Neutron-rich matter from chiral EFT interaction

Kai Hebeler
(TU Darmstadt)

From Few-Nucleon Forces to Many-Body Structure

in collaboration with R. Furnstahl, J. Lattimer, C. Pethick, A. Schwenk

Trento, June 11, 2013

TECHNISCHE UNIVERSITÄT DARMSTADT

EMMI

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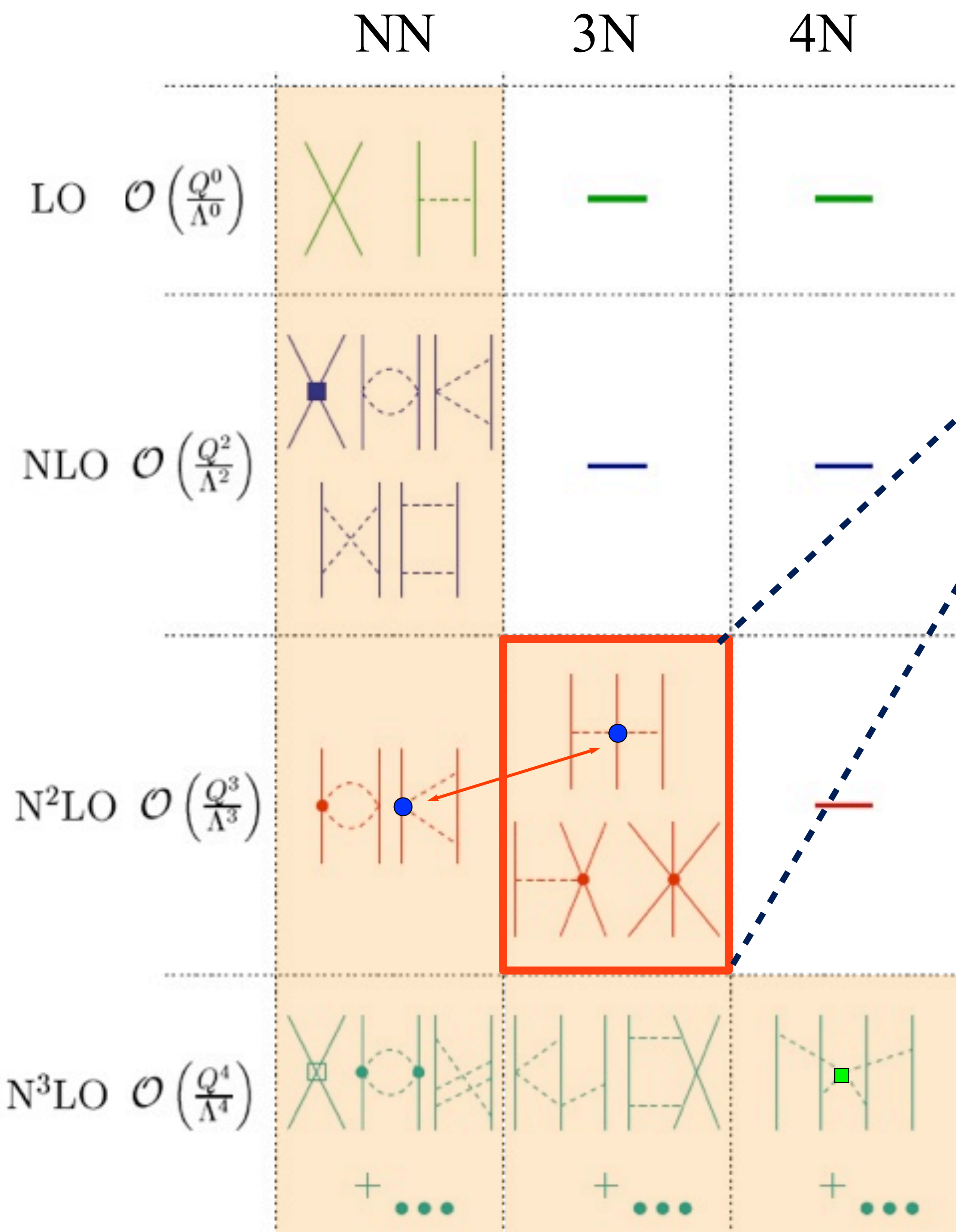
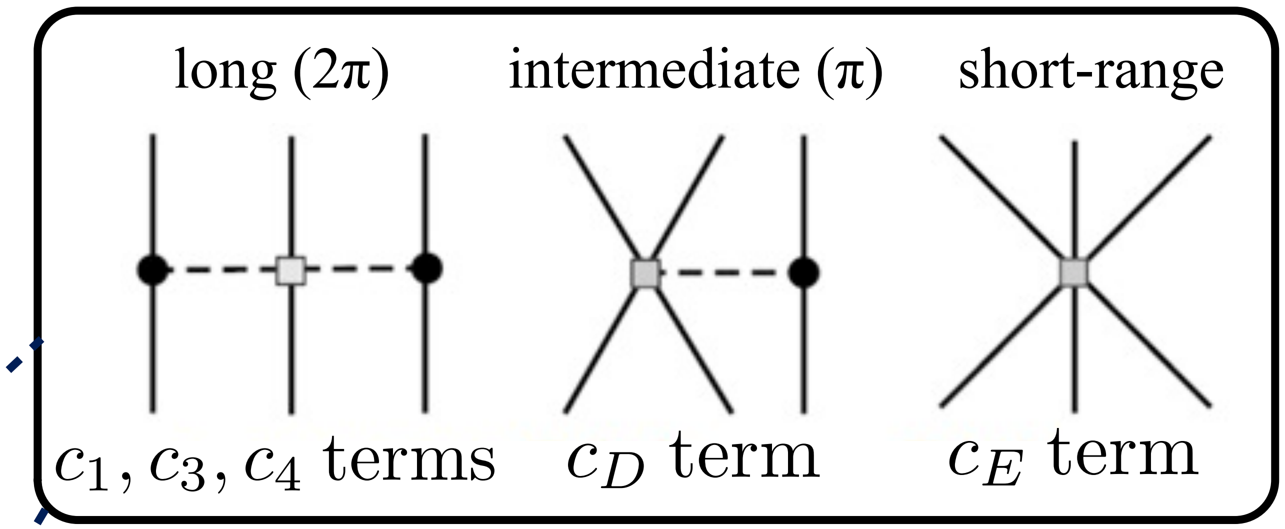
Chiral EFT for nuclear forces, leading order 3N forces





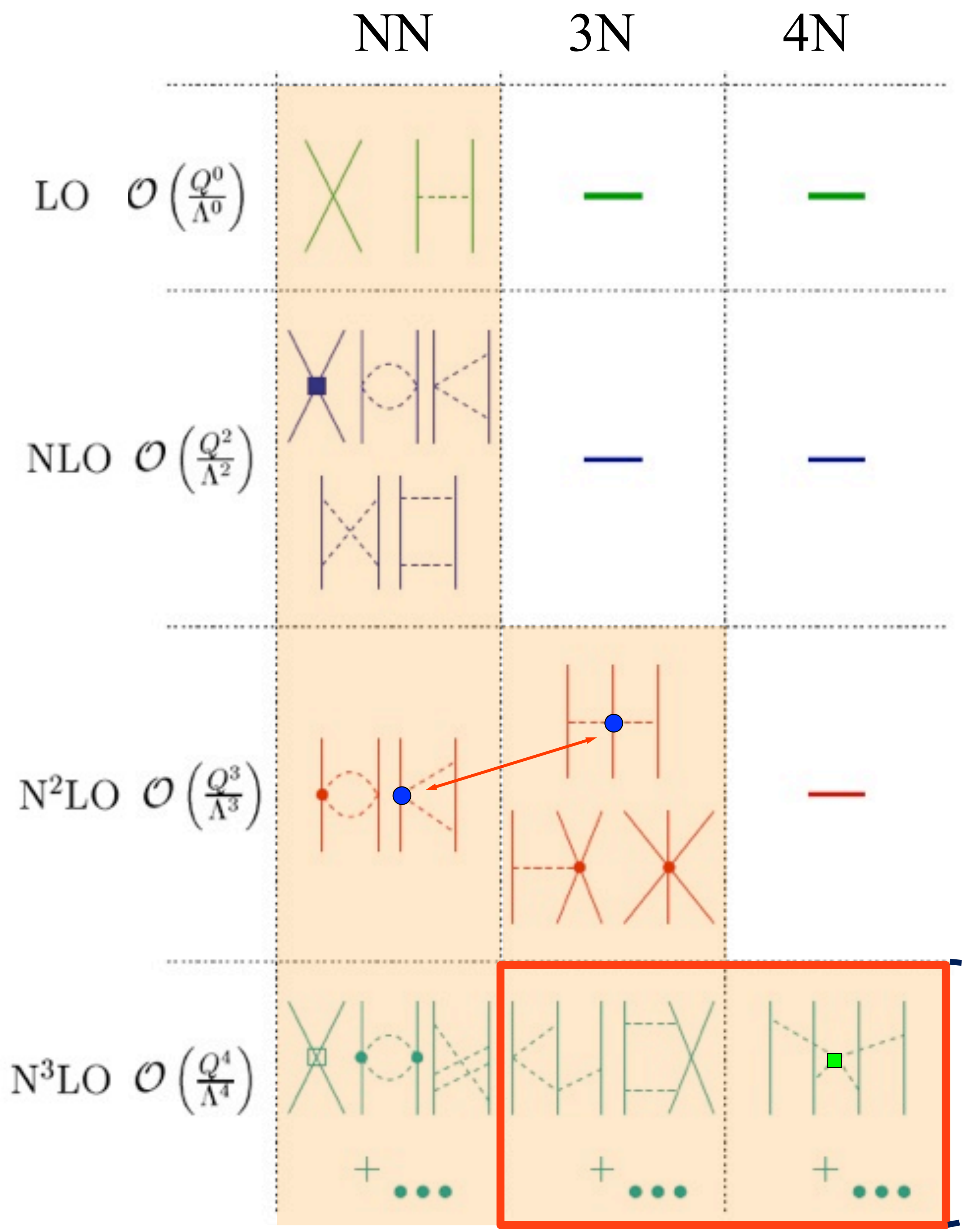
large uncertainties in coupling constants at present:

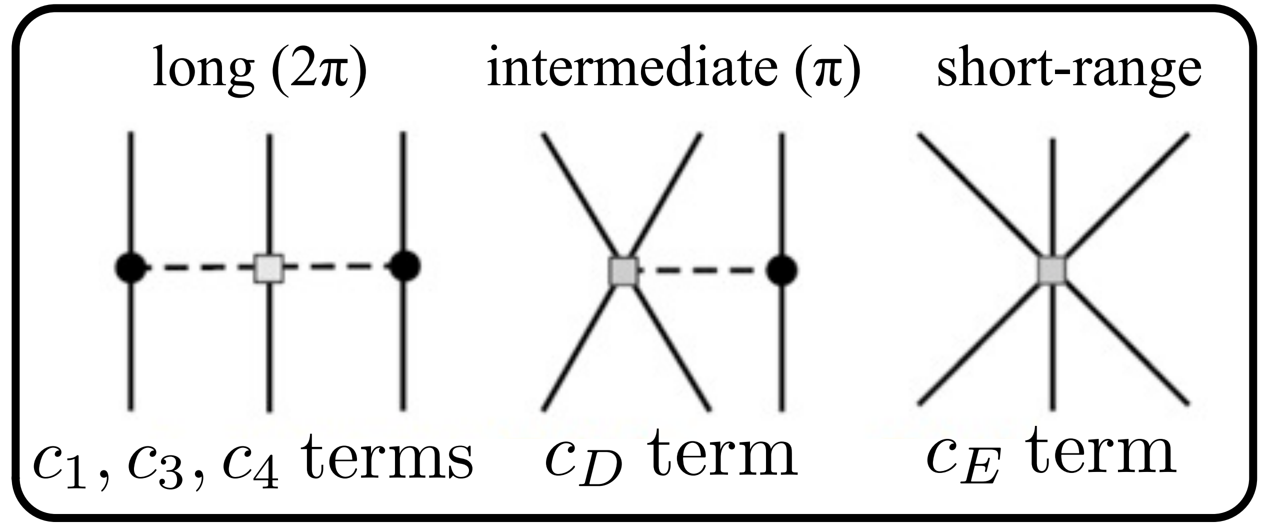
$$c_1 = -0.9^{+0.2}_{-0.5}\,,\quad c_3 = -4.7^{+1.5}_{-1.0}\,,\quad c_4 = 3.5^{+0.5}_{-0.2}$$

lead to theoretical uncertainties in many-body observables

use chiral interactions as input for RG evolution

Chiral EFT for nuclear forces, leading order 3N forces





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first incorporation in calculations of neutron matter

Tews, Krueger, KH, Schwenk
PRL 110, 032504 (2013)

Changing the resolution: The (Similarity) Renormalization Group

- goal: generate unitary transformation of "hard" Hamiltonian

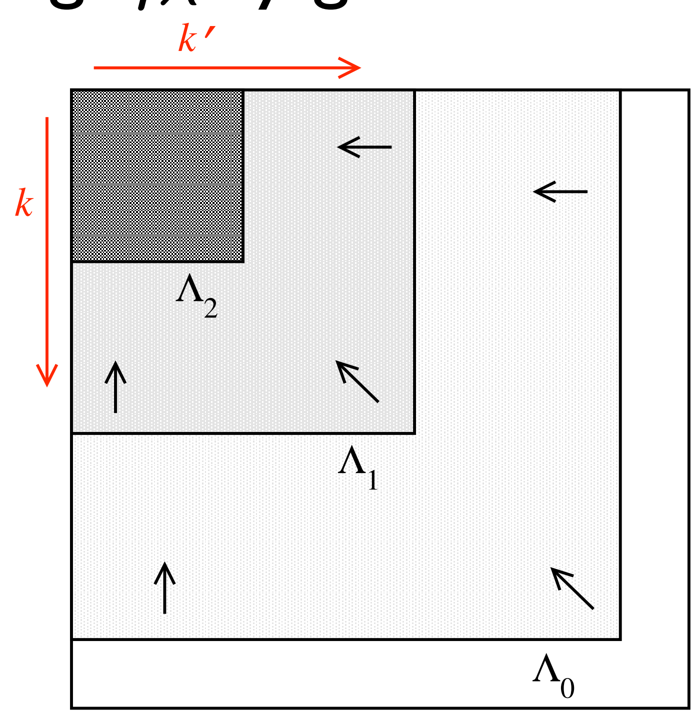
$$H_\lambda = U_\lambda H U_\lambda^\dagger \qquad \text{with the resolution parameter } \lambda$$

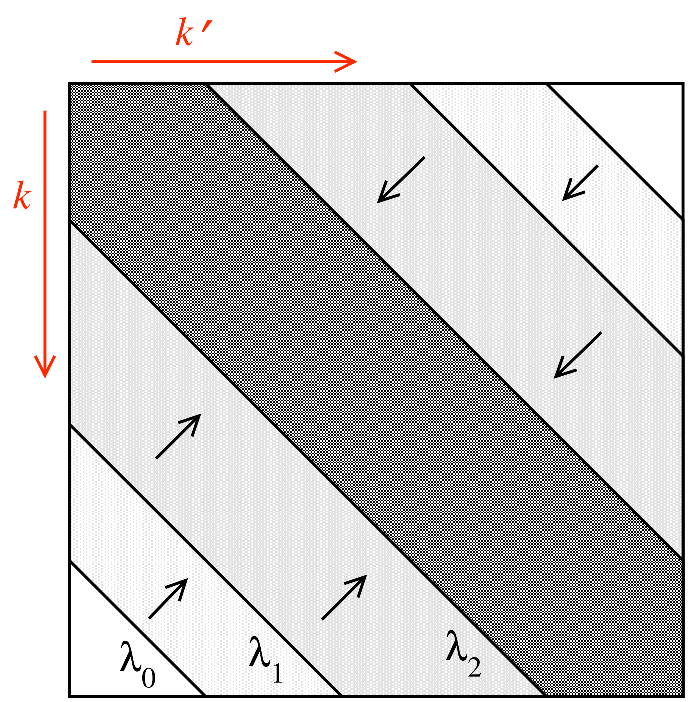
- change resolution in small steps: $$\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$$

- transformed wave functions and operators

$$|\psi_\lambda\rangle = U_\lambda |\psi\rangle \qquad O_\lambda = U_\lambda O U_\lambda^\dagger \quad \Rightarrow \quad \langle\psi| O |\psi\rangle = \langle\psi_\lambda| O_\lambda |\psi_\lambda\rangle$$

- specifying η_λ by generator G_λ: $\quad \eta_\lambda = [G_\lambda, H_\lambda]$

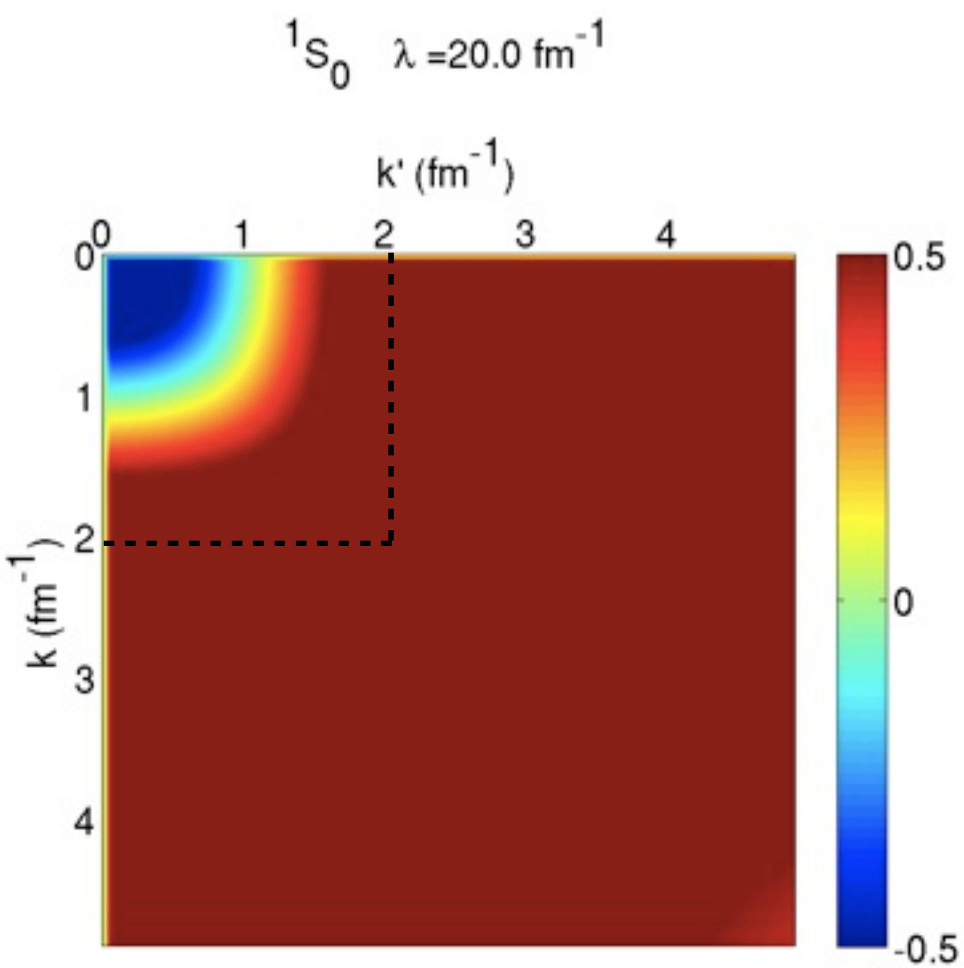


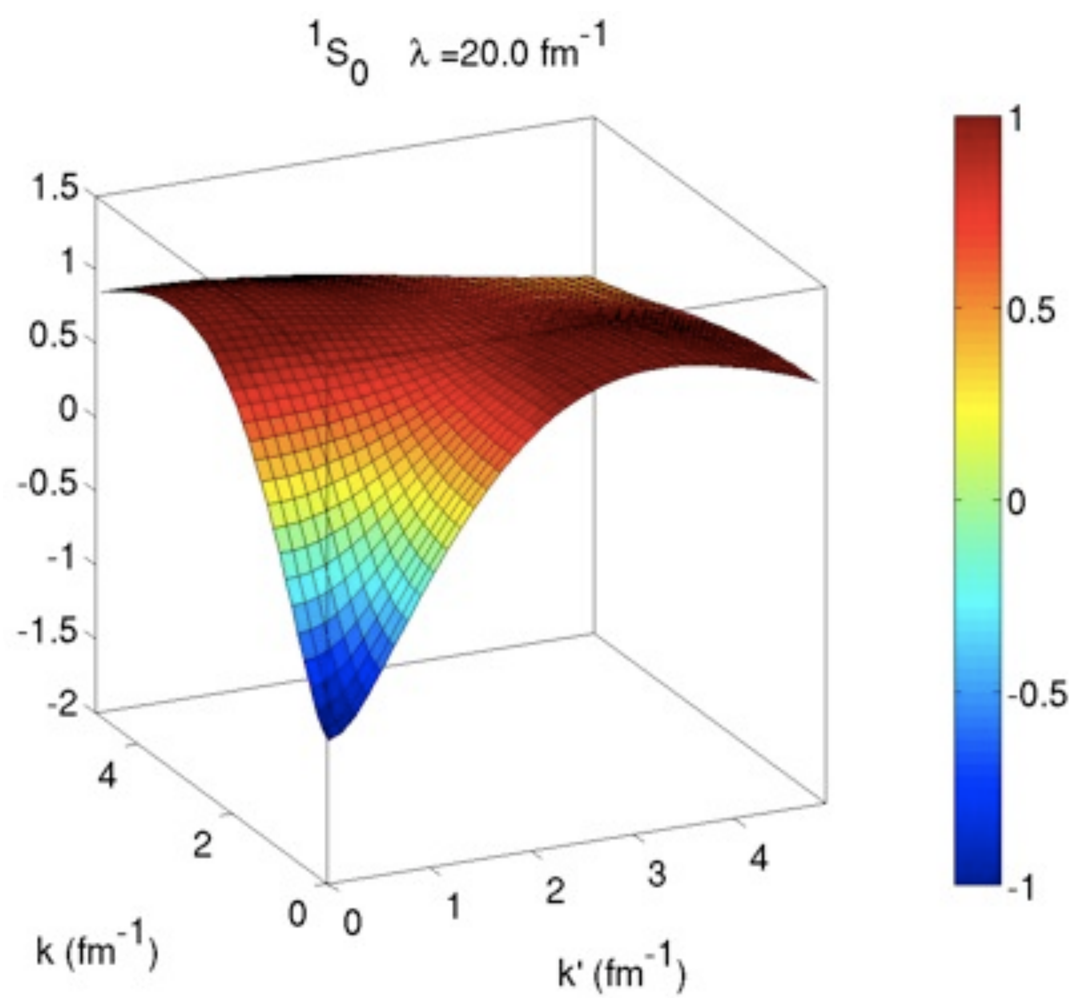


Changing the resolution: The (Similarity) Renormalization Group

- common choice for generator

  relative kinetic energy operator $G_\lambda = T$:





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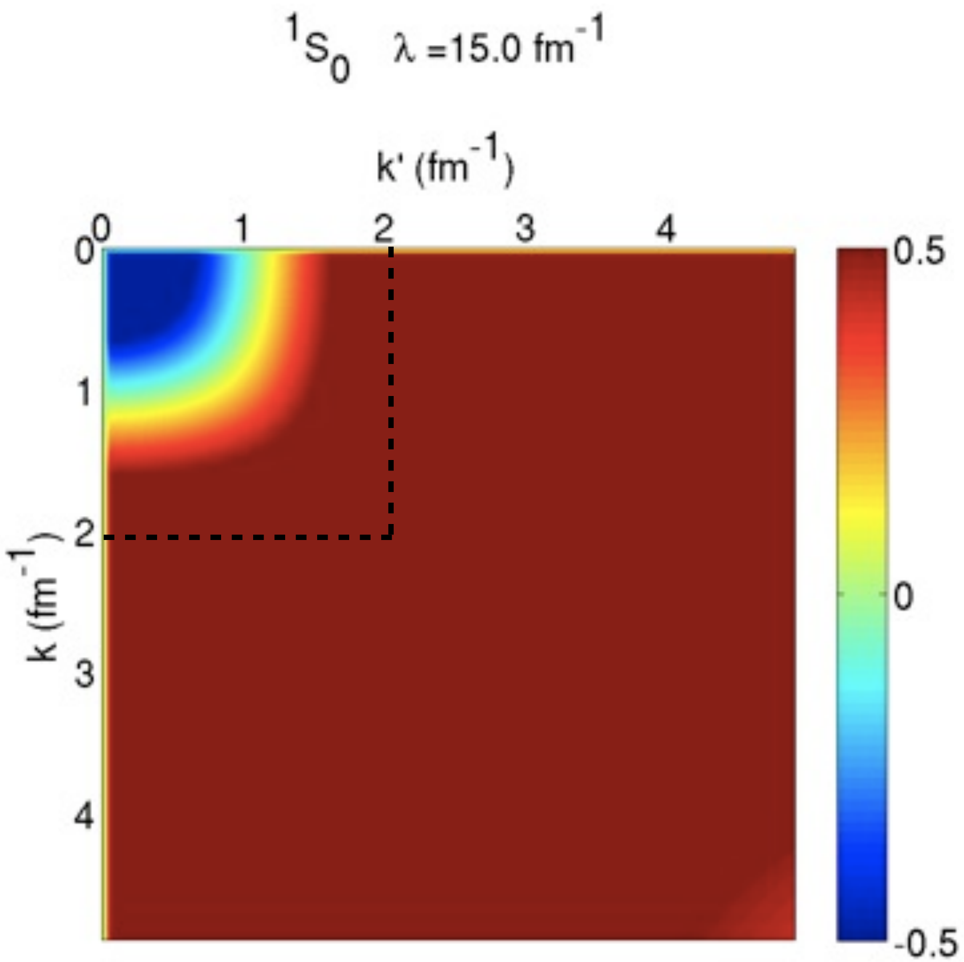

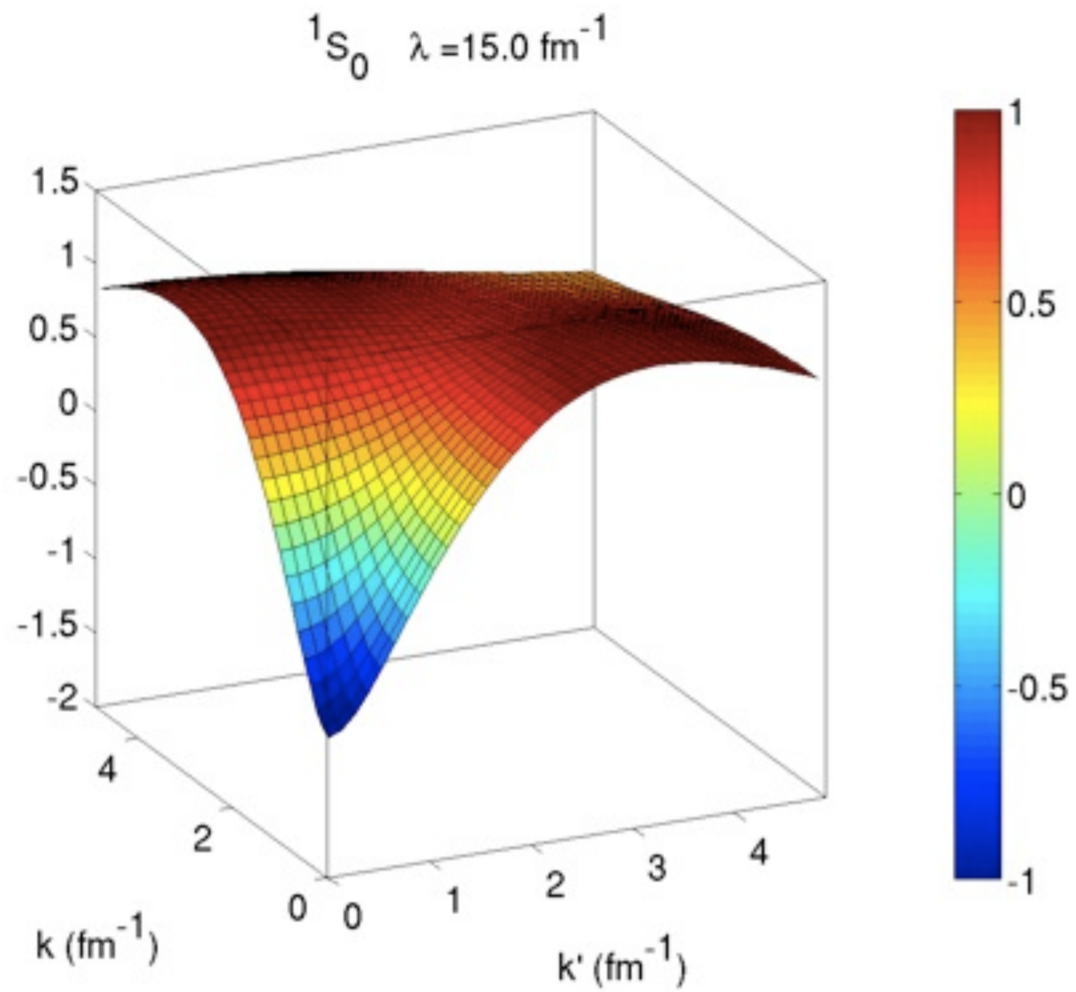

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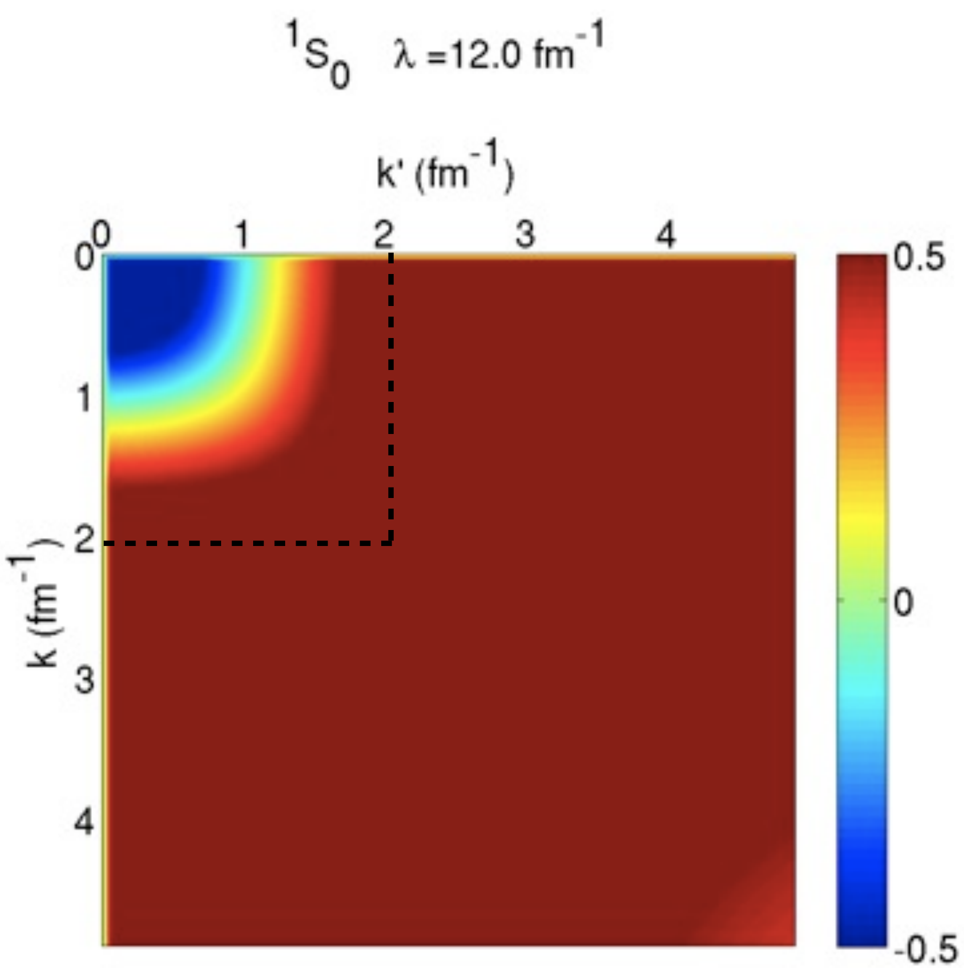

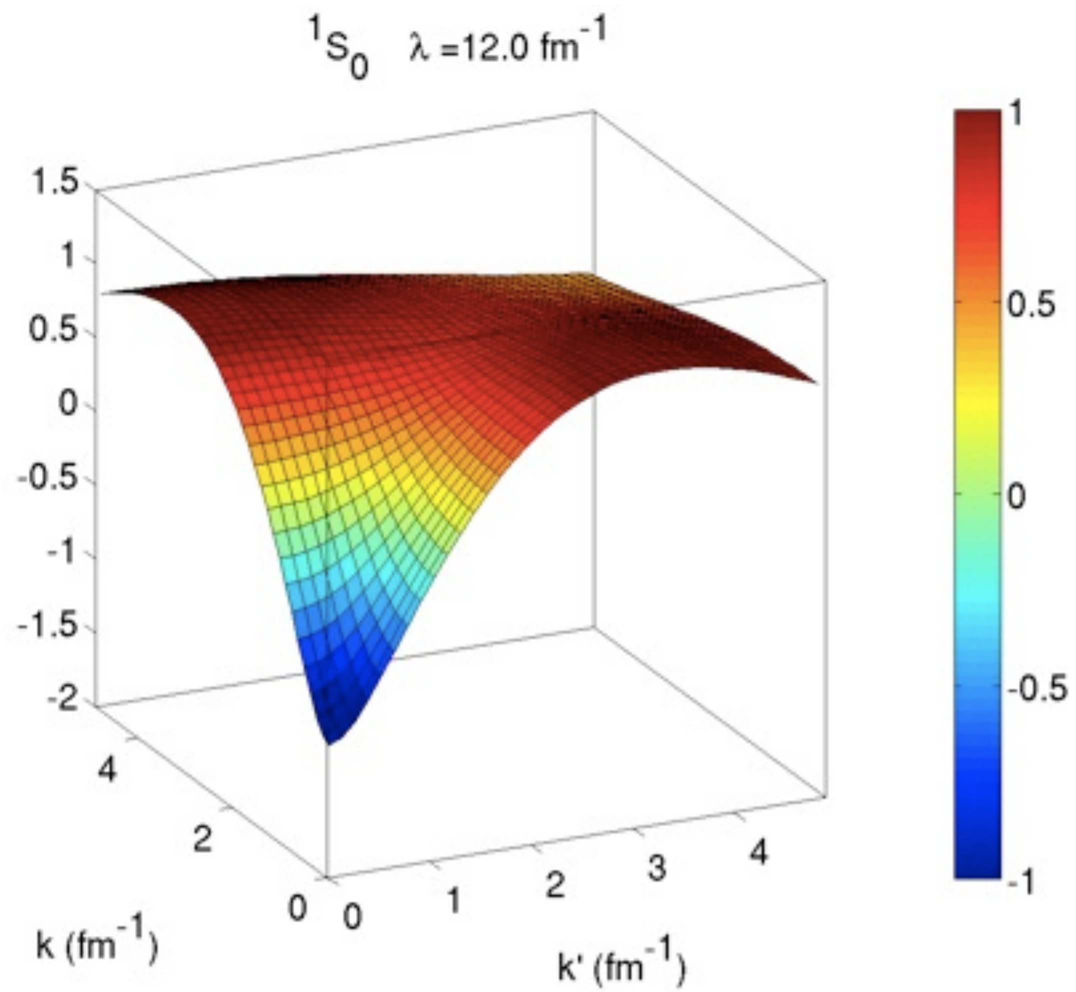

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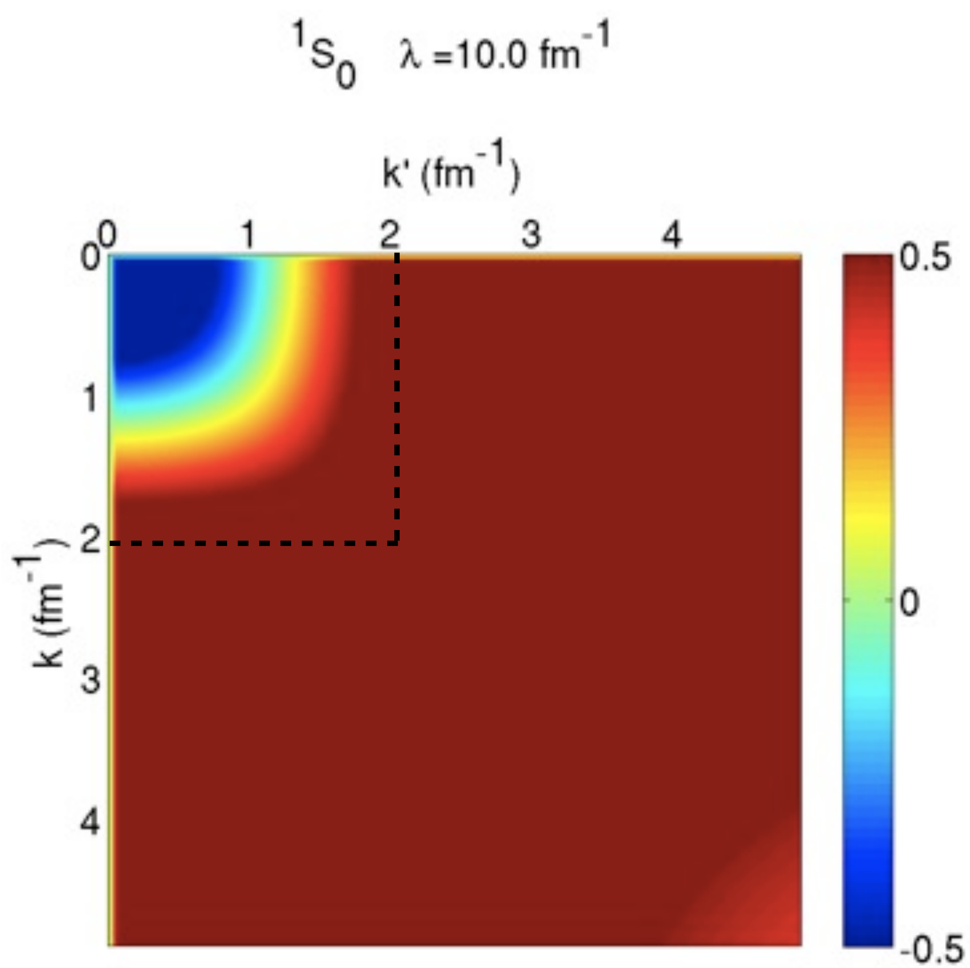

