{V} + \mathcal{V}(\mathcal{G} - g_{BbS})\mathcal{V}(\mathcal{G} - g_{BbS})\mathcal{V} + \dots$$

as $\sqrt{s} = 2E_q$

$$\mathcal{M}(\vec{q}',\vec{q}) = \mathcal{W}(\vec{q}',\vec{q}) + \int \frac{d^3k}{(2\pi)^3} \mathcal{W}(\vec{q}',\vec{k}) \frac{M_N^2}{E_k} \frac{\Lambda_+^{(1)}(\vec{k})\Lambda_+^{(2)}(-\vec{k})}{\vec{q}^2 - \vec{k}^2 + i\epsilon} \mathcal{M}(\vec{k},\vec{q})$$

Taking matrix elements (and turning to the more common notation $q = p$ **and** $q' = p'$**)**

$$\mathcal{T}(\vec{p}',\vec{p}) = V(\vec{p}',\vec{p}) + \int \frac{d^3k}{(2\pi)^3} V(\vec{p}',\vec{k}) \frac{M_N^2}{E_k} \frac{1}{\vec{p}^2 - \vec{k}^2 + i\epsilon} \mathcal{T}(\vec{k},\vec{p})$$

Definition of the potential

$$\mathcal{T}(\vec{p}\,',\vec{p}) \quad = \quad V(\vec{p}\,',\vec{p}) + \int \frac{d^3k}{(2\pi)^3} V(\vec{p}\,',\vec{k}) \frac{M_N^2}{E_k} \frac{1}{\vec{p}^2 - \vec{k}^2 + i\epsilon} \mathcal{T}(\vec{k},\vec{p})$$

So if we take the definition ('minimal relativity')

$$\hat{T}(\vec{p}\,',\vec{p}) \quad = \quad \frac{1}{(2\pi)^3}\sqrt{\frac{M_N}{E_p'}}\,\mathcal{T}(\vec{p}\,',\vec{p})\sqrt{\frac{M_N}{E_p}}$$

$$\hat{V}(\vec{p}\,',\vec{p}) \quad = \quad \frac{1}{(2\pi)^3}\sqrt{\frac{M_N}{E_p'}}\,V(\vec{p}\,',\vec{p})\sqrt{\frac{M_N}{E_p}}$$

we end up with Lippman-Schwinger Eq.

$$\hat{T}(\vec{p}\,',\vec{p}) \quad = \quad \hat{V}(\vec{p}\,',\vec{p}) + \int d^3k\, \hat{V}(\vec{p}\,',\vec{k}) \frac{M_N}{\vec{p}^2 - \vec{k}^2 + i\epsilon} \hat{T}(\vec{k},\vec{p})$$

Definition of the potential

Notice that the energy-momentum relation is the relativistic one so

$$p^2 = \frac{M_p^2 T_L (T_L + 2M_n)}{(M_p + M_n)^2 + 2T_L M_p}$$

and we use

$$M_N = \frac{2M_p M_n}{M_p + M_n}$$

Also notice that now the box diagram is

$$\mathcal{W}_{box} = \mathcal{V}_{1\pi}(\mathcal{G} - g_{BbS})\mathcal{V}_{1\pi}$$

This makes an slight difference of our irreducible box contribution and Kaiser *et al.* **convention**
Details in Phys. Rep. 503, 1 (2011)

Singular potentials

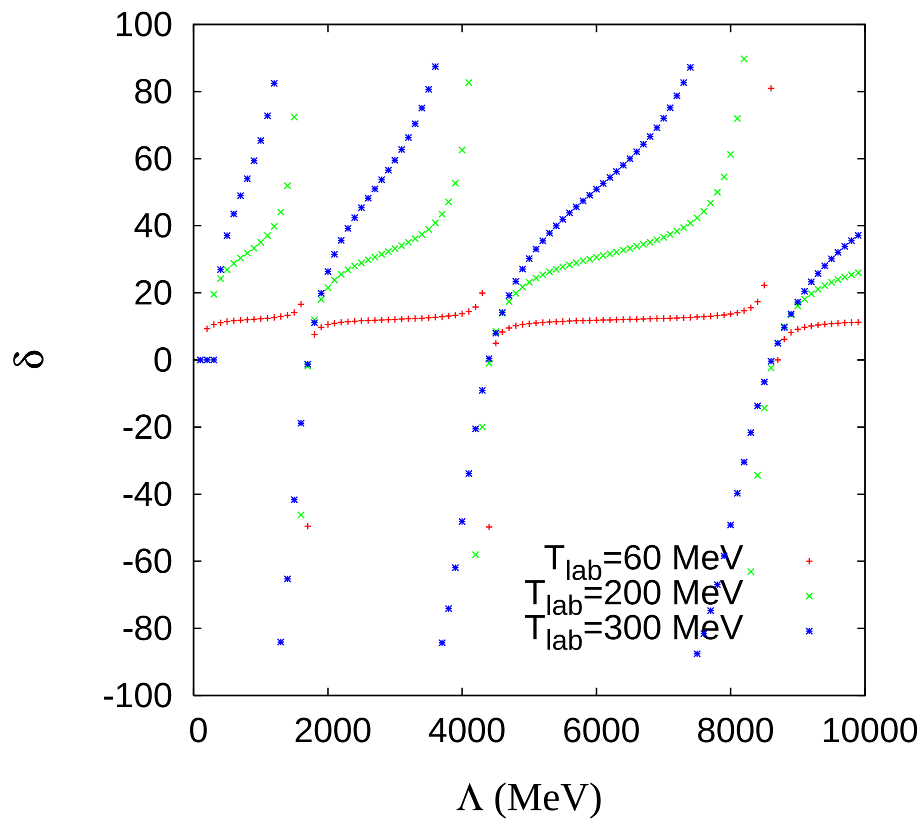
- Higher orders in the Chiral expansion produces singular potentials
  - More singular than $1/r^2$ when $r \to 0$.
  - Divergent when $q \to \infty$.
- The Lippman-Schwinger Eq. has to be regularized
- In order to obtain regularization independence there are two points of view
  - Use $\Lambda \gg \Lambda_\chi$
  - Lepage plots point of view, use Λ between the low energy and the high energy scales so $\Lambda < \Lambda_\chi$.

The tensor part of the LO potential is already singular

Nogga, Timmermans and van Kolck in momentum space

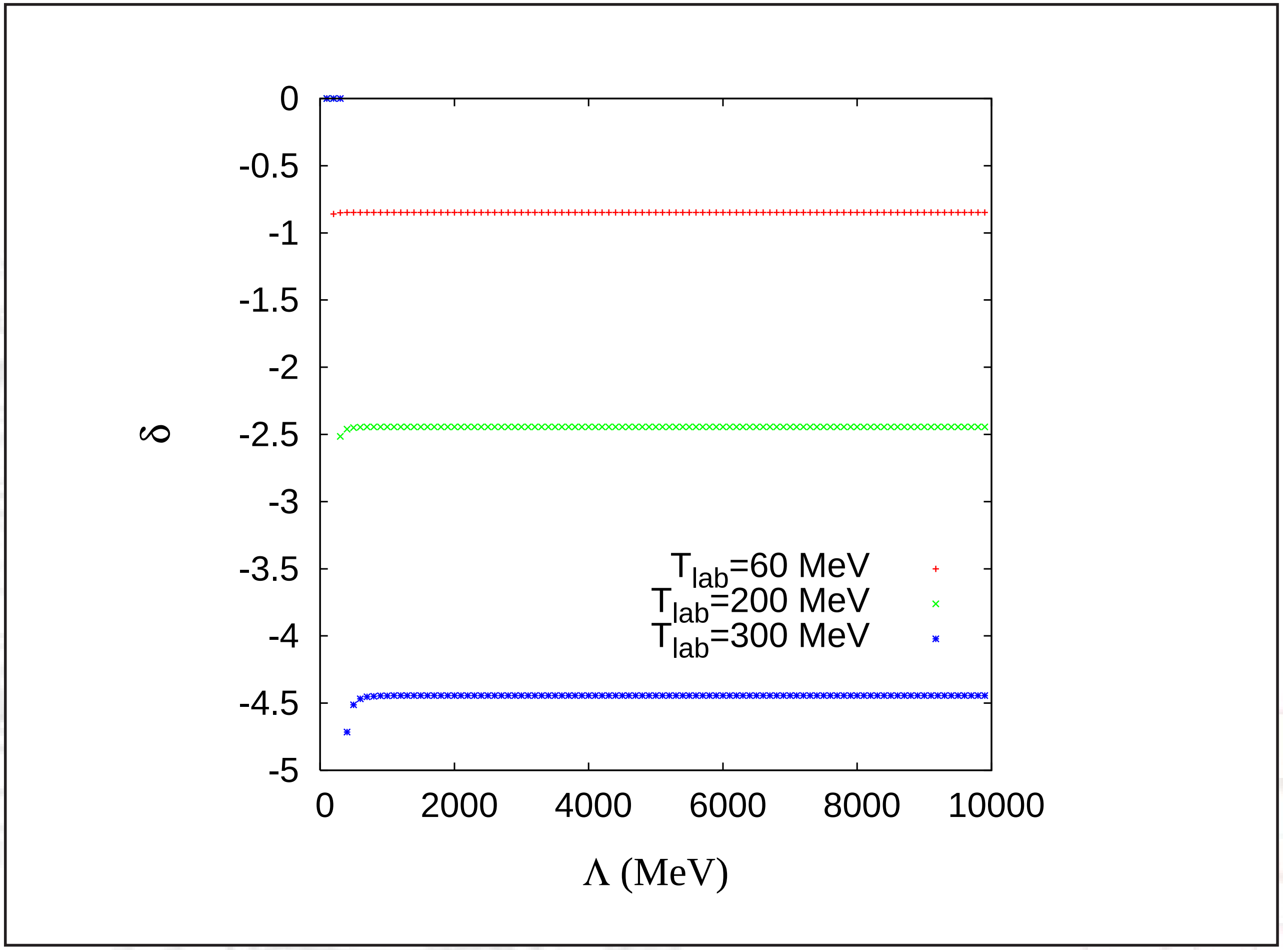
Pavón-Valderrama and Ruiz-Arriola in coordinate space

Singular tensor force



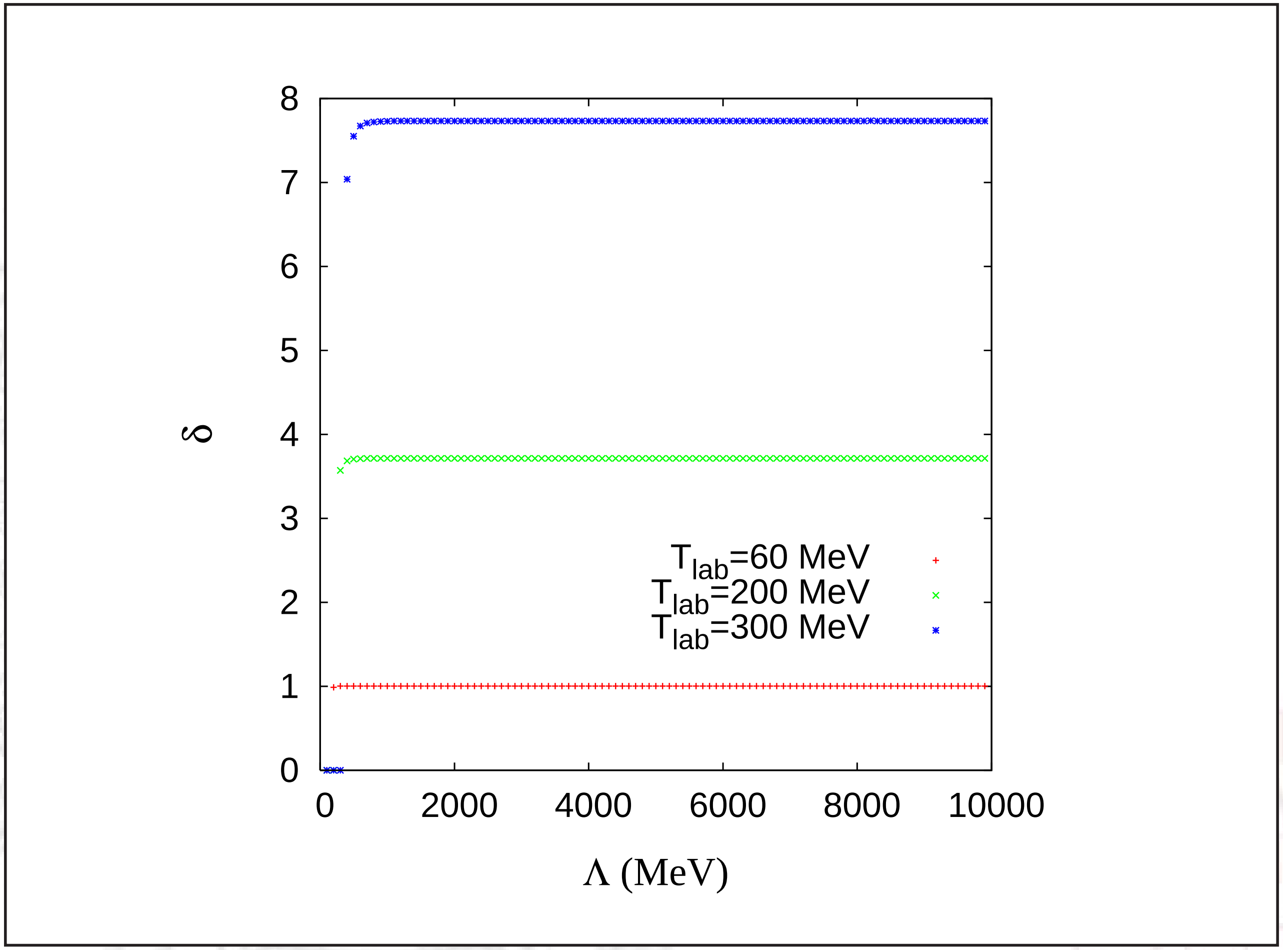
$^3D_2$ **LO** **Tensor goes as** $-\frac{1}{r^3}$