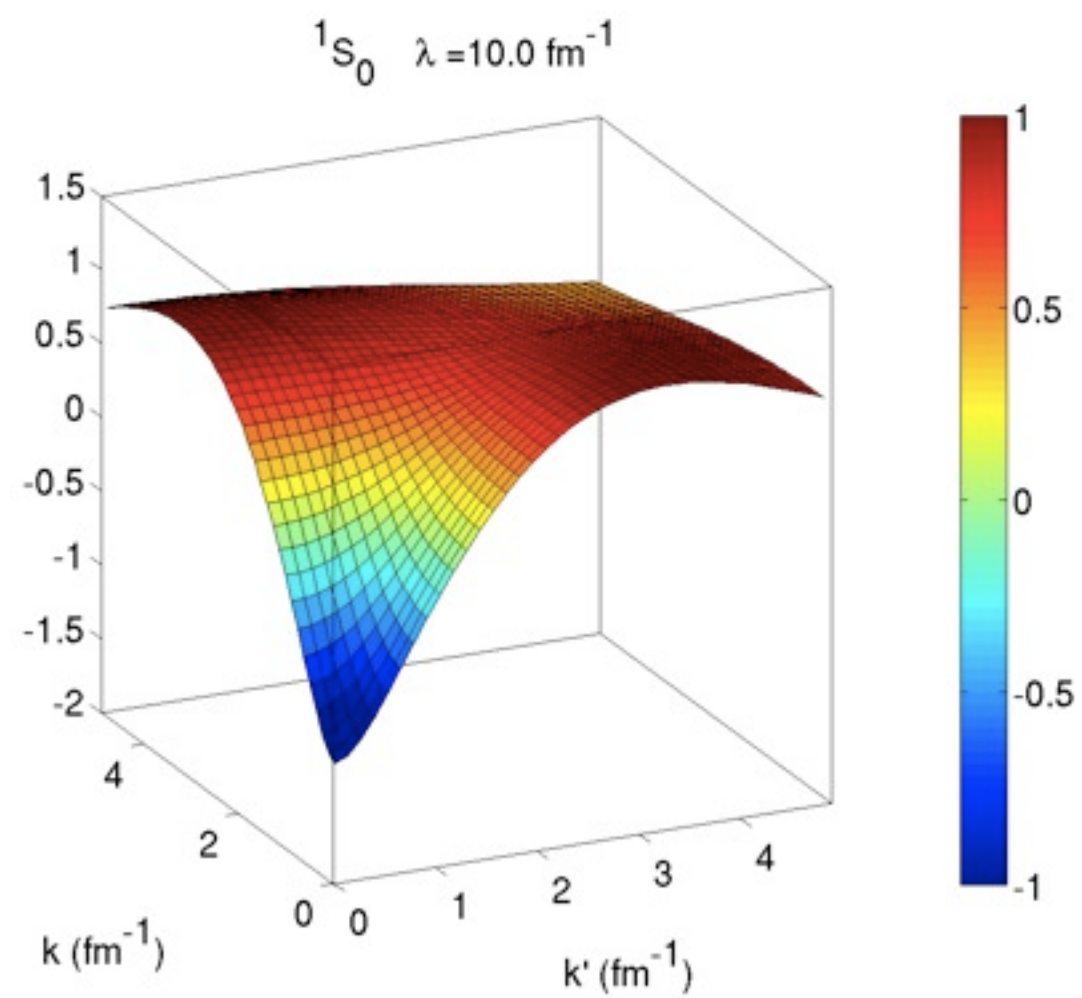

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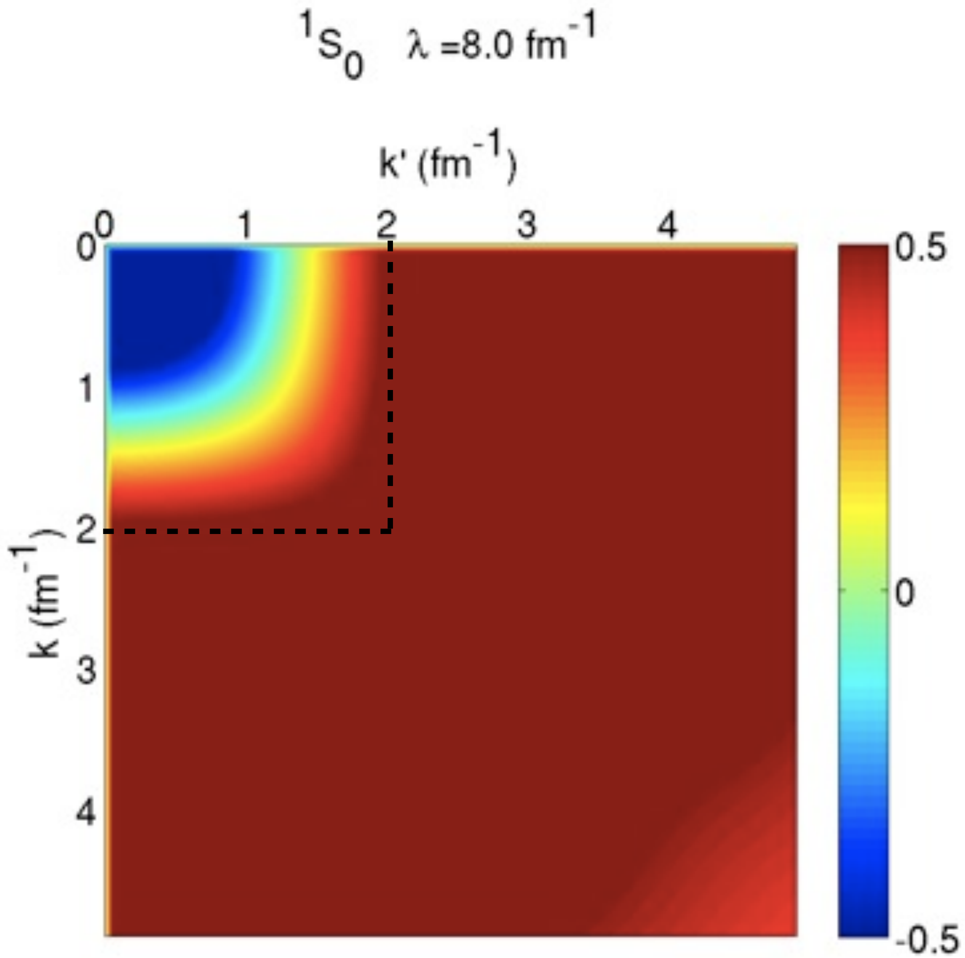

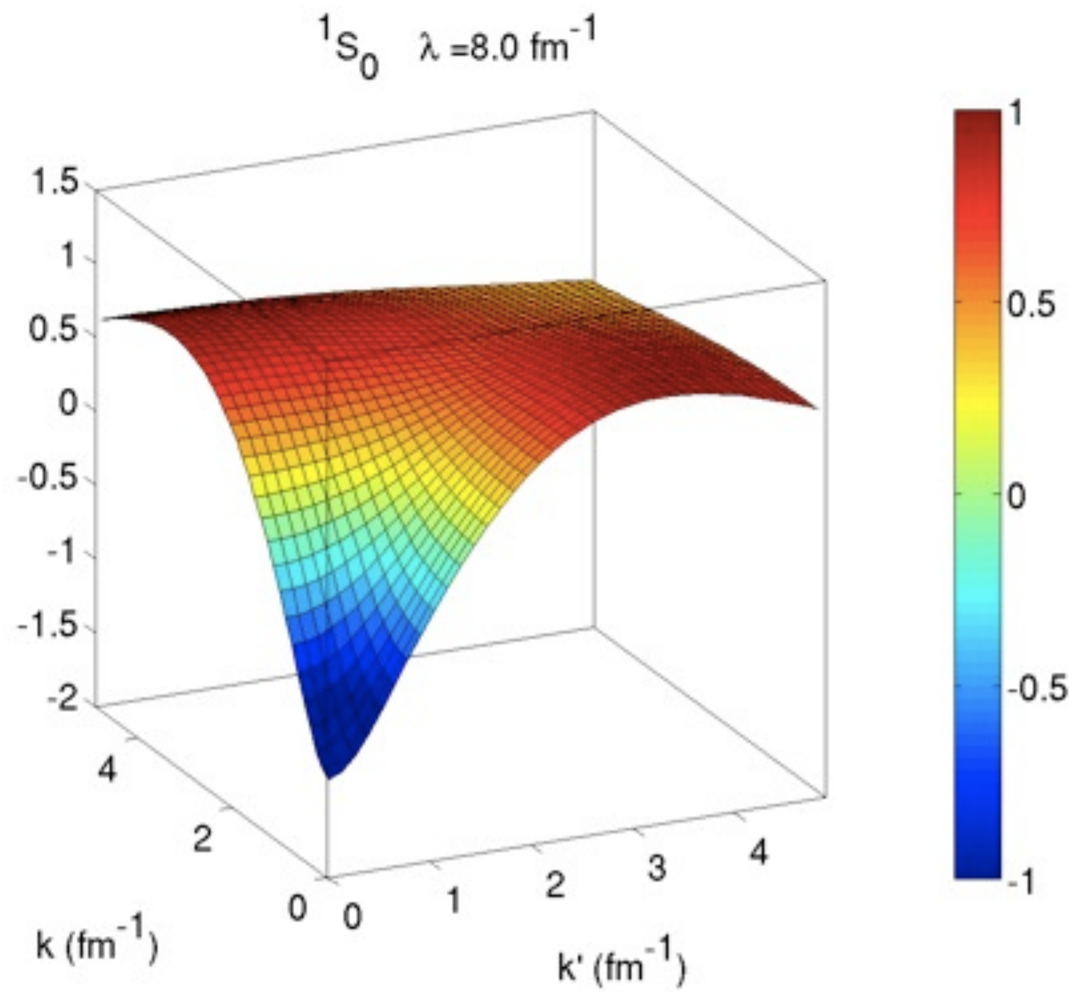

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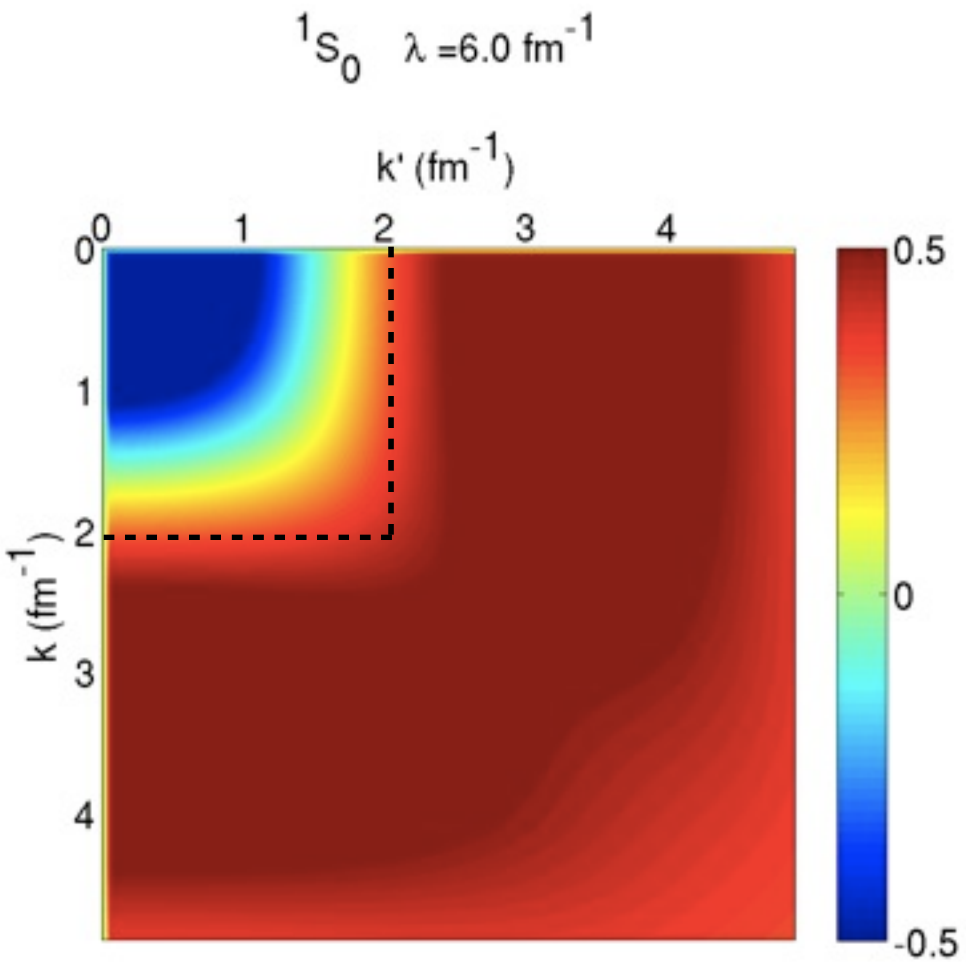

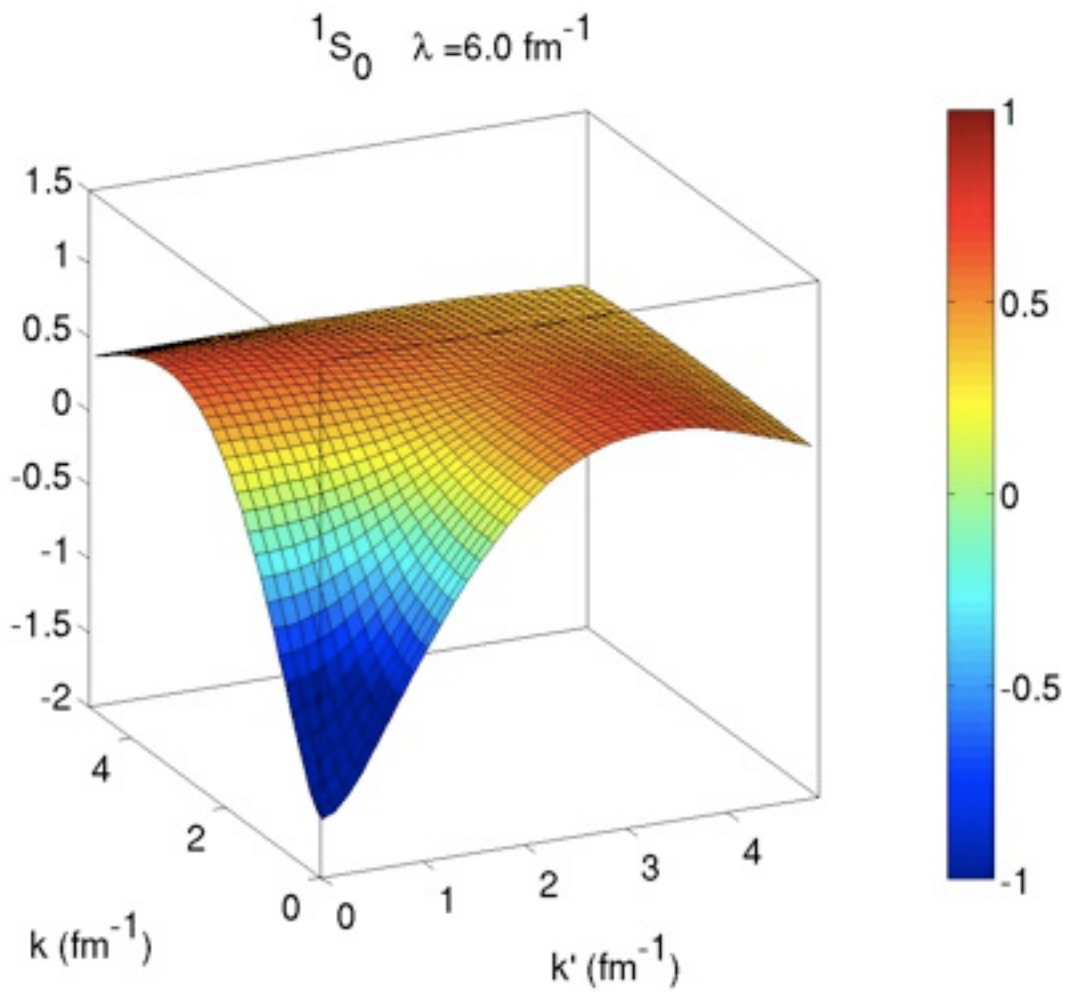

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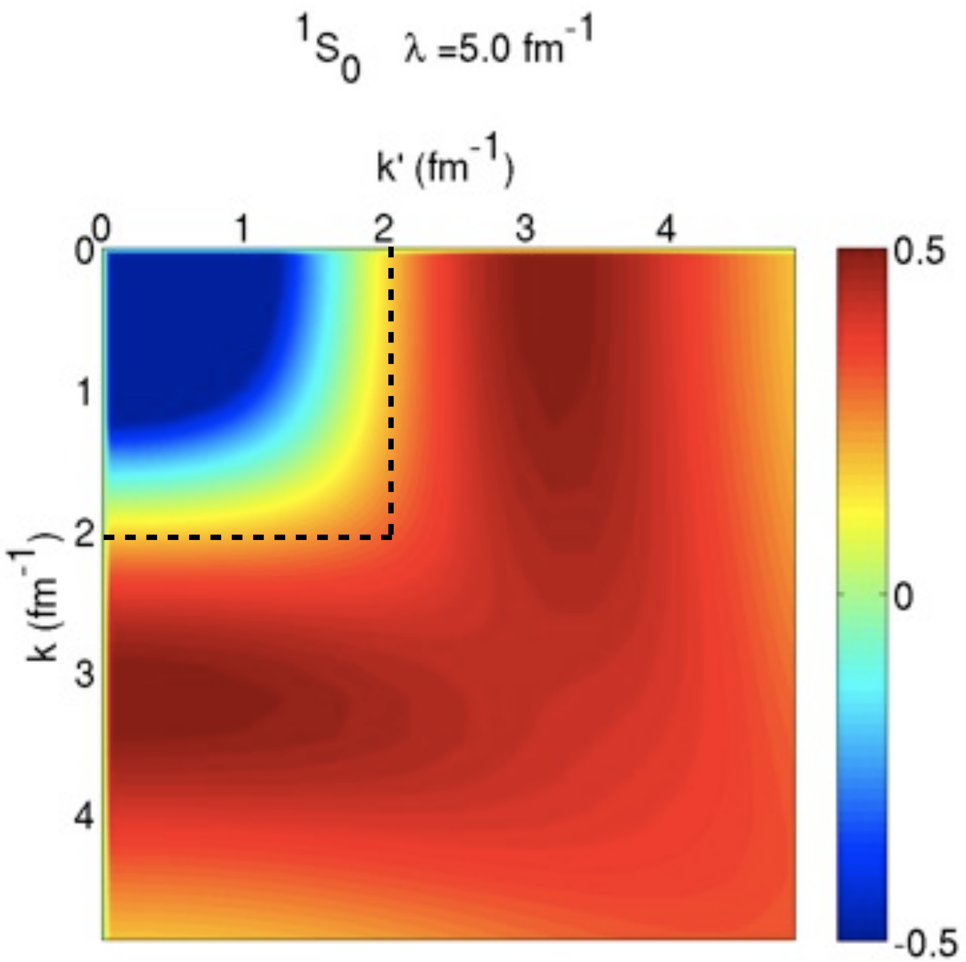

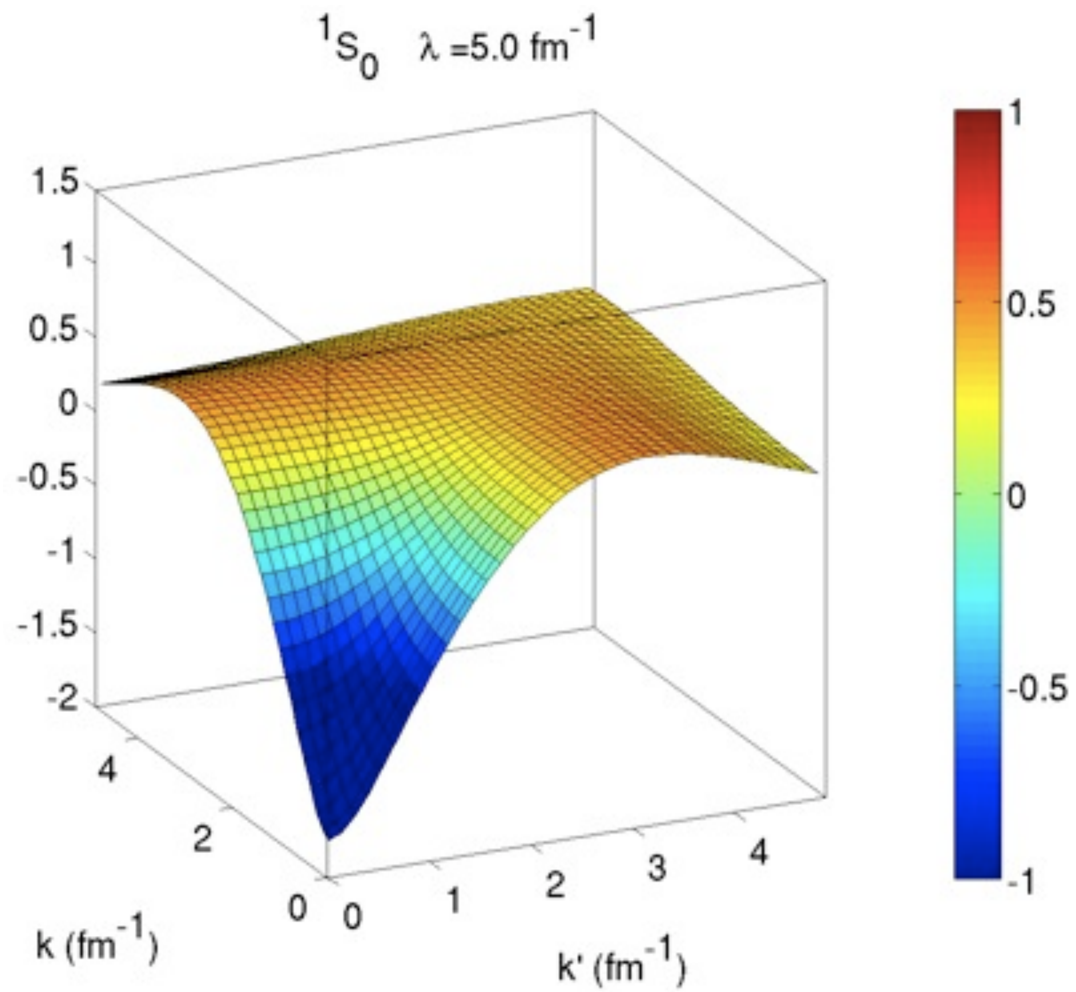

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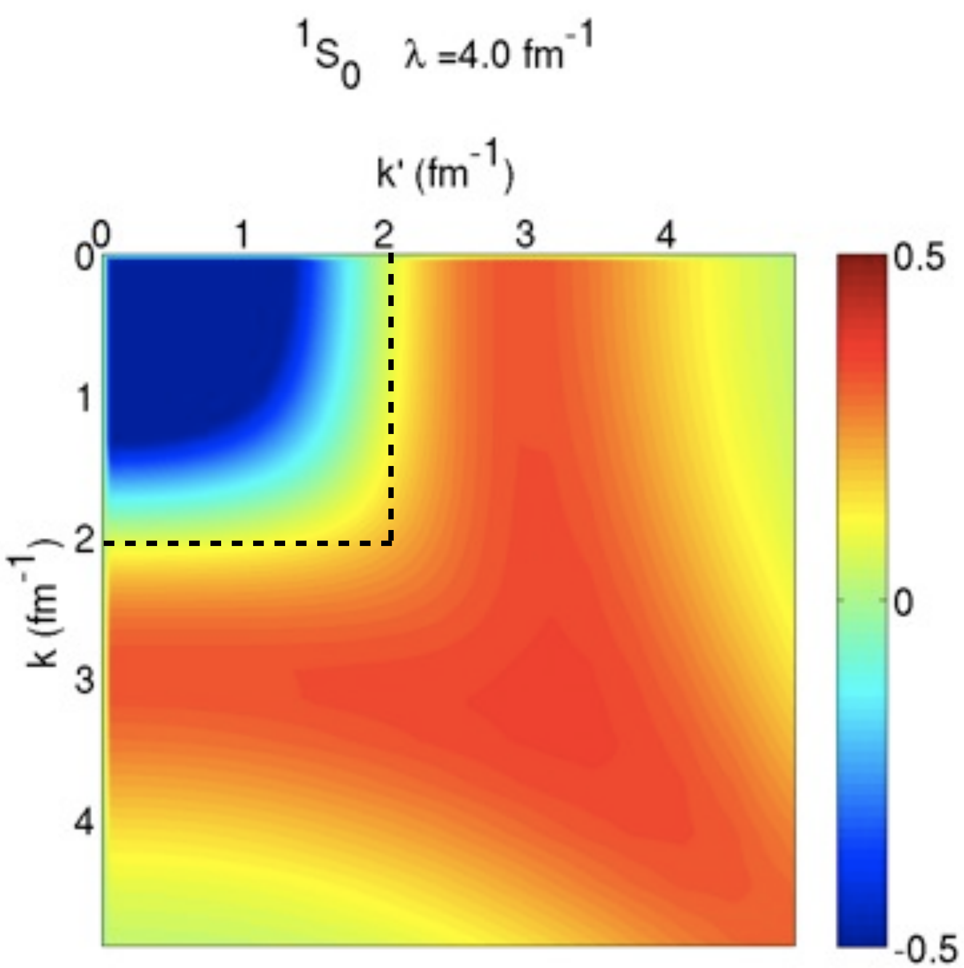

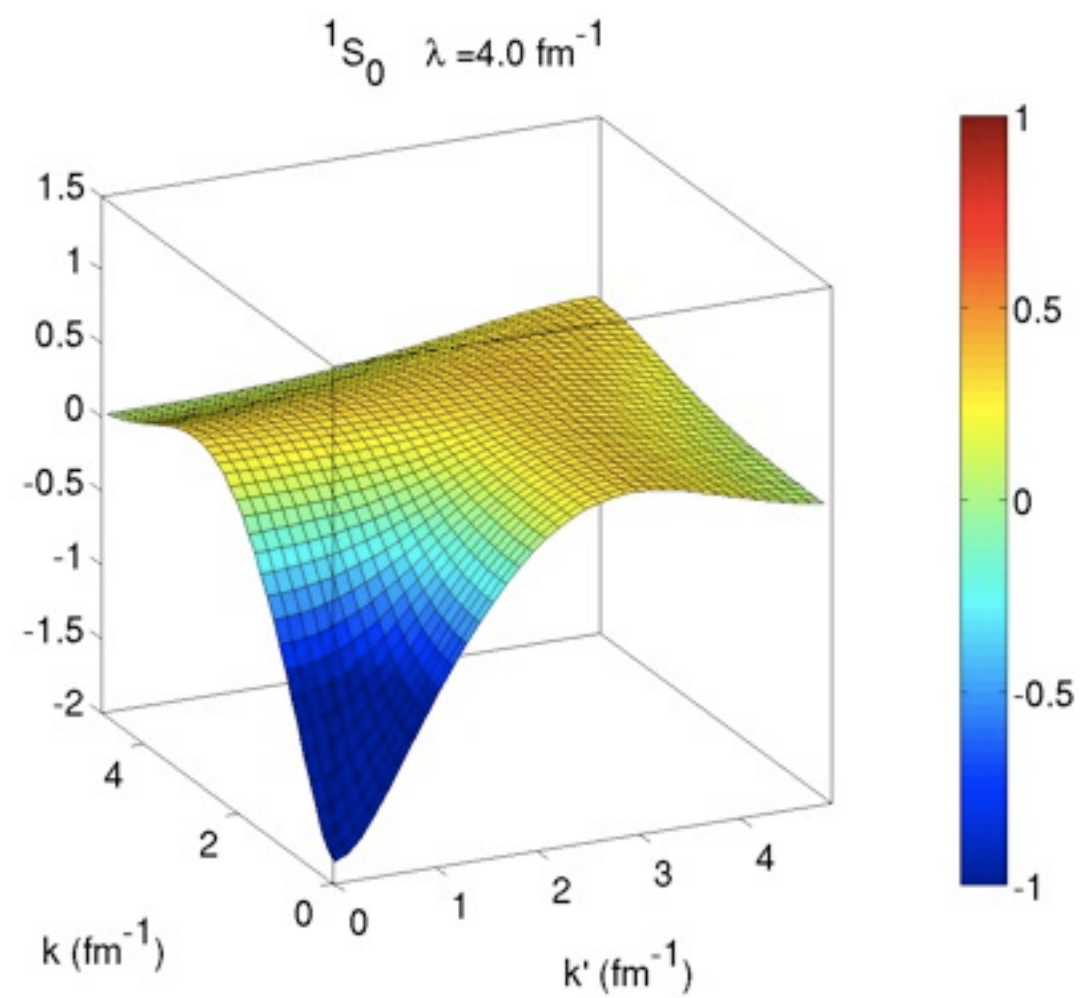

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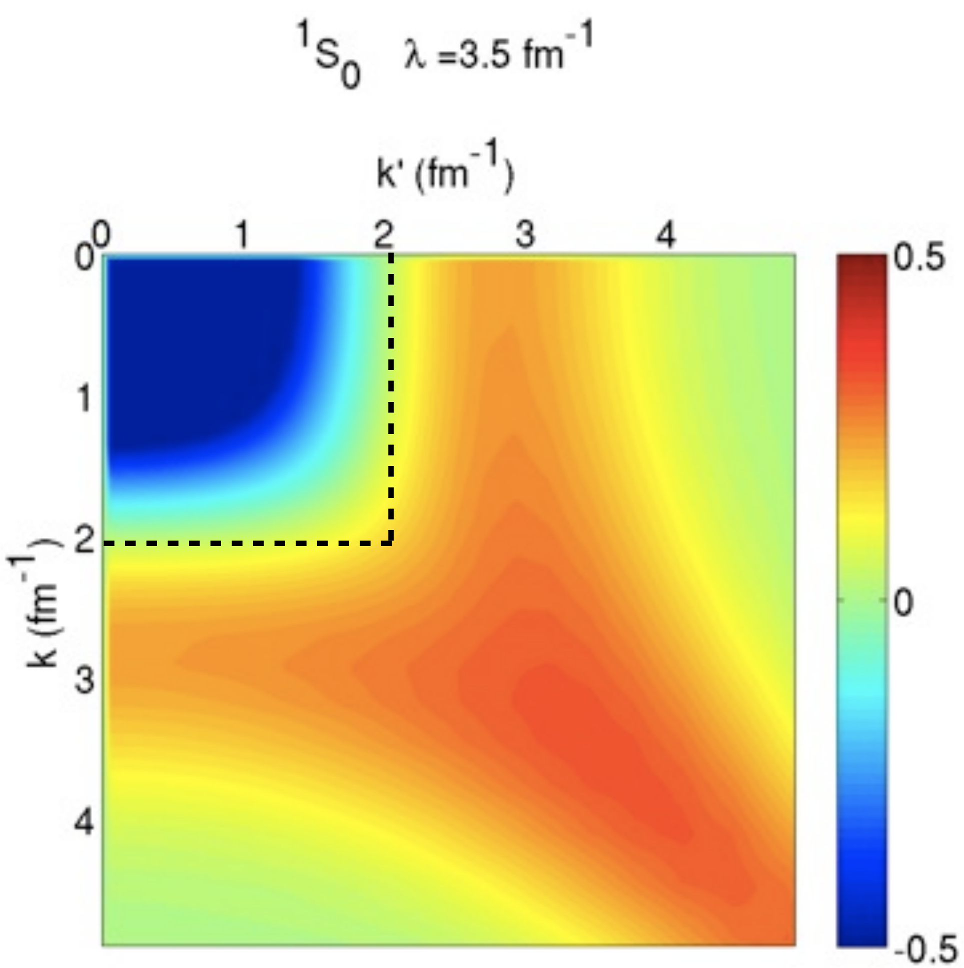

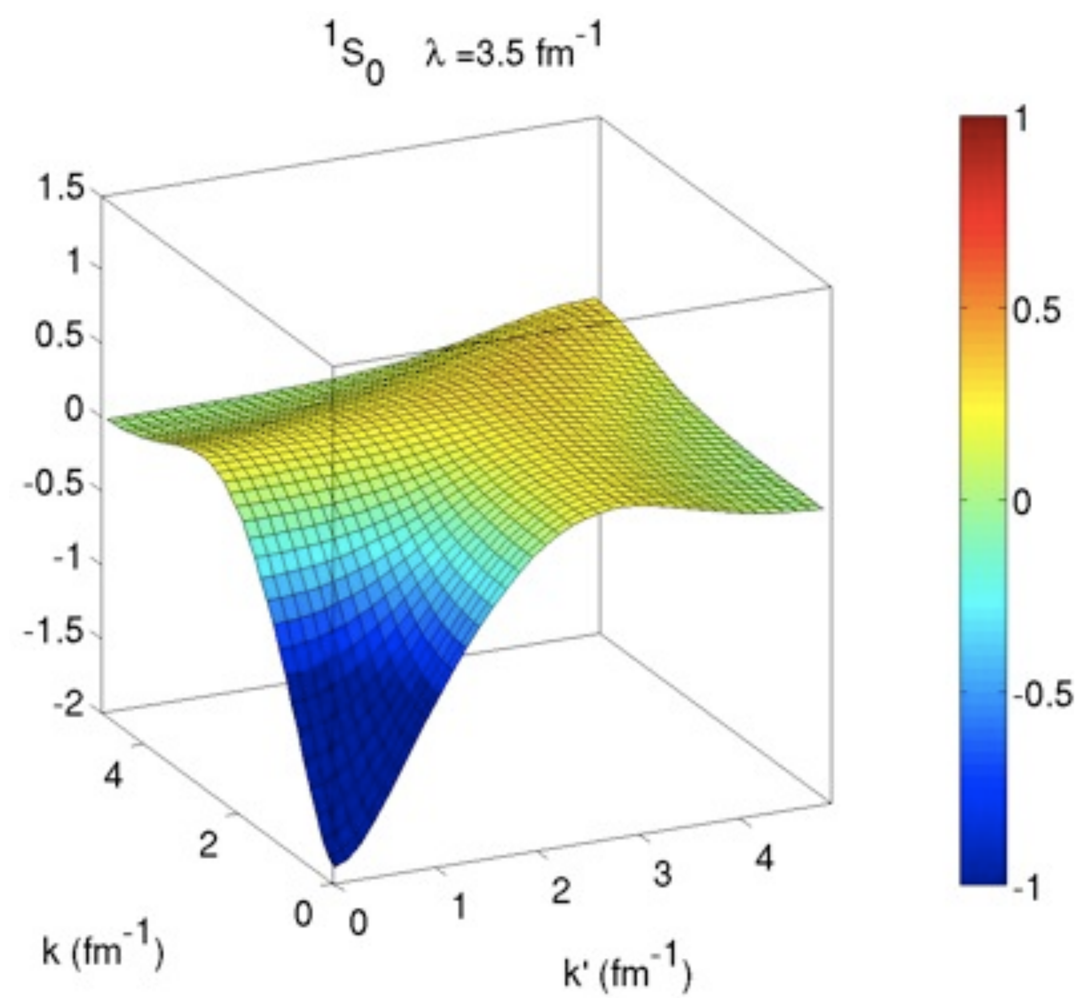

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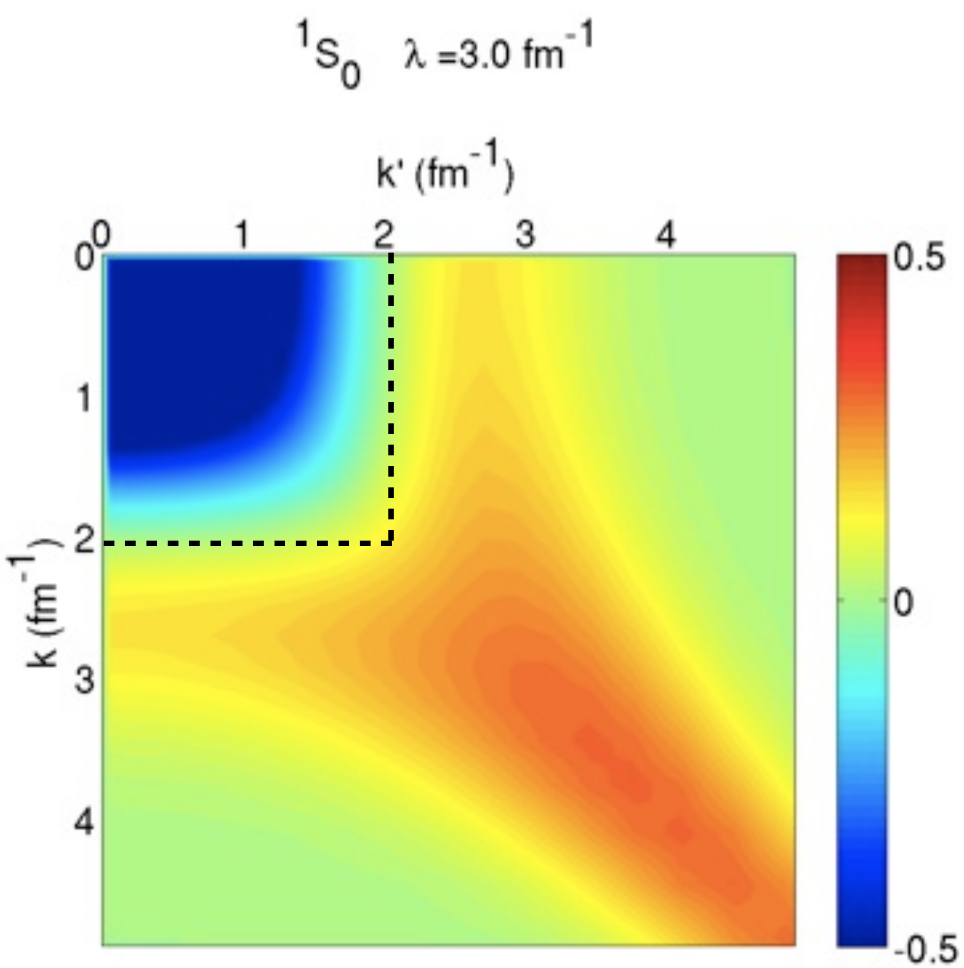

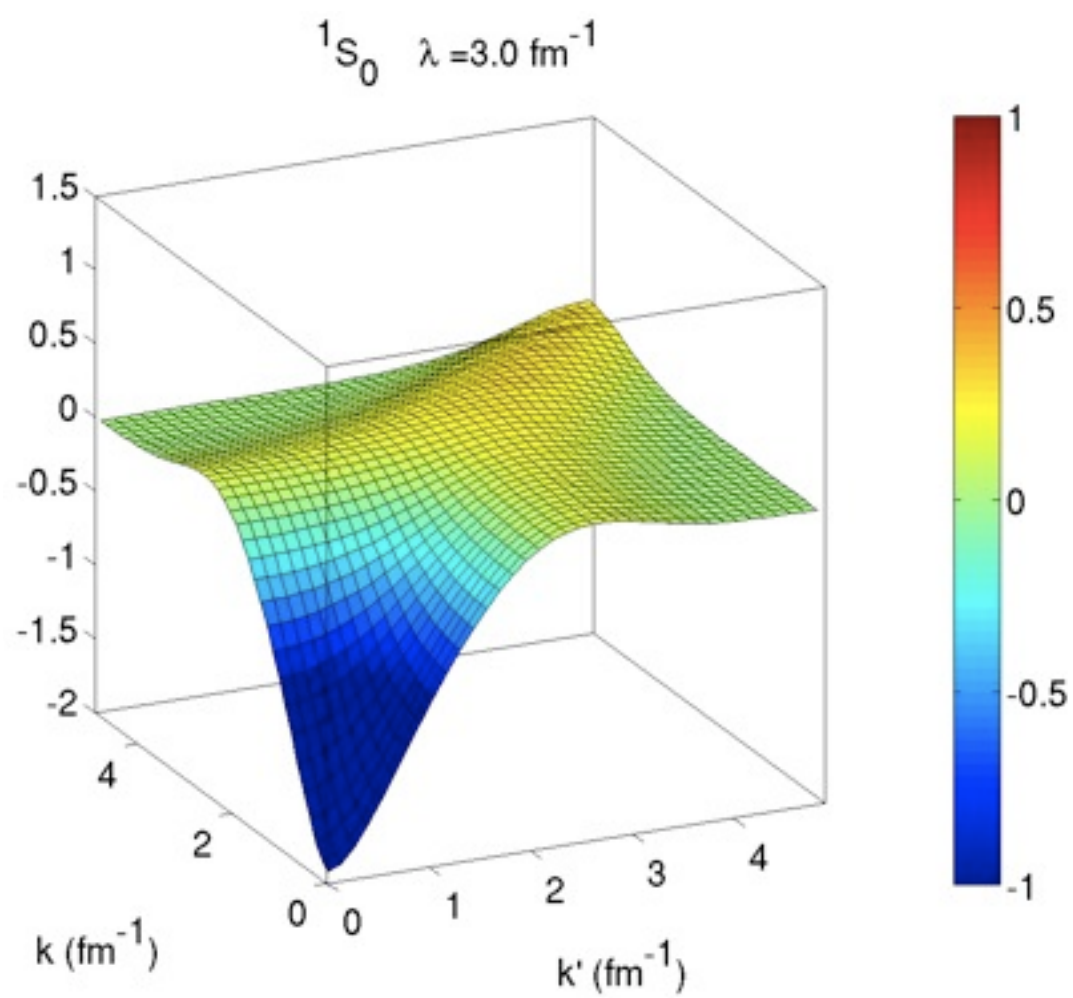

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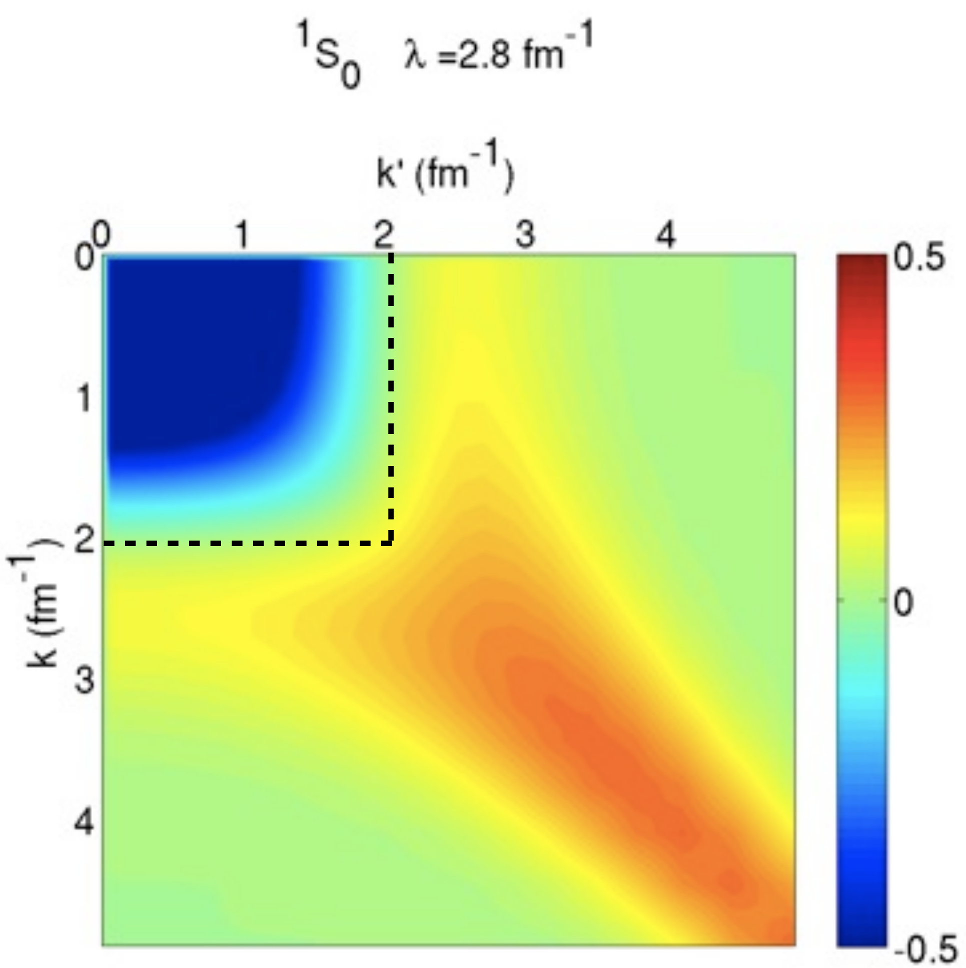

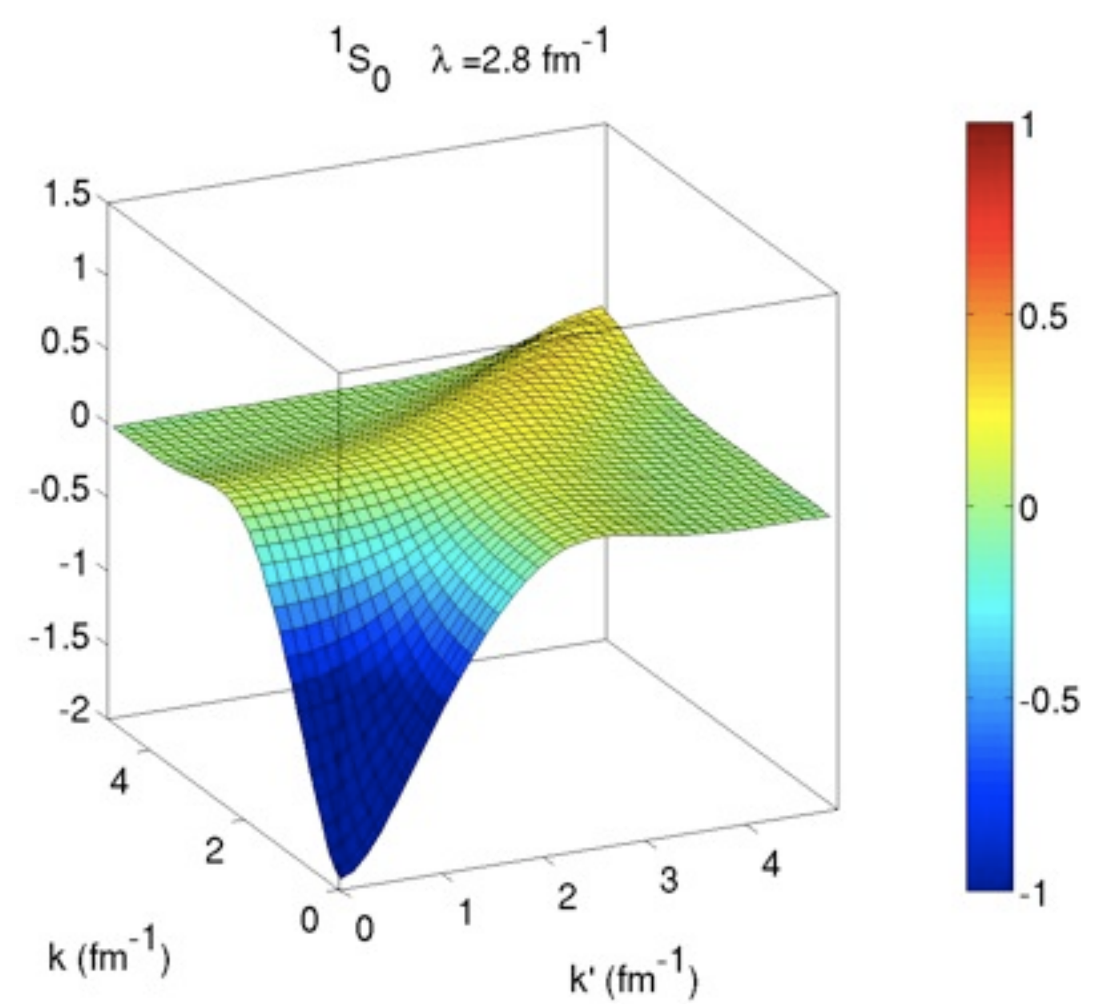

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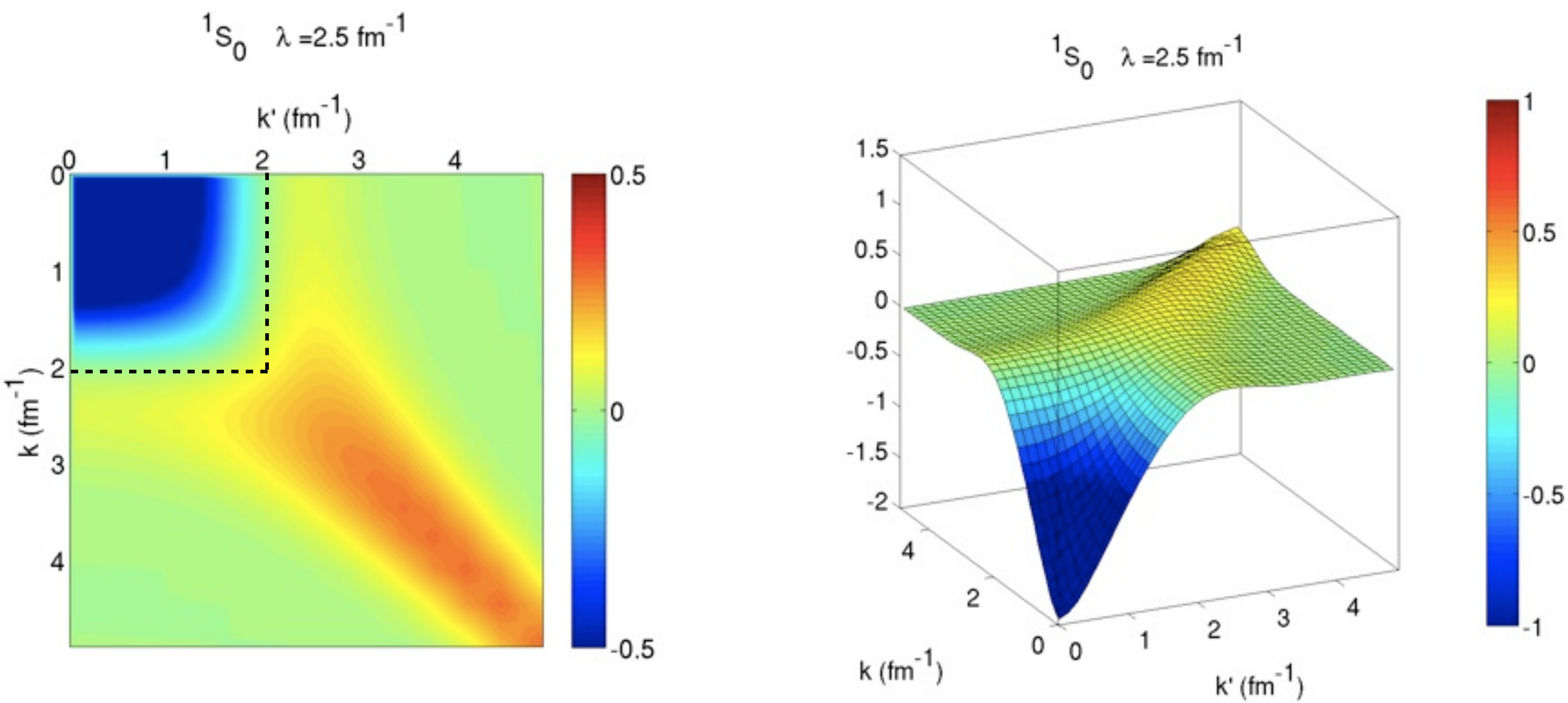

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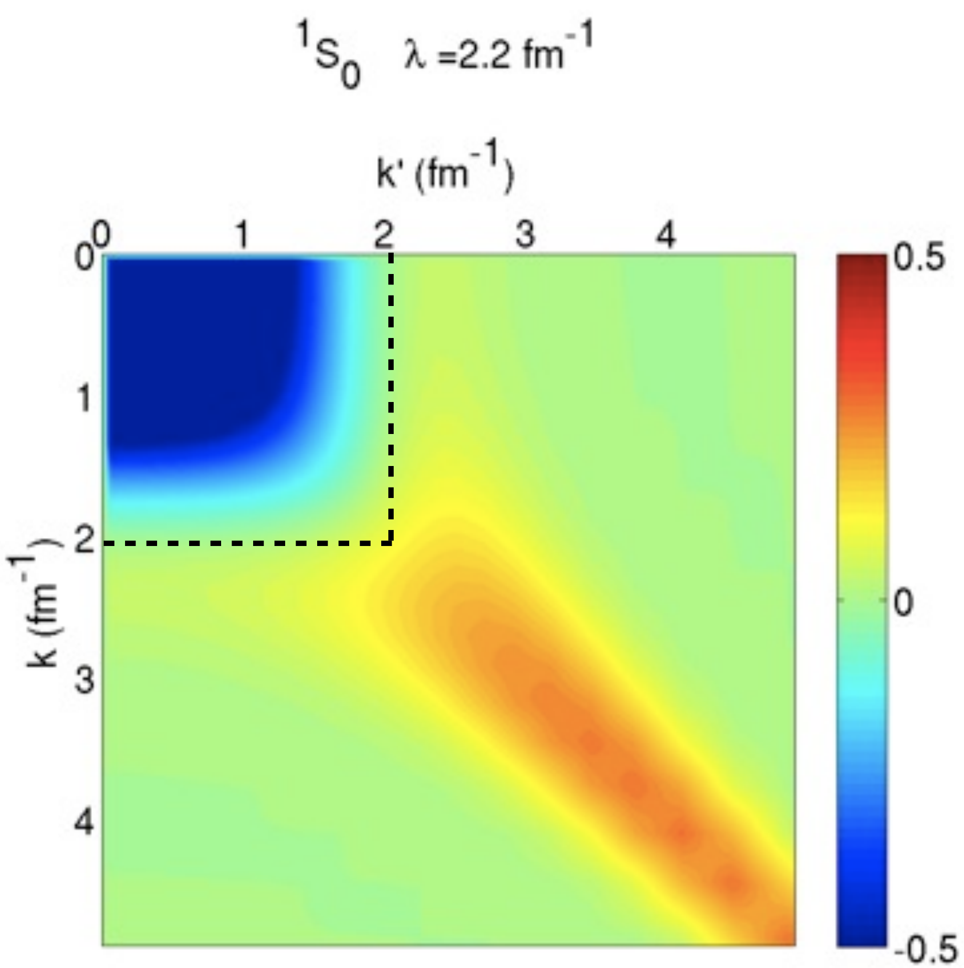

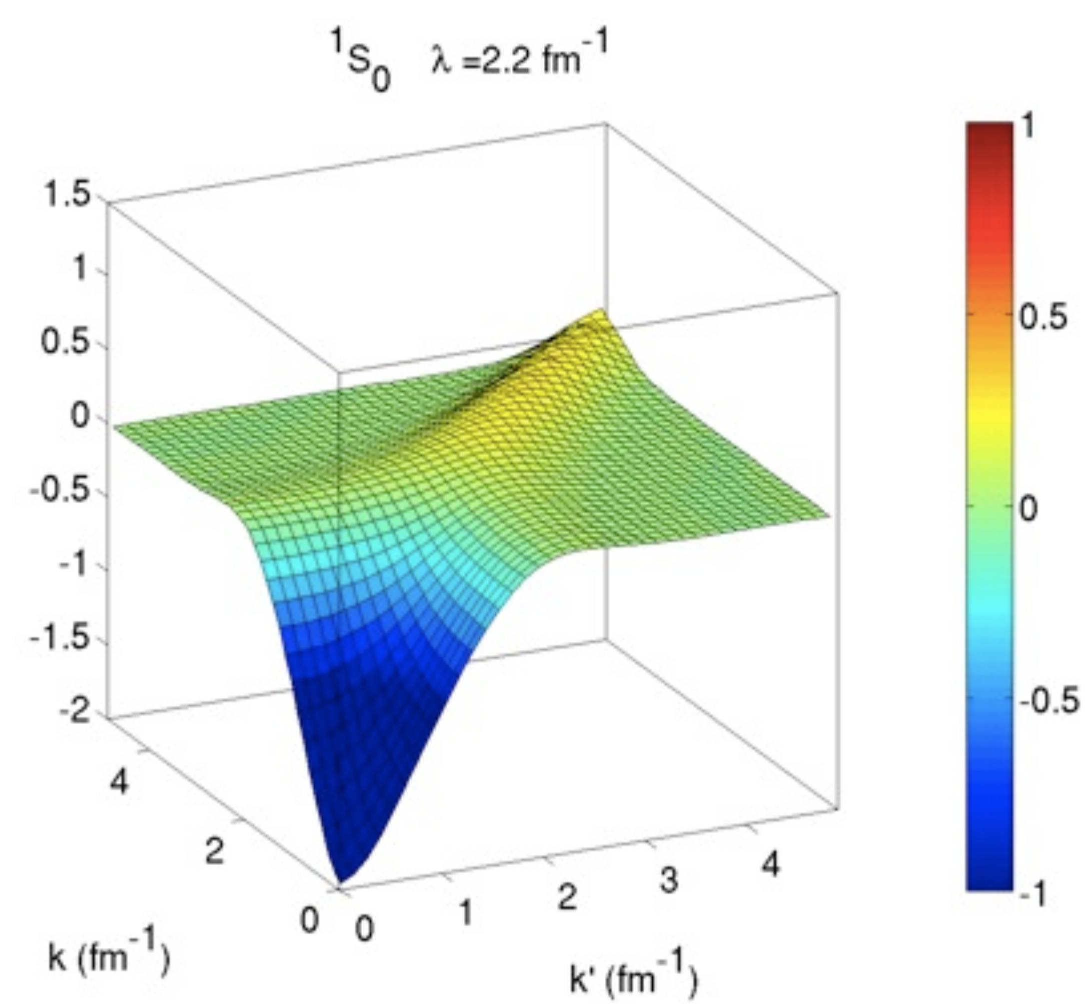

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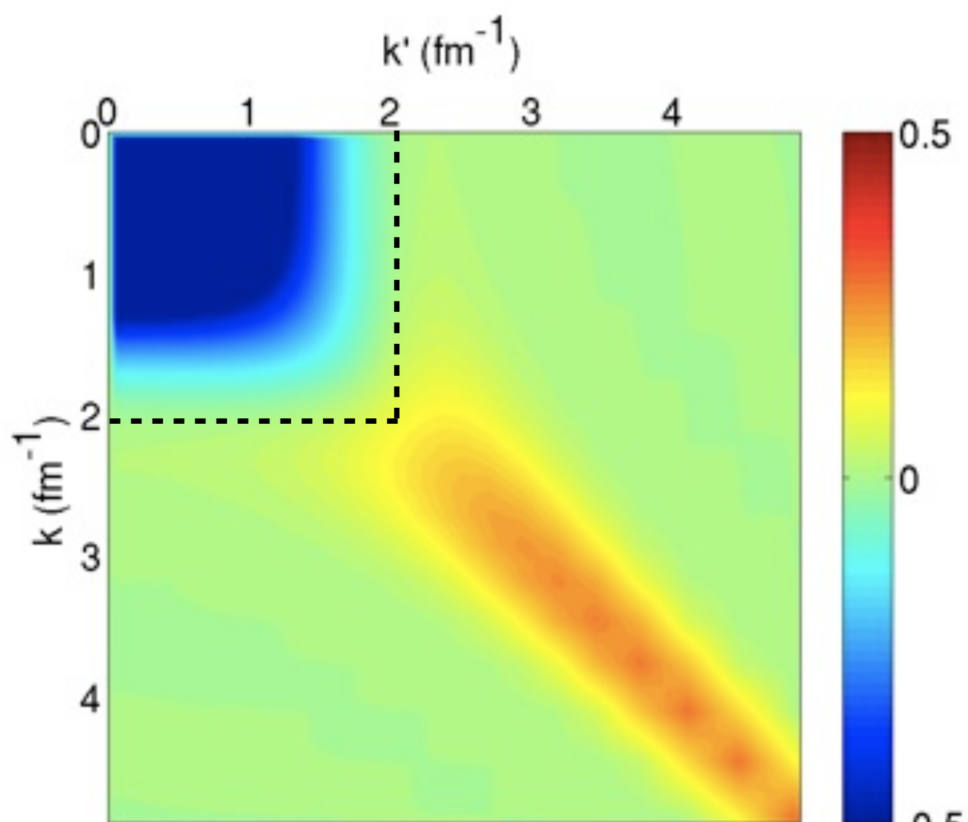
$^1S_0$ $\lambda = 2.0\,\mathrm{fm}^{-1}$



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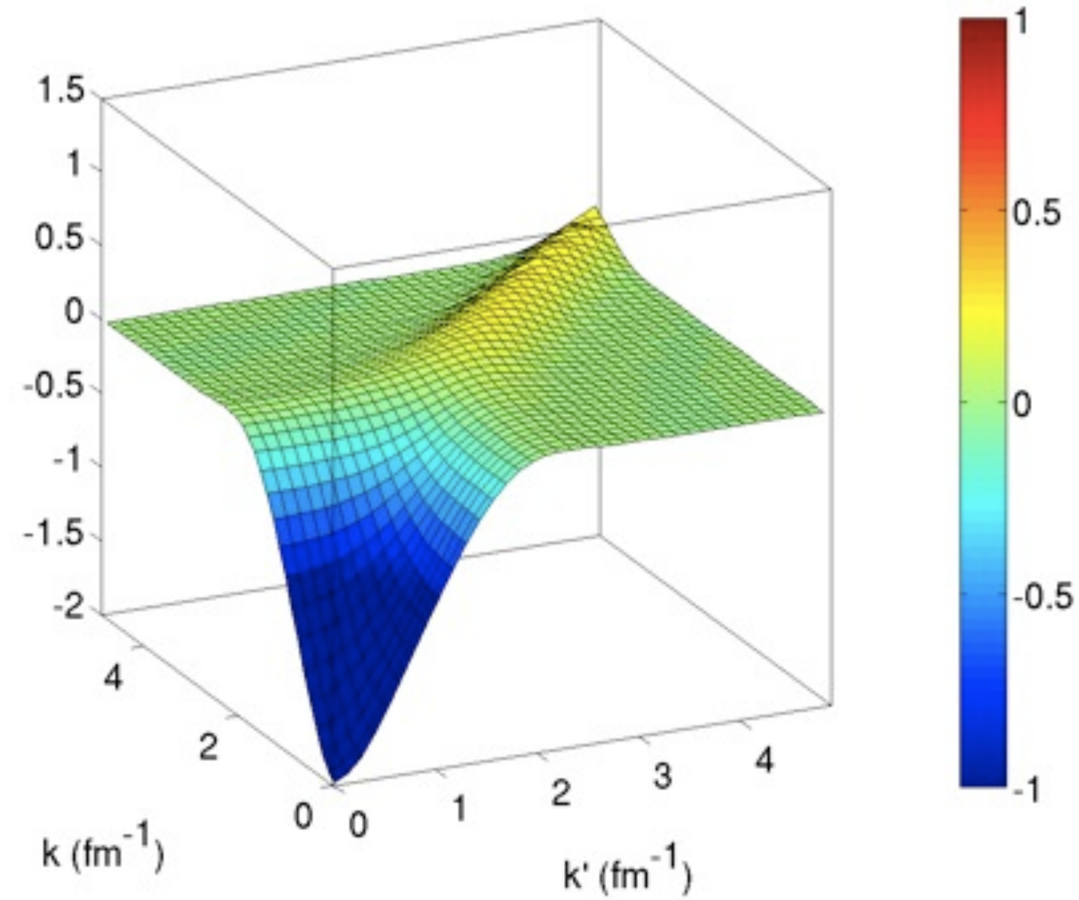

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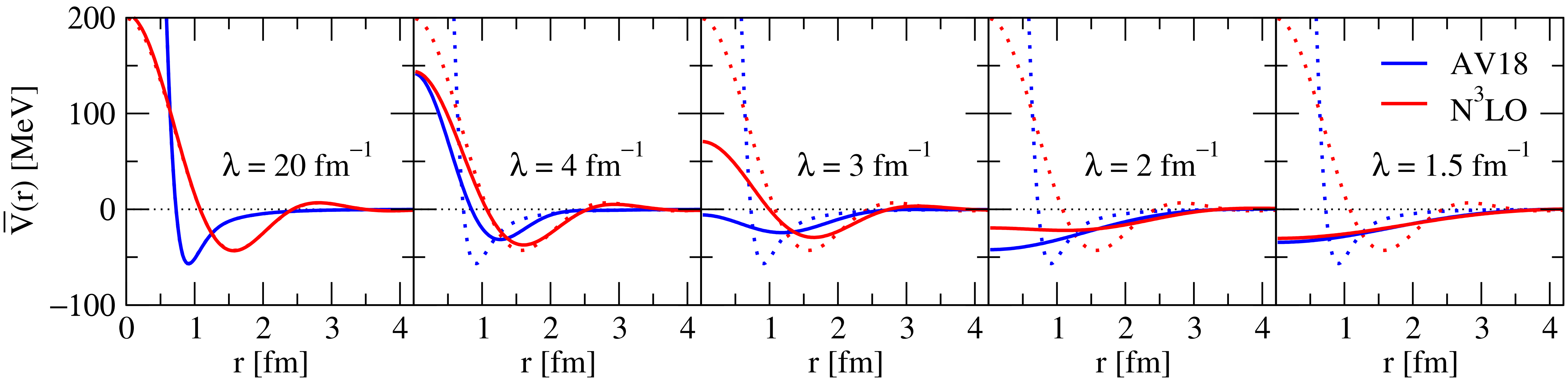
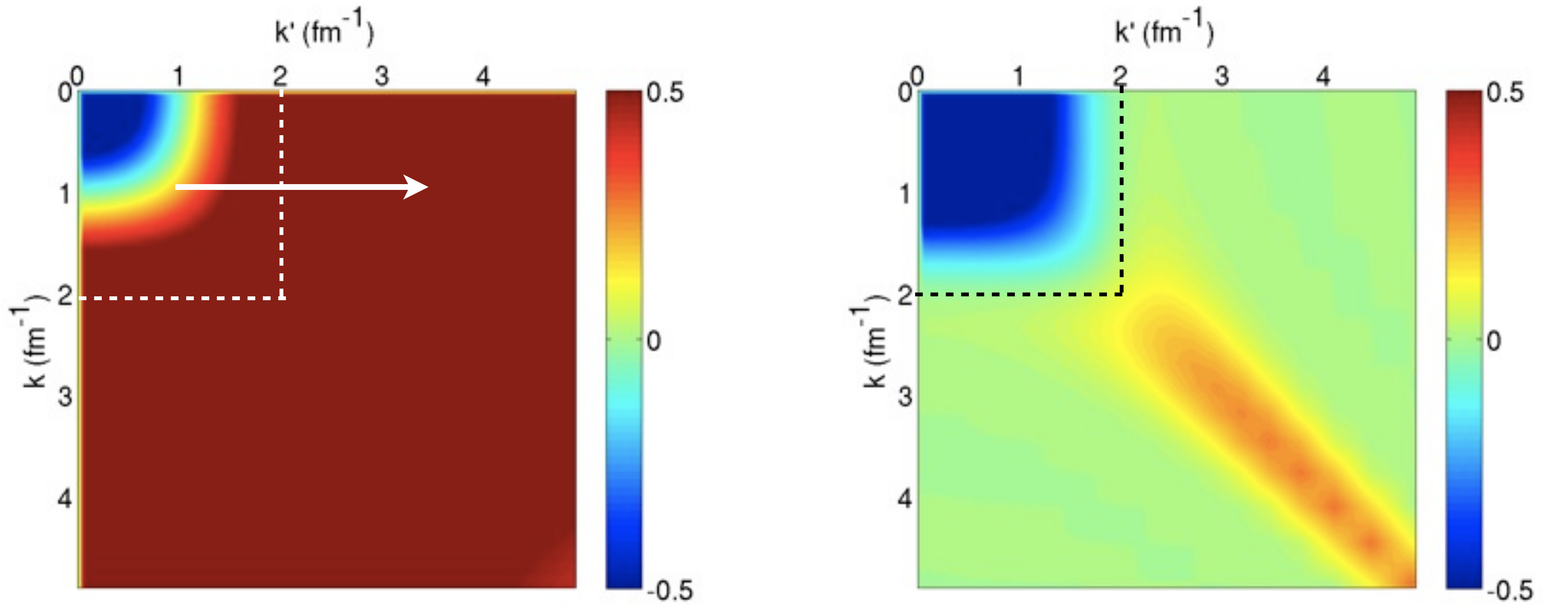


K. Wendt et al, PRC 86, 014003 (2012)

using local projection for representation:

$$\overline{V}_\lambda(r) = \int dr' r'^2 V_\lambda(r, r')$$

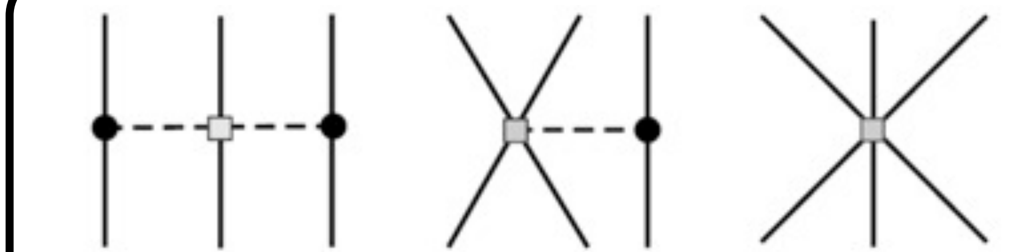
Changing the resolution: The Similarity Renormalization Group



- elimination of coupling between low- and high momentum components, many-body problem more perturbative and tractable
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformation also changes three-body (and higher-body) interactions.

RG evolution of 3N interactions



- So far (in momentum basis):

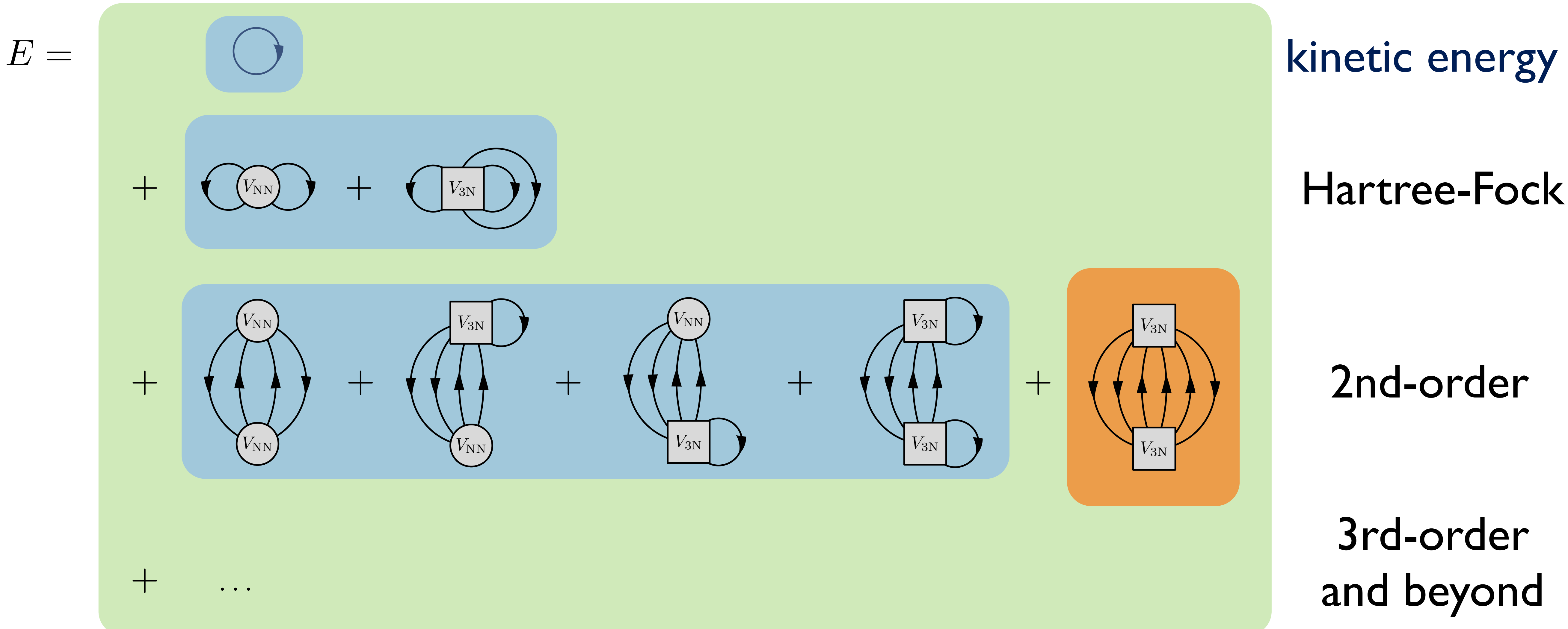
intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:

$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV}$ and $r_{^4\mathrm{He}} = 1.464\,\mathrm{fm}$

$\longrightarrow$ coupling constants of natural size

- in neutron matter contributions from c_D, c_E and c_4 terms vanish
- long-range 2π contributions assumed to be invariant under RG evolution
- at low resolution scales nuclear many-body problem more perturbative

Application to infinite nuclear matter: Equation of state



- calculate EOS by taking all blue-boxed contributions into account
- in this approximation NN and 3NF contributions factorize

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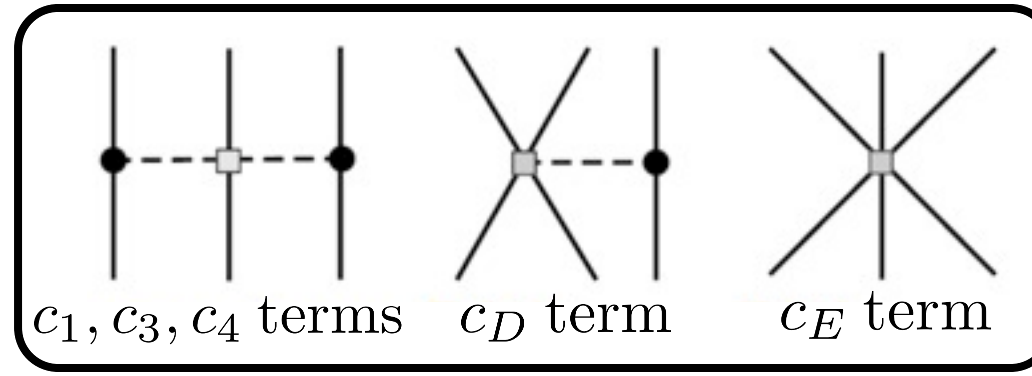