Singular tensor force



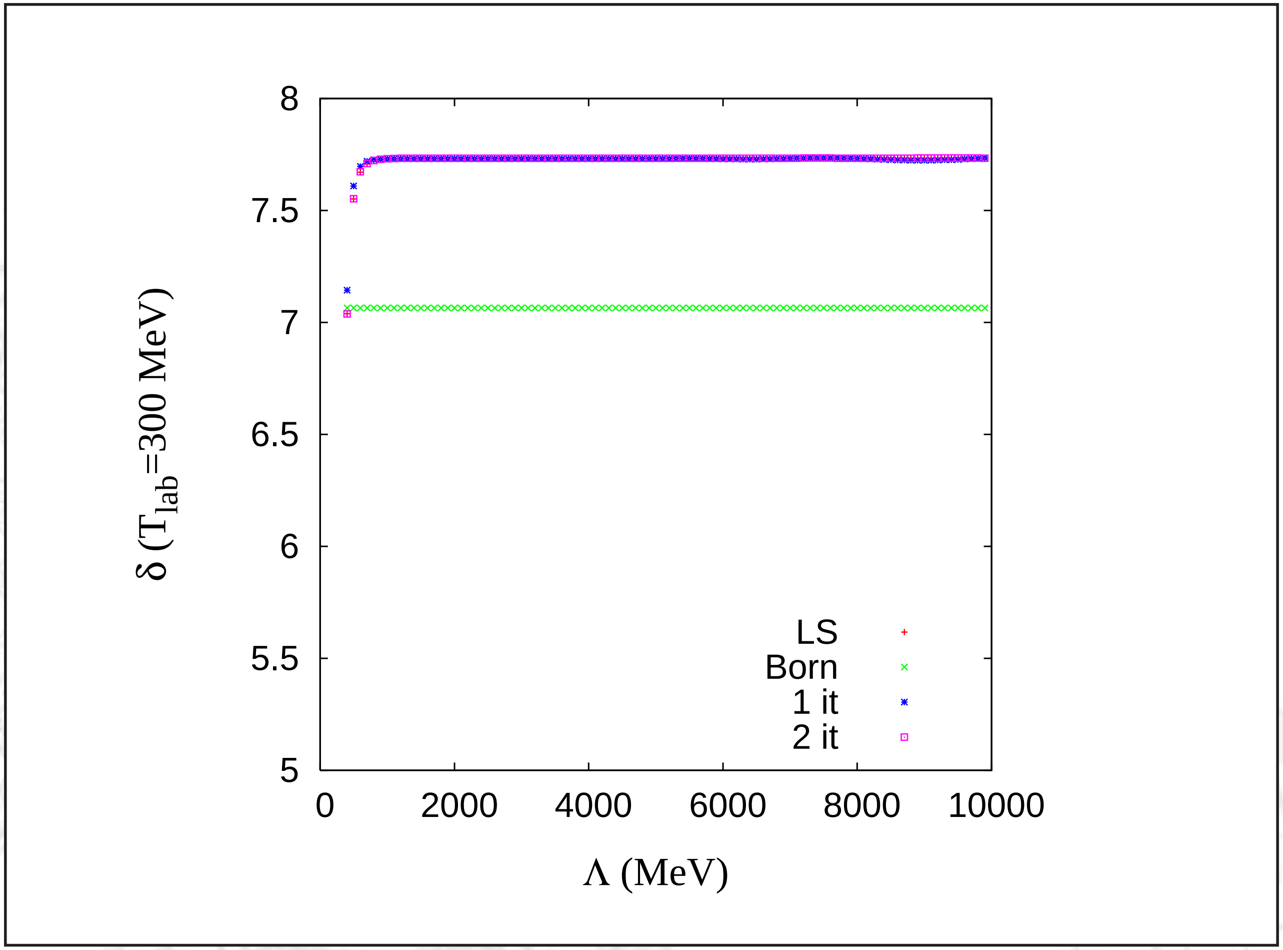
$^3F_3$ **LO** **Tensor goes as** $\frac{1}{r^3}$

Singular tensor force



$^3G_4$ **LO** **Tensor goes as** $-\frac{1}{r^3}$

Singular tensor force



$^3G_4$ **LO** **Tensor goes as** $-\frac{1}{r^3}$

Pionless EFT

$$T_\Lambda(\vec{k}', \vec{k}; k) = C + \frac{m_N}{2\pi^2}\int_0^\Lambda dqC\frac{q^2}{k^2 - q^2 + i\epsilon}T_\Lambda(\vec{k}', \vec{k}; k)$$

Solution

$$T_\Lambda(\vec{k}', \vec{k}; k) = \frac{C(\Lambda)}{1 - C(\Lambda)I(k, \Lambda)}$$

with

$$I(k, \Lambda) = \frac{m_N}{2\pi^2}\int_0^\Lambda dq\frac{q^2}{k^2 - q^2 + i\epsilon} = \frac{m_N}{2\pi^2}(-\Lambda + \frac{k}{2}\log\frac{\Lambda + k}{\Lambda - k}) - i\frac{m_N}{4\pi}k$$

Fix $C(\Lambda)$ at a certain energy scale μ

$$T(\mu) = T_\Lambda(\vec{k}', \vec{k}; \mu) = \frac{C(\Lambda, \mu)}{1 - C(\Lambda, \mu)I(\mu, \Lambda)} \Rightarrow C^{-1}(\Lambda, \mu) = T^{-1}(\mu) + I(\mu, \Lambda)$$

Pionless EFT

$$T_\Lambda(\vec{k}', \vec{k}; k) = \frac{C(\Lambda)}{1 - C(\Lambda) I(k,\Lambda)} = \frac{1}{C^{-1}(\Lambda,\mu) - I(k,\Lambda)} = \frac{1}{T^{-1}(\mu) + I(\mu,\Lambda) - I(k,\Lambda)}$$

But

$$I(\mu,\Lambda) - I(k,\Lambda) = \frac{m_N}{2\pi^2}\left(\frac{\mu}{2}\log\frac{\Lambda+\mu}{\Lambda-\mu} - \frac{k}{2}\log\frac{\Lambda+k}{\Lambda-k}\right) - i\frac{m_N}{4\pi}(\mu - k)$$

The result is finite for any value of Λ

$$\lim_{\Lambda\to\infty} I(\mu,\Lambda) - I(k,\Lambda) = -i\frac{m_N}{4\pi}(\mu - k)$$

$$\lim_{\Lambda\to\infty} T_\Lambda(\vec{k}', \vec{k}; k) = \frac{1}{T^{-1}(\mu) - i\frac{m_N}{4\pi}(\mu - k)} = \frac{T(\mu)}{1 + m_N T(\mu)(ik - i\mu)/4\pi}$$

Substractive Renormalization

T. Frederico, V.S. Timoteo, L. Tomio

C.-J. Yang, Ch. Elster, D.R. Phillips

Zero Energy

$$T_\Lambda(p',0;0) = V(p',0) + C + \frac{2}{\pi} M \int_0^\Lambda dp''\, p''^2 \left(\frac{V(p',p'') + C}{-p''^2} \right) T_\Lambda(p'',0;0)$$

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$T_\Lambda(0,0;0) = T(0,0;0) = \frac{a}{M}$

$$T_\Lambda(p',0;0) = V(p',0) + \frac{a}{M} + \frac{2}{\pi} M \int_0^\Lambda dp''\, p''^2 \left(\frac{V(p',p'') - V(0,p'')}{-p''^2} \right) T_\Lambda(p'',0;0)$$

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$$T_\Lambda(p', p; 0) = V(p', p) + C + \frac{2}{\pi} M \int_0^\Lambda dp''\, p''^2 \left(\frac{V(p', p'') + C}{-p''^2} \right) T_\Lambda(p'', p; 0)$$

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Zero Energy

$$
\begin{aligned}
T_\Lambda(p', p; 0) &= V(p', p) + C + \frac{2}{\pi} M \int_0^\Lambda dp''\, p''^2 \left(\frac{V(p', p'') + C}{-p''^2} \right) T_\Lambda(p'', p; 0) \\
T_\Lambda(0, p; 0) &= V(0, p) + C + \frac{2}{\pi} M \int_0^\Lambda dp''\, p''^2 \left(\frac{V(0, p'') + C}{-p''^2} \right) T_\Lambda(p'', p; 0)
\end{aligned}
$$

$T_\Lambda(0,0;0) = T(0,0;0) = \frac{a}{M}$

$$
T_\Lambda(p', 0; 0) = V(p', 0) + \frac{a}{M} + \frac{2}{\pi} M \int_0^\Lambda dp''\, p''^2 \left(\frac{V(p', p'') - V(0, p'')}{-p''^2} \right) T_\Lambda(p'', 0; 0)
$$

$T_\Lambda(p, 0; 0) = T_\Lambda(0, p; 0)$

$$
\begin{aligned}
T_\Lambda(p', p; 0) - T_\Lambda(0, p; 0) &= V(p', p) - V(0, p) \\
&+ \frac{2}{\pi} M \int_0^\Lambda dp''\, p''^2 \left(\frac{V(p', p'') - V(0, p'')}{-p''^2} \right) T_\Lambda(p'', p; 0)
\end{aligned}
$$

Substractive Renormalization

T. Frederico, V.S. Timoteo, L. Tomio
C.-J. Yang, Ch. Elster, D.R. Phillips

Non-zero energy

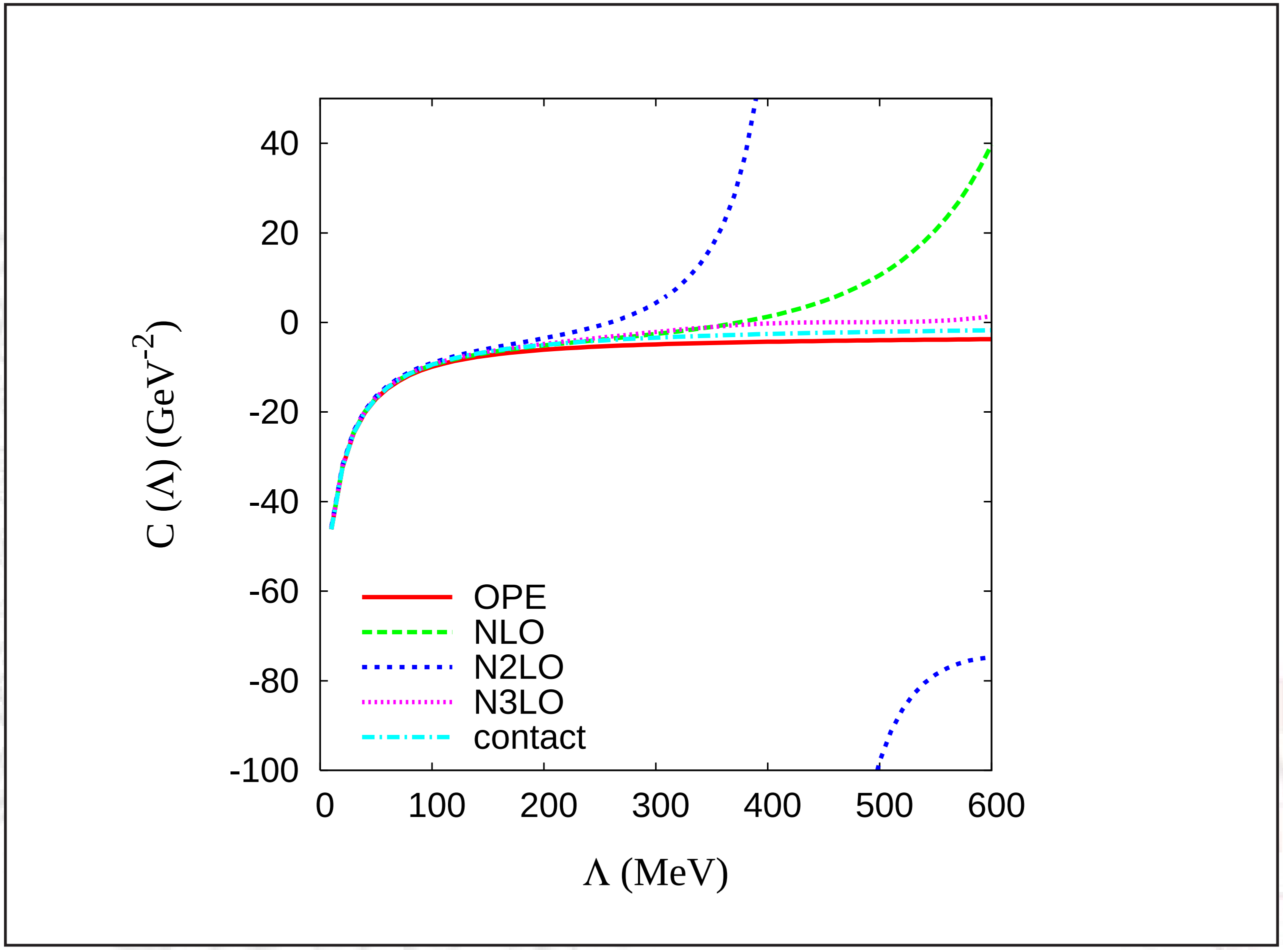
$$
\begin{aligned}
T_\Lambda(0) &= (V+C) + T_\Lambda(0)G_\Lambda(0)(V+C) \quad \Rightarrow \quad (V+C) = T_\Lambda(0)(1+T_\Lambda(0)G_\Lambda(0))^{-1} \\
T_\Lambda(E) &= (V+C) + (V+C)G_\Lambda(E)T_\Lambda(E) \quad \Rightarrow \quad (V+C) = T_\Lambda(E)(1+G_\Lambda(E)T_\Lambda(E))^{-1}
\end{aligned}
$$

$$
T_\Lambda(E) = T_\Lambda(0) + T_\Lambda(0)(G_\Lambda(E) - G_\Lambda(0))T_\Lambda(E)
$$