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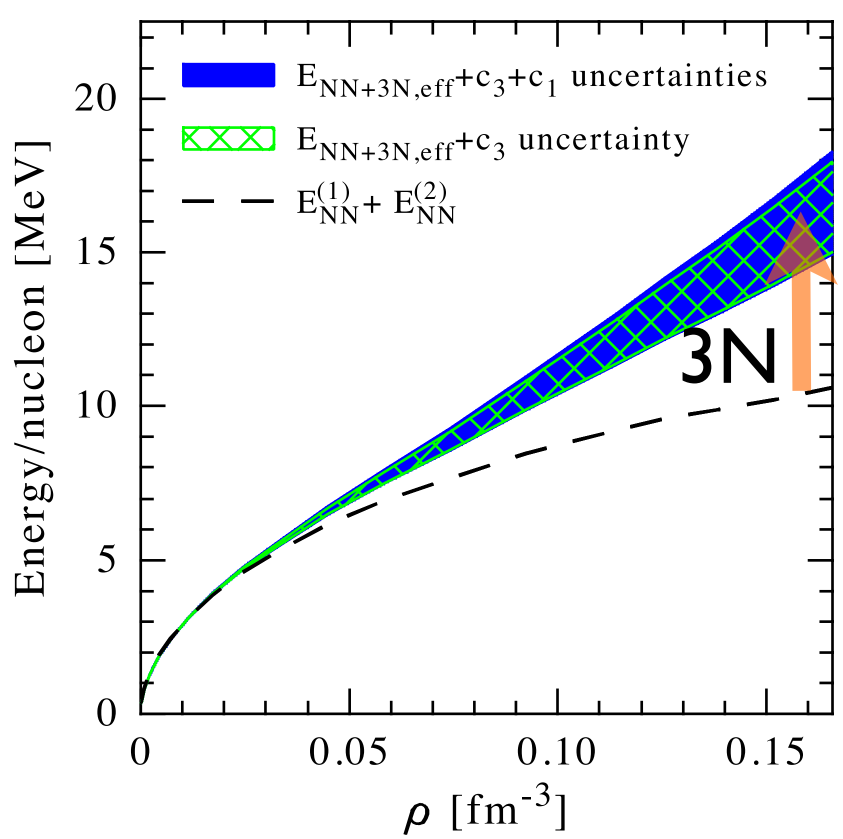
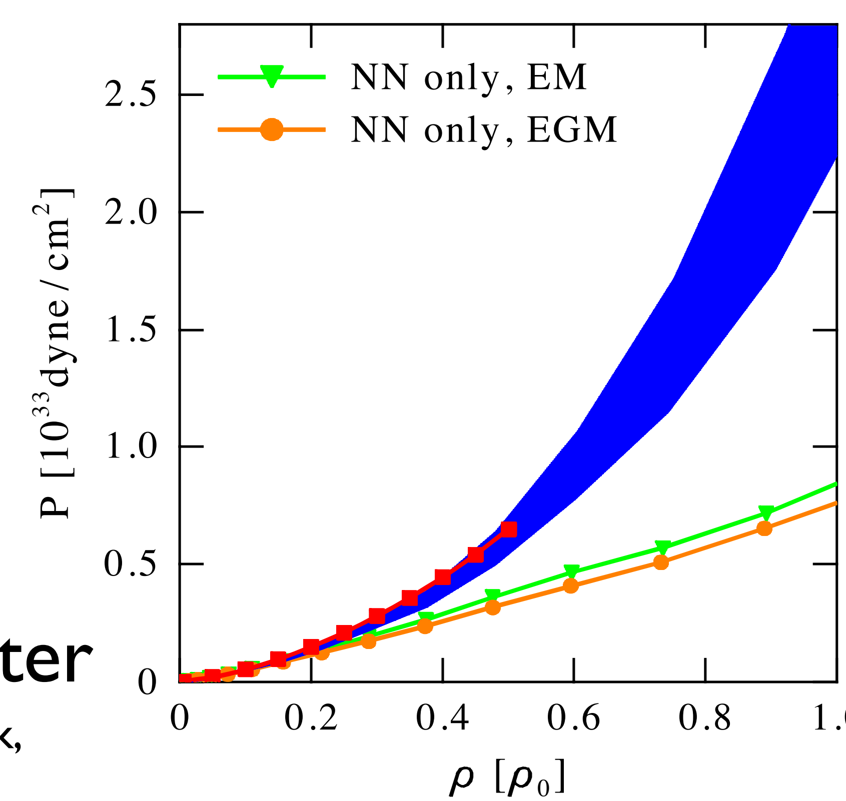


pure neutron matter

KH and Schwenk PRC 82, 014314 (2010)



neutron star matter

KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

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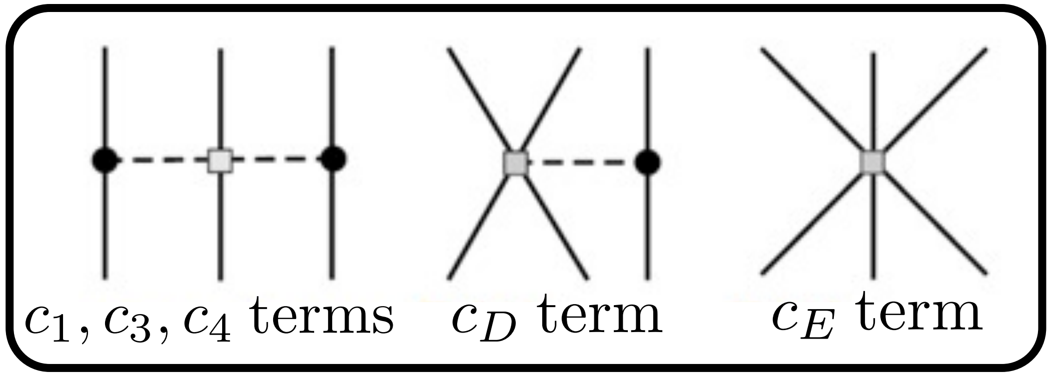

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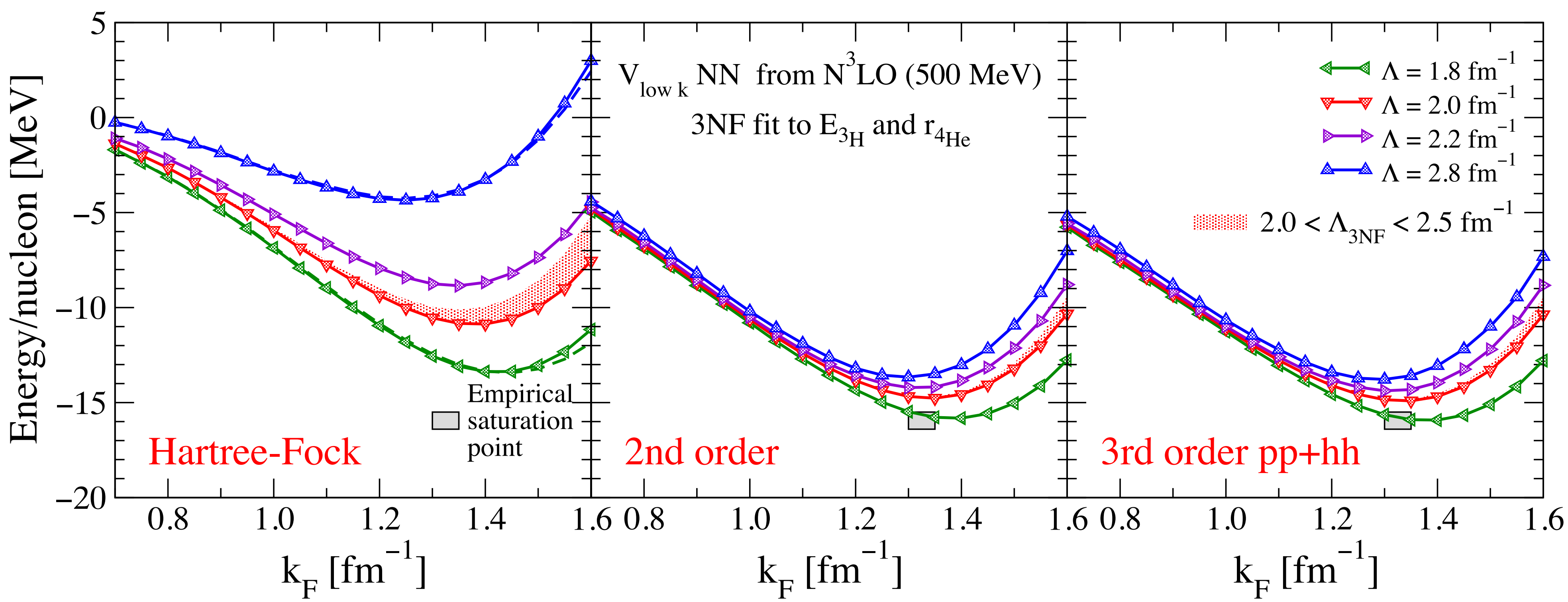
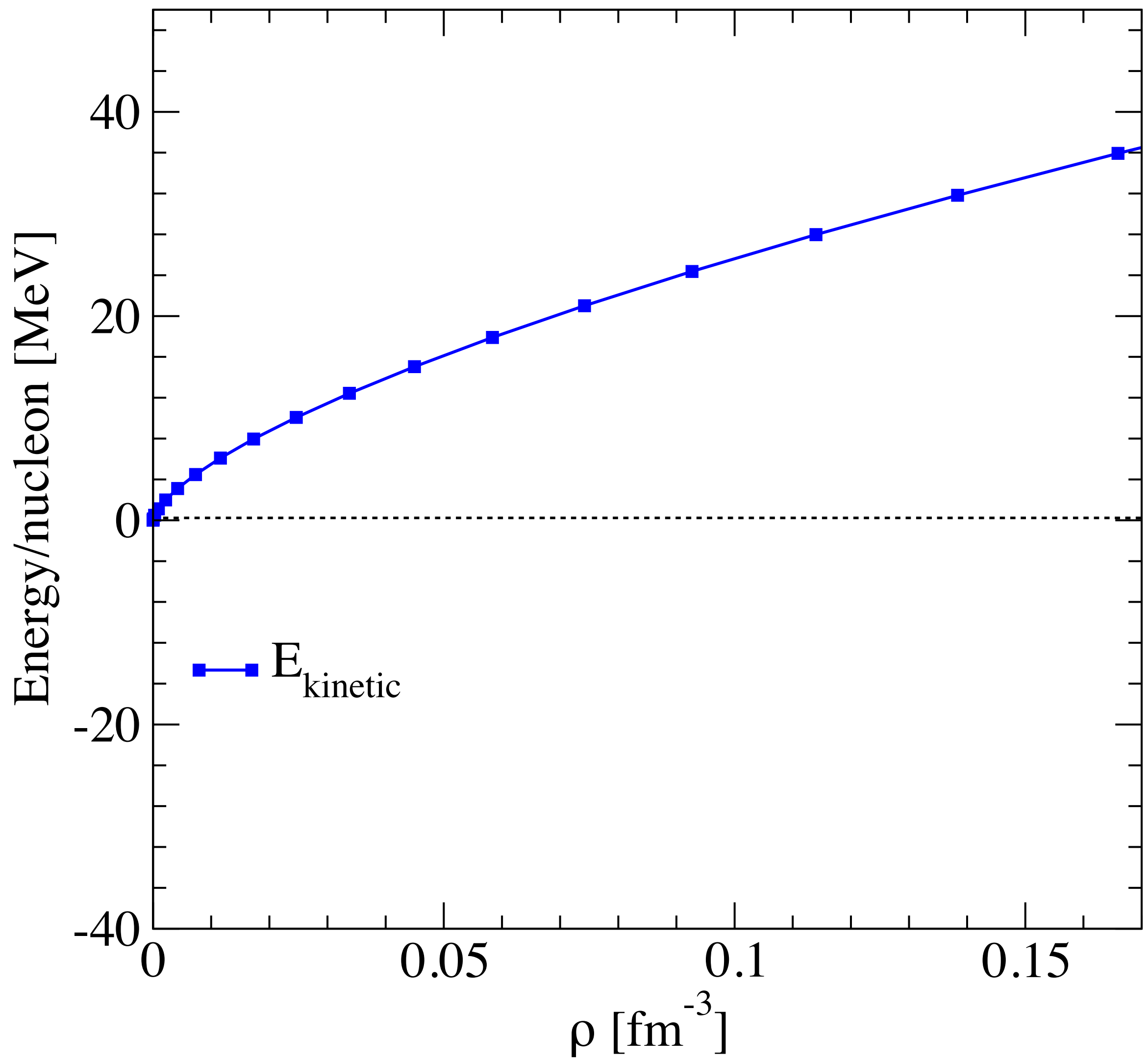


symmetric nuclear matter

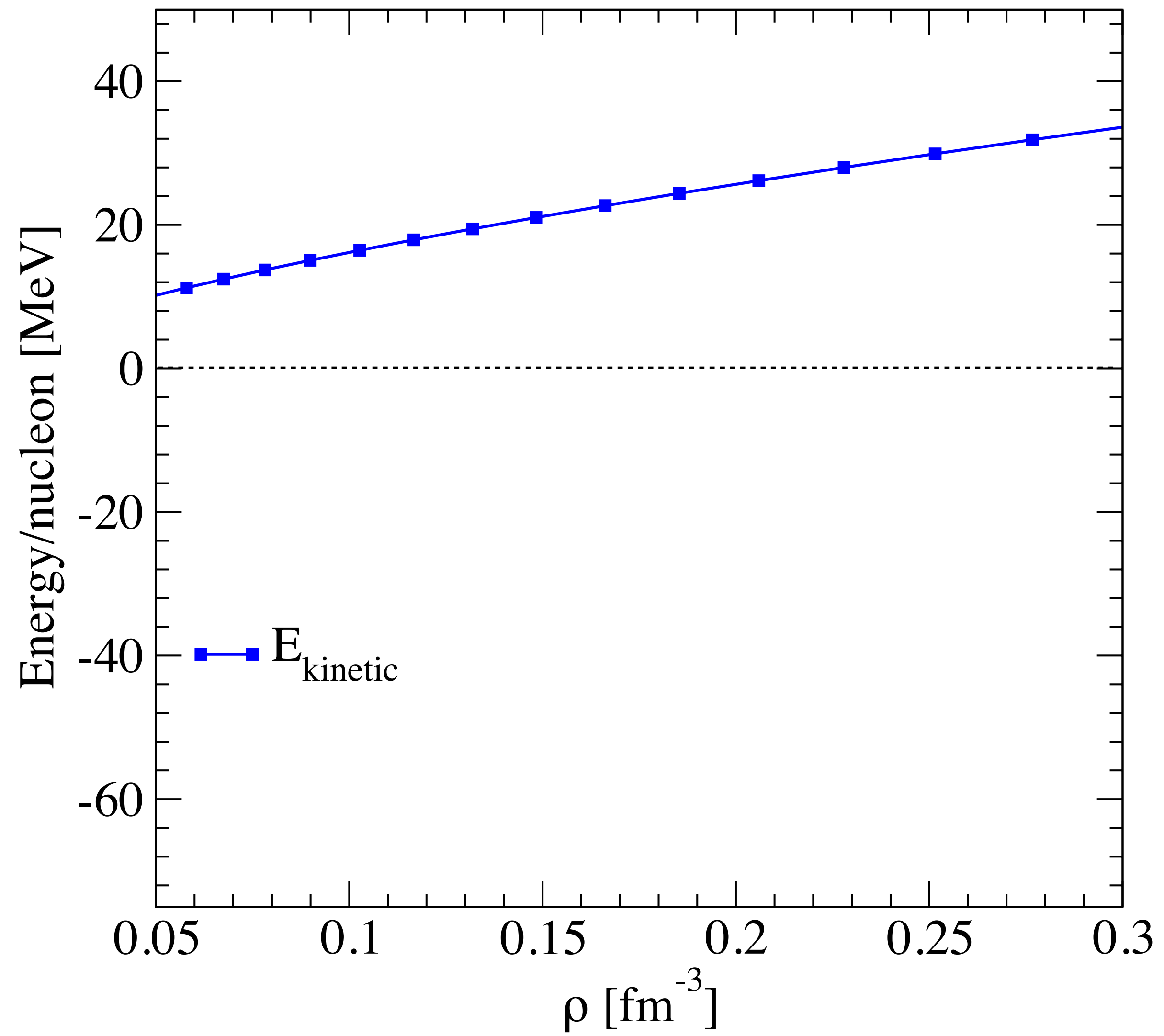
KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

Hierarchy of many-body contributions

neutron matter

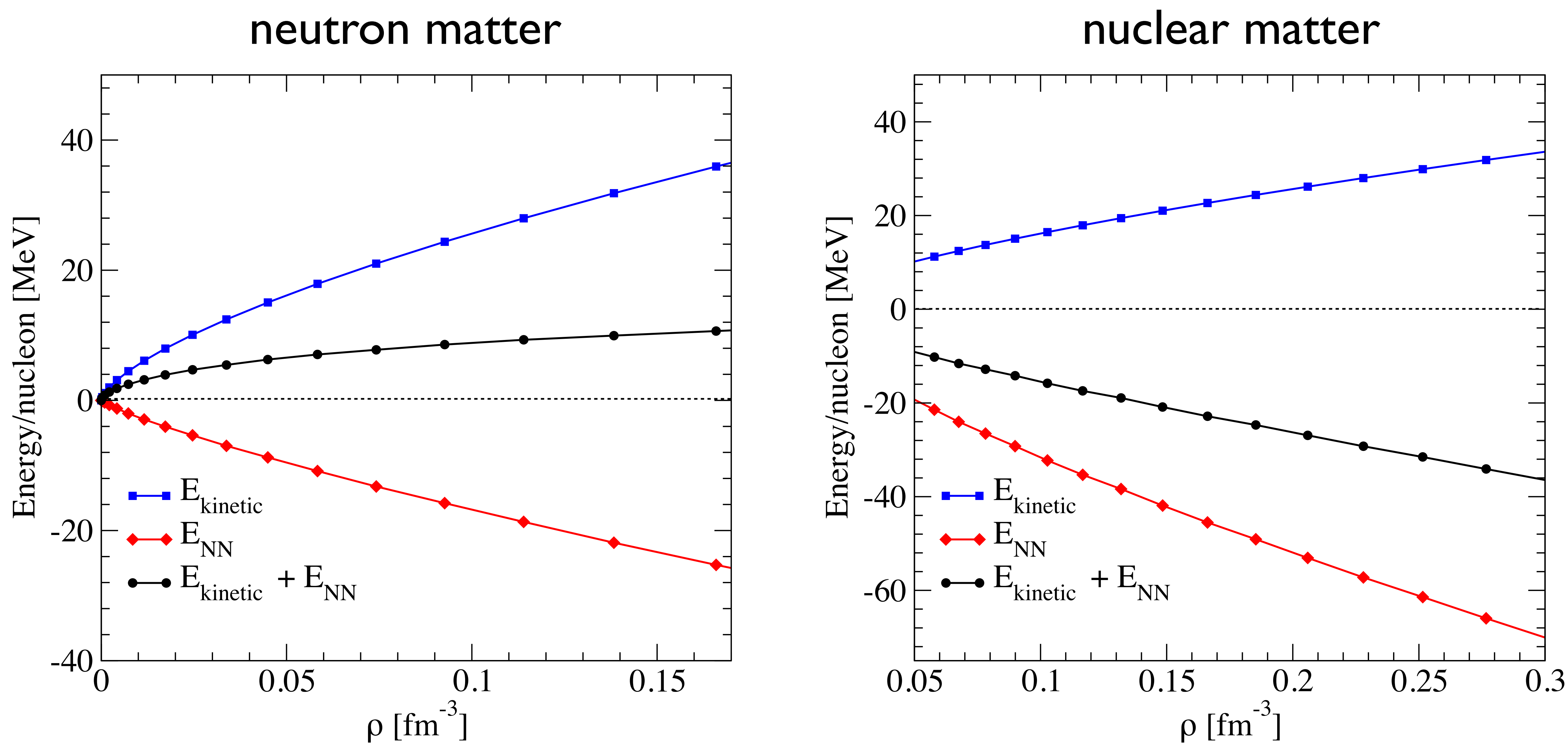


nuclear matter



- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

Hierarchy of many-body contributions



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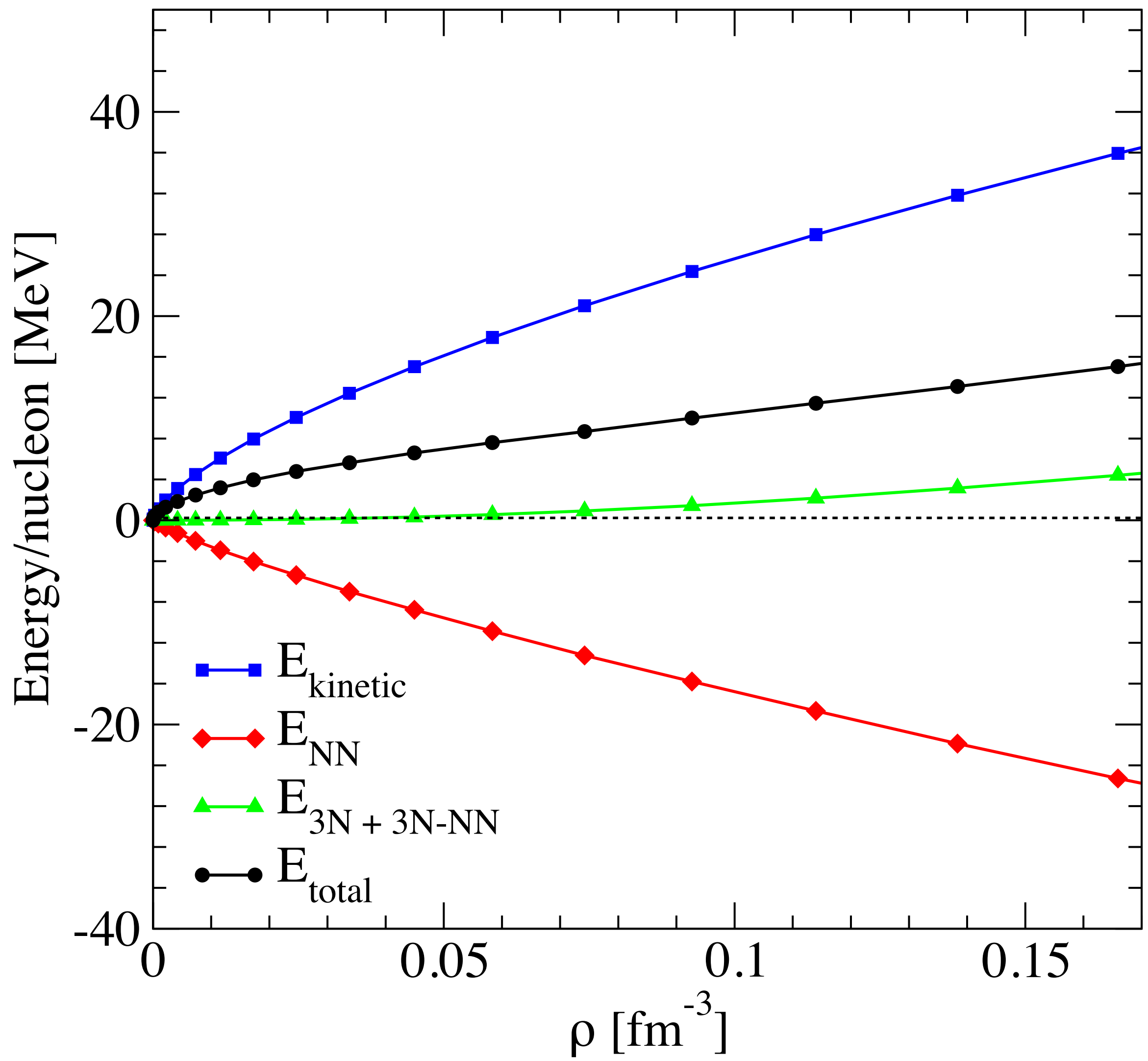

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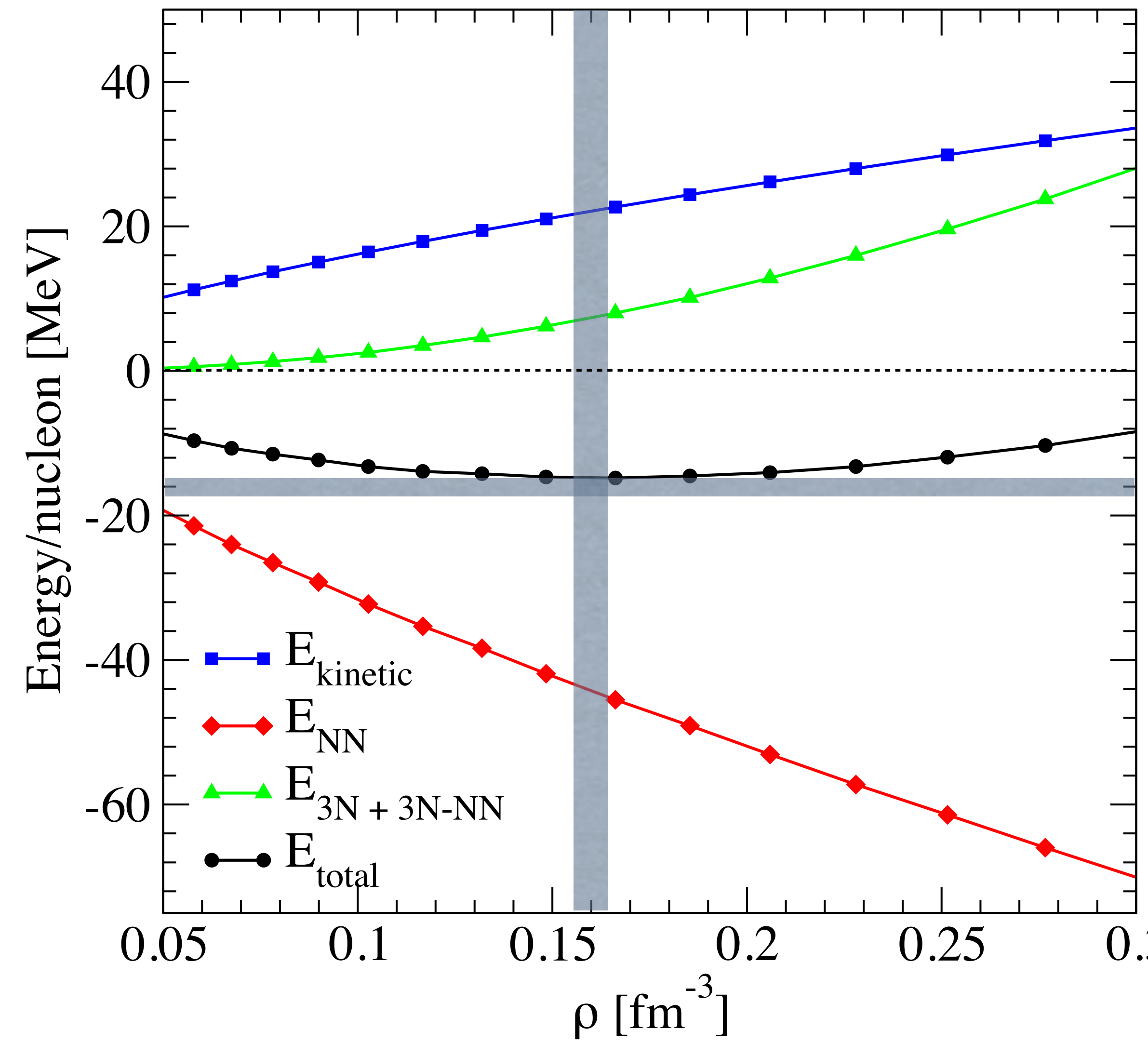

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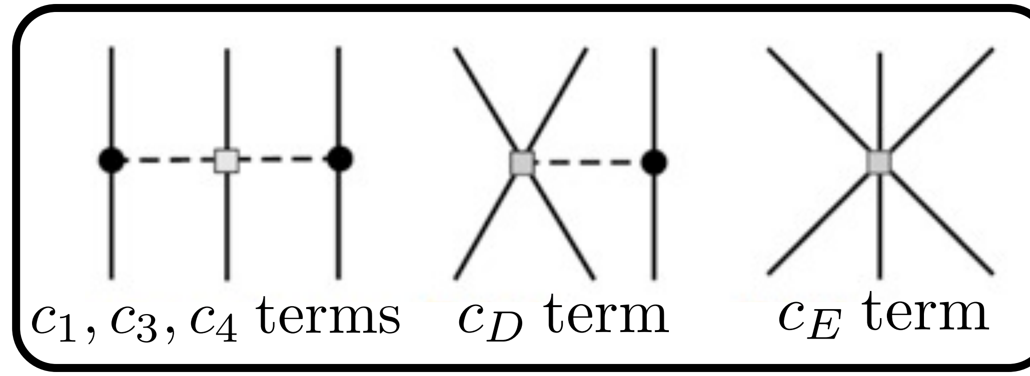

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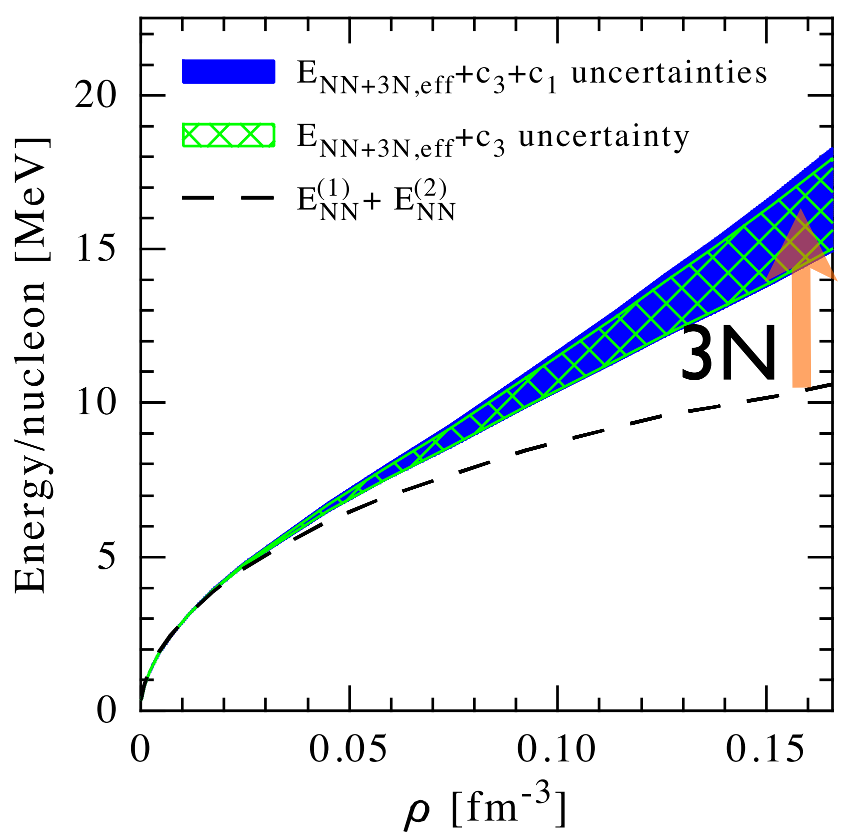

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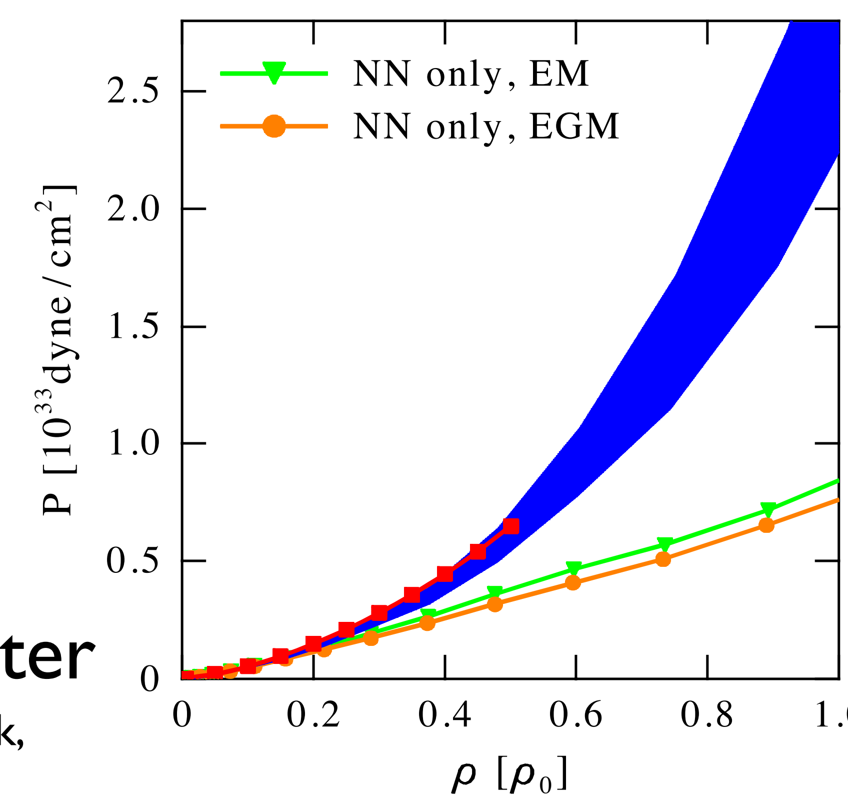
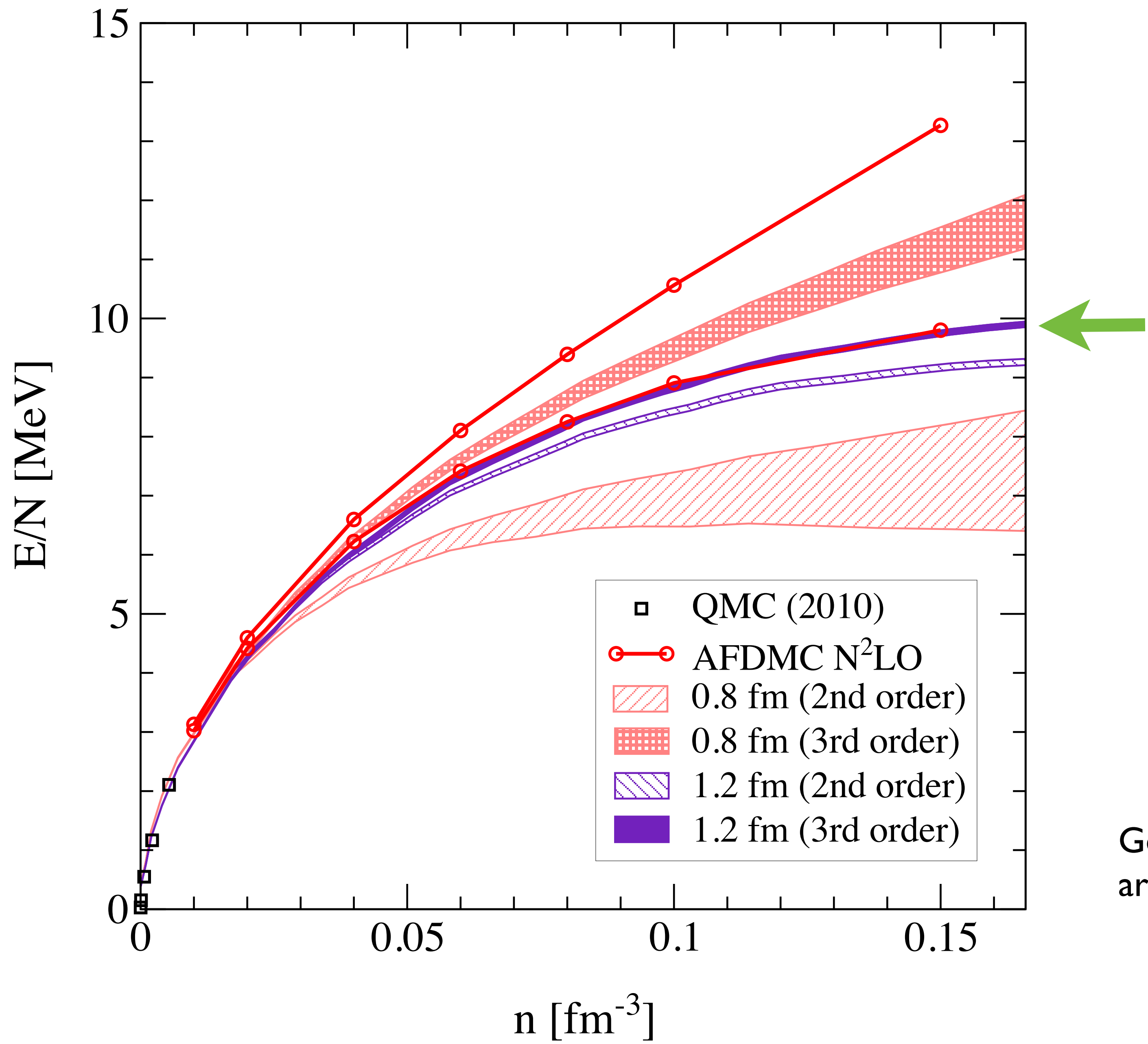


neutron star matter

KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

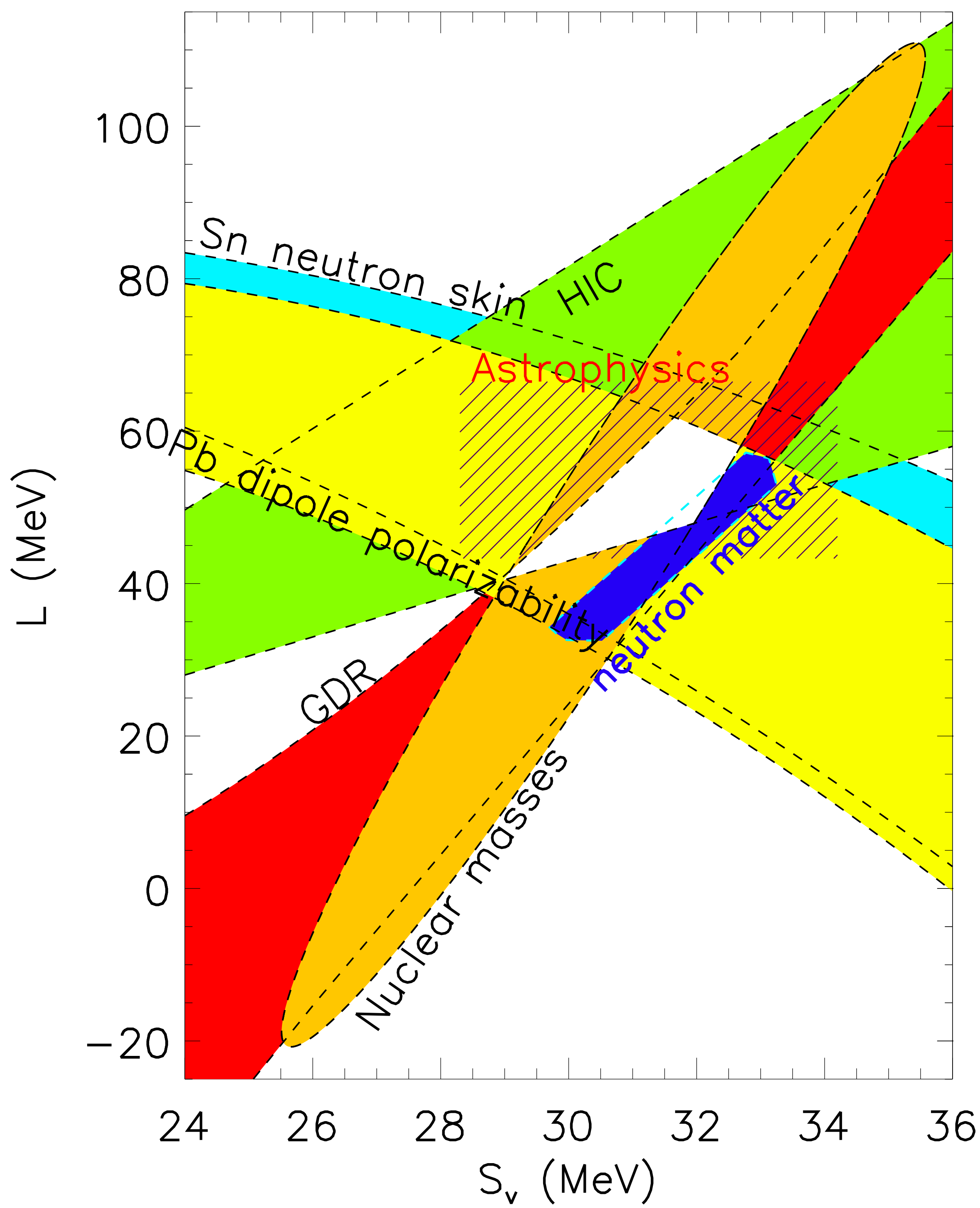
Non-perturbative Quantum Monte Carlo validation of perturbative calculations