- Half off-shell T-Matrix at $E = 0$
- Full off-shell T-Matrix at $E = 0$
- Full off-shell T-Matrix at E
- The value of $T(0,0;0)$ is fixed

$^1S_0$ partial wave

We fit the contact term to the np scattering length $a_{np} = -23{,}75\, fm$



D.R. Entem, E.R. Arriola, M. Pavon-Valderrama, R. Machleidt, PRC 77, 044006 (2008)

$^1S_0$ partial wave

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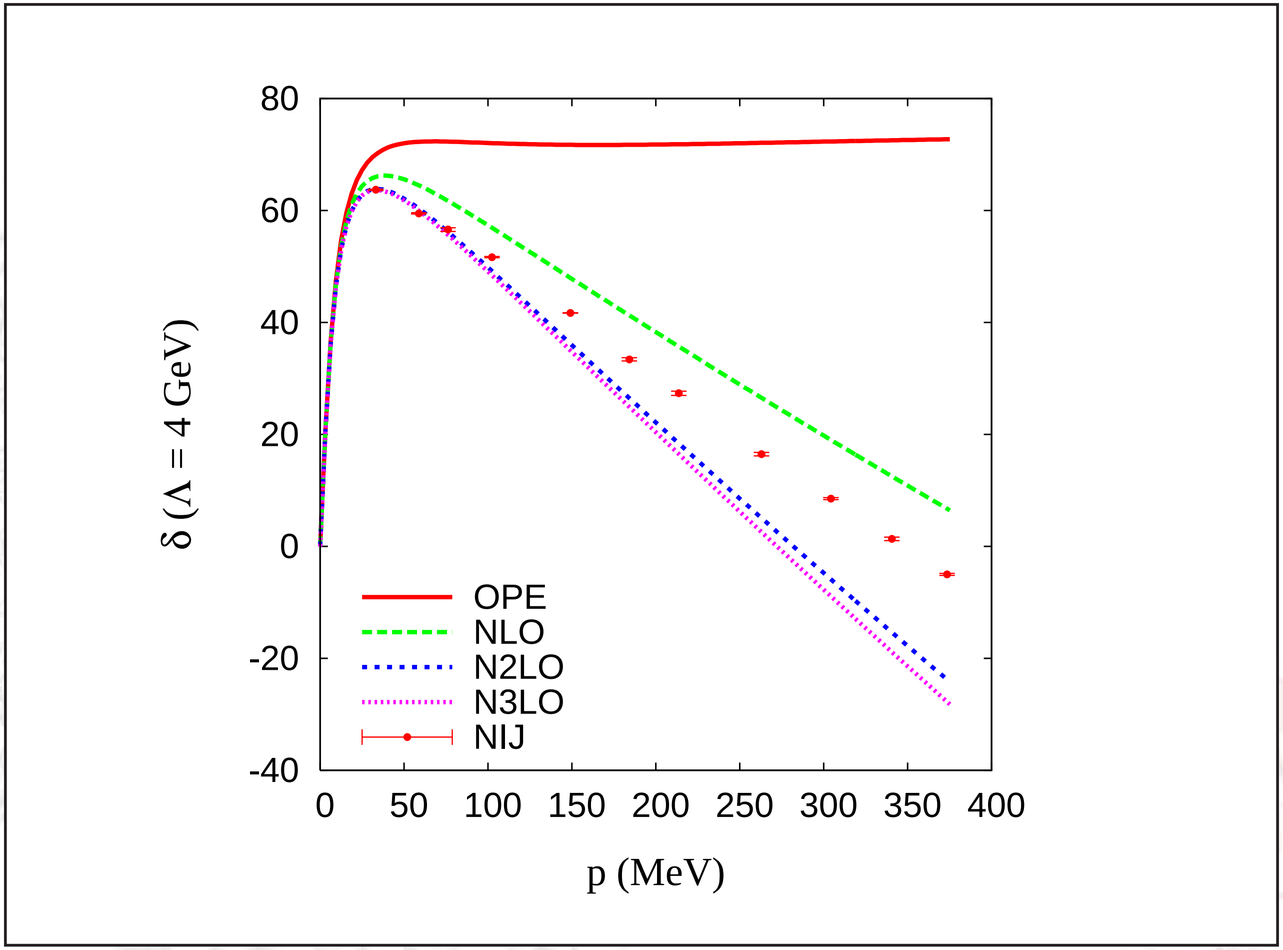
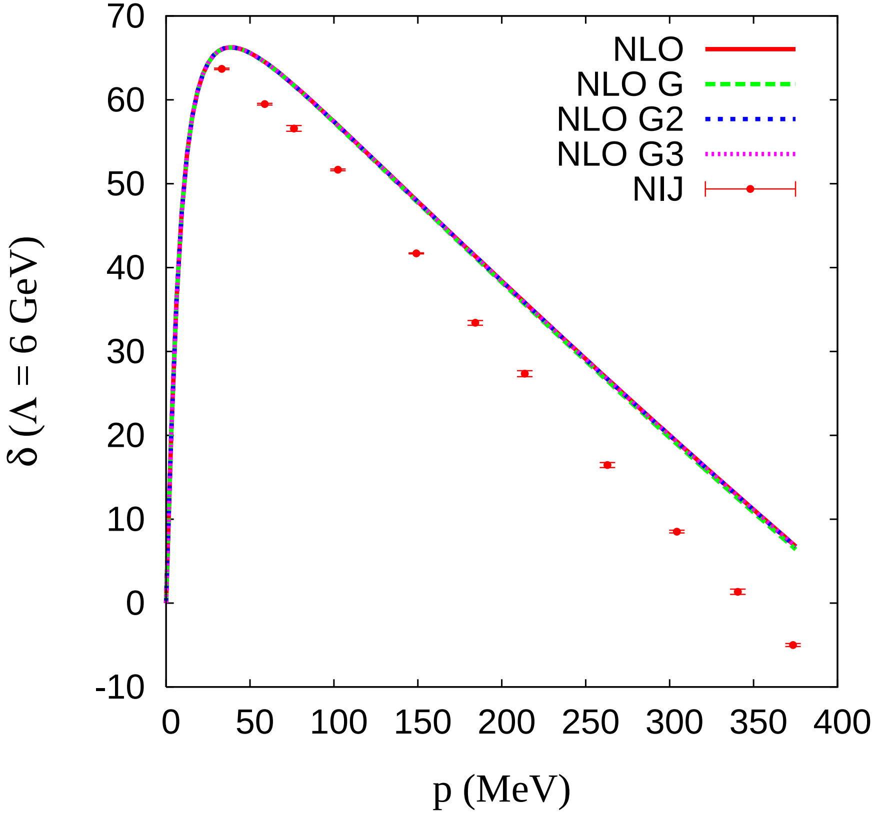
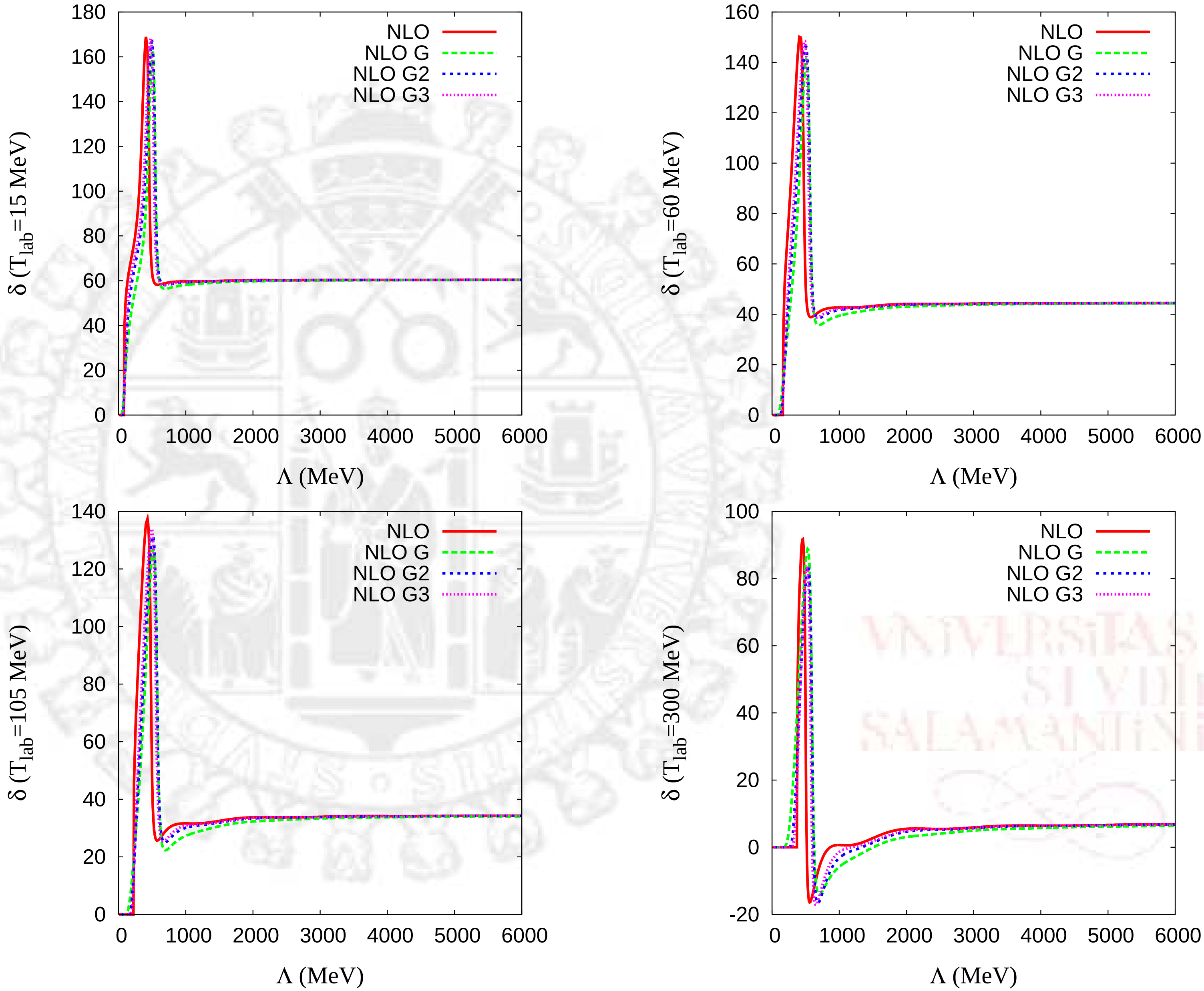


D.R. Entem, E.R. Arriola, M. Pavon-Valderrama, R. Machleidt, PRC 77, 044006 (2008)