Gezerlis, Tews, Epelbaum, Gandolfi, KH, Nogga, Schwenk
arXiv: 1303:6243

- first QMC calculations based on chiral EFT forces (regulator ranges: 0.8-1.2 fm)
- perfect agreement for soft interactions

Symmetry energy constraints



extend EOS to finite proton fractions x

and extract symmetry energy parameters

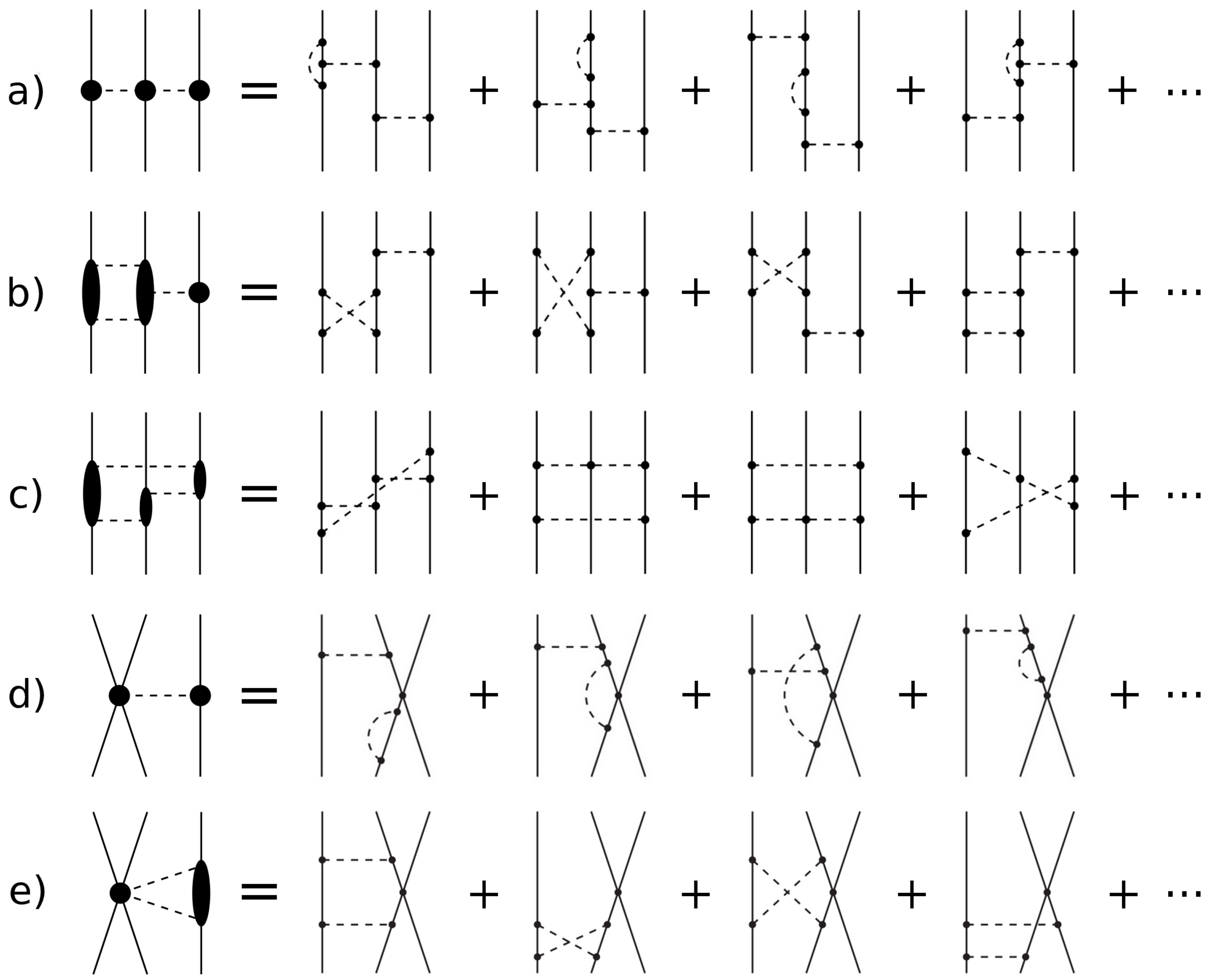
$$S_v = \left. \frac{\partial^2 E/N}{\partial^2 x} \right|_{\rho=\rho_0, x=1/2}$$

$$L = \frac{3}{8} \left. \frac{\partial^3 E/N}{\partial\rho\partial^2 x} \right|_{\rho=\rho_0, x=1/2}$$

KH, Lattimer, Pethick and Schwenk, arXiv:1303.4662

symmetry energy parameters consistent with other constraints

3N interactions at N3LO



2π exchange

2π-1π exchange

pion ring (involved)

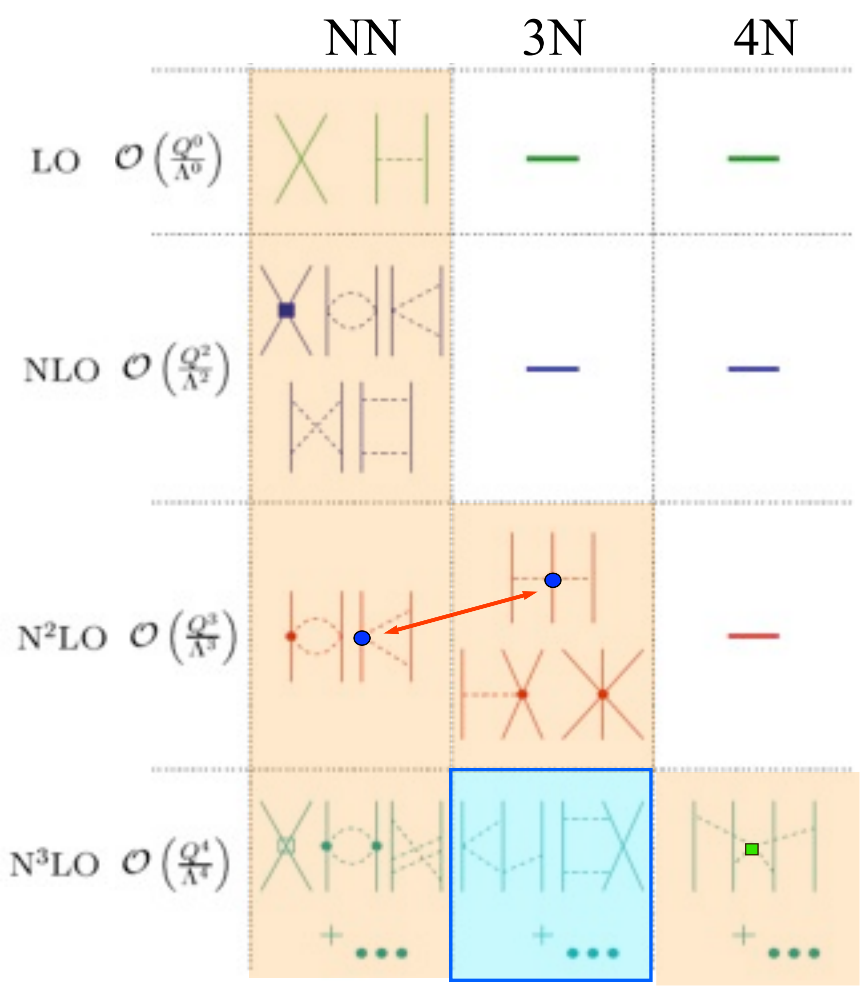
1π-exchange contact
(vanishes)

2π-exchange contact
(2-body-contact C_T)

Bernard et. al (2007, 2011)

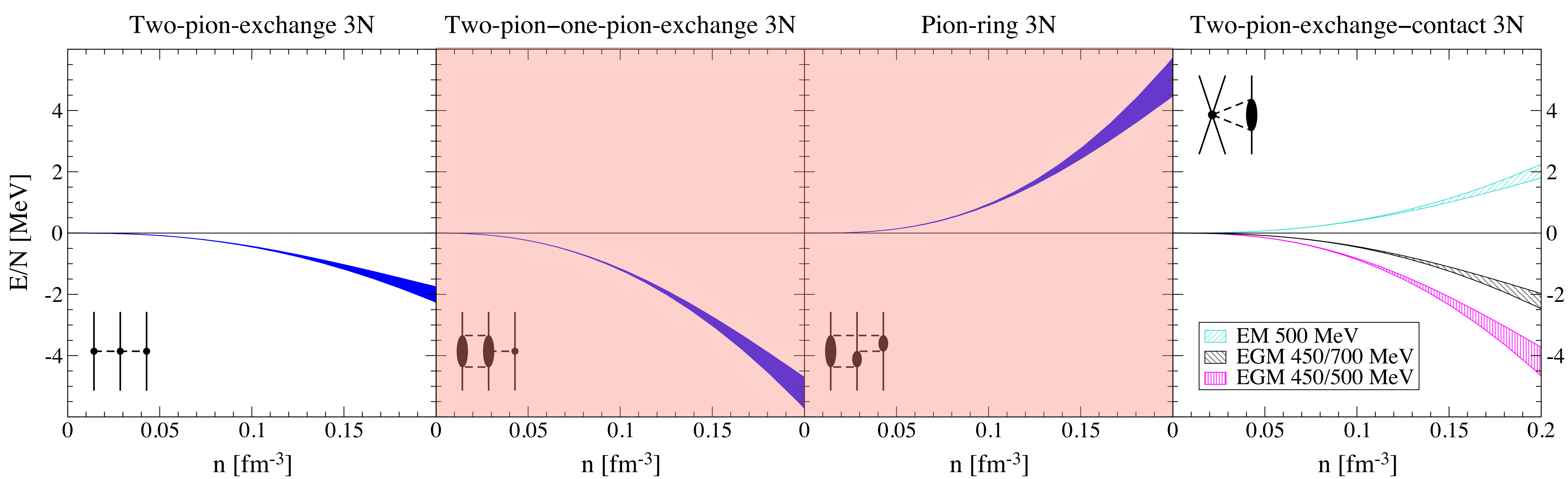
relativistic corrections
(2-body-contacts C_T, C_S)

Contributions of many-body forces at N^3LO



- study chiral power counting in nuclear systems
- first calculations of N^3LO 3NF and 4NF contributions to EOS of neutron matter
- found unnaturally large 3NF contributions, comparable to size of N^2LO contributions
- 4NF contributions of natural size

Tews, Krueger, KH, Schwenk
PRL 110, 032504 (2013)



Calculation of three-body forces at N^3LO

Low
Energy
Nuclear
Physics
International
Collaboration

J. Golak, R. Skibinski,
K. Tolponicki, H. Witala

RUB — E. Epelbaum, H. Krebs

JÜLICH FORSCHUNGSZENTRUM — A. Nogga

THE OHIO STATE UNIVERSITY — R. Furnstahl

TECHNISCHE UNIVERSITÄT DARMSTADT — S. Binder, A. Calci, K. Hebeler,
J. Langhammer, R. Roth

STATE — P. Maris, J. Vary

Kyutech — H. Kamada

Goal

Calculate matrix elements of 3NF in a partial-wave decomposed form which is suitable for different few- and many-body frameworks

---

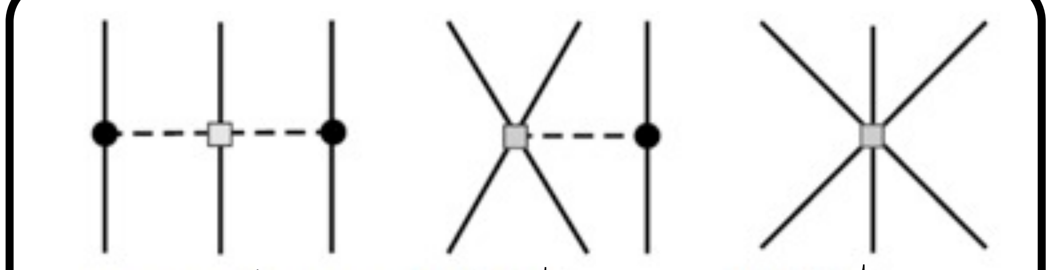
Challenge

Due to the large number of matrix elements, the calculation is extremely expensive.

Strategy

Develop an efficient code which allows to treat arbitrary local 3N interactions.
(Krebs and Hebeler)

RG evolution of 3N interactions



- So far (in momentum basis):

intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:

$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.464\,\mathrm{fm}$$

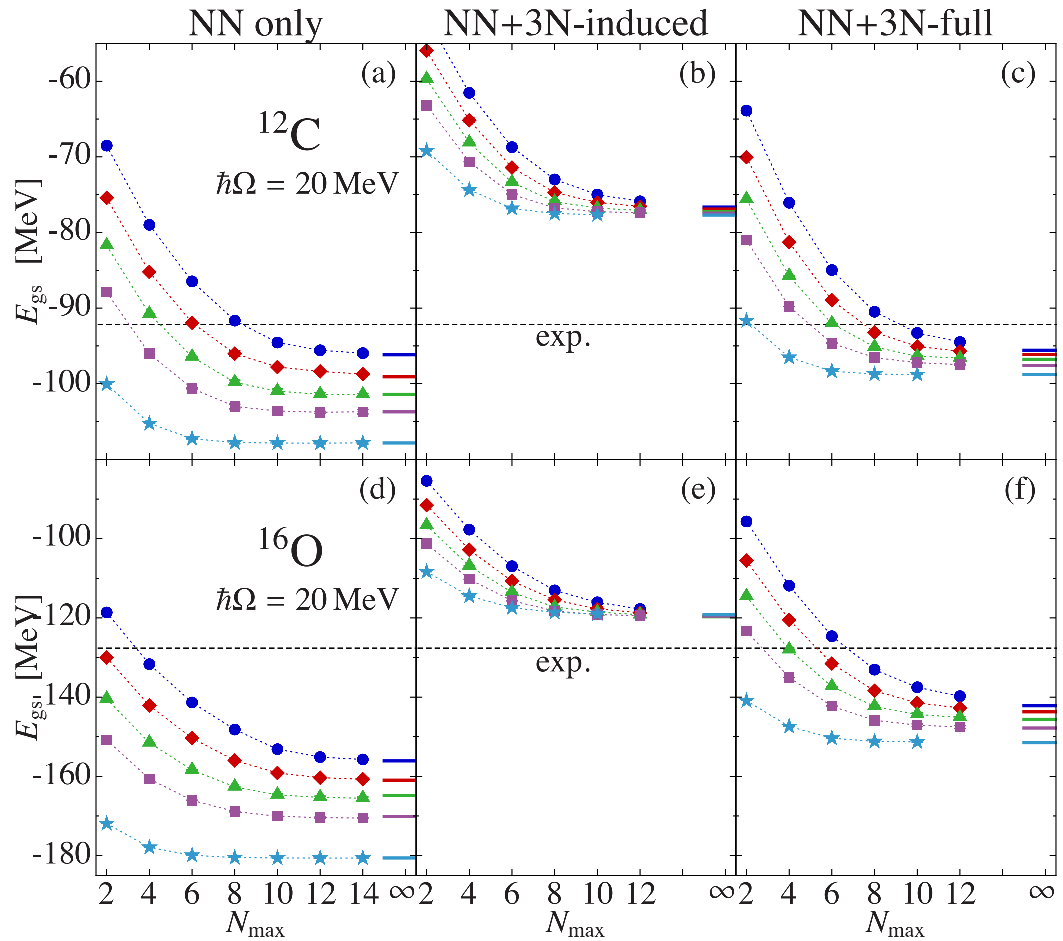
$\longrightarrow$ coupling constants of natural size

- in neutron matter contributions from c_D, c_E and c_4 terms vanish
- long-range 2π contributions assumed to be invariant under RG evolution

---

- Ideal case: evolve 3NF consistently with NN interactions within the SRG
  - has been achieved in oscillator basis (Jurgenson, Roth)
  - promising results in very light nuclei
  - puzzling effects in heavier nuclei (higher-body forces?)
  - not immediately applicable to infinite systems
  - limitations on $\hbar\Omega$

RG evolution of 3N interactions



Roth et al. PRL 107, 072501 (2011)

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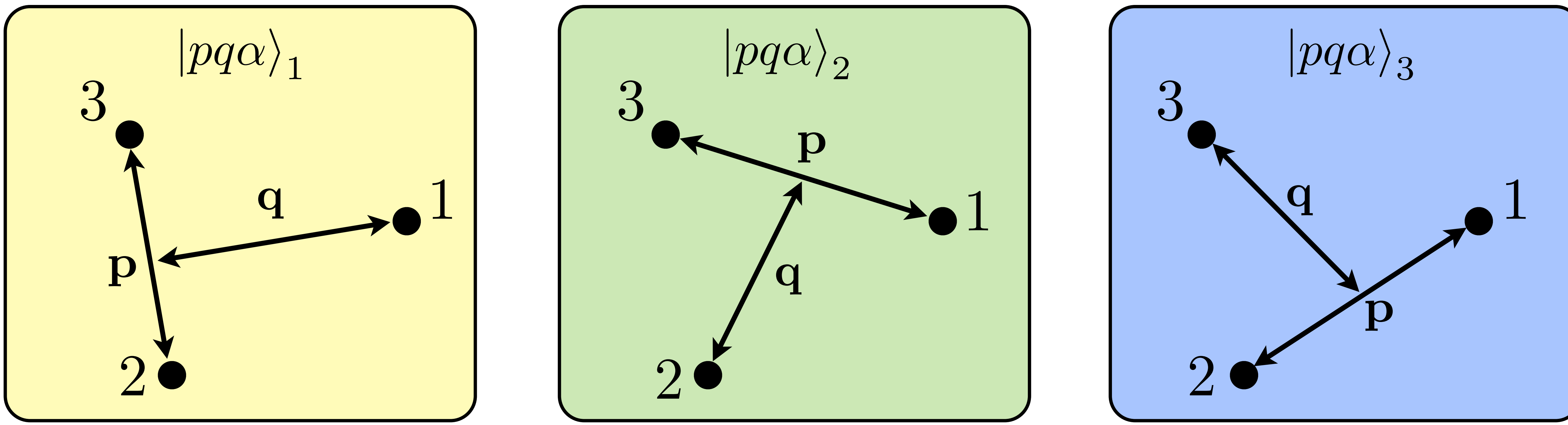
RG evolution of 3N interactions in momentum space

Three-body Faddeev basis:

$$|pq\alpha\rangle_i \equiv |p_i q_i; [(LS)J(ls_i)j]\,\mathcal{J}\mathcal{J}_z(Tt_i)\mathcal{T}\mathcal{T}_z\rangle$$



$$_i\langle pq\alpha|P|p'q'\alpha'\rangle_i = {}_i\langle pq\alpha|p'q'\alpha'\rangle_j$$

Faddeev bound-state equation:

$$|\psi_i\rangle = G_0\left[2t_iP + (1 + t_iG_0)\,V_{3N}^i(1+2P)\right]|\psi_i\rangle$$

SRG flow equations of NN and 3N forces in momentum basis

$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \eta_s = [T_{\rm rel}, H_s]$$

$$H = T + V_{12} + V_{13} + V_{23} + V_{123}$$

- spectators correspond to delta functions, matrix representation of H_s ill-defined
- solution: explicit separation of NN and 3N flow equations

$$\begin{aligned}
\frac{dV_{ij}}{ds} &= \left[\left[T_{ij}, V_{ij}\right], T_{ij} + V_{ij}\right], \\
\frac{dV_{123}}{ds} &= \left[\left[T_{12}, V_{12}\right], V_{13} + V_{23} + V_{123}\right] \\
&+ \left[\left[T_{13}, V_{13}\right], V_{12} + V_{23} + V_{123}\right] \\
&+ \left[\left[T_{23}, V_{23}\right], V_{12} + V_{13} + V_{123}\right] \\
&+ \left[\left[T_{\rm rel}, V_{123}\right], H_s\right]
\end{aligned}$$

- only connected terms remain in $\frac{dV_{123}}{ds}$, 'dangerous' delta functions cancel

see Bogner, Furnstahl, Perry PRC 75, 061001(R) (2007)

SRG evolution in momentum space

- evolve the antisymmetrized 3N interaction

special thanks to
J. Golak, R. Skibinski, K. Topolnicki

$$\overline{V}_{123} = {}_i\langle pq\alpha|\,(1+P_{123}+P_{132})V^{(i)}_{123}(1+P_{123}+P_{132})\,|p'q'\alpha'\rangle_i$$

- embed NN interaction in 3N basis:

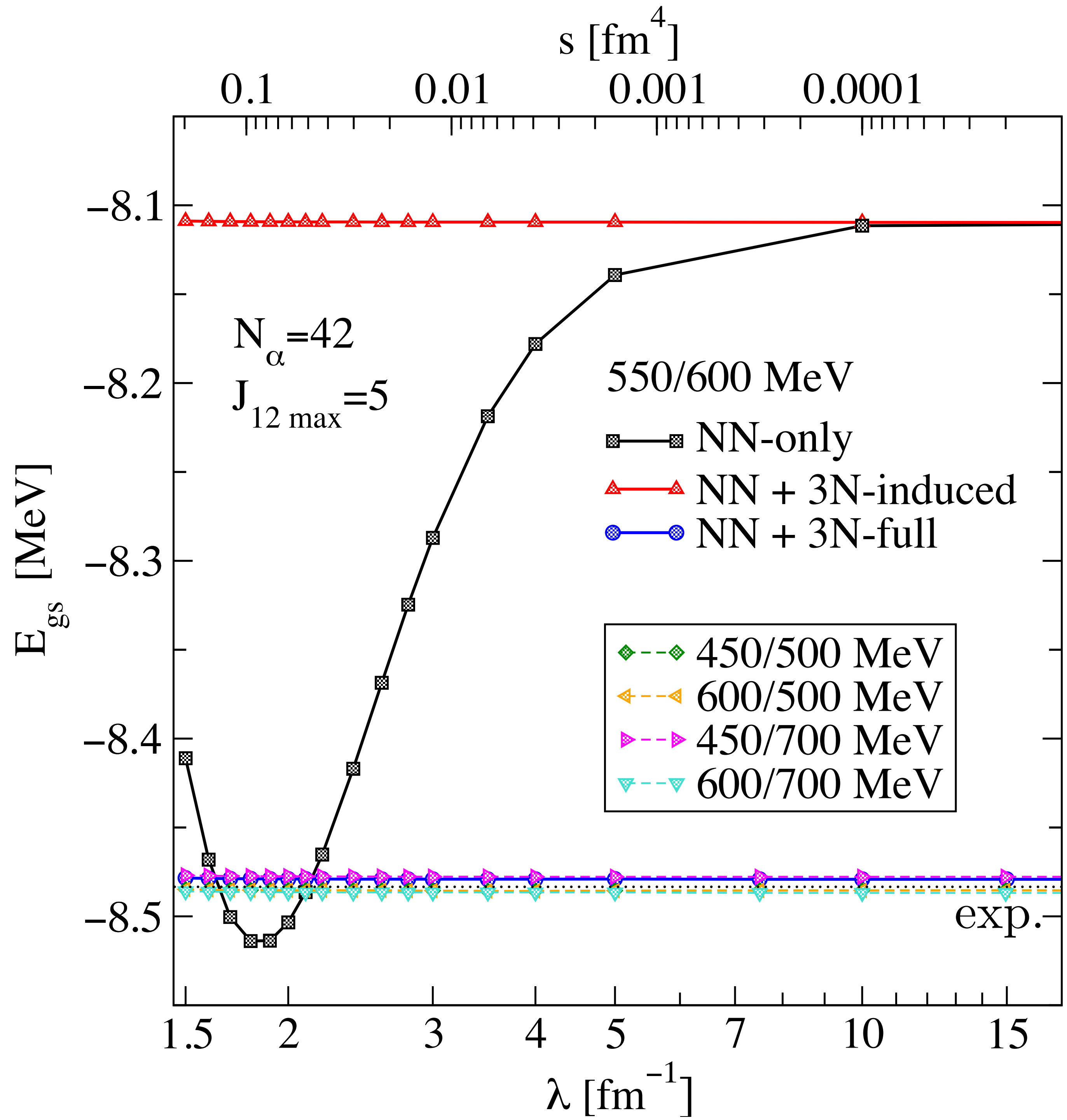
$$V_{13} = P_{123}V_{12}P_{132}, \quad V_{23} = P_{132}V_{12}P_{123}$$

with $\;{}_3\langle pq\alpha|V_{12}|p'q'\alpha'\rangle_3 = \langle p\tilde{\alpha}|V_{\rm NN}|p'\tilde{\alpha}'\rangle\,\delta(q-q')/q^2$

- use $P_{123}\overline{V}_{123} = P_{132}\overline{V}_{123} = \overline{V}_{123}$

$$\Rightarrow \quad \begin{aligned} d\overline{V}_{123}/ds &= C_1(s,T,V_{\rm NN},P) \\ &\quad + C_2(s,T,V_{\rm NN},\overline{V}_{123},P) \\ &\quad + C_3(s,T,\overline{V}_{123}) \end{aligned}$$

SRG evolution of 3N interactions in momentum space: Results for the Triton



Hebeler PRC(R) 85, 021002 (2012)

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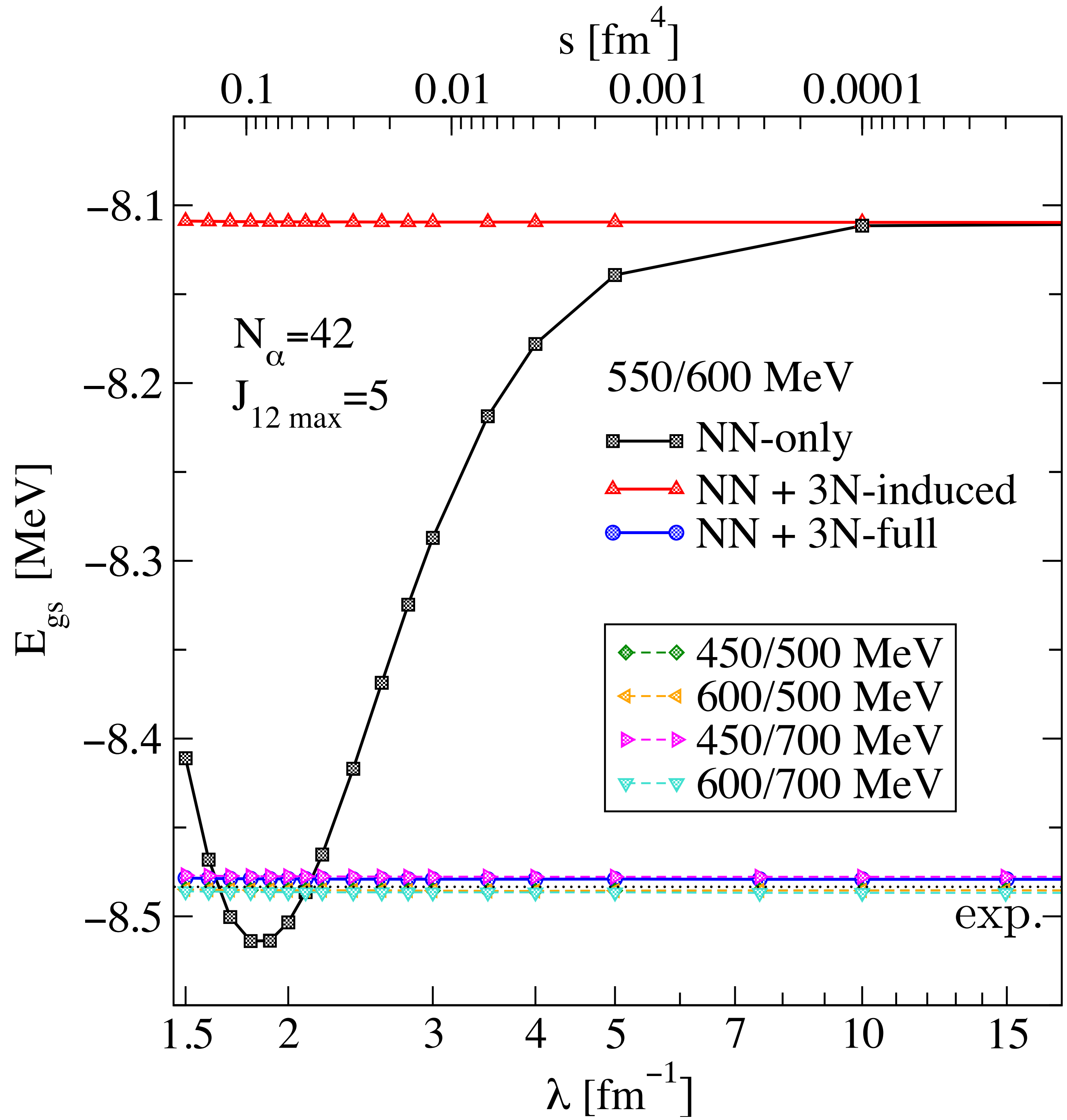


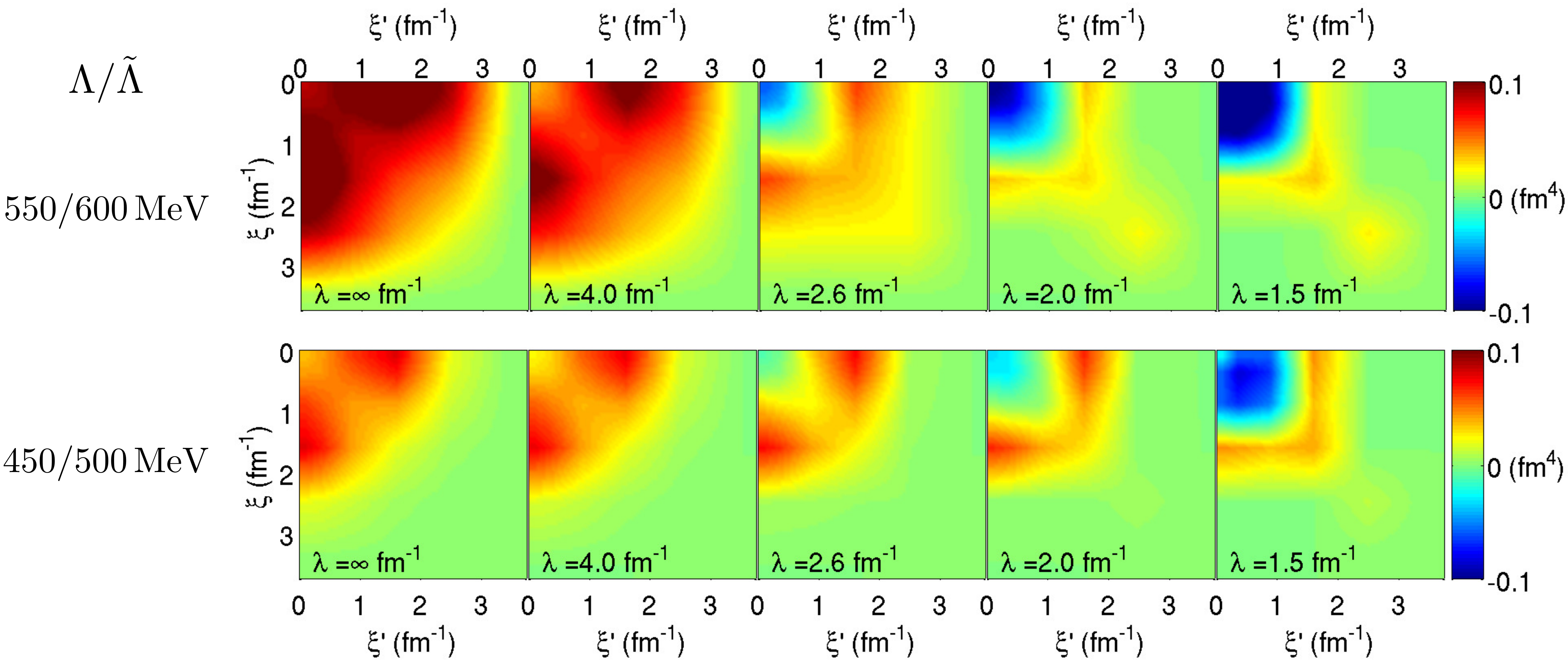
Hebeler PRC(R) 85, 021002 (2012)

It works:

Invariance of $E_{\rm gs}^{^3H}$ within $\leq 1\rm keV$ for consistent chiral interactions at $\rm N^2LO$

Decoupling in 3NF matrix elements

$$\theta = \frac{\pi}{12} \qquad \mathcal{T} = \mathcal{J} = \frac{1}{2}$$



KH, PRC(R) 85, 021002 (2012)

see also KH, Furnstahl, PRC(R) 87, 031302 (2013)

hyperradius: $\xi^2 = p^2 + \frac{3}{4}q^2$ hyperangle: $\tan\theta = \frac{2\,p}{\sqrt{3}\,q}$

same decoupling patterns like in NN interactions

3NF evolution in momentum basis: Current developments and applications

- application to infinite systems
  - equation of state
  - systematic study of induced many-body contributions, scaling behavior
  - include initial N3LO 3N interactions

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  (no core shell model, coupled cluster, valence shell model, DFT, self-consistent Green's function, IM-SRG)

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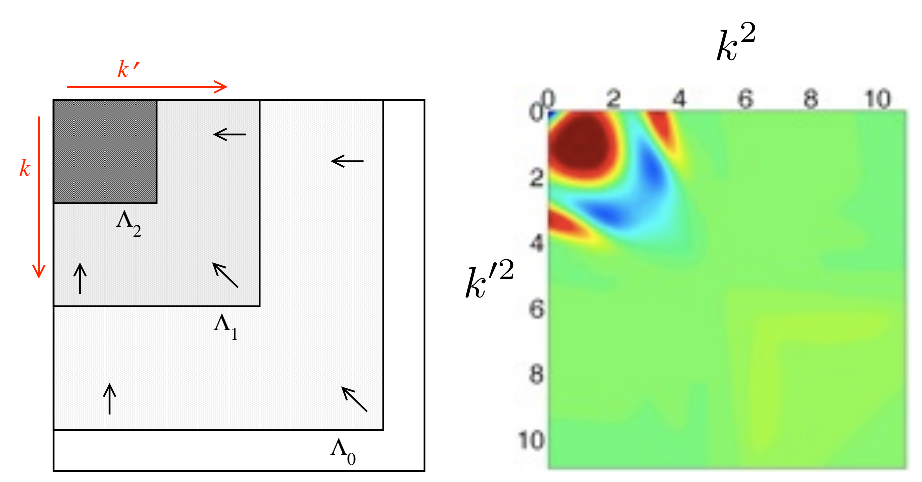
  - improved efficiency of evolution
  - suppression of many-body forces?