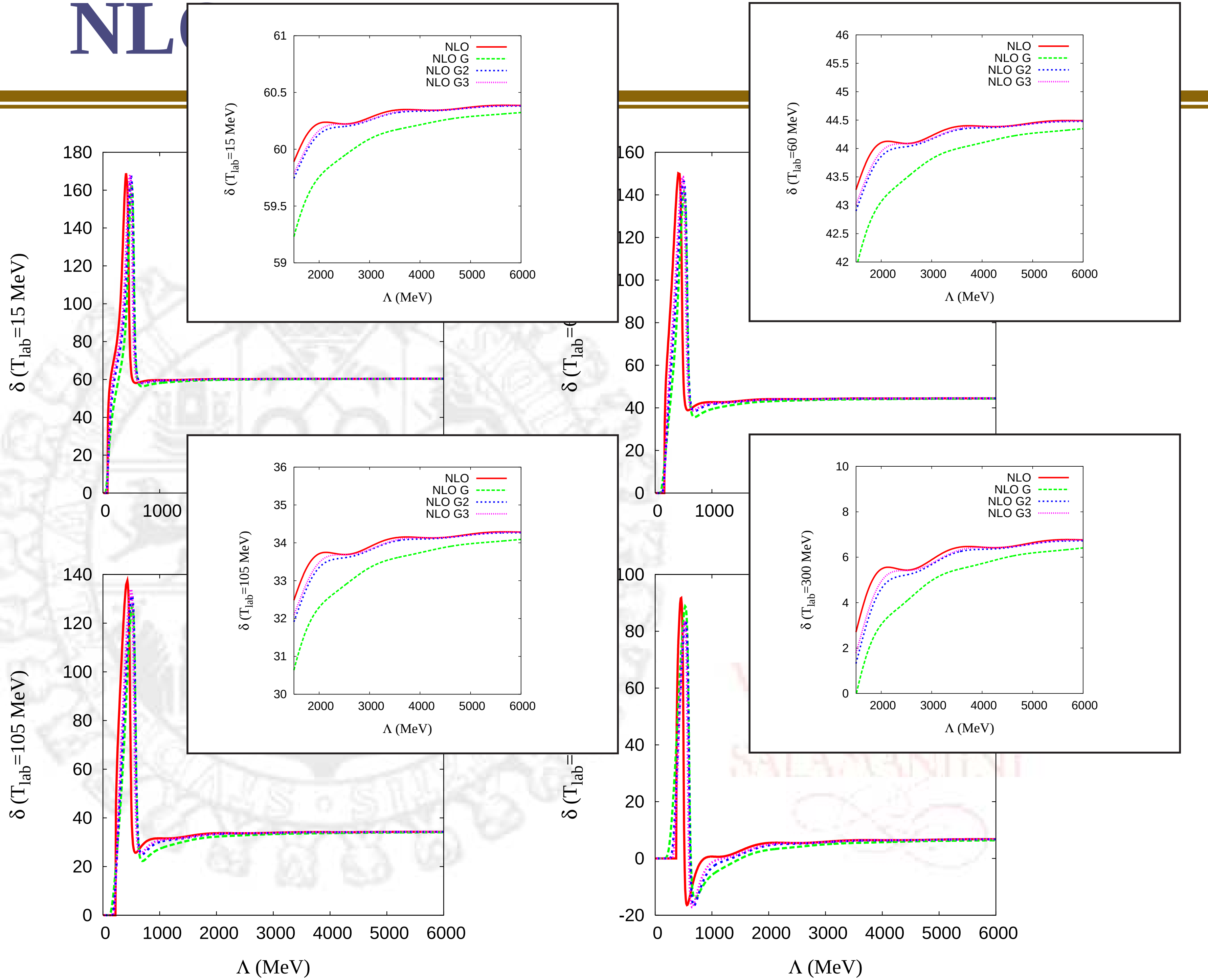
Regularization dependence - NLO



NLO

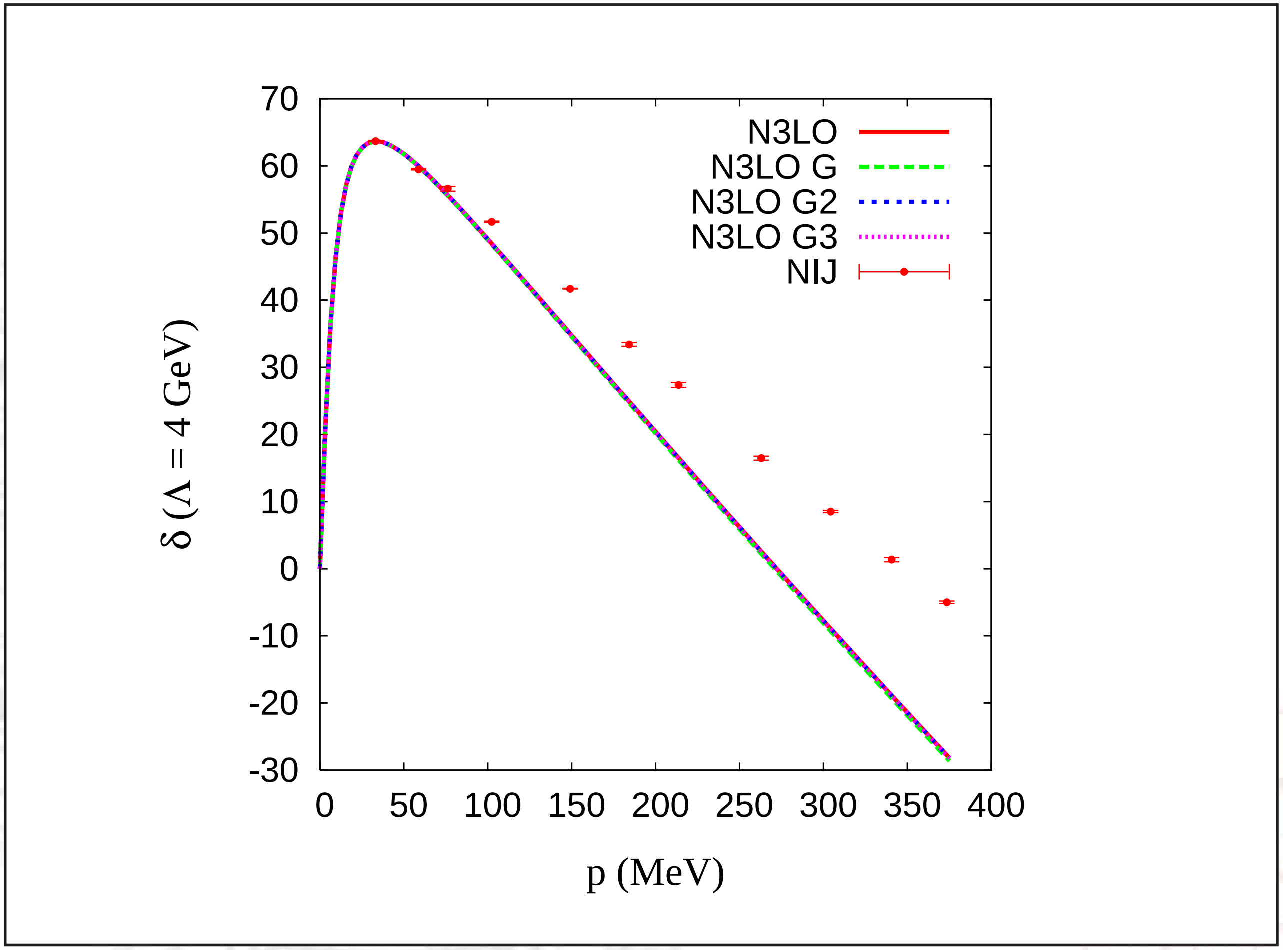

NLO
NLO G
NLO G2
NLO G3
δ (T_lab=15 MeV)
δ (T_lab=60 MeV)
δ (T_lab=105 MeV)
δ (T_lab=300 MeV)
Λ (MeV)

NLO

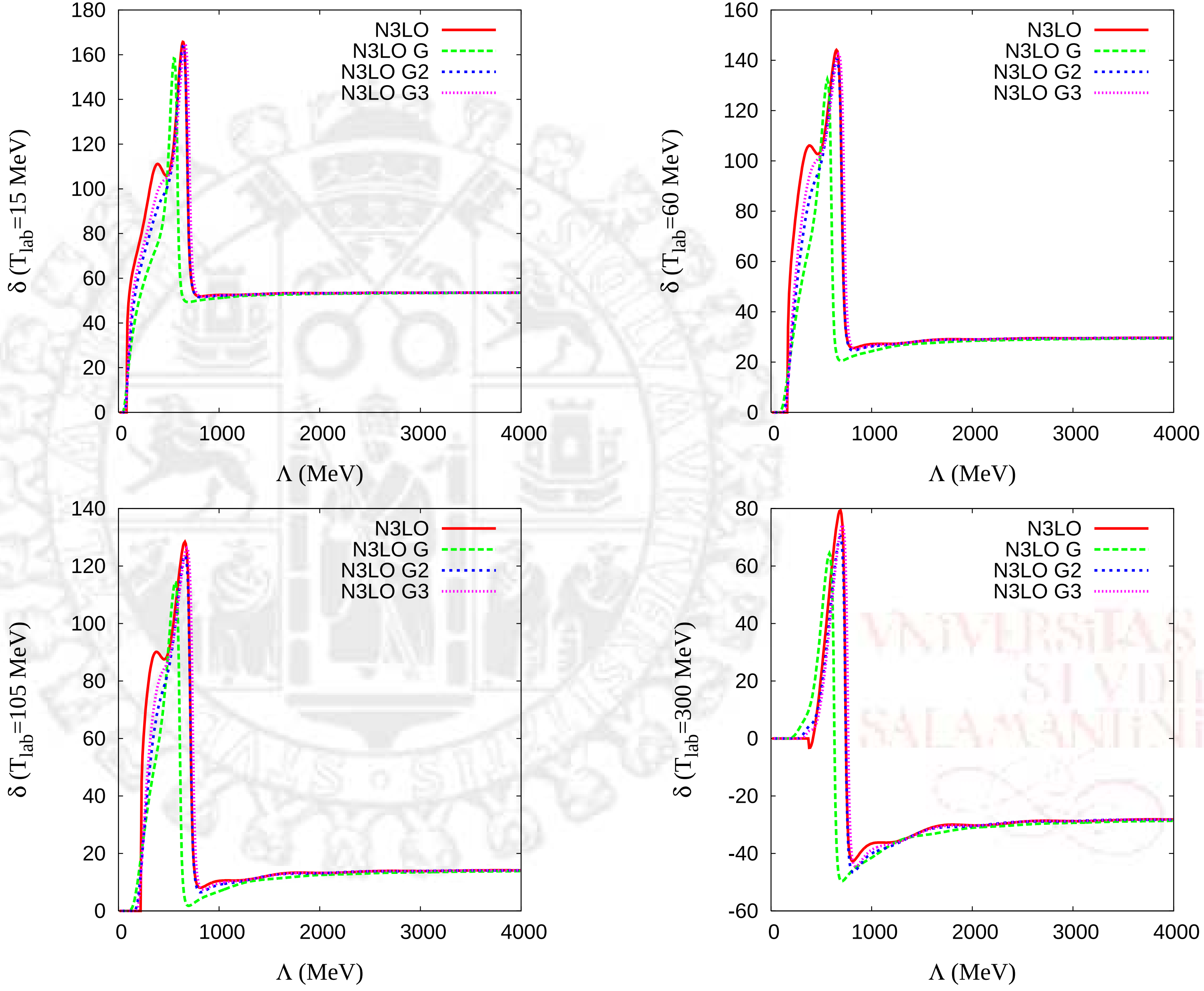


NLO
NLO G
NLO G2
NLO G3
δ (T_lab=15 MeV)
δ (T_lab=60 MeV)
δ (T_lab=105 MeV)
δ (T_lab=300 MeV)
Λ (MeV)

Regularization dependence - N^3LO

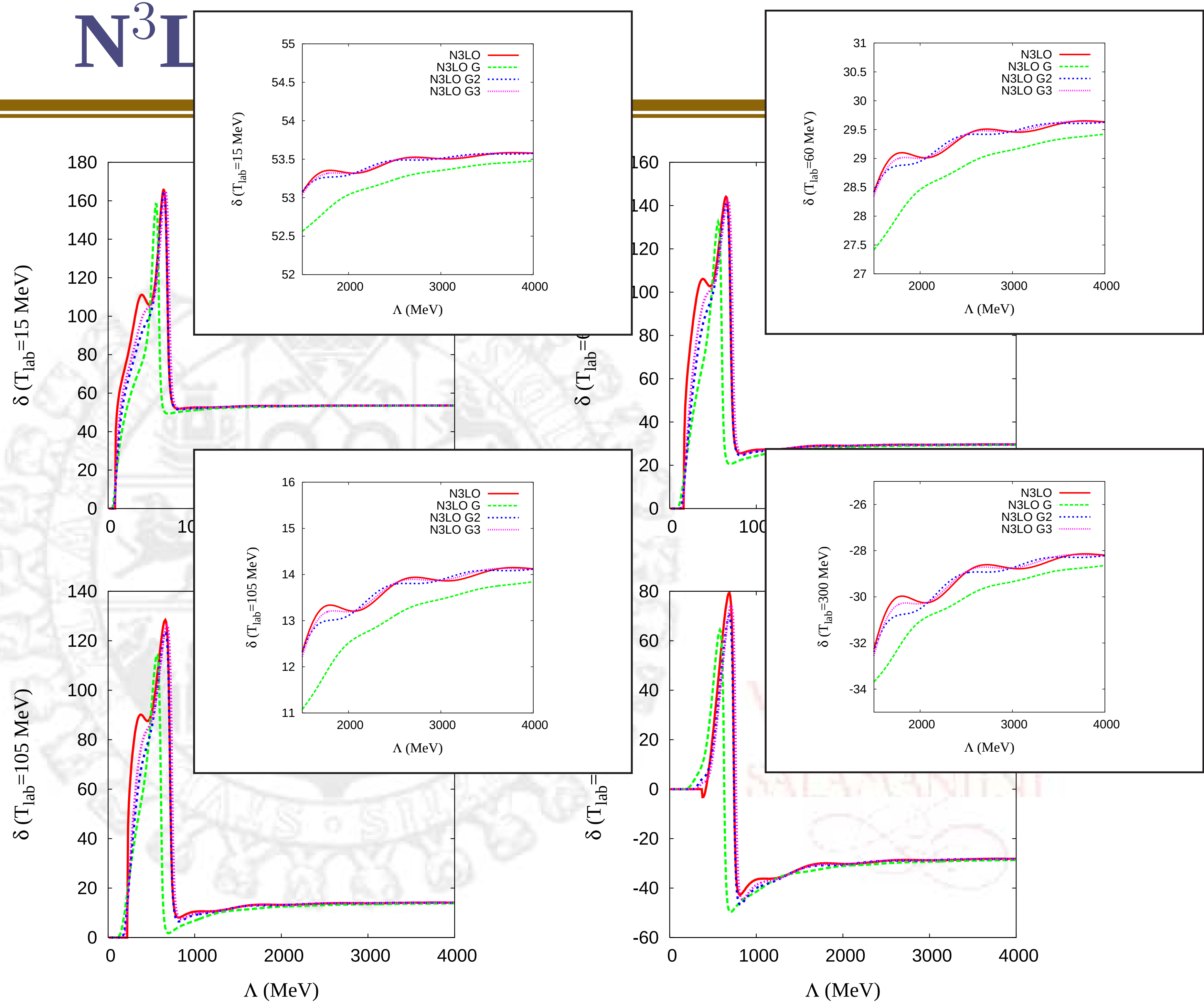


N^3LO

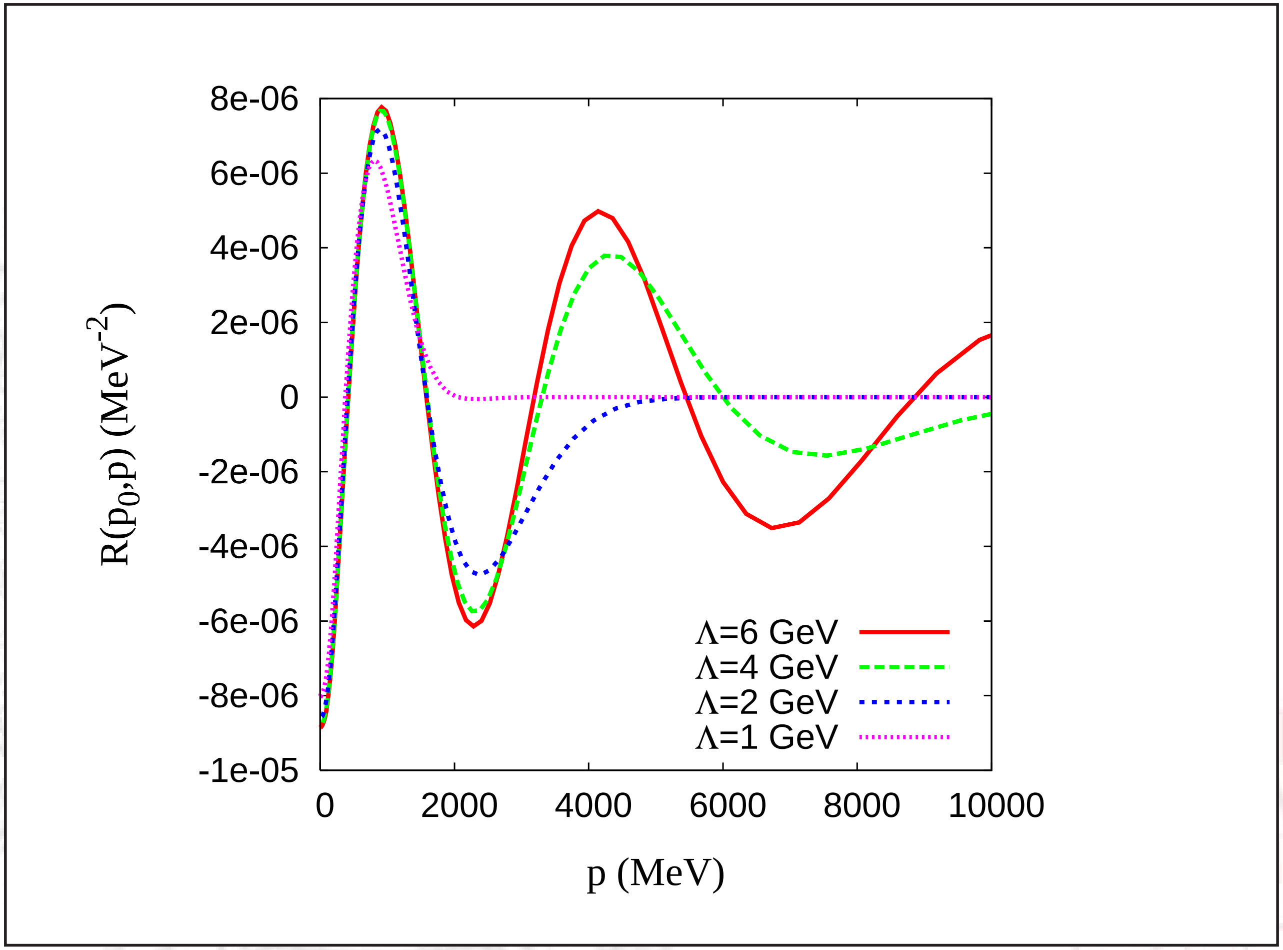


N3LO
N3LO G
N3LO G2
N3LO G3
δ (T_lab=15 MeV)
δ (T_lab=60 MeV)
δ (T_lab=105 MeV)
δ (T_lab=300 MeV)
Λ (MeV)

N^{3}L

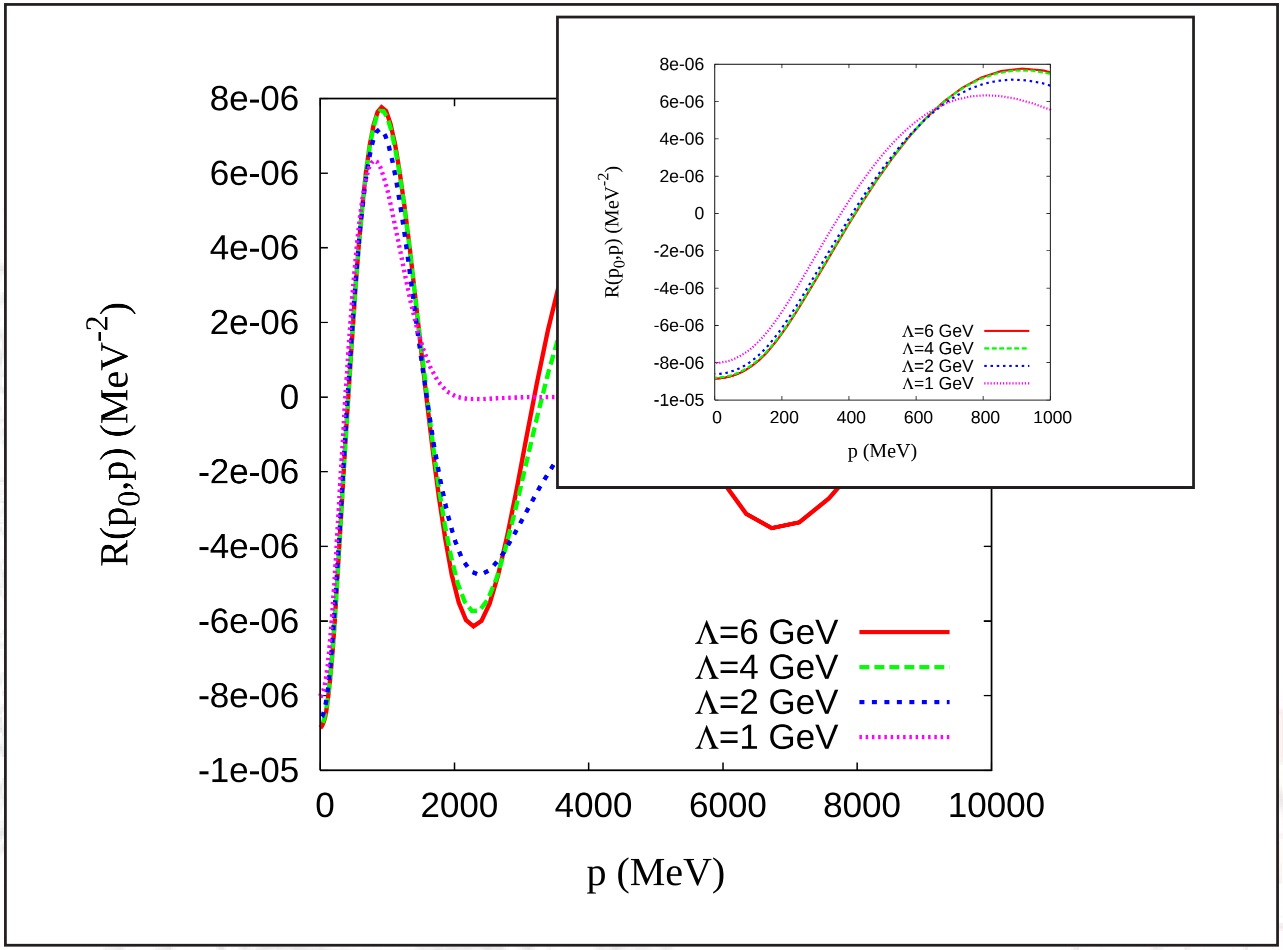

δ (T_lab=15 MeV)
δ (T_lab=60 MeV)
δ (T_lab=105 MeV)
δ (T_lab=300 MeV)
N3LO
N3LO G
N3LO G2
N3LO G3
Λ (MeV)

Half-offshell K-matrix



NLO Gaussian cutoff $T_{lab} = 50\, MeV$

Half-offshell K-matrix



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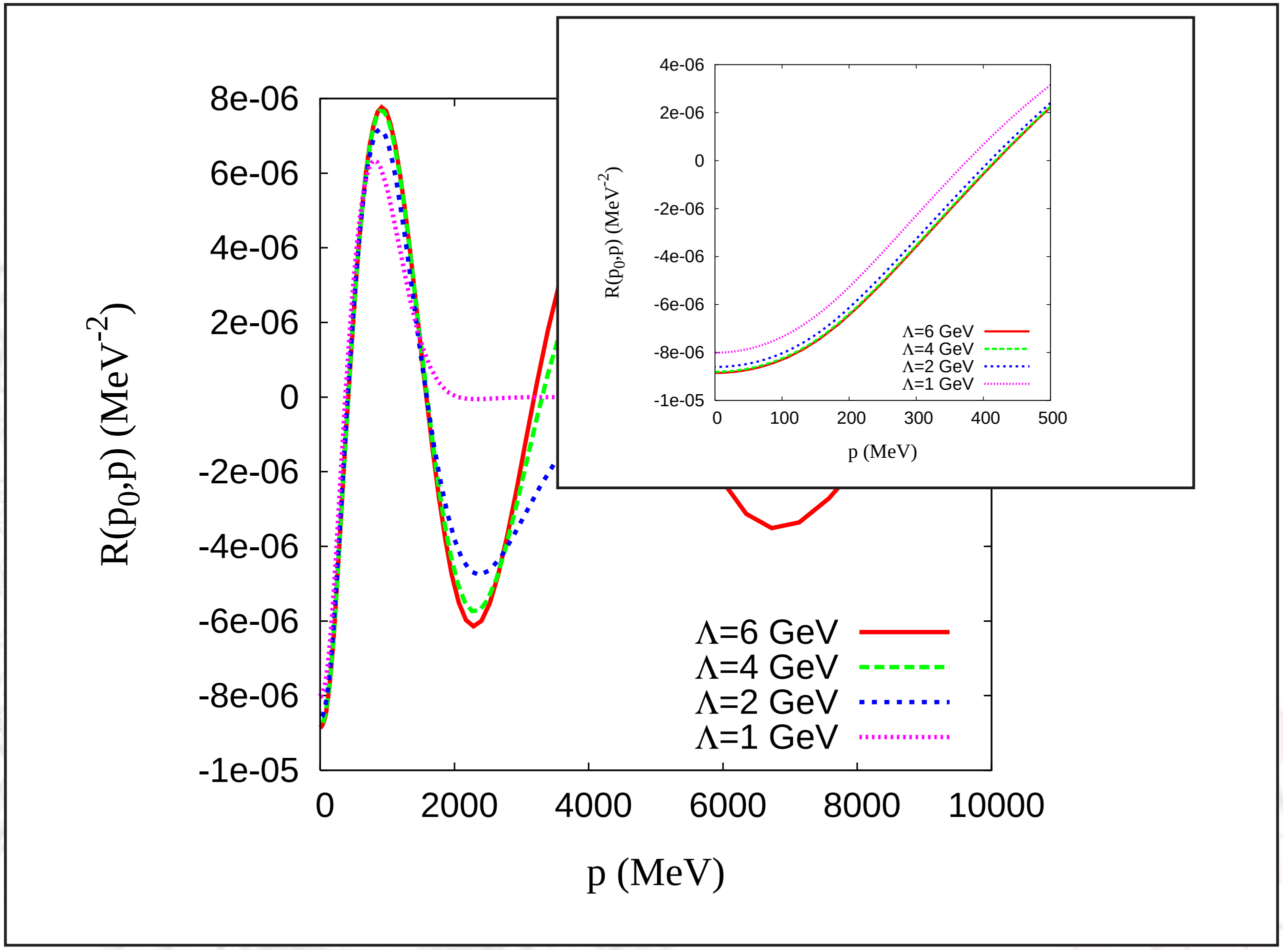
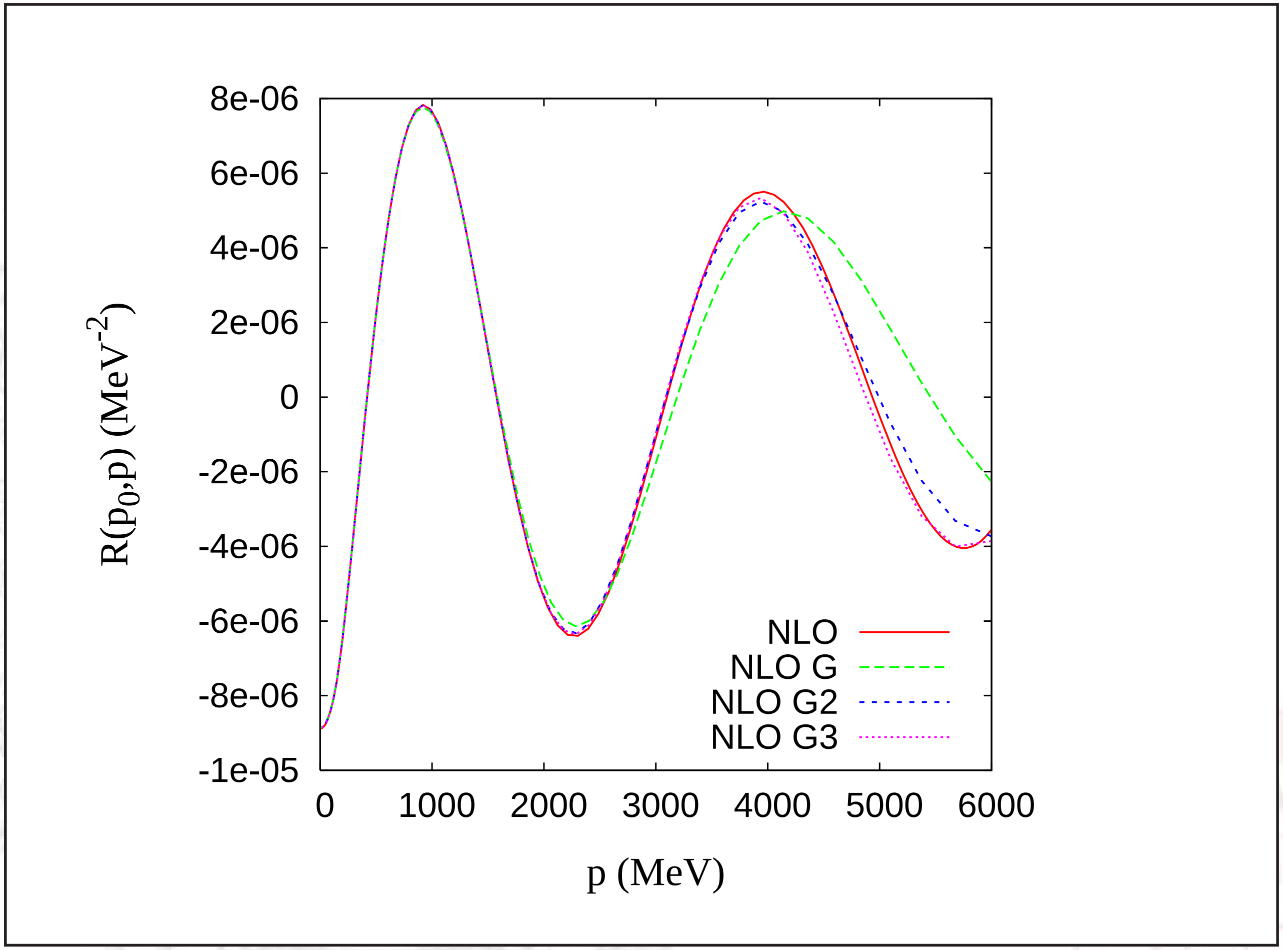