Anderson et al. , PRC 77, 037001 (2008)

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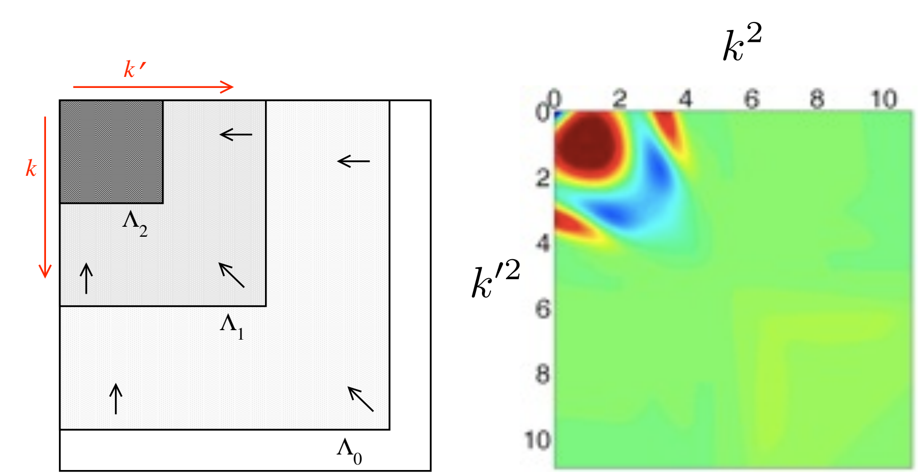


Anderson et al. , PRC 77, 037001 (2008)

- explicit calculation of unitary transformation
  - RG evolution of operators
  - study of correlations in nuclear systems ⟶ factorization

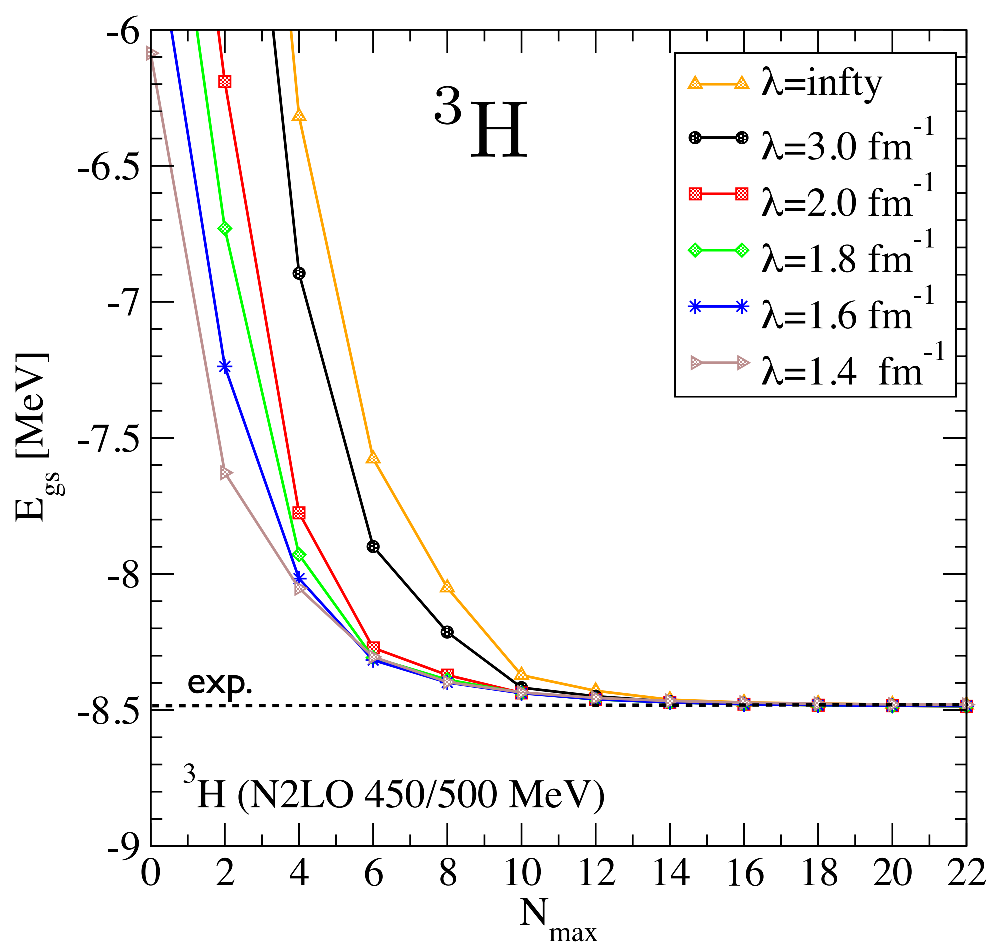
Application of evolved 3N forces to finite nuclei

1.) evolve NN and 3N interactions in all relevant 3N partial waves, (rather expensive, but only needs to be done once for a given interaction)

2.) transform matrix elements to HO basis for different $N_{\max}$ and $\hbar\Omega$ (fast)

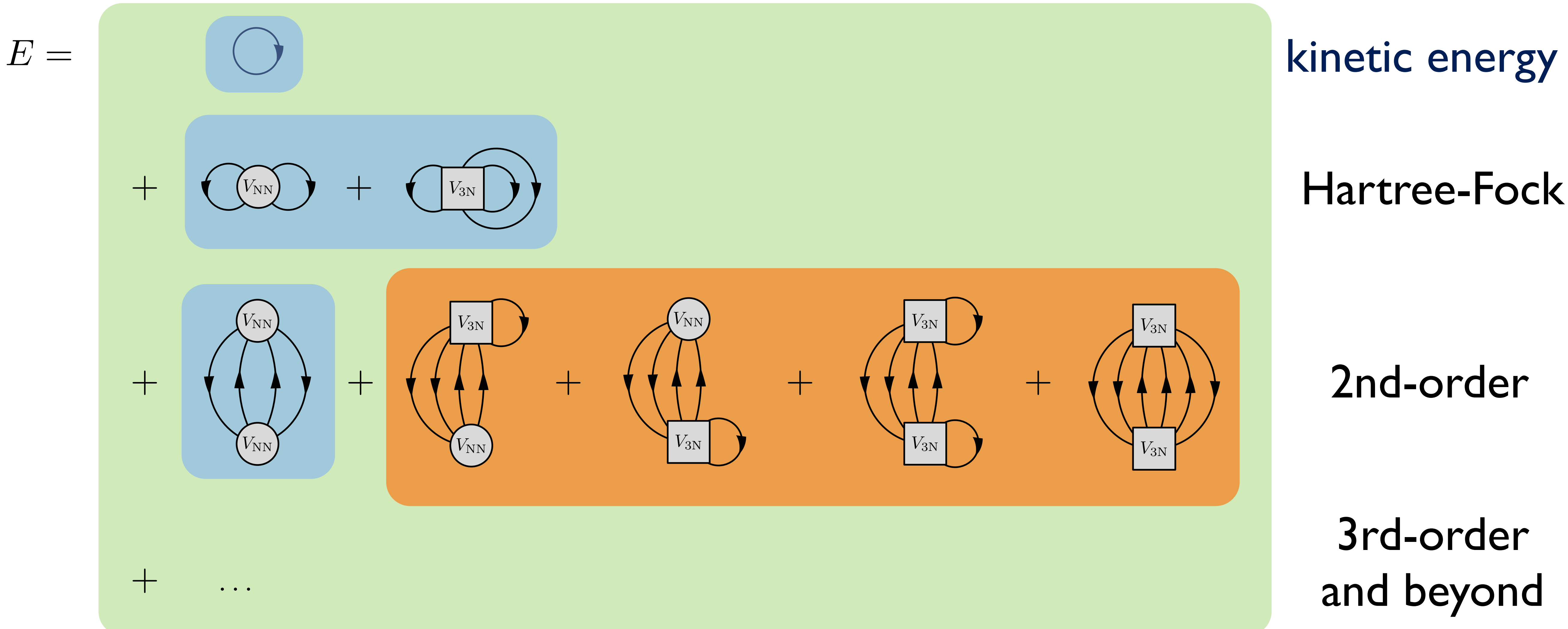
$$\langle p'q'\alpha'|\, V_{123}\, |pq\alpha\rangle \rightarrow \langle N'_{12}L'_{12}J'_{12}n'_3l'_3j'_3\ldots|\, V_{123}\, |N_{12}L_{12}J_{12}n_3l_3j_3\ldots\rangle$$

3.) anti-symmetrization, transformation to lab-frame (and possibly normal-ordering) of 3NF is performed in many-body frameworks (coupled cluster, no-core/valence shell model, Green's functions, IM-SRG,...)



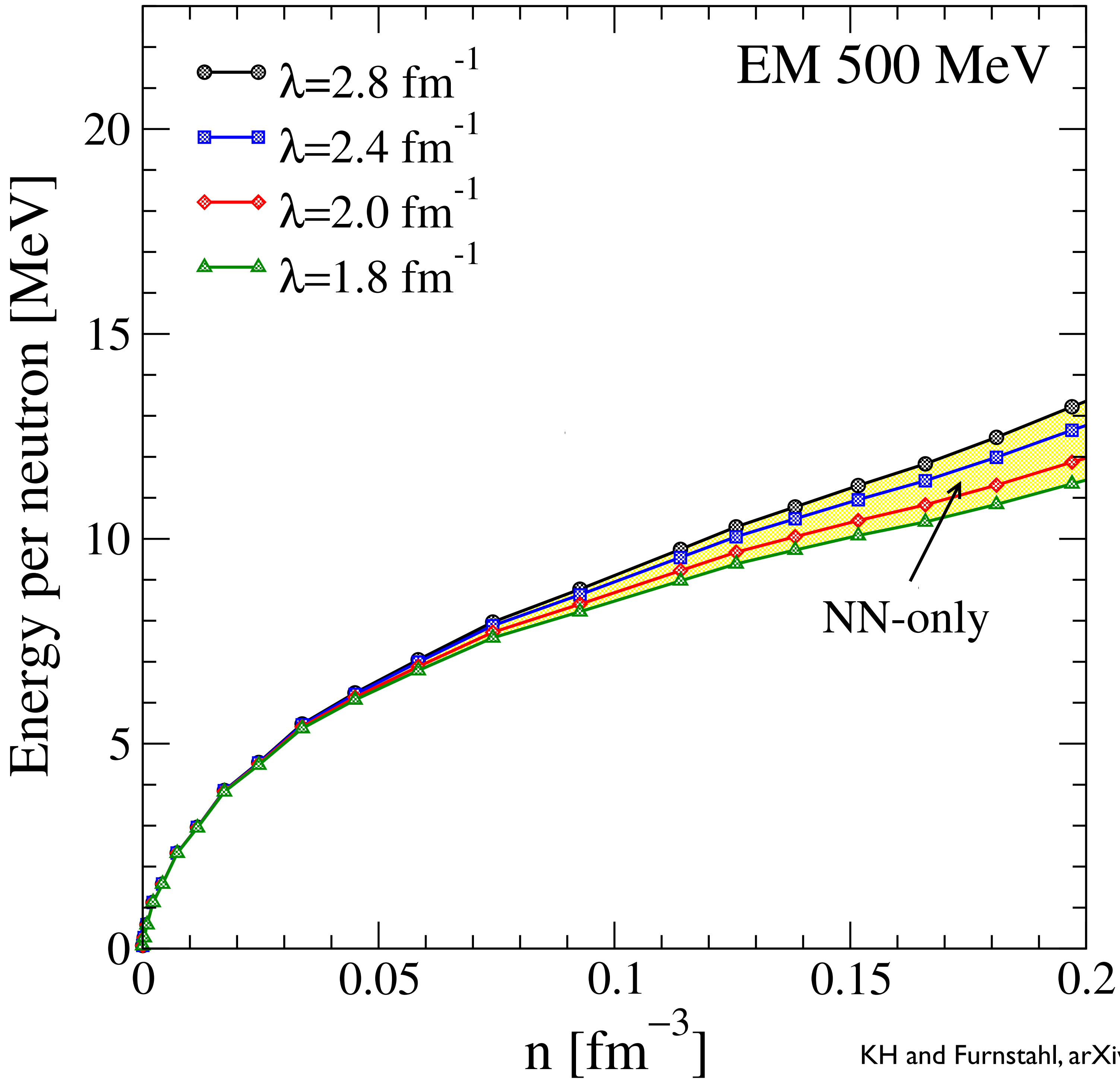
with A. Ekstroem, G. Hagen

Equation of state based on consistently evolved 3NF



- calculate EOS by taking all blue-boxed contributions into account
- in this approximation NN and 3NF contributions factorize

First results for neutron matter



KH and Furnstahl, arXiv: 1301.xxxx

- all partial waves included up to $\mathcal{J} = 9/2$ in SRG evolution and EOS calculation
- consistent 3NF with $c_1 = -0.81\ \mathrm{GeV}^{-1}$ and $c_3 = -3.2\ \mathrm{GeV}^{-1}$

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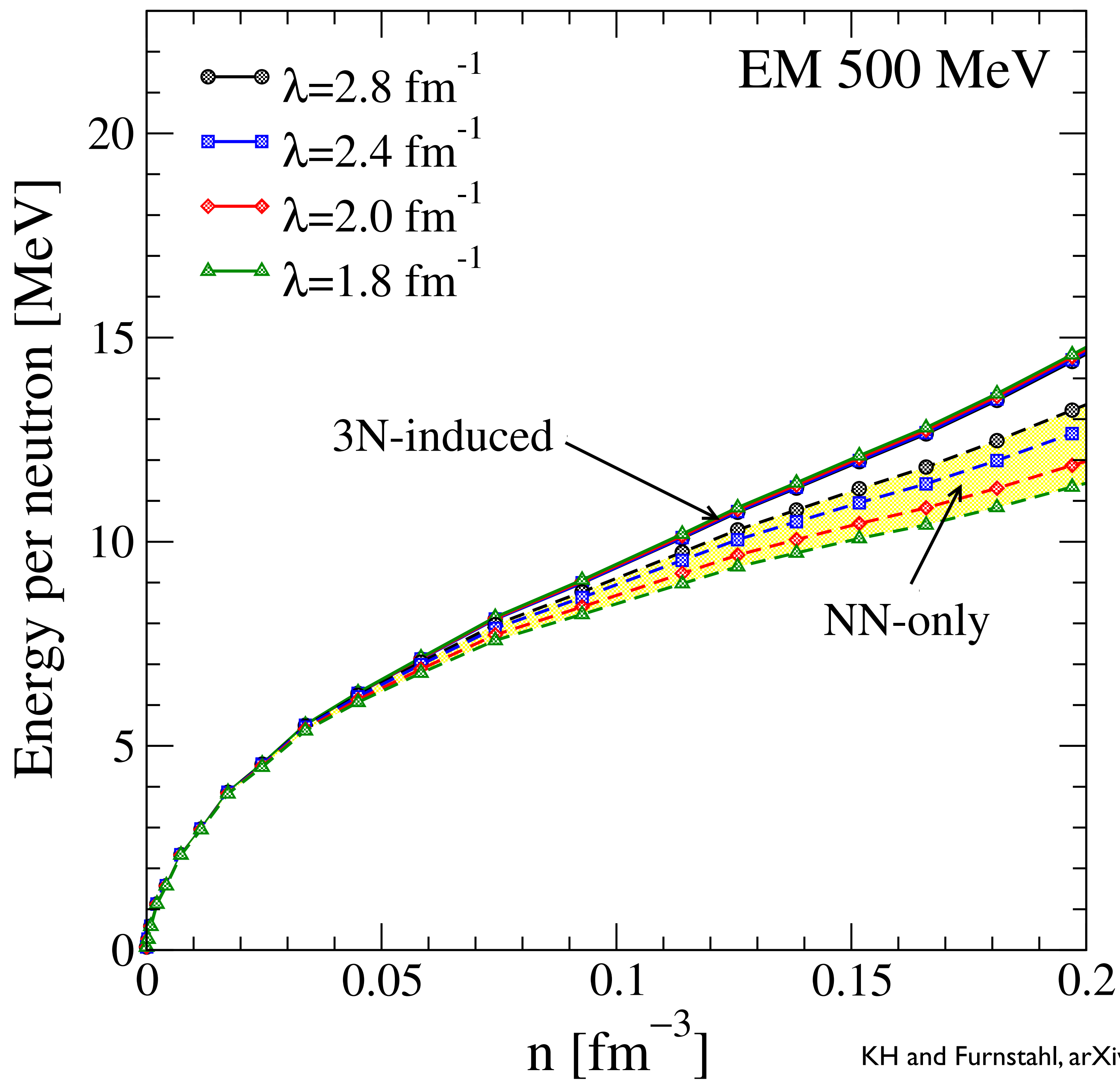

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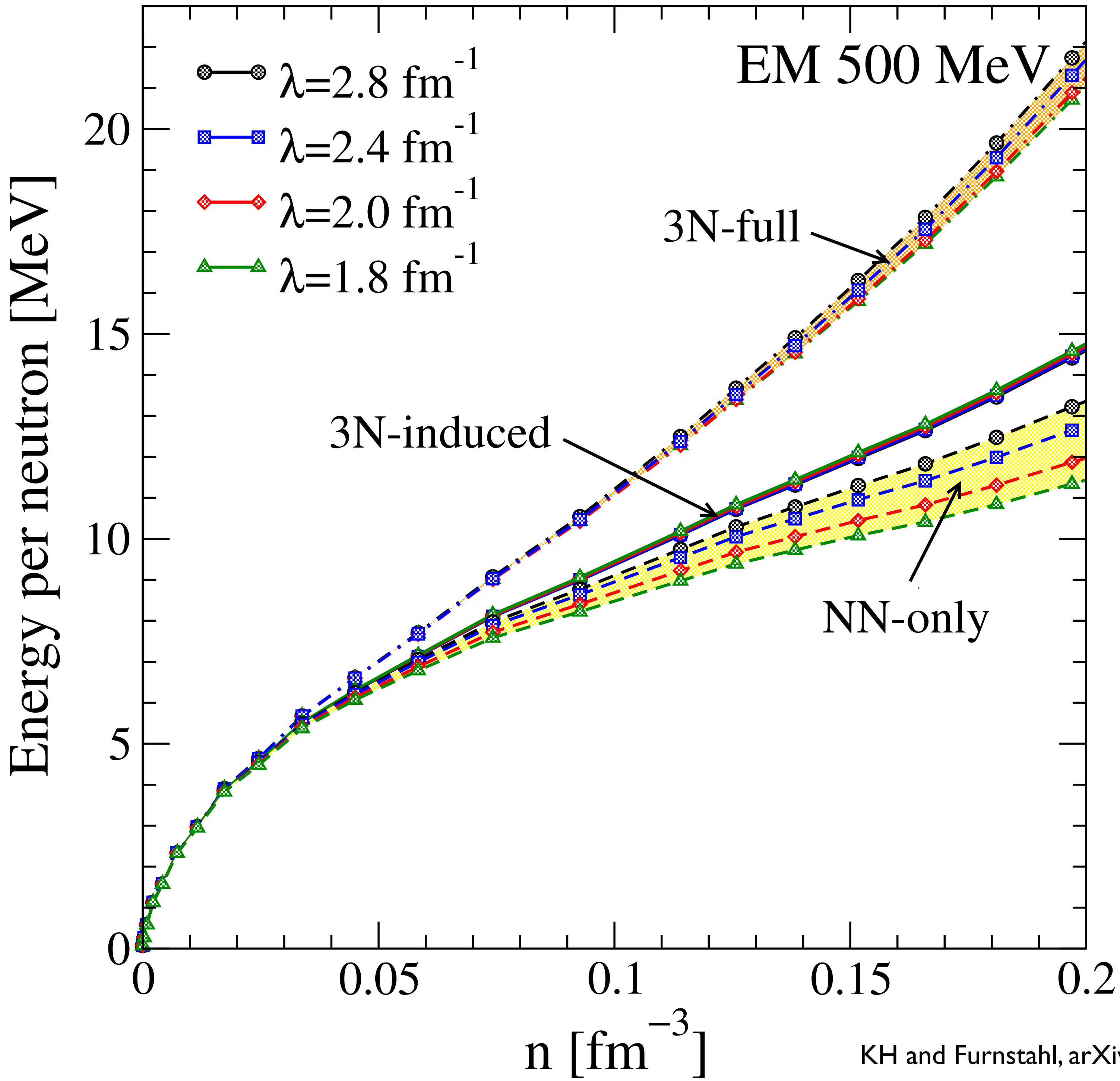

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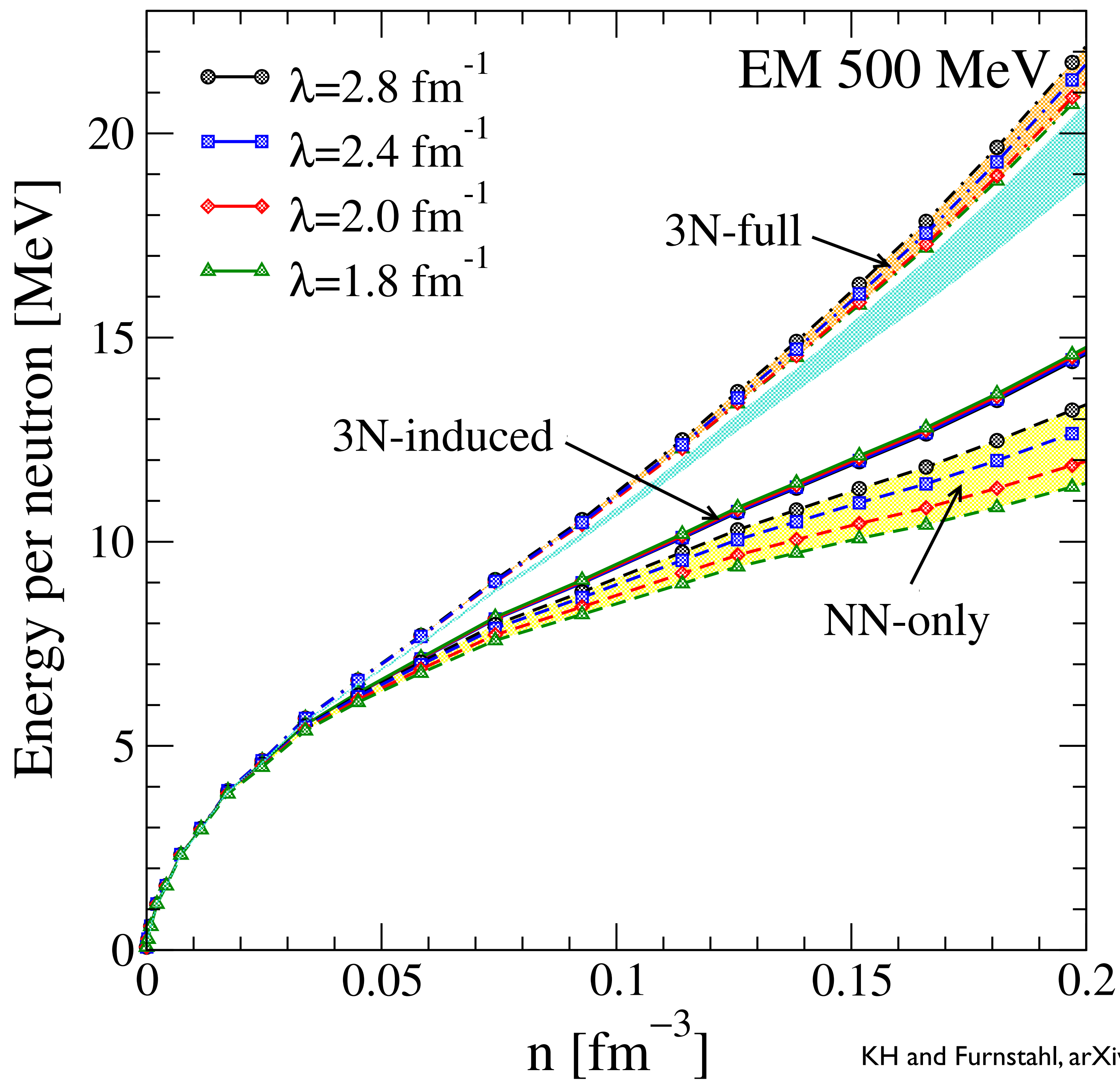

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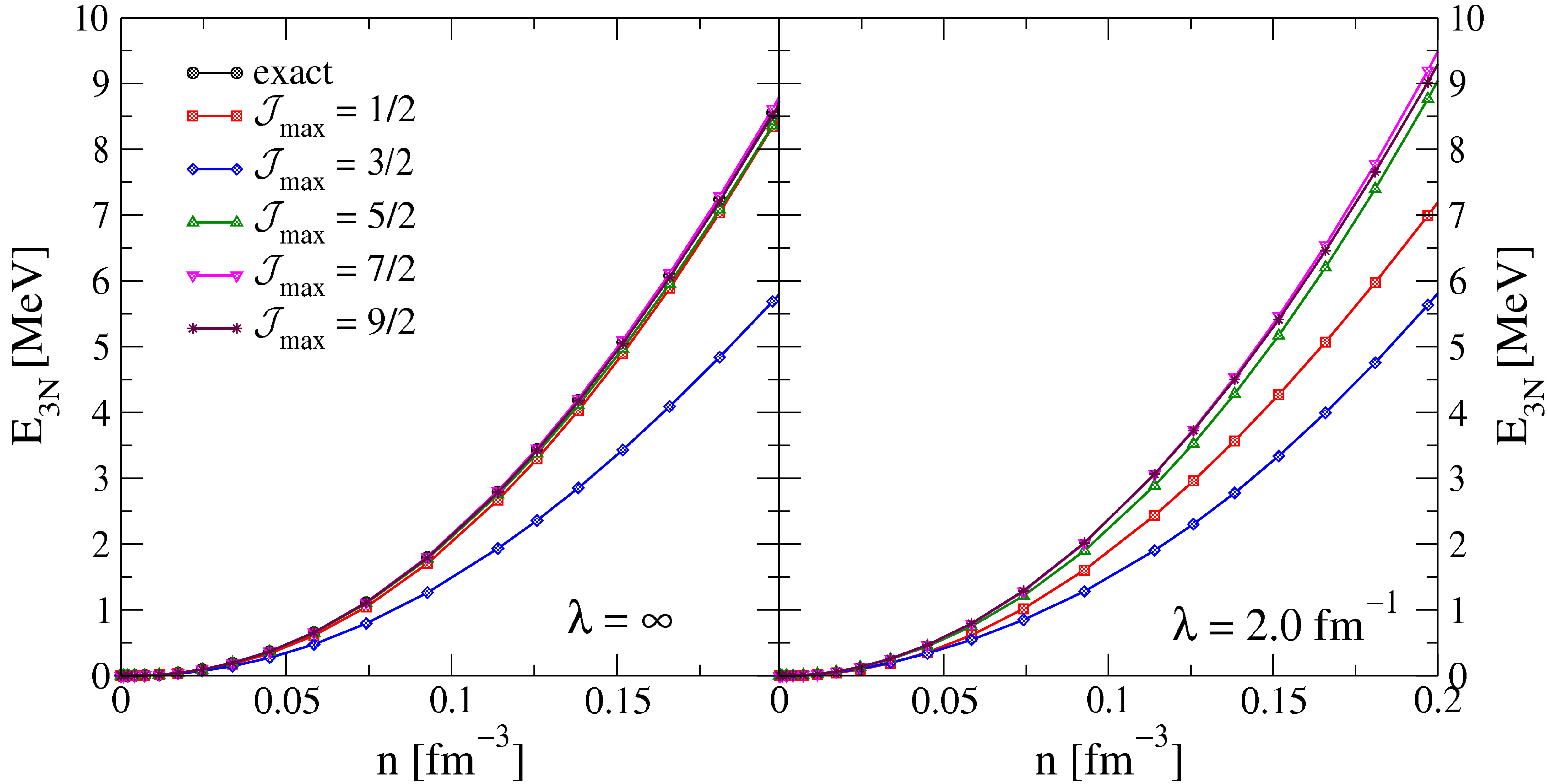
Partial-wave convergence of 3NF contributions



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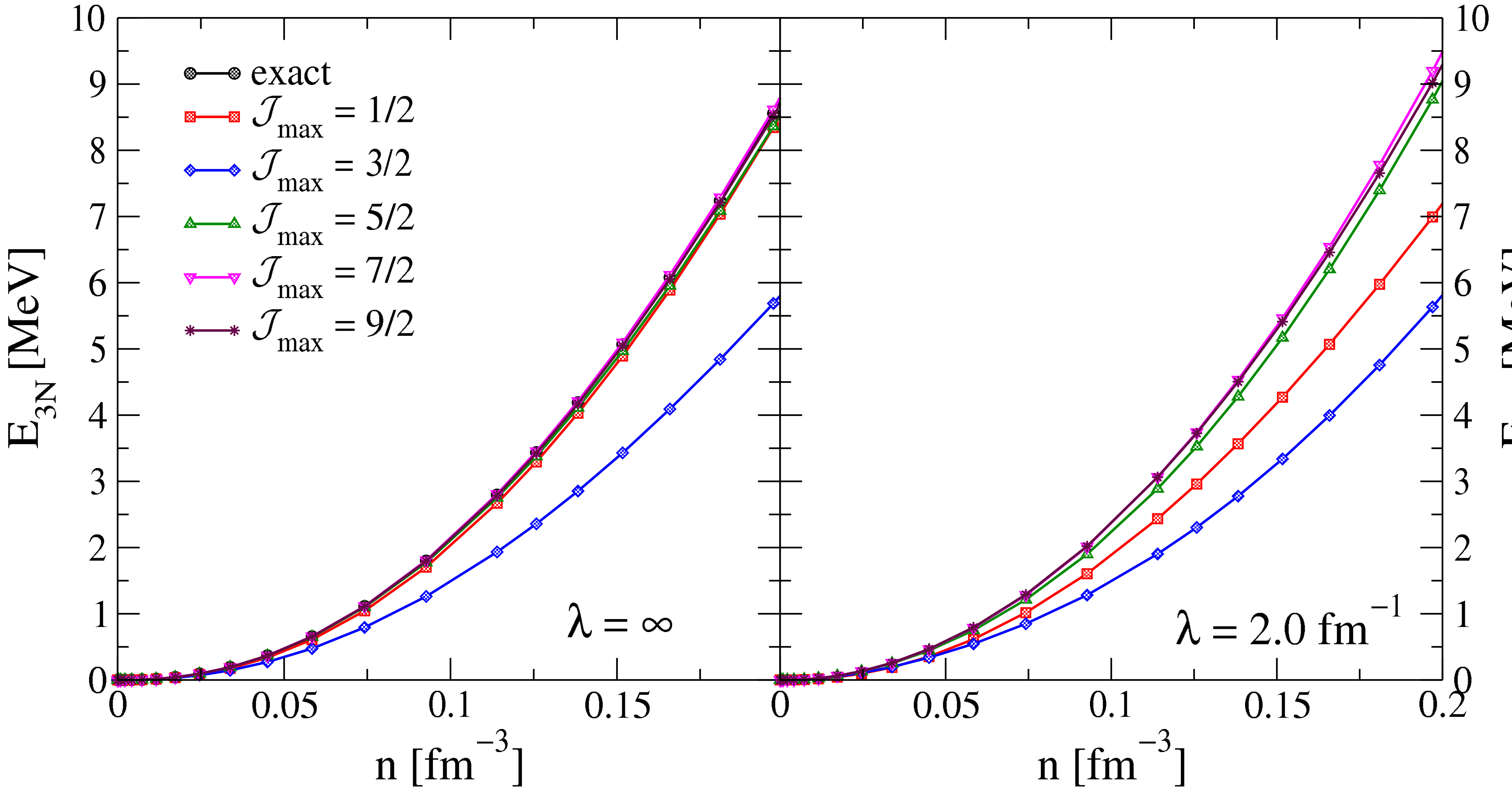
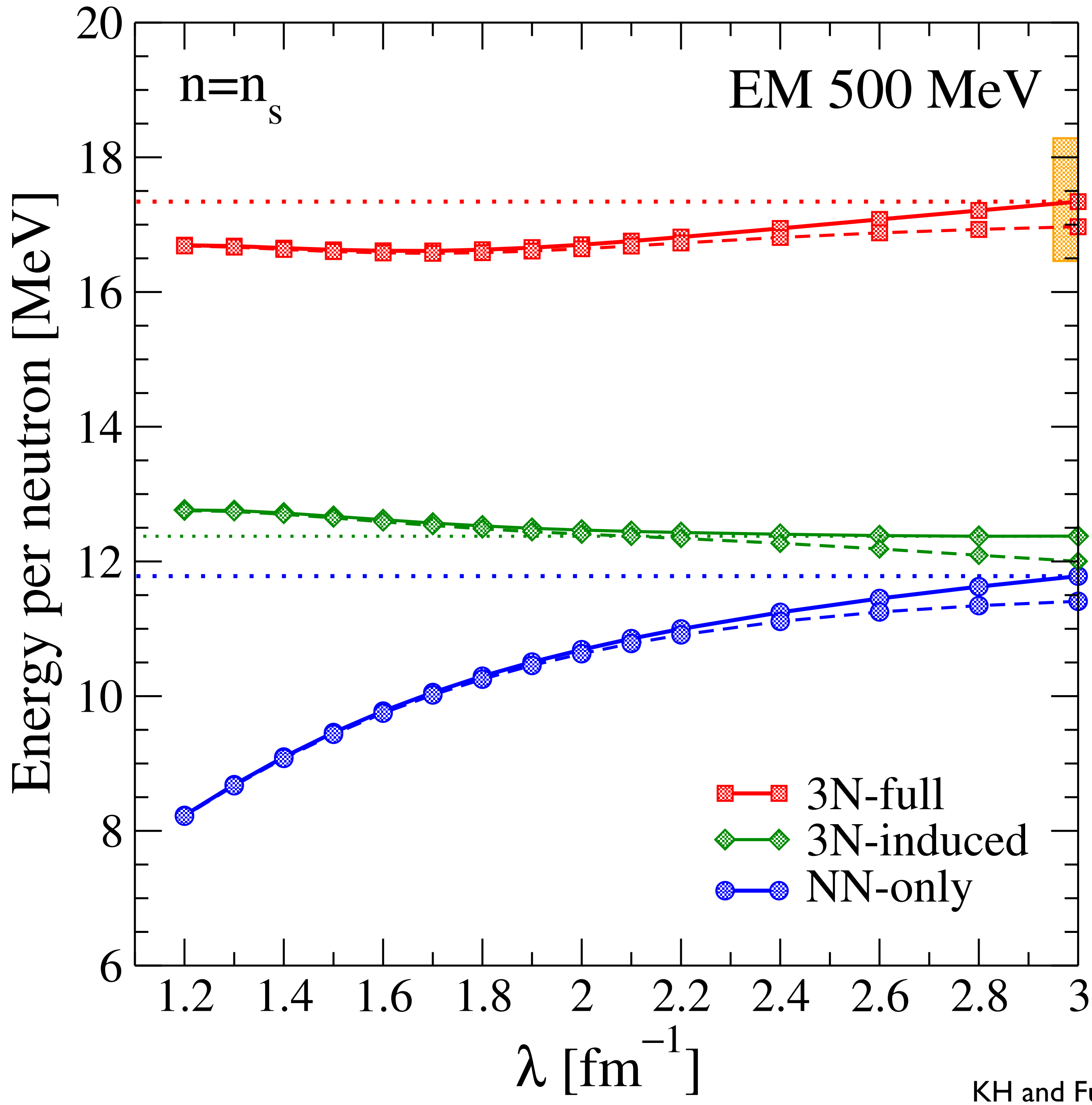


KH and Furnstahl, arXiv: 1301/02.xxxx

- E_{3N} agrees within 0.4 % with the exact result at saturation density
- E_{3N} converged in partial waves at both scales, $\lambda = \infty$ and $\lambda = 2.0\ \mathrm{fm}^{-1}$

Resolution-scale dependence at saturation density

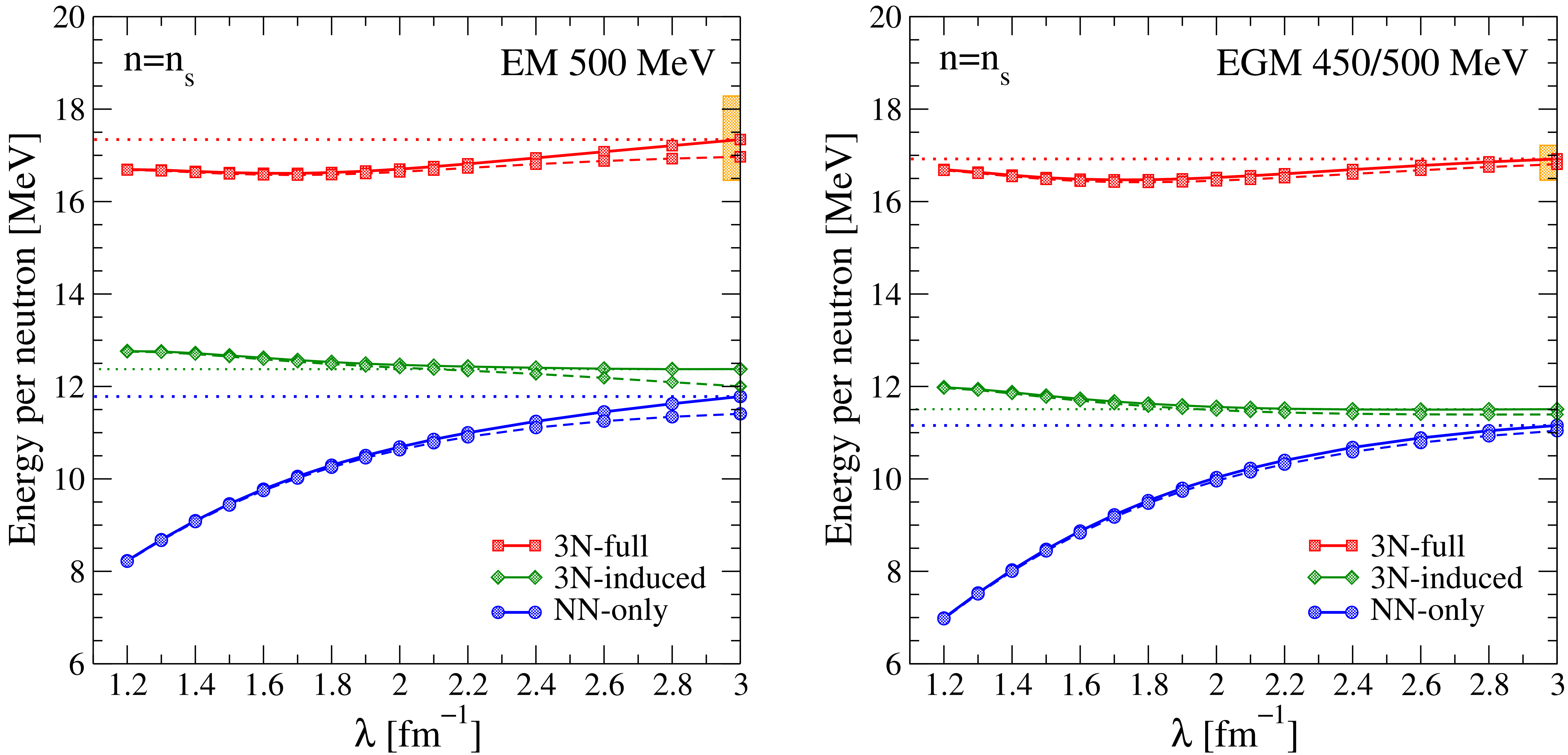


2nd order NN-3N contributions at $\lambda = \infty$

KH and Furnstahl, arXiv: 1301/02.xxxx

- solid lines: NN resummed, dashed lines: 2nd order
- variations: NN-only 3.6 MeV, 3N-induced: 390 keV, 3N-full: 650 keV
- indications for 4N forces at small λ?

Resolution-scale dependence at saturation density



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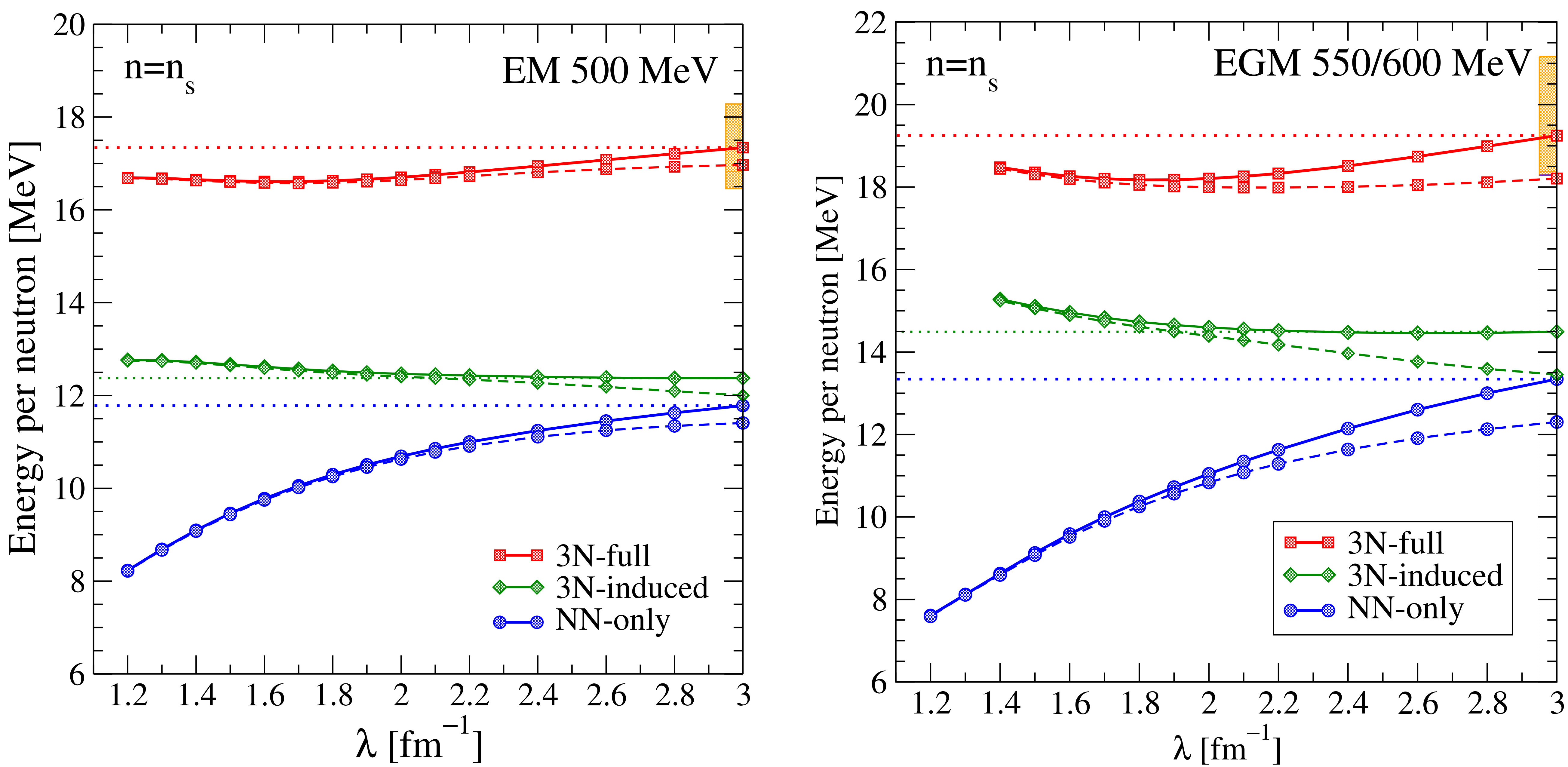

KH and Furnstahl, arXiv: 1301/02.xxxx

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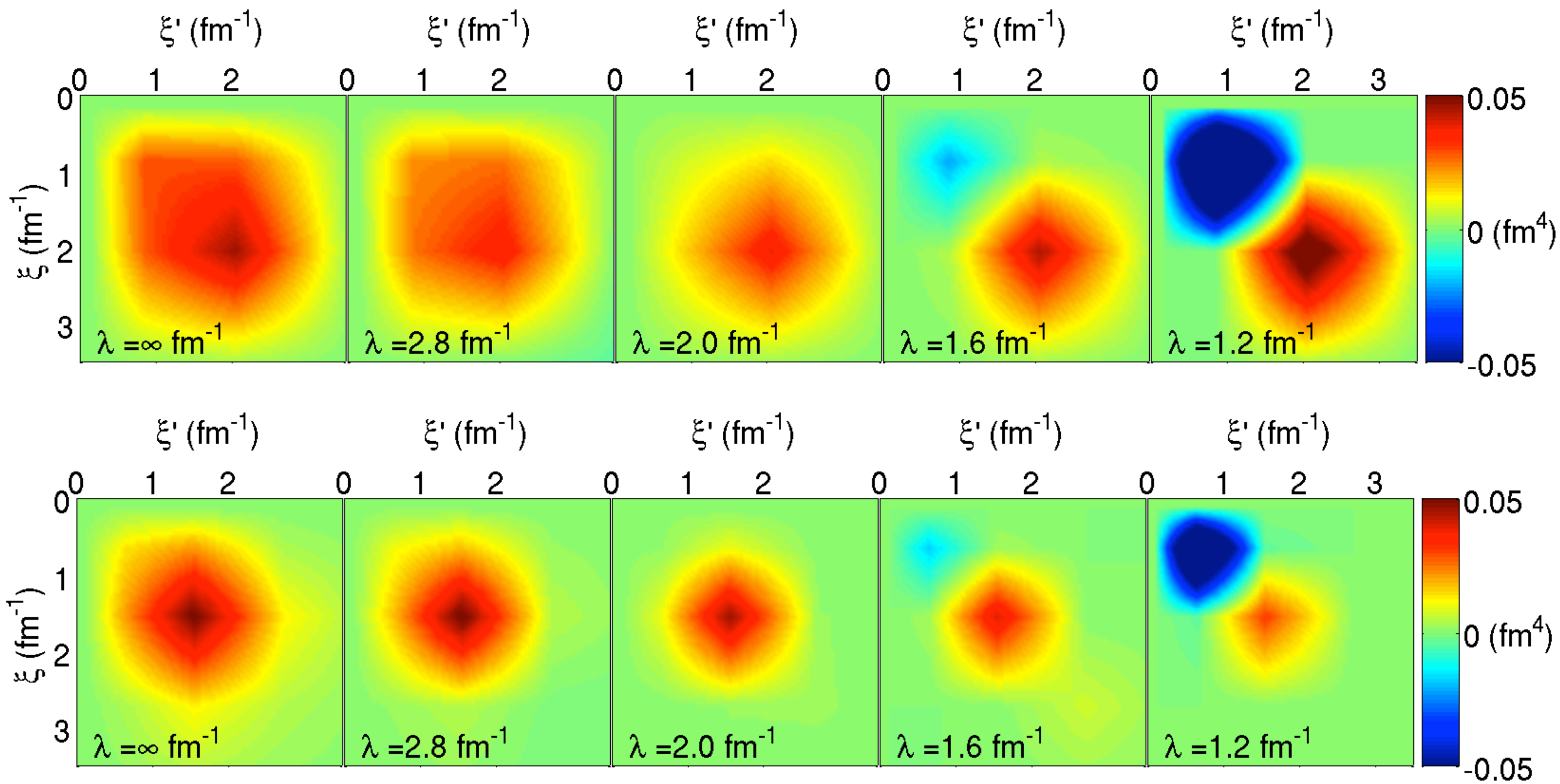
Matrix elements of evolved 3-neutron interactions

$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$\theta = \frac{\pi}{20}$

$\theta = \frac{\pi}{15}$



- strong renormalization effects of long-range two-pion exchange
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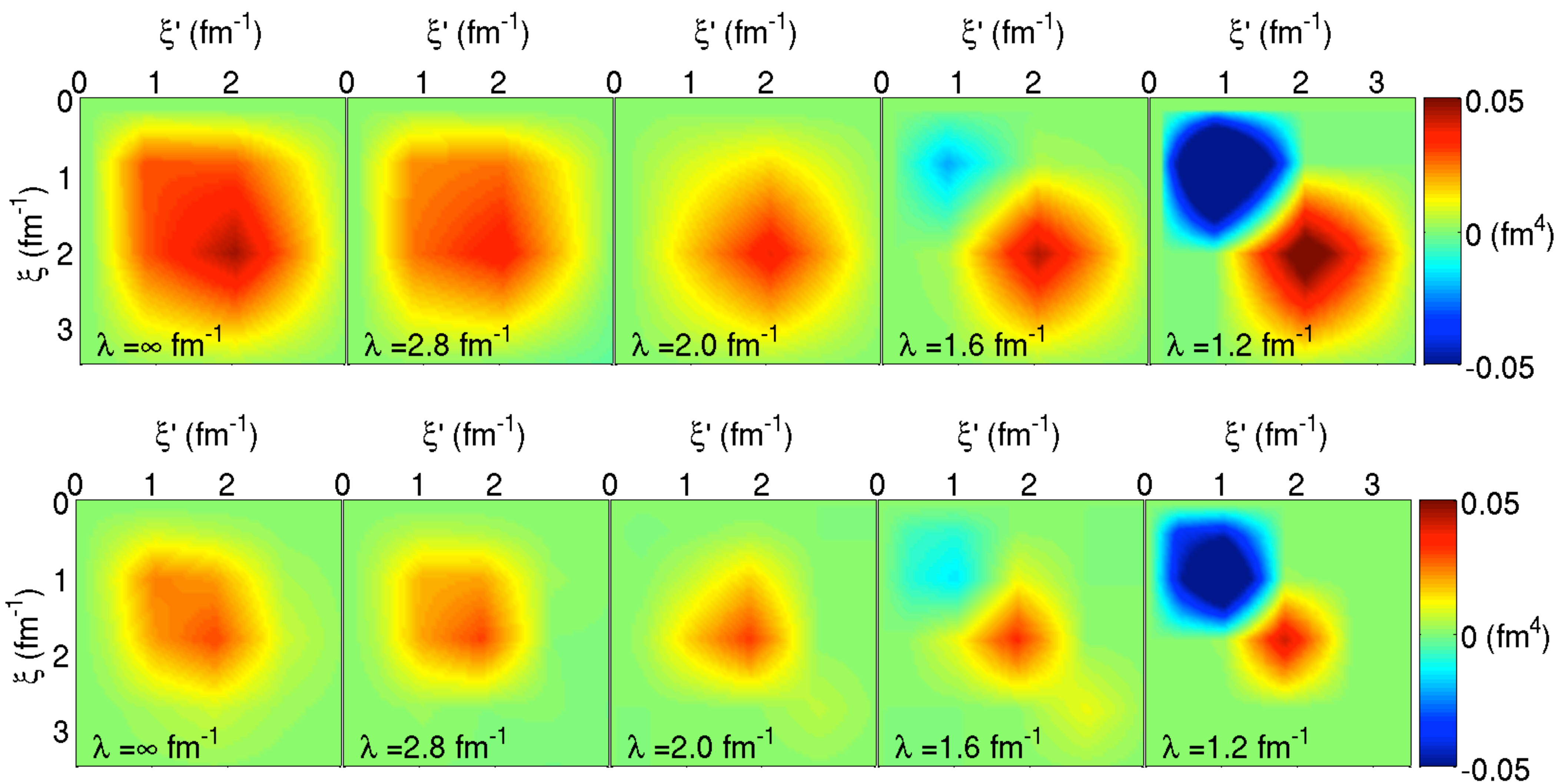

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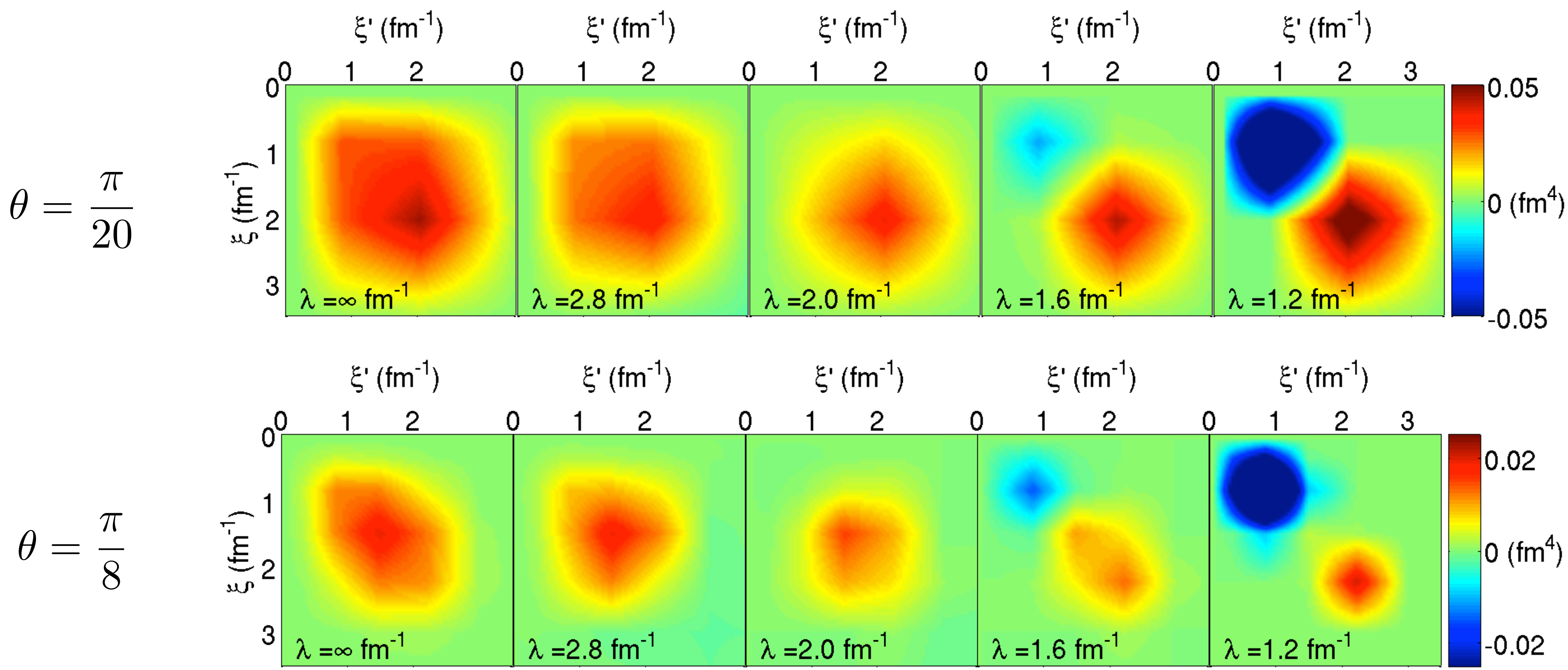

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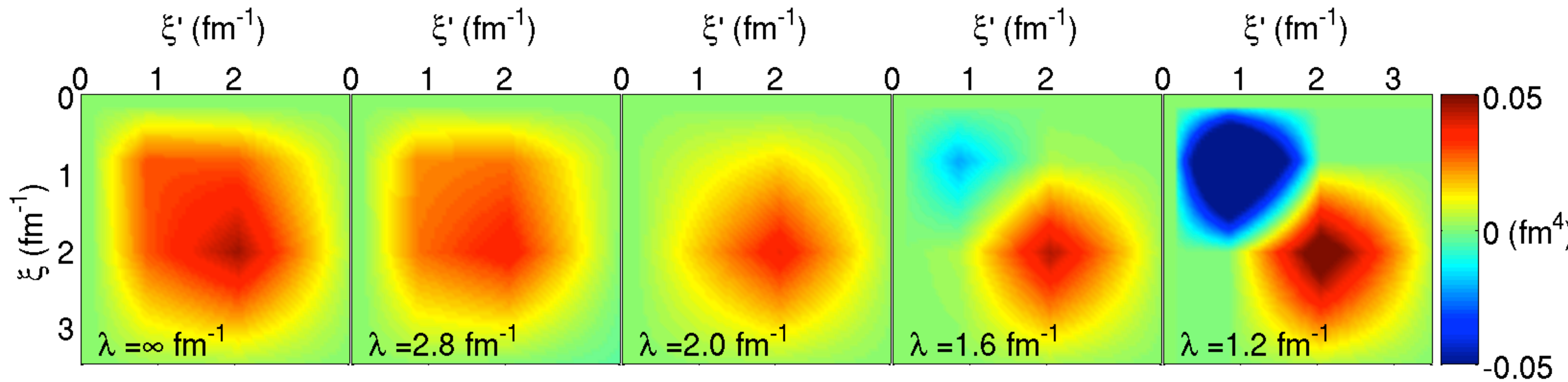
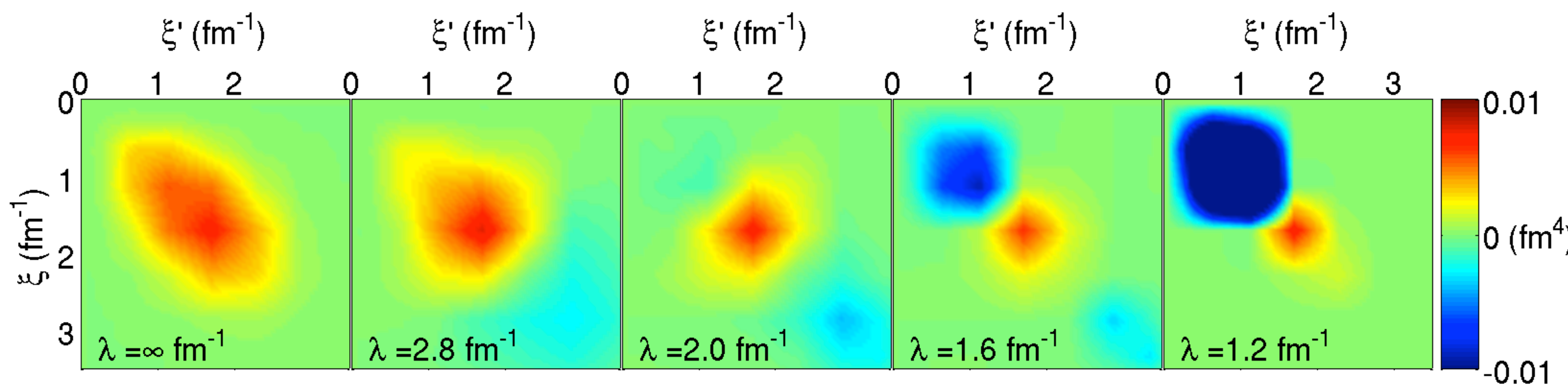


$\theta = \frac{\pi}{6}$



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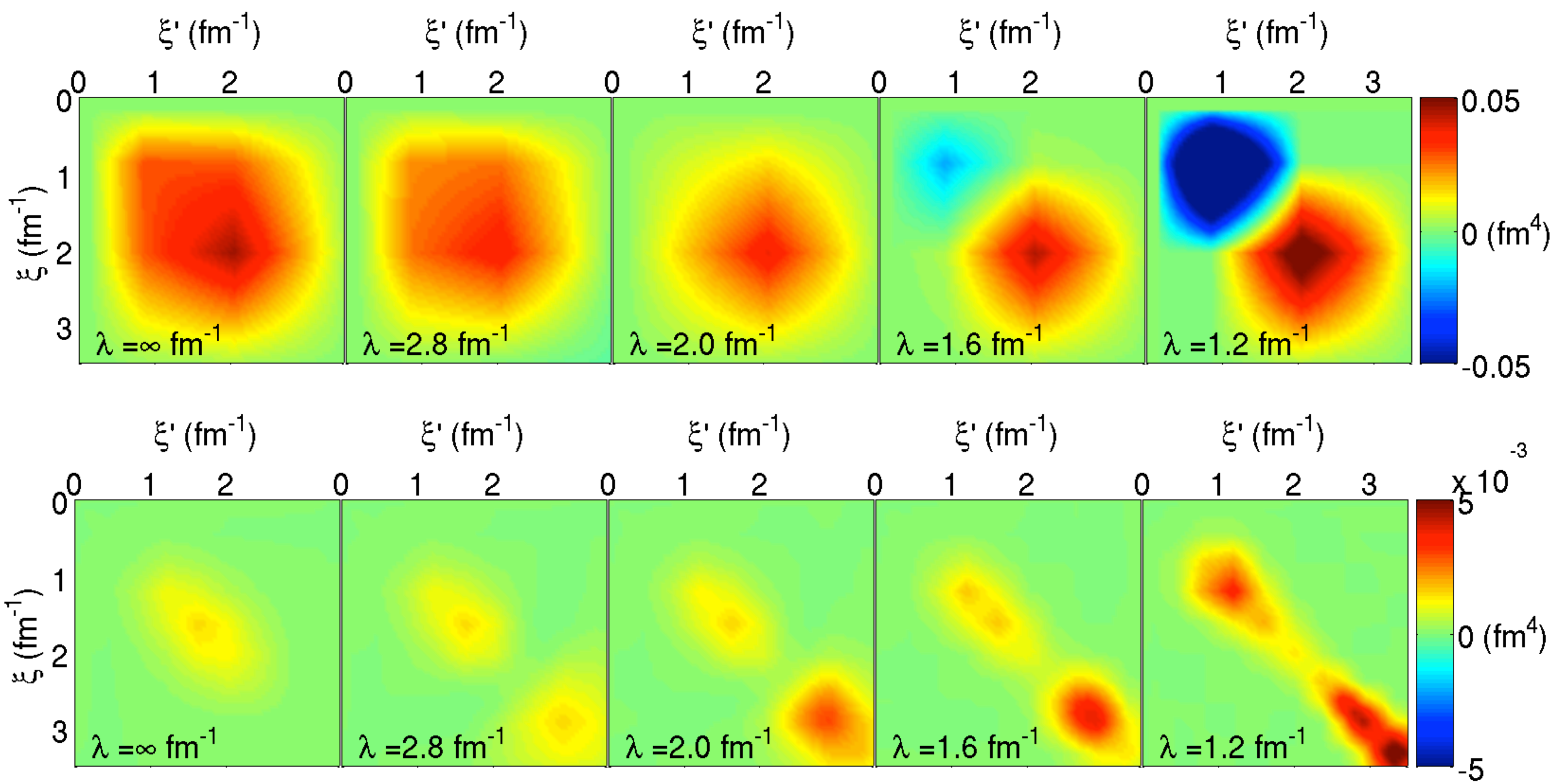
$\theta = \frac{\pi}{4}$



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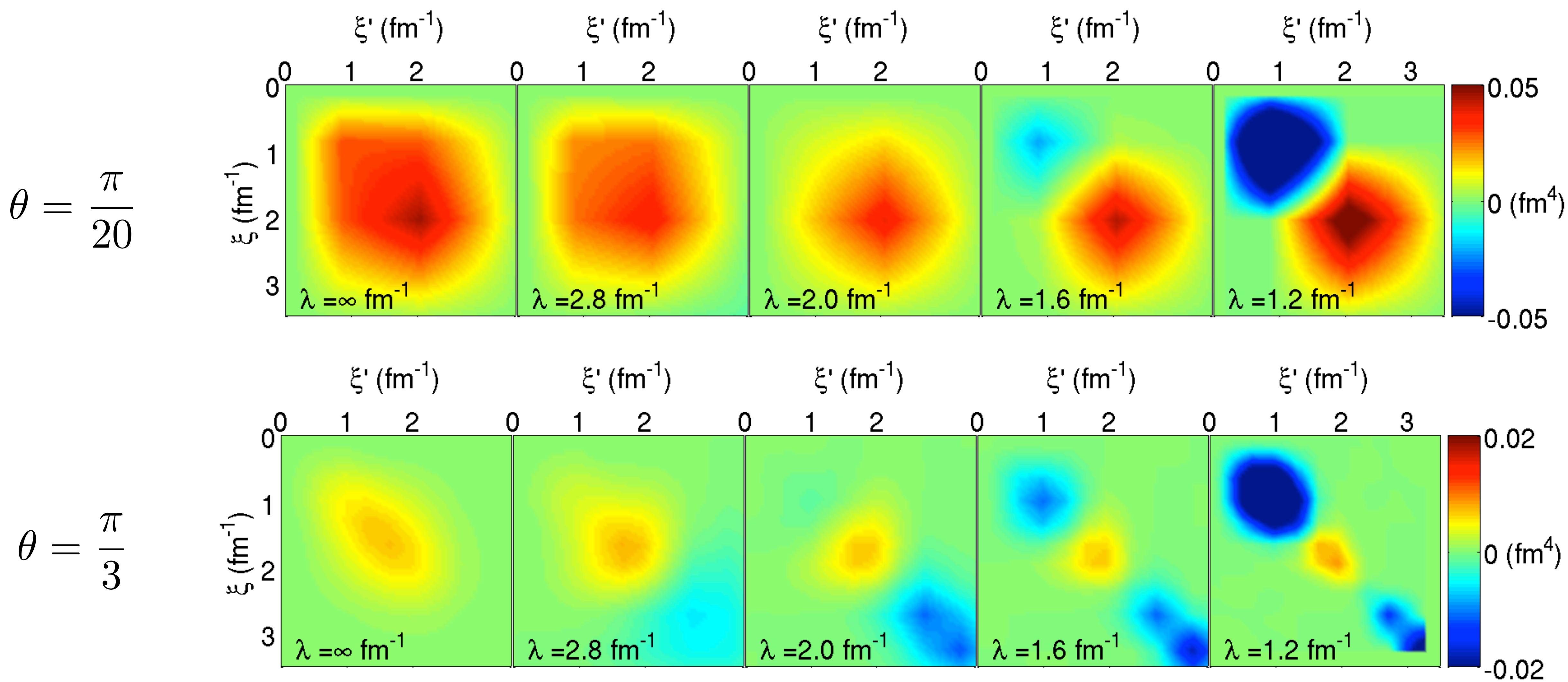

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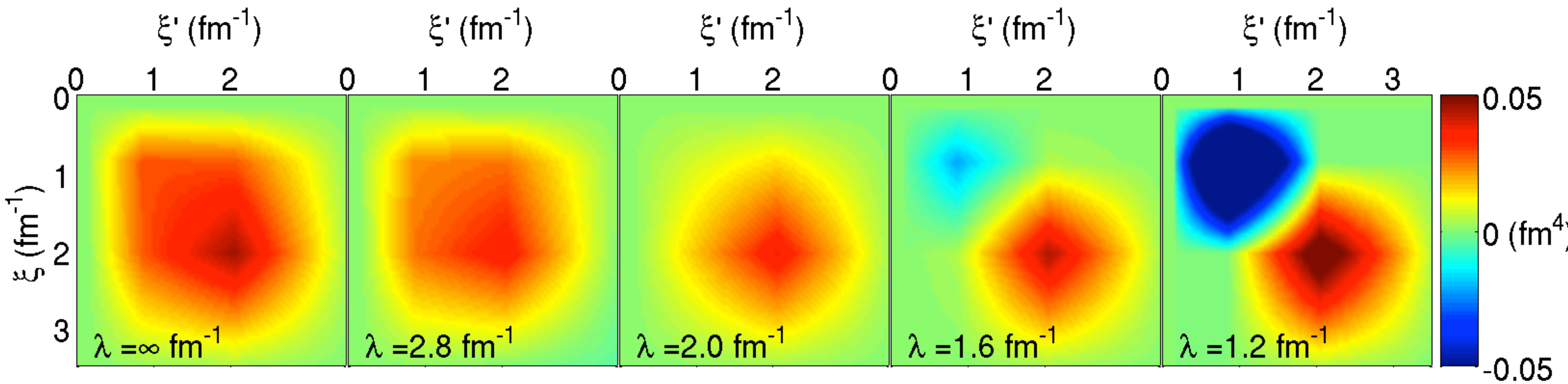
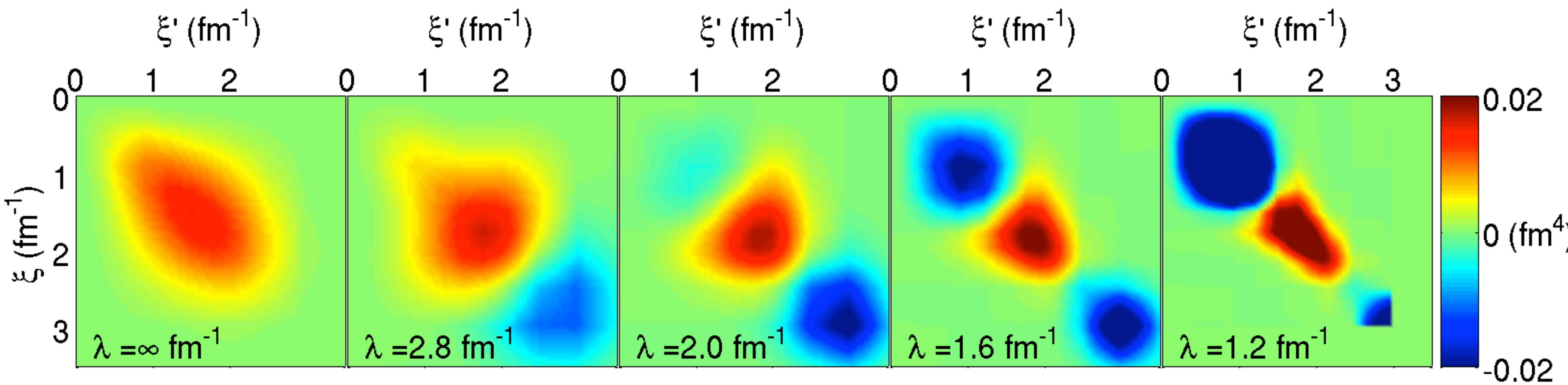


$\theta = \frac{\pi}{2.5}$



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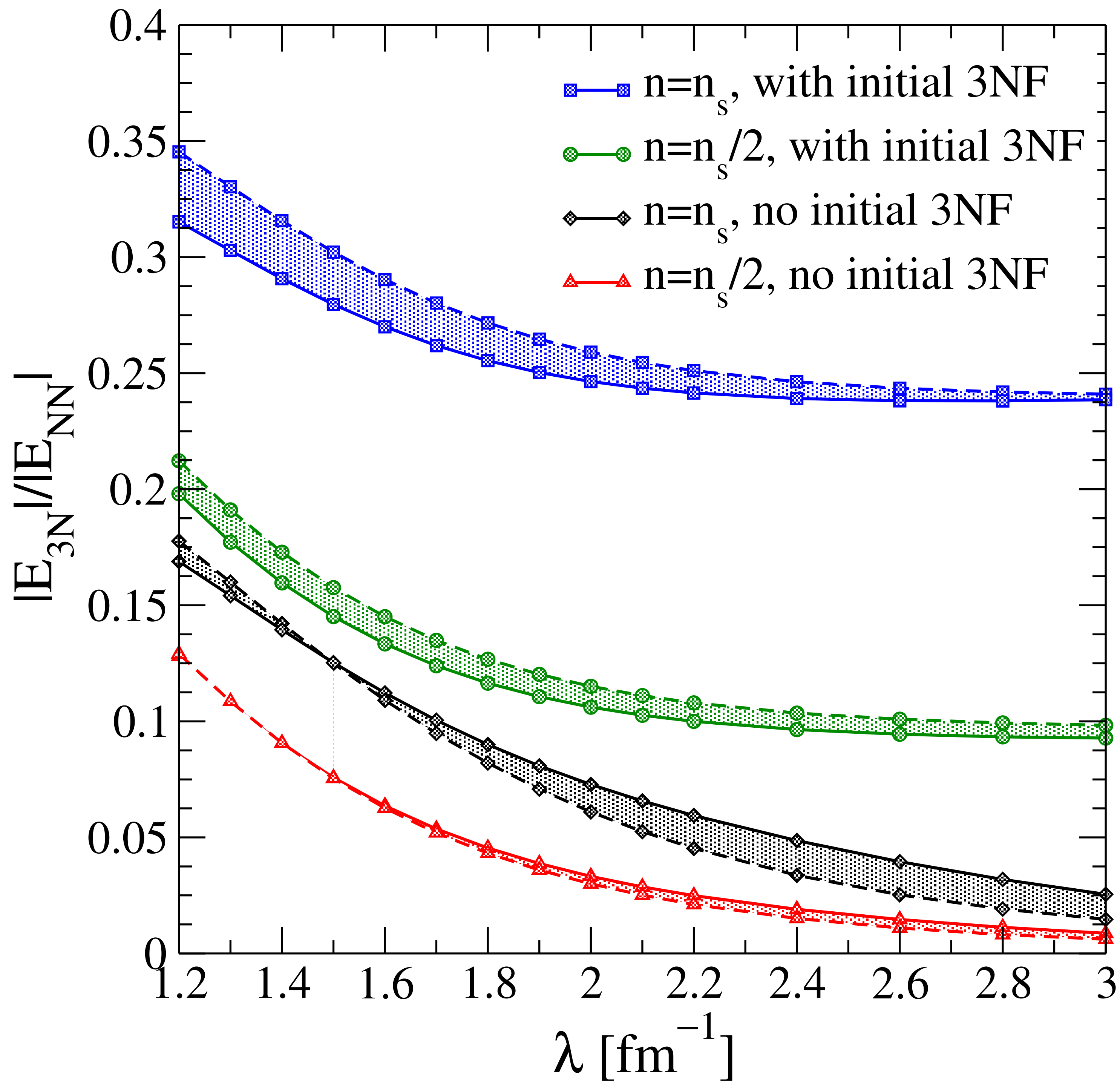
Scaling of three-body contributions



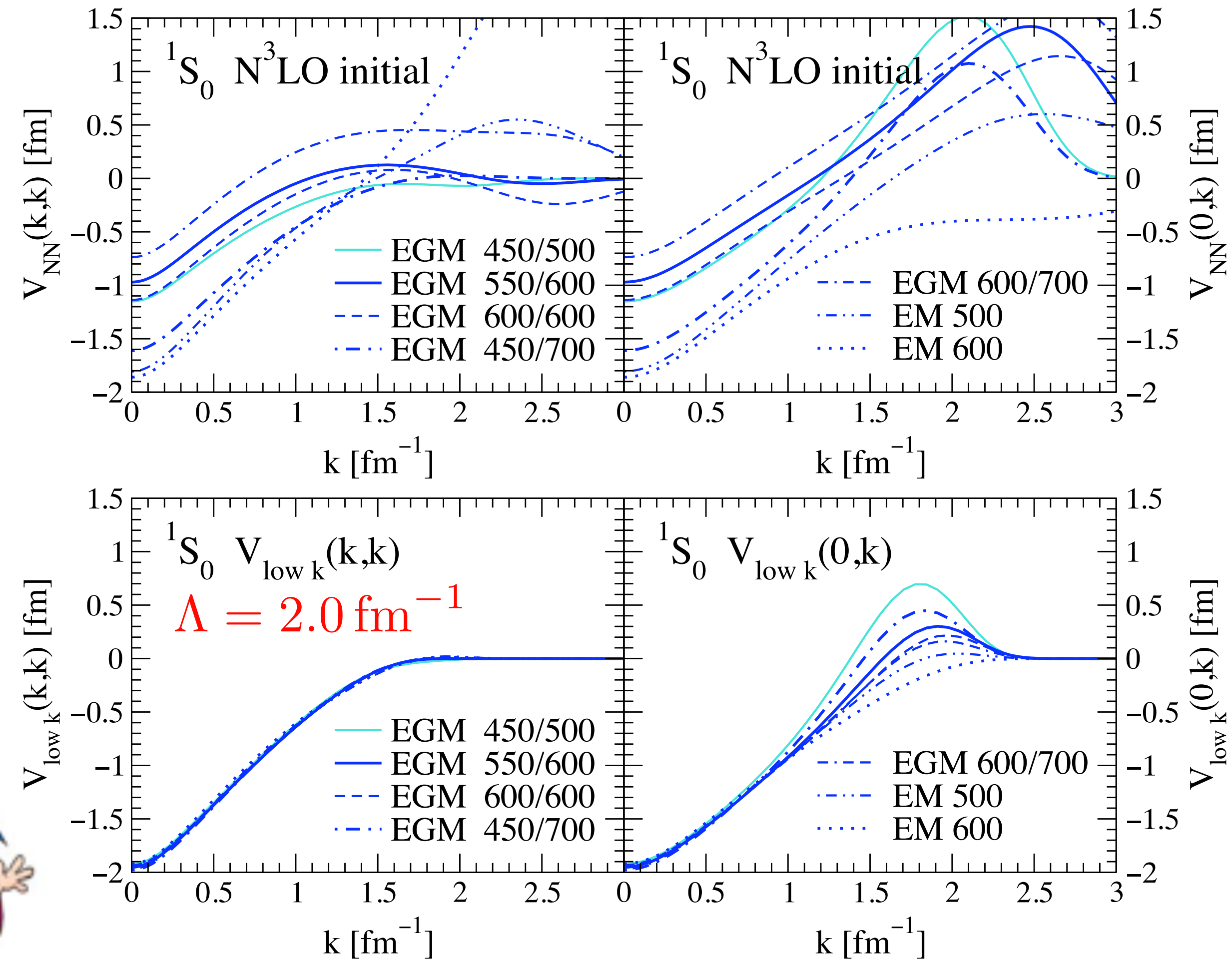
- relative size of 3N contribution grows systematically towards smaller λ
- no obvious trend with density (may be obscured by cancellations among contributions)

Universality of nuclear interactions at low resolution

phase-shift equivalence

common long-range physics

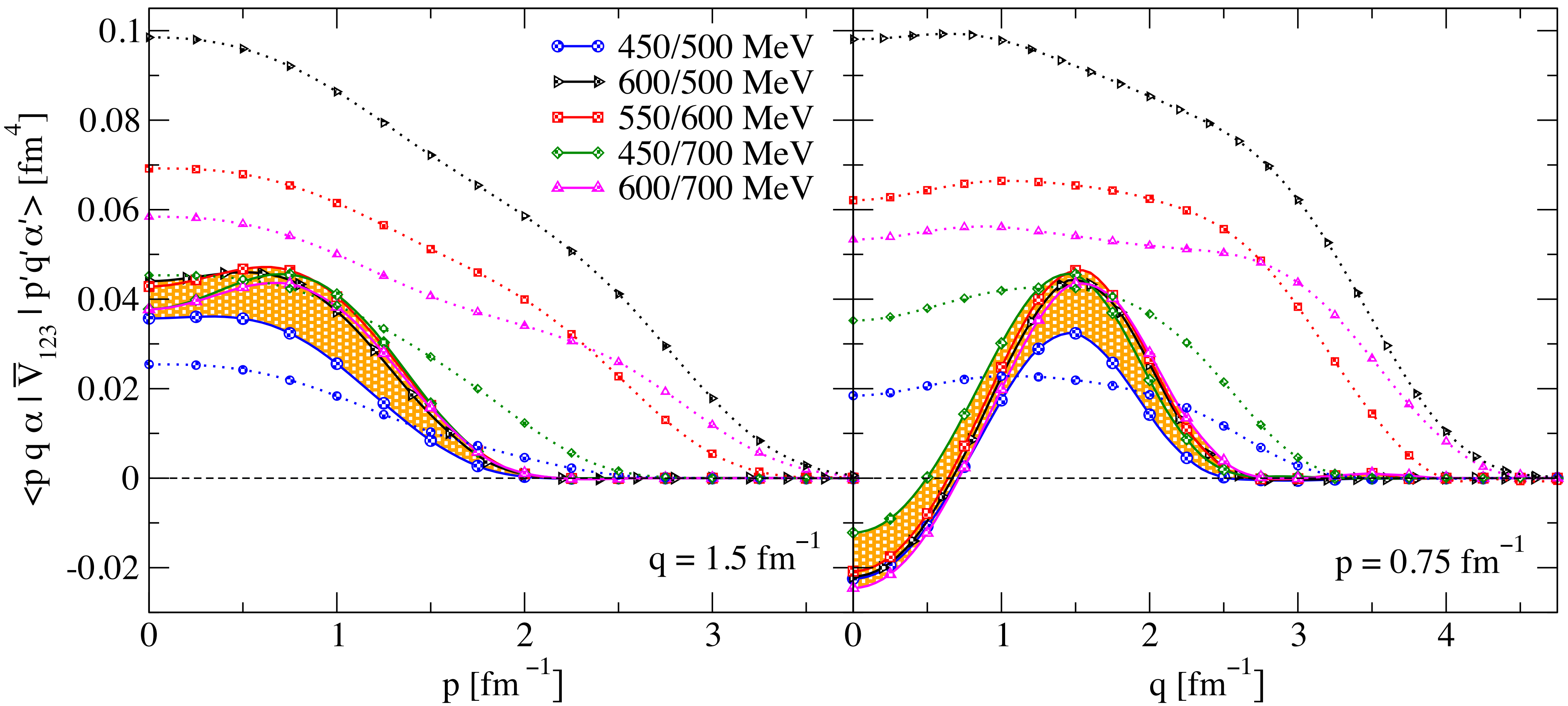
(approximate) universality of low-resolution NN interactions



To what extent are 3N interactions constrained at low resolution?