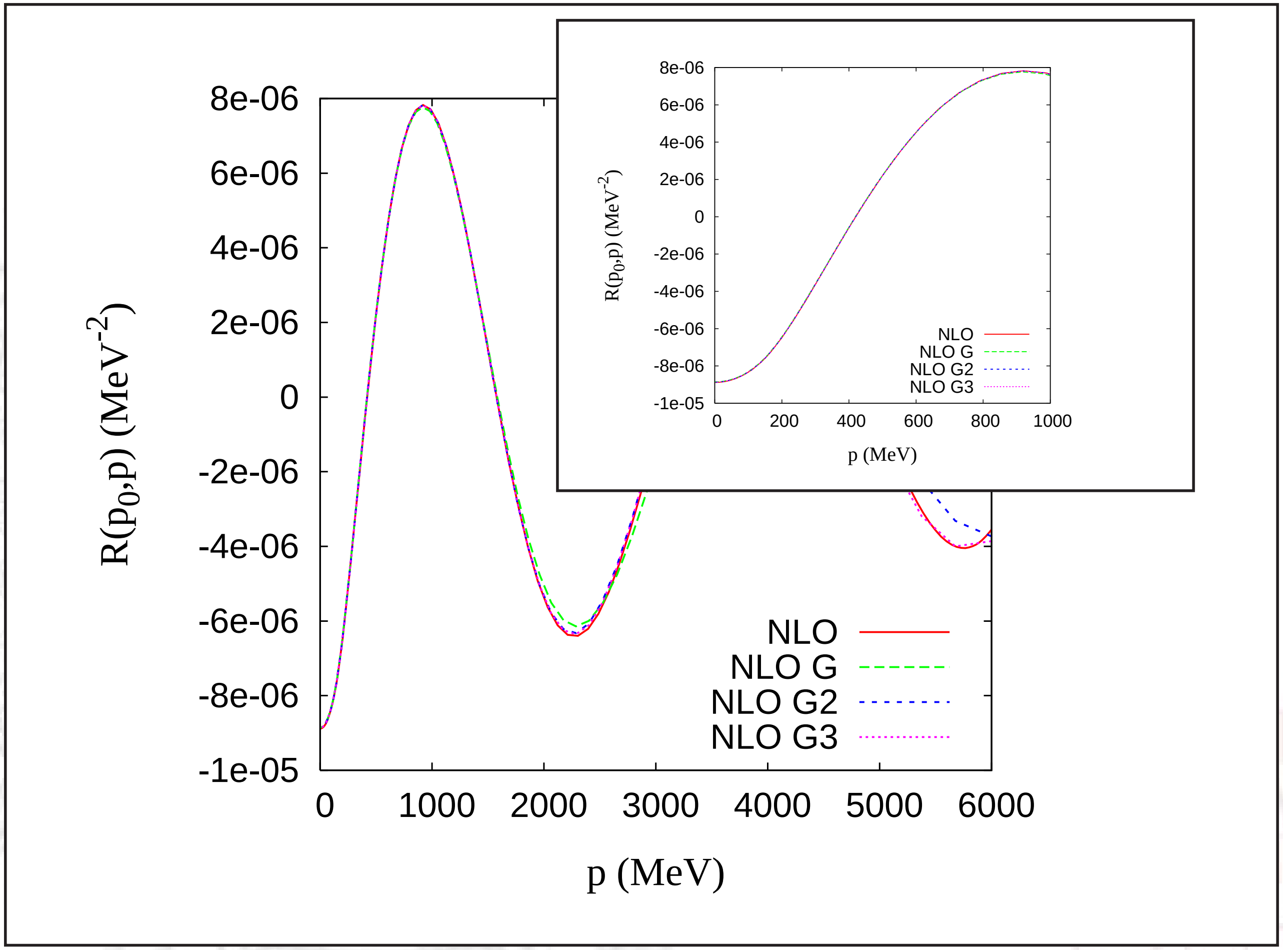
NLO Gaussian cutoff $T_{lab} = 50\, MeV$

Half-offshell K-matrix



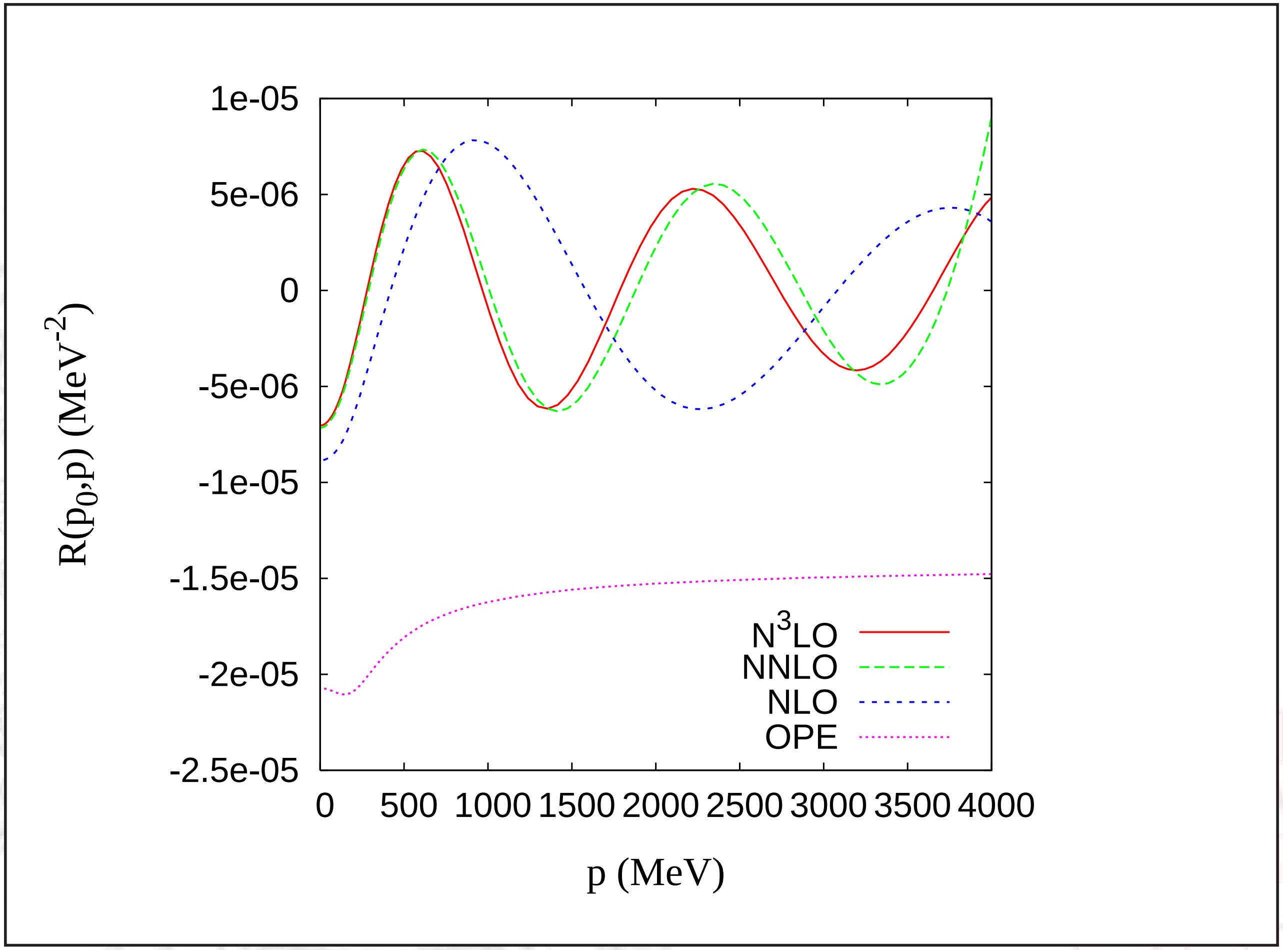
NLO Regularization dependence ($\Lambda = 6\,GeV$) $T_{lab} = 50\,MeV$

Half-offshell K-matrix



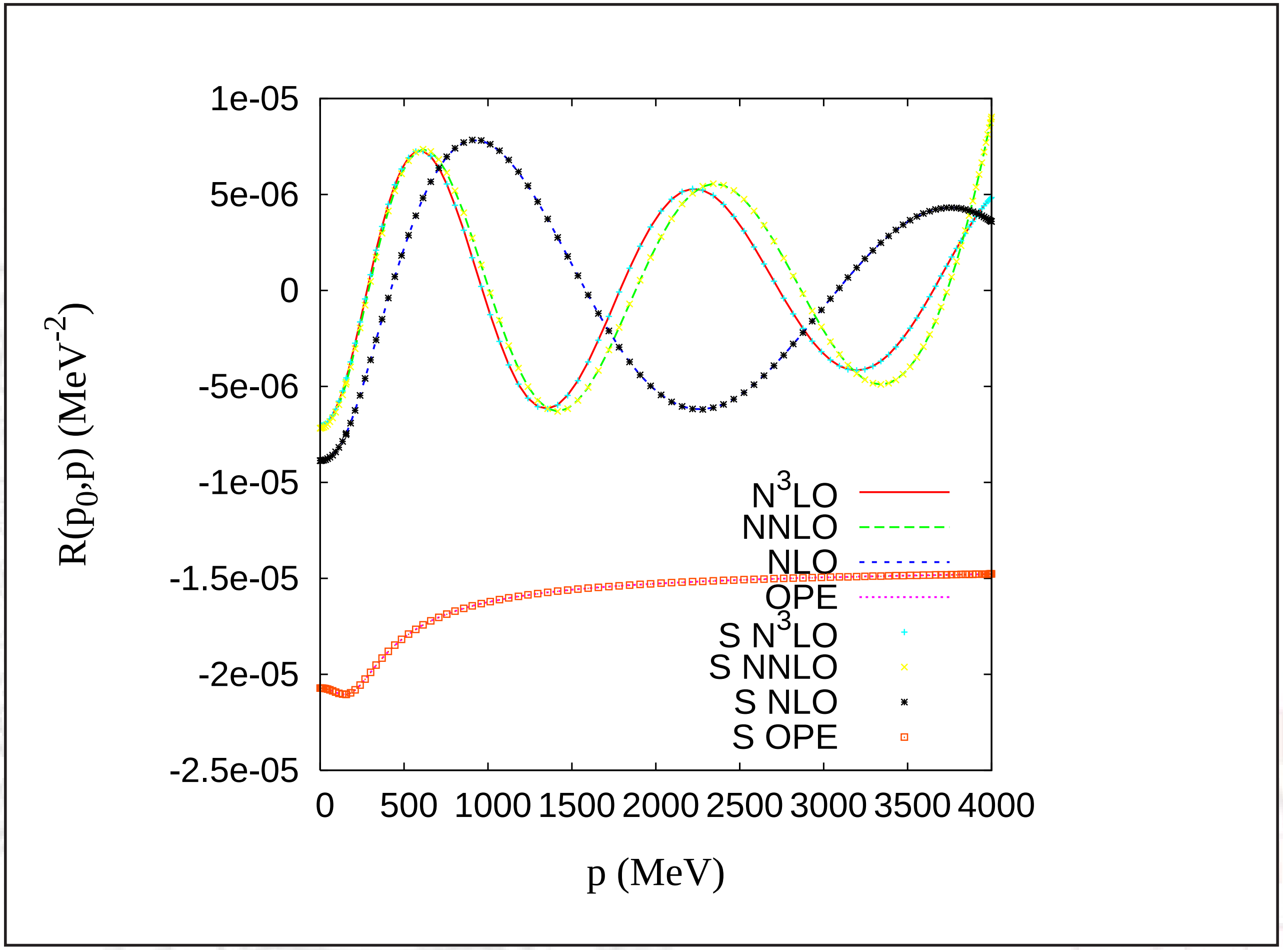
NLO Regularization dependence ($\Lambda = 6\,GeV$) $T_{lab} = 50\,MeV$

Half-offshell K-matrix



Sharp cutoff ($\Lambda = 4\,GeV$) $T_{lab} = 50\,MeV$

Half-offshell K-matrix



Sharp cutoff ($\Lambda = 4\,GeV$) $T_{lab} = 50\,MeV$

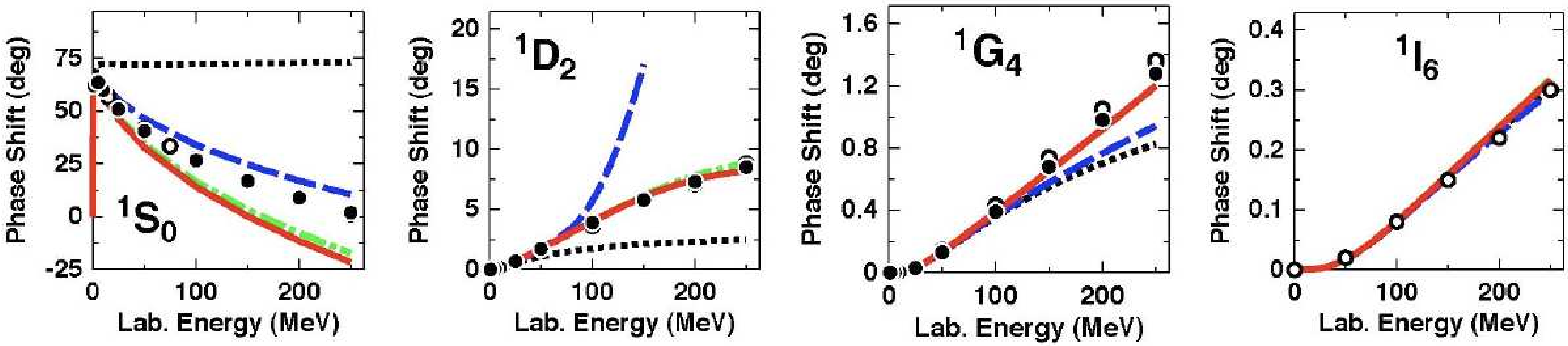
$\Lambda \gg \Lambda_\chi$ up to N^3LO

'Infinite-cutoff renormalization of the chiral nucleon-nucleon interaction at N^3LO

Ch. Zeoli, R. Machleidt, D.R. Entem, Few Body Systems (2012) 1-15 (arXiv:1208.2657)

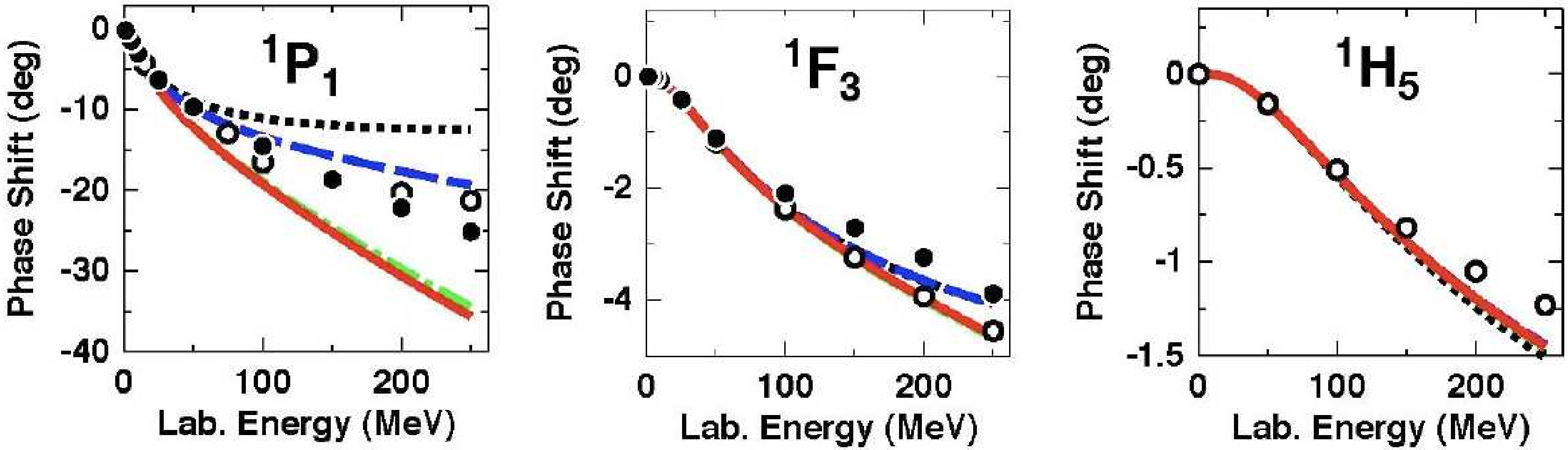
- We include one counter term for short range attractive partial waves
- No counter term is needed for short range repulsive partial waves
- A second counter terms is ineffective for $\Lambda \gg \Lambda_\chi$.
- We vary Λ between 0.5 and 10 GeV
- We fit $a_s = 23{,}740$ fm $a_t = 5{,}417$ fm and the phase shift at $T_L = 50$ MeV ($J \leq 2$)

$S = 0$ **and** $T = 1$



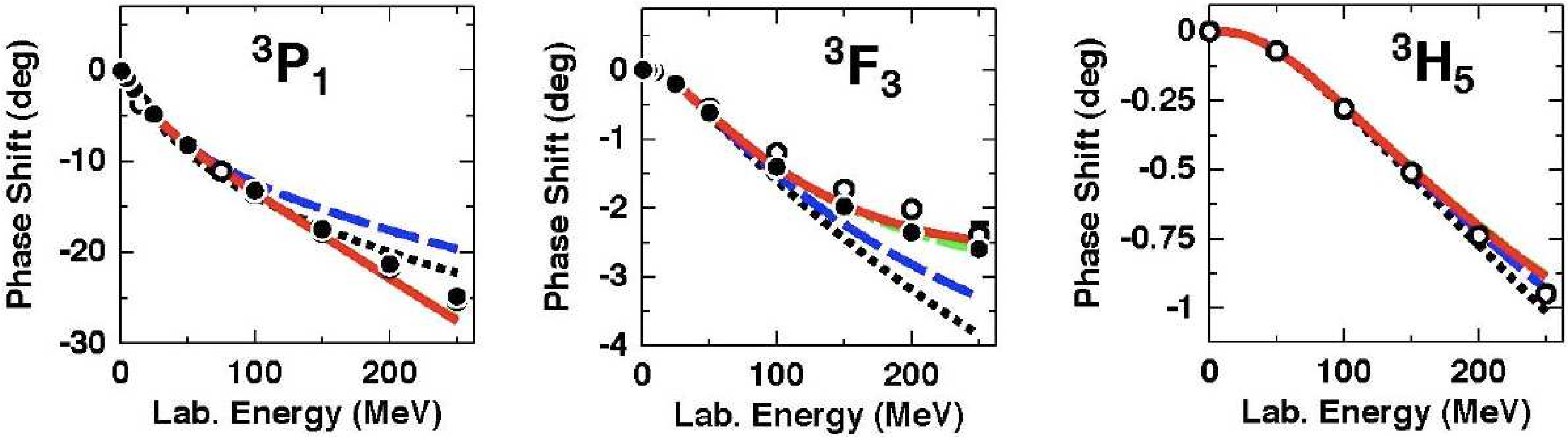
Partial wave	LO	NLO	NNLO	N^{3}LO
$^1S_0$	**1**	**1**	**1**	**1**
$^1D_2$	**0**	**1**	**1**	**1**
$^1G_4$	**0**	**0**	**1**	**1**

$S = 0$ and $T = 0$



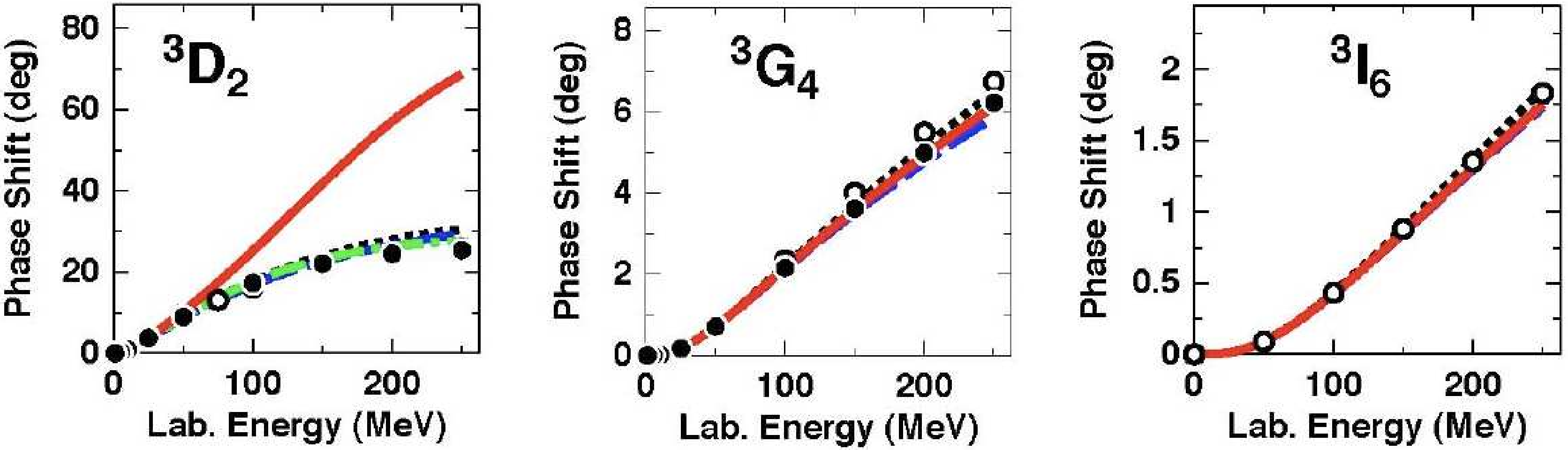
Partial wave	LO	NLO	NNLO	N^3LO
$^1P_1$	0	0	0	0
$^1F_3$	0	0	0	0

$S = 1$ and $T = 1$ uncoupled



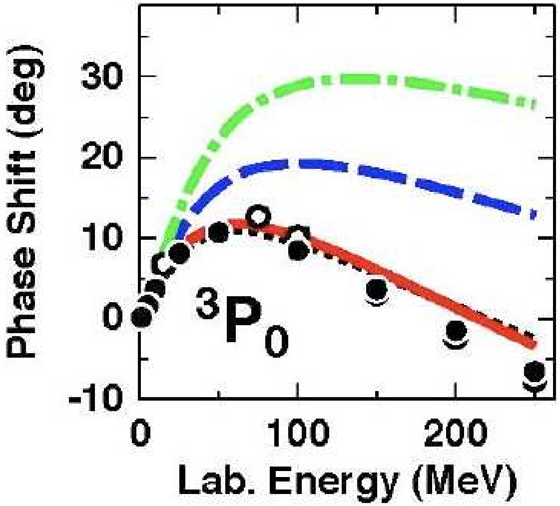
Partial wave	LO	NLO	NNLO	N^3LO
$^3P_1$	0	1	1	1
$^3F_3$	0	0	1	1

$S = 1$ and $T = 0$ uncoupled



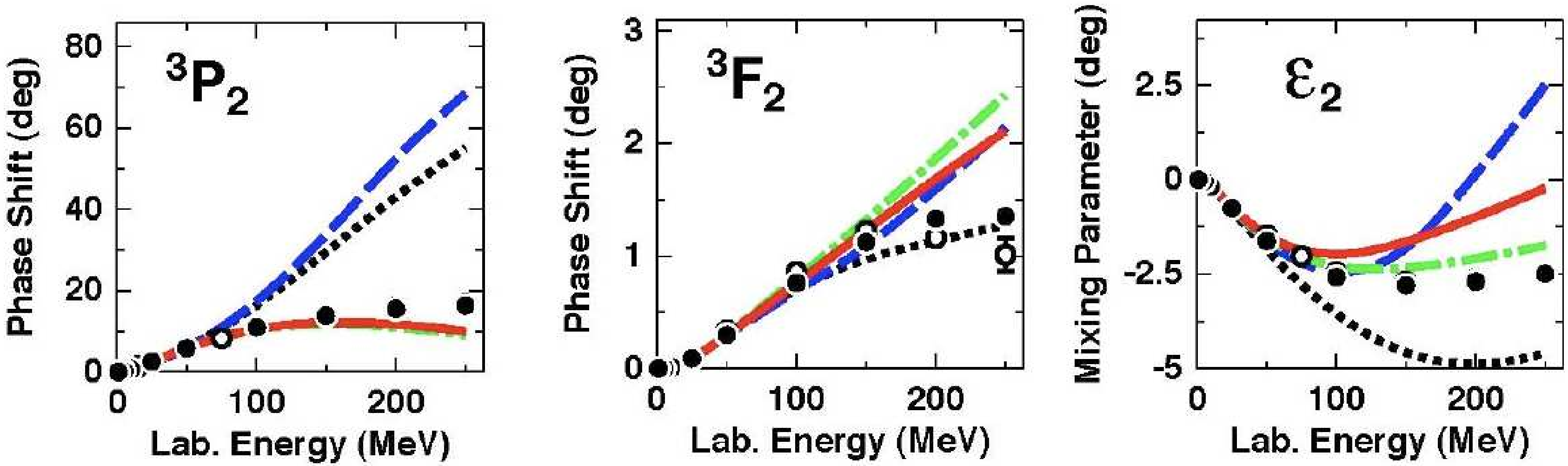
Partial wave	LO	NLO	NNLO	N^{3}LO
$^3D_2$	1	0	1	0
$^3G_4$	0	0	0	0