- only two low-energy constants c_D and c_E
- 3N interactions give only subleading contributions to observables

Universality in 3N interactions at low resolution



KH, PRC(R) 85, 021002 (2012)

- remarkably reduced scheme dependence for typical momenta $\sim 1\,\mathrm{fm}^{-1}$, matrix elements with significant phase space well constrained at low resolution
- new momentum structures induced at low resolution
- study based on $\mathrm{N^2LO}$ chiral interactions, improved universality at $\mathrm{N^3LO}$?

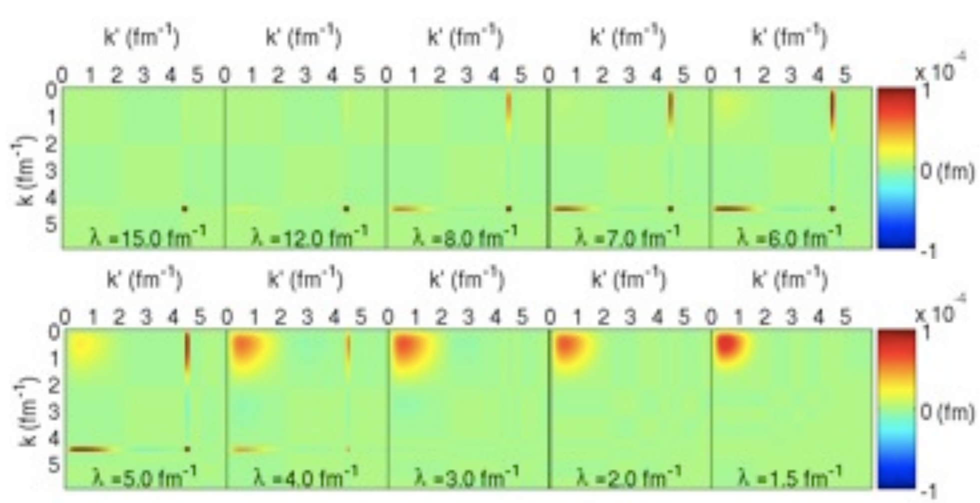
Summary

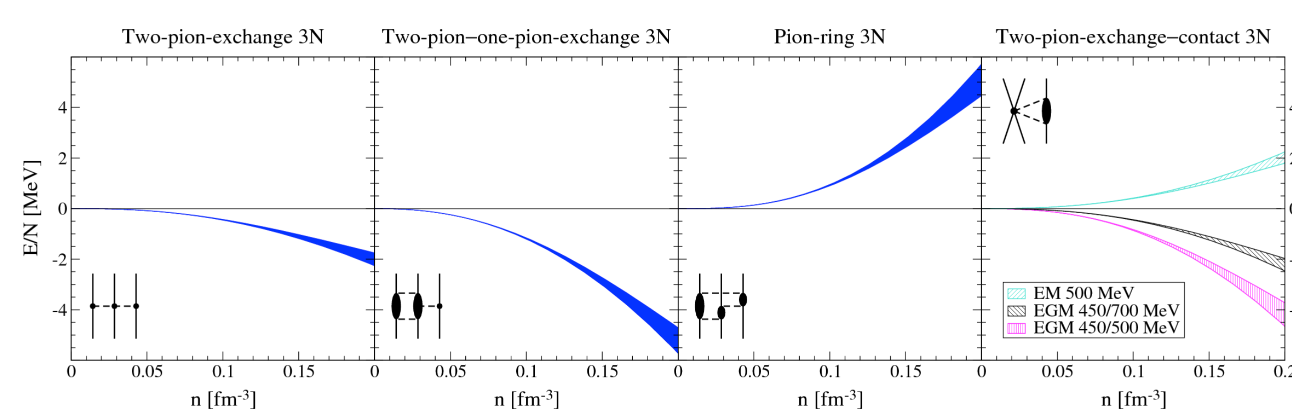
- demonstrated the feasibility of SRG evolution of NN+3NF in momentum space
- first results of neutron matter based on consistently evolved NN+3NF interactions
- strong renormalization effects of chiral two-pion exch. interaction in neutron matter
- no indications of significant contributions from 4N forces down to $\lambda = 1.2\ \mathrm{fm}^{-1}$

in neutron matter

Outlook

- inclusion of 3NF N3LO contributions in RG evolution
- extend RG evolution to $\mathcal{T} = 1/2$ channels, application to nuclear matter
- transformation to HO basis, application to finite nuclei (CC, NCSM)
- RG evolution of operators: nuclear scaling and correlations in nuclear systems



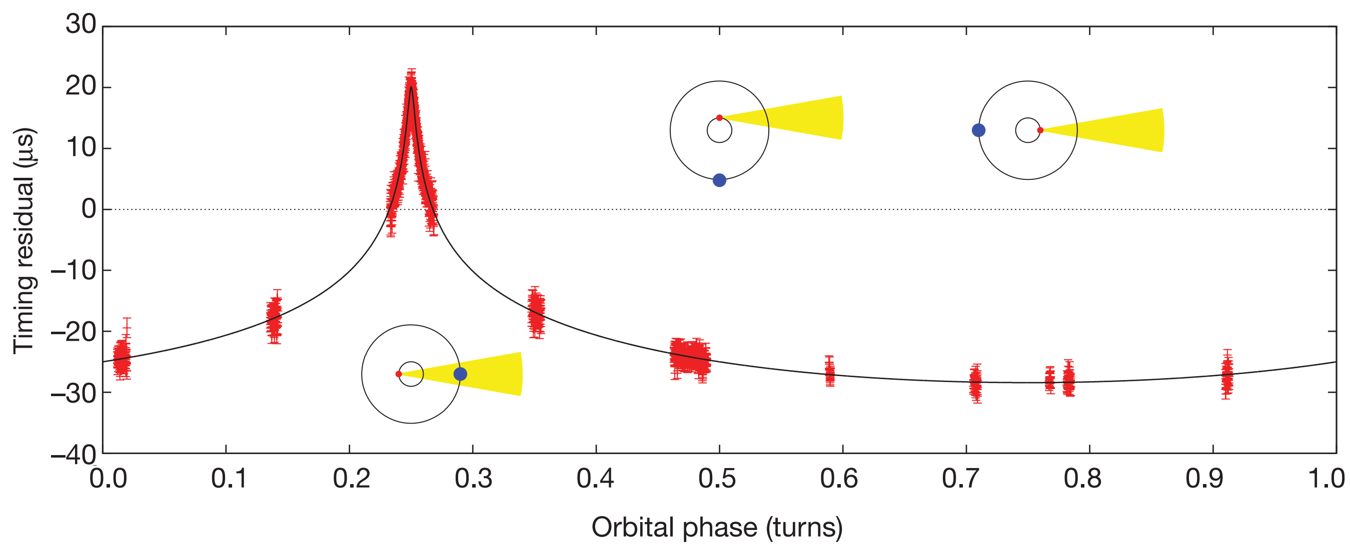


Constraints on the nuclear equation of state (EOS)

nature

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]

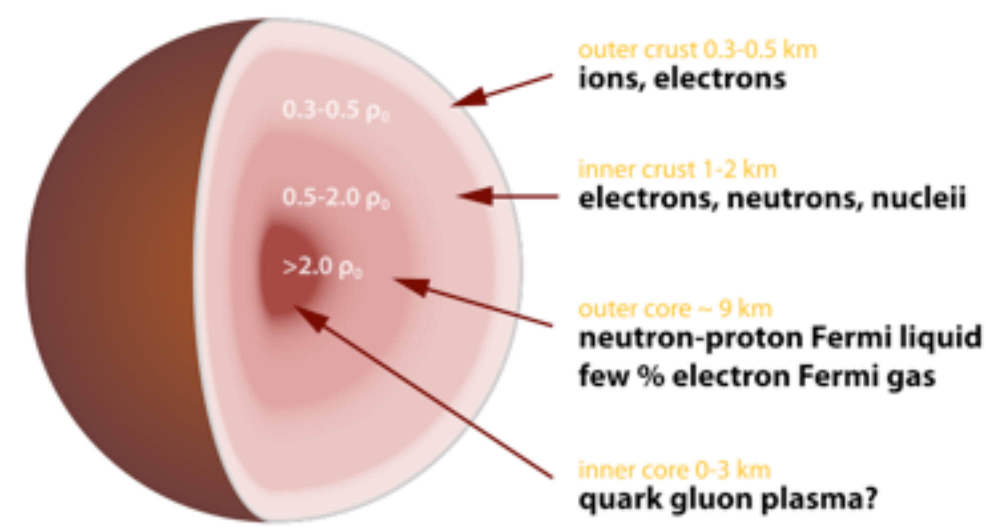


Demorest et al., Nature 467, 1081 (2010)

$$M_{\max} = 1.65 M_\odot \rightarrow 1.97 \pm 0.04\, M_\odot$$

Credit: NASA/Dana Berry

Calculation of neutron star properties requires EOS up to high densities.



Strategy:

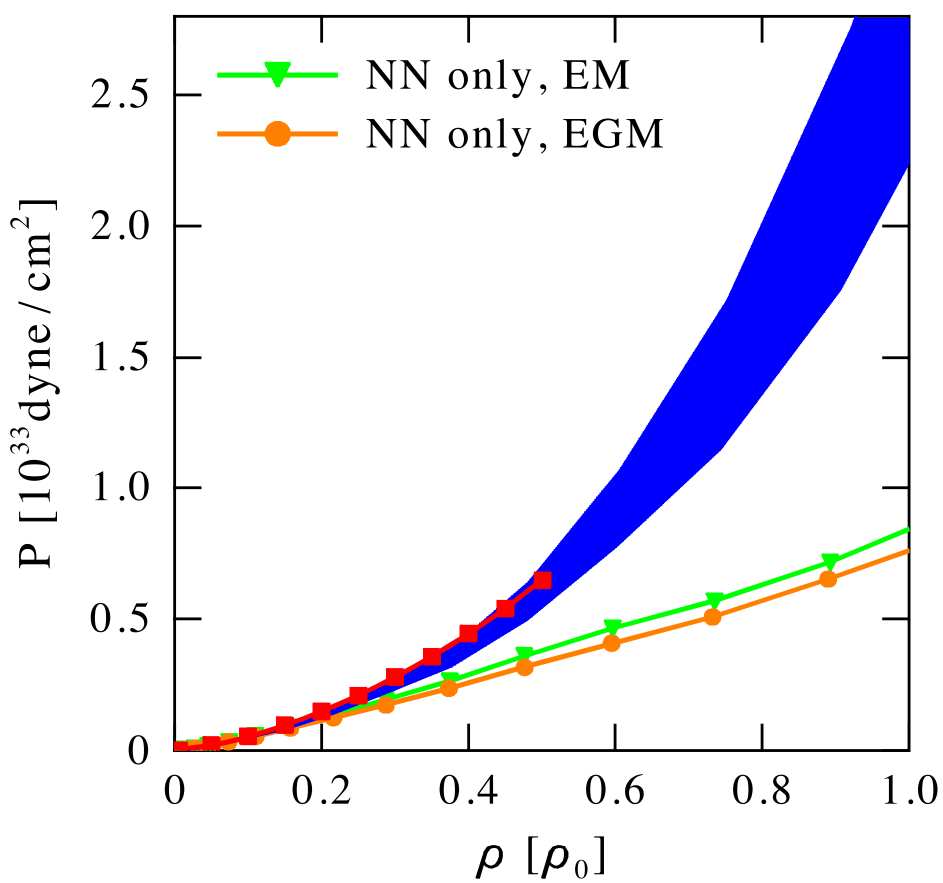
Use observations to constrain the high-density part of the nuclear EOS.

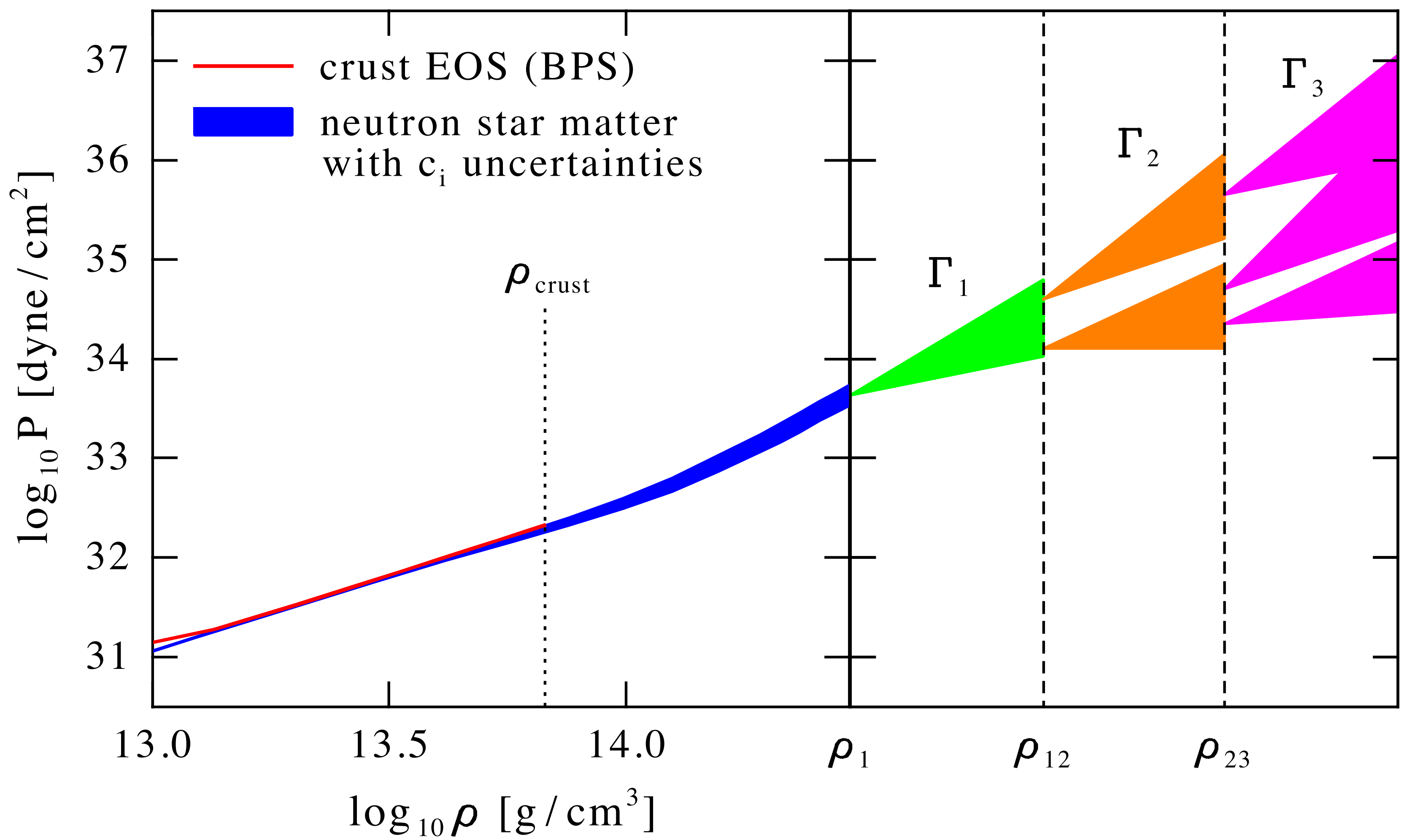
Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter $\longrightarrow$ neutron star matter

parametrize piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^\Gamma$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics!





KH, Lattimer, Pethick, Schwenk, arXiv:1303.4662
KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

Constraints on the nuclear equation of state

use the constraints:

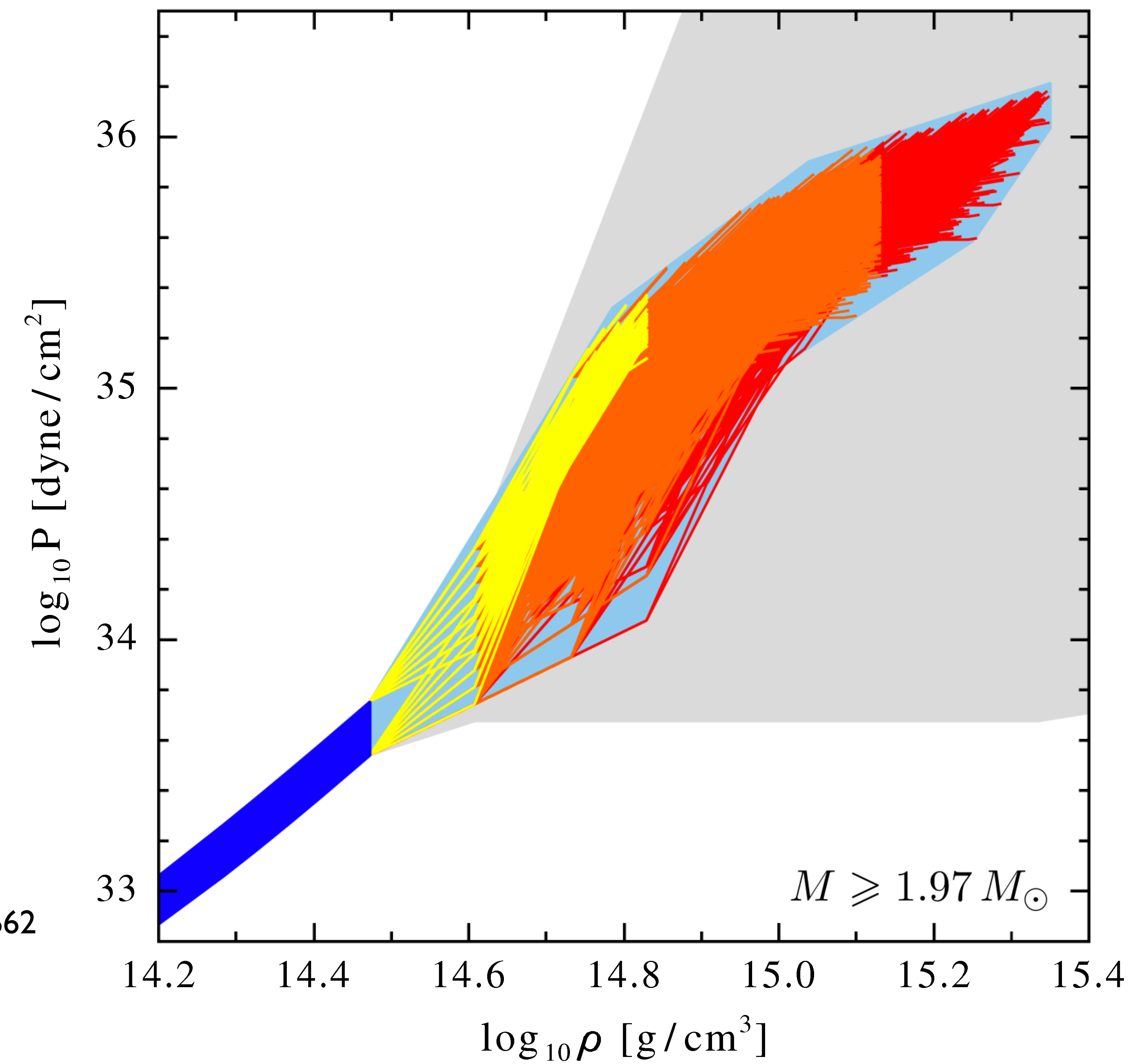
recent NS observation

$$M_{\max} > 1.97\, M_\odot$$

causality

$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH, Lattimer, Pethick, Schwenk, arXiv:1303.4662



significant reduction of uncertainty band

Constraints on the nuclear equation of state

use the constraints:

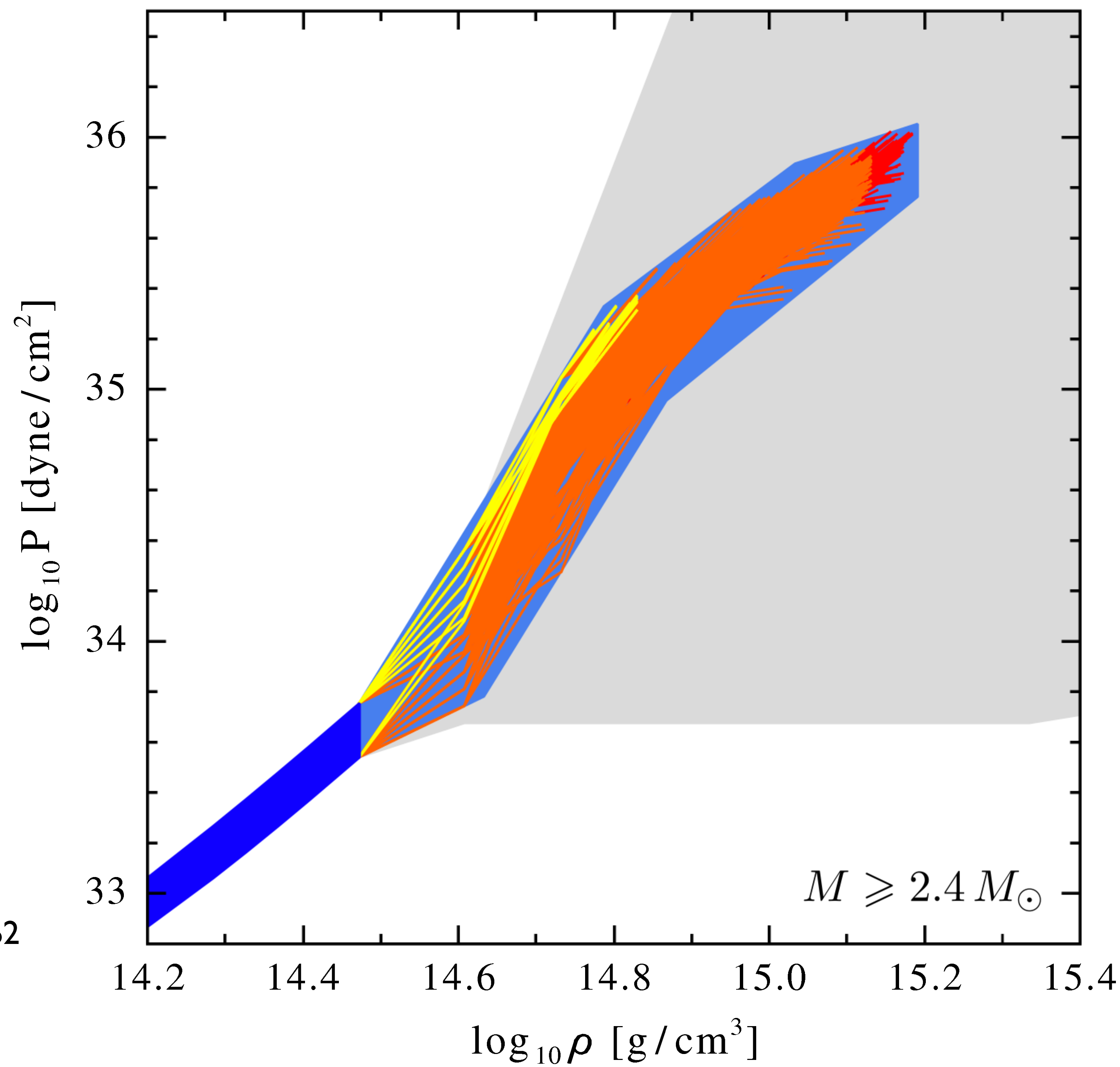
NS mass

$$M_{\rm max} > 2.4\, M_\odot$$

causality

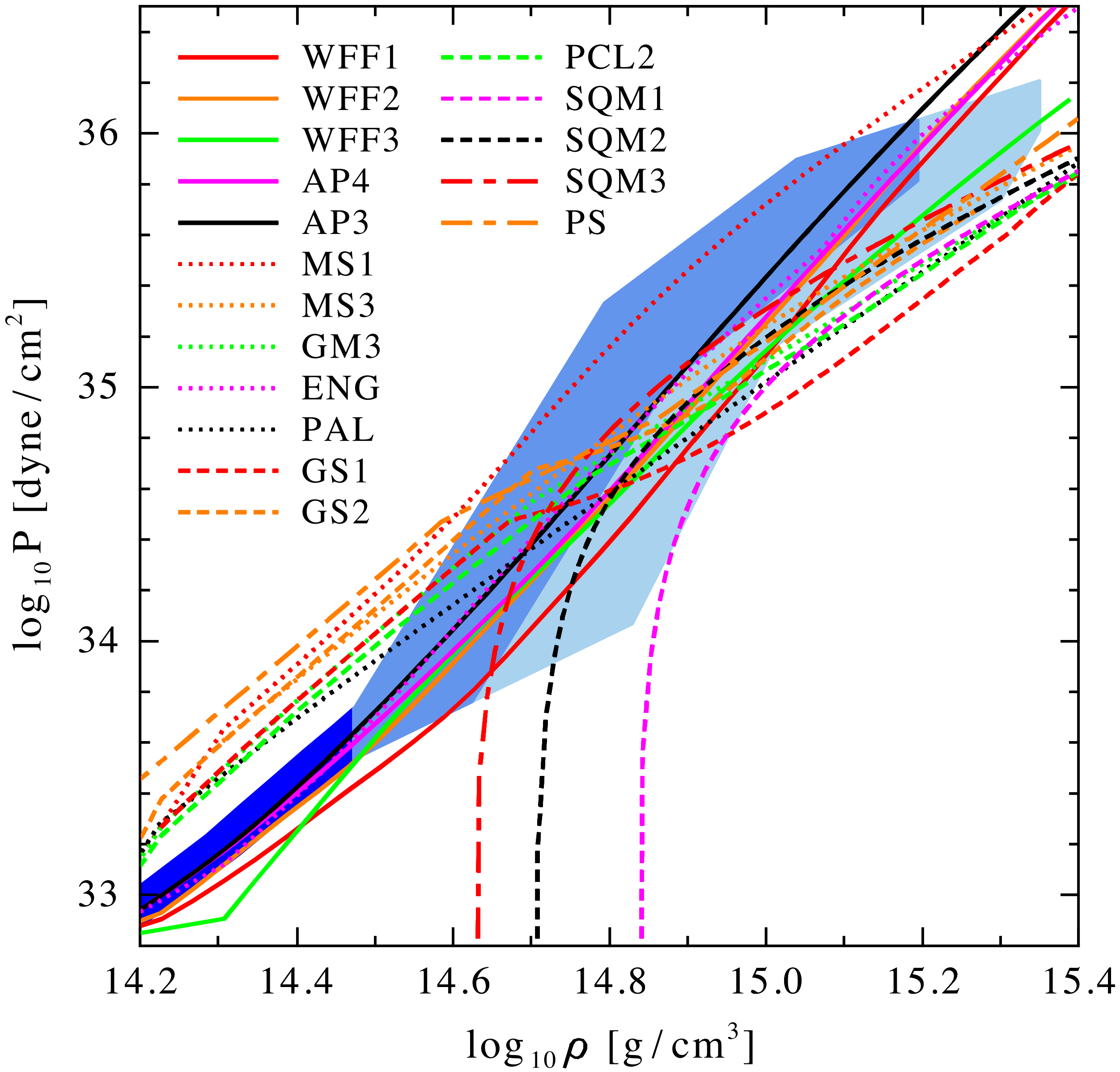
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

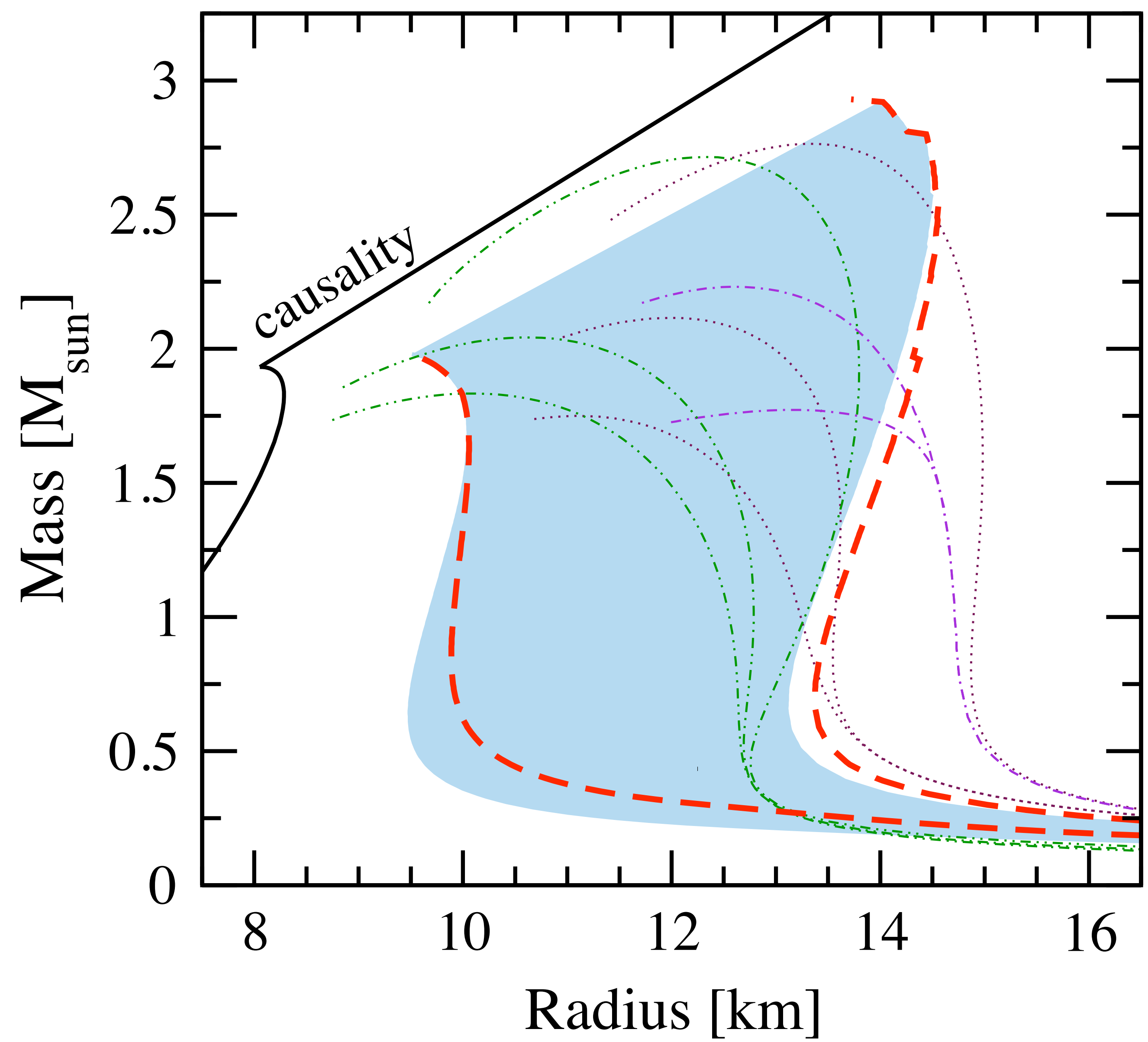
KH, Lattimer, Pethick, Schwenk, arXiv:1303.4662



increased $M_{\rm max}$ systematically reduces width of band

Constraints on neutron star radii

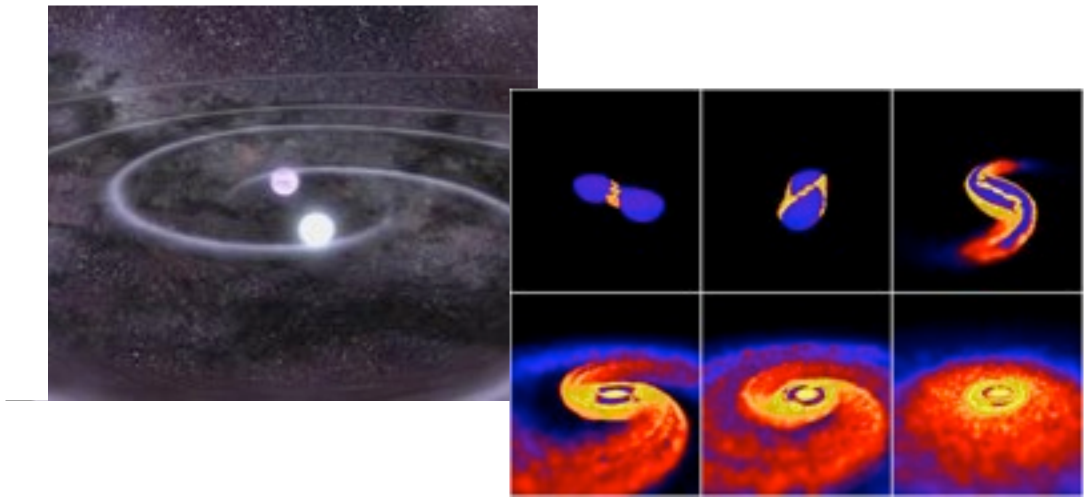


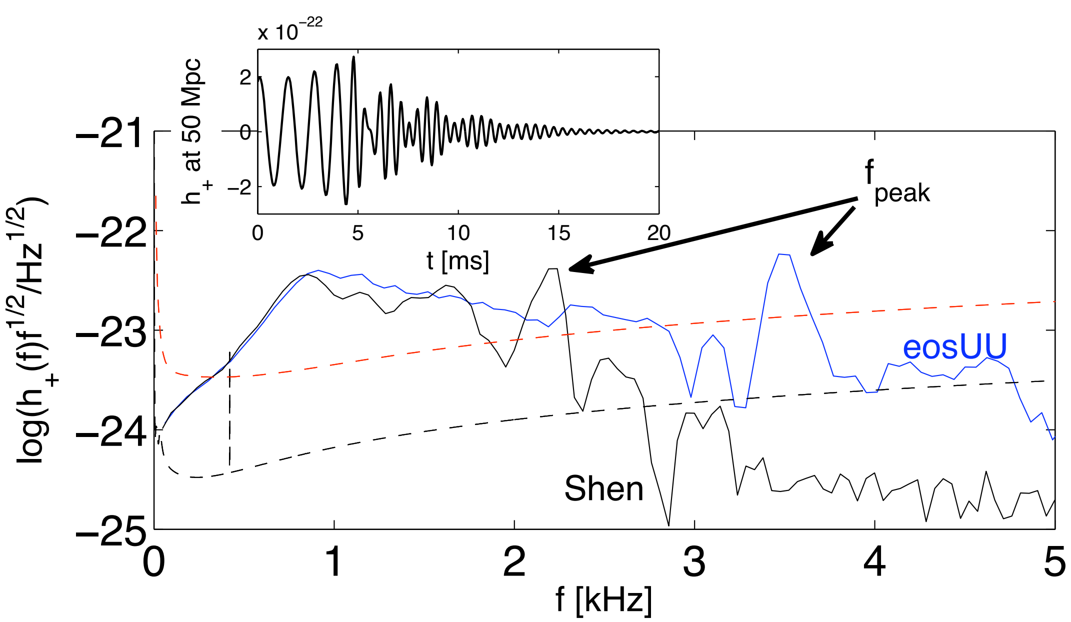