$S = 1$ and $T = 1$ coupled



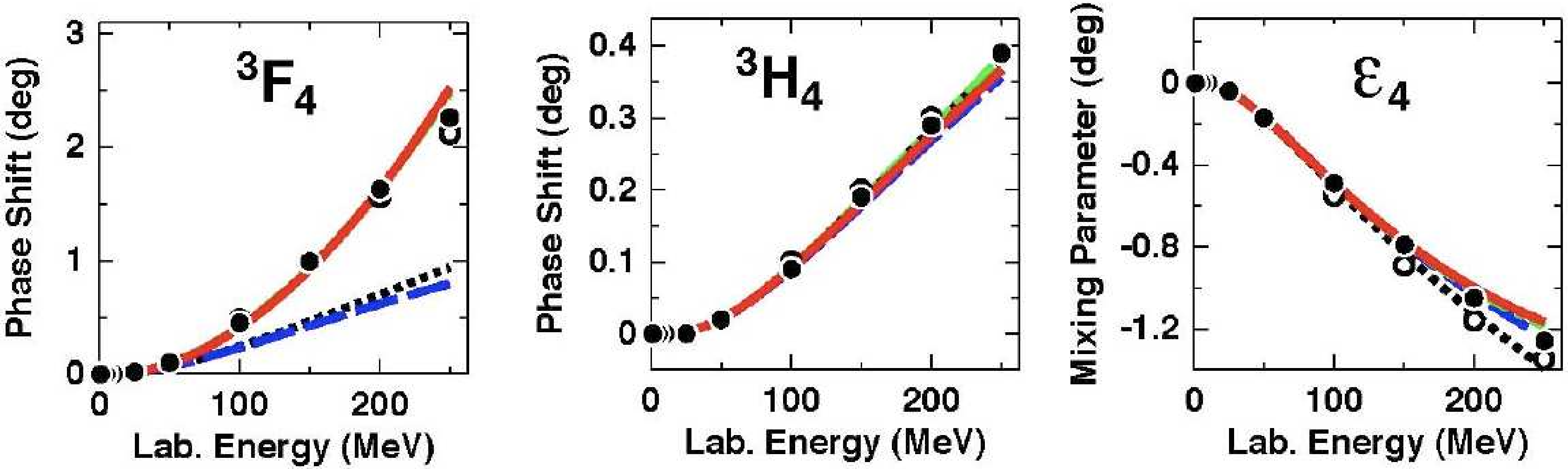
Partial wave	LO	NLO	NNLO	N^3LO
$^3P_0$	1	0	0	0

$S = 1$ and $T = 1$ coupled



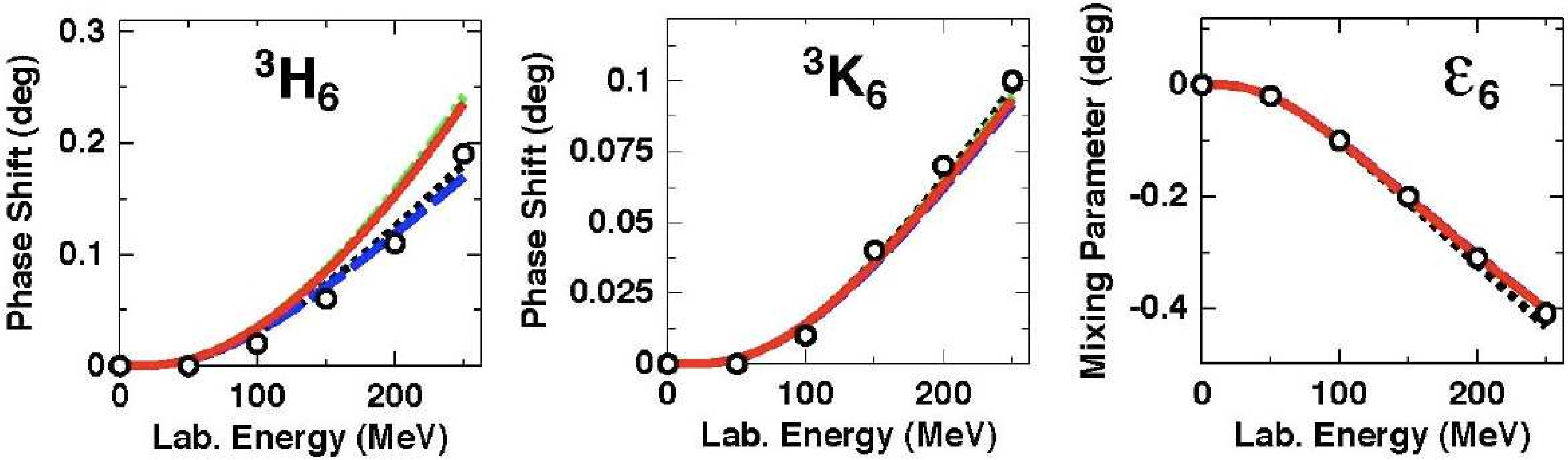
Partial wave	LO	NLO	NNLO	N^{3}LO
$^3P_2$	1	1	1	1
$^3P_2 -^3 F_2$	0	0	0	0
$^3F_2$	0	0	0	0

$S = 1$ and $T = 1$ coupled

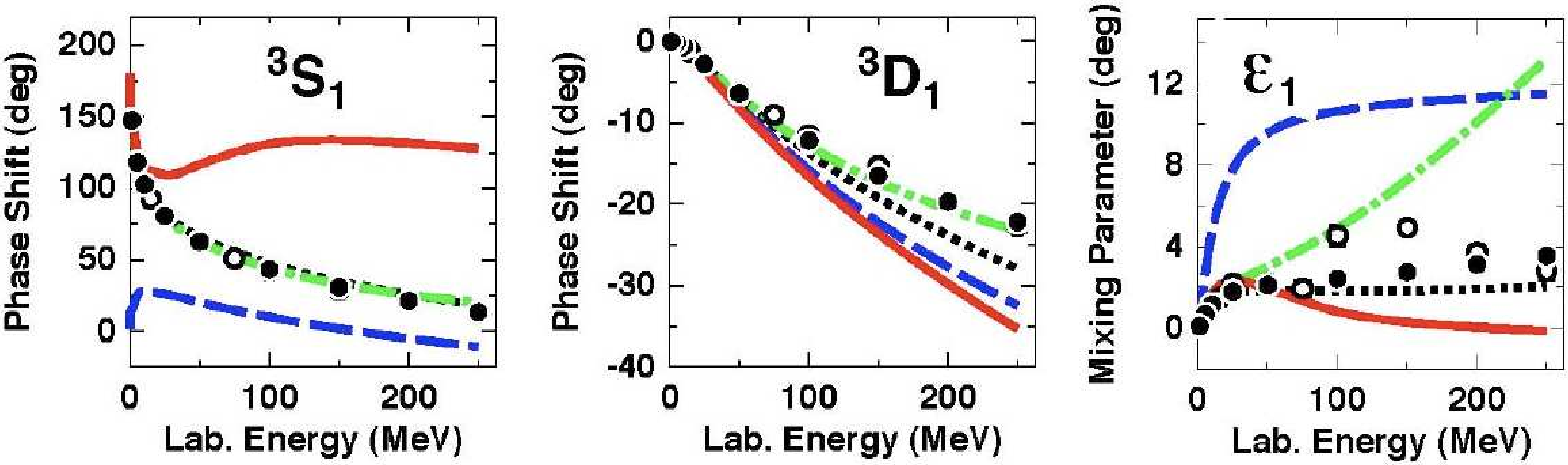


Partial wave	LO	NLO	NNLO	N^{3}LO
$^3F_4$	0	0	1	1

$S = 1$ and $T = 1$ coupled

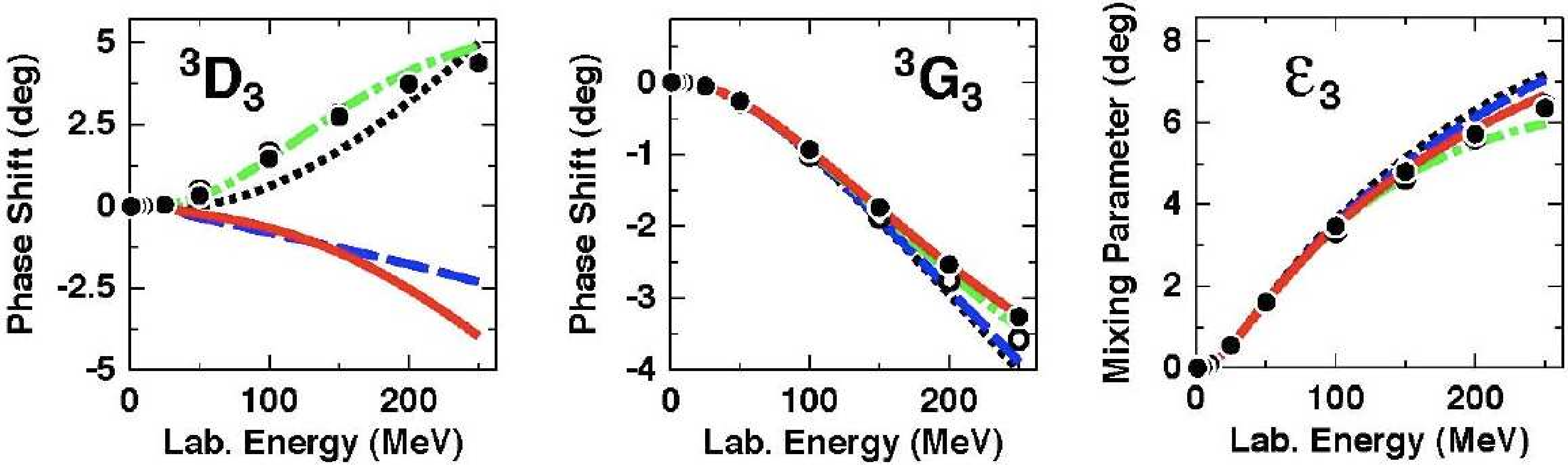


$S = 1$ and $T = 0$ coupled



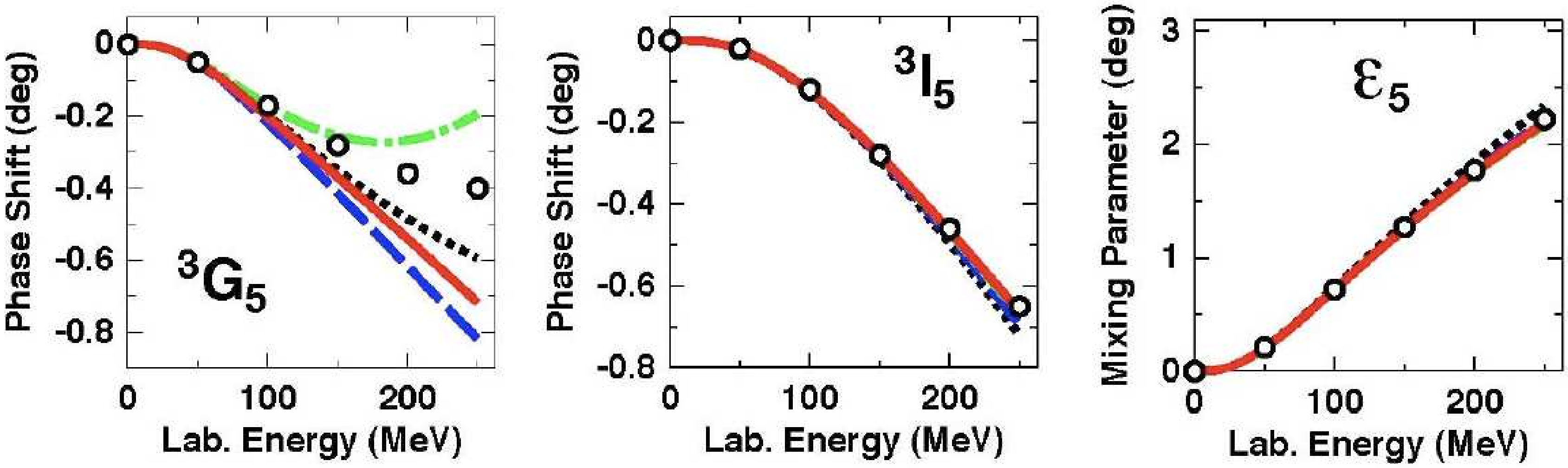
Partial wave	LO	NLO	NNLO	N^3LO
$^3S_1$	1	0	1	1
$^3S_1 -^3 D_1$	0	0	1	0
$^3D_1$	0	0	0	0

$S=1$ and $T=0$ coupled



Partial wave	LO	NLO	NNLO	N^{3}LO
$^3D_3$	0	0	1	0
$^3D_3 - ^3G_3$	0	0	0	0
$^3G_3$	0	0	0	0

$S = 1$ and $T = 0$ coupled



Final remarks

- $^1S_0$ and $^1P_1$ show convergence but not to the empiral phase-shifts
- $^3P_0$ agrees with empiral phase-shifts but shows no convergence
- $^3S_1 -^3 D_1$, $^3D_2$ and $^3D_3$ show no convergence and disagrees with empiral phase-shifts
- Cut-off independence is achivied with $\Lambda > 5$ GeV
- Order by order improvements of the predictions do not occur in several partial waves

$\Lambda < \Lambda_\chi$

'Recent Progress in the Theory of Nuclear Forces'

R. Machleidt, Q. MacPherson, E. Marji, R. Winzer, Ch. Zeoli, D.R. Entem, arXiv:1210.0992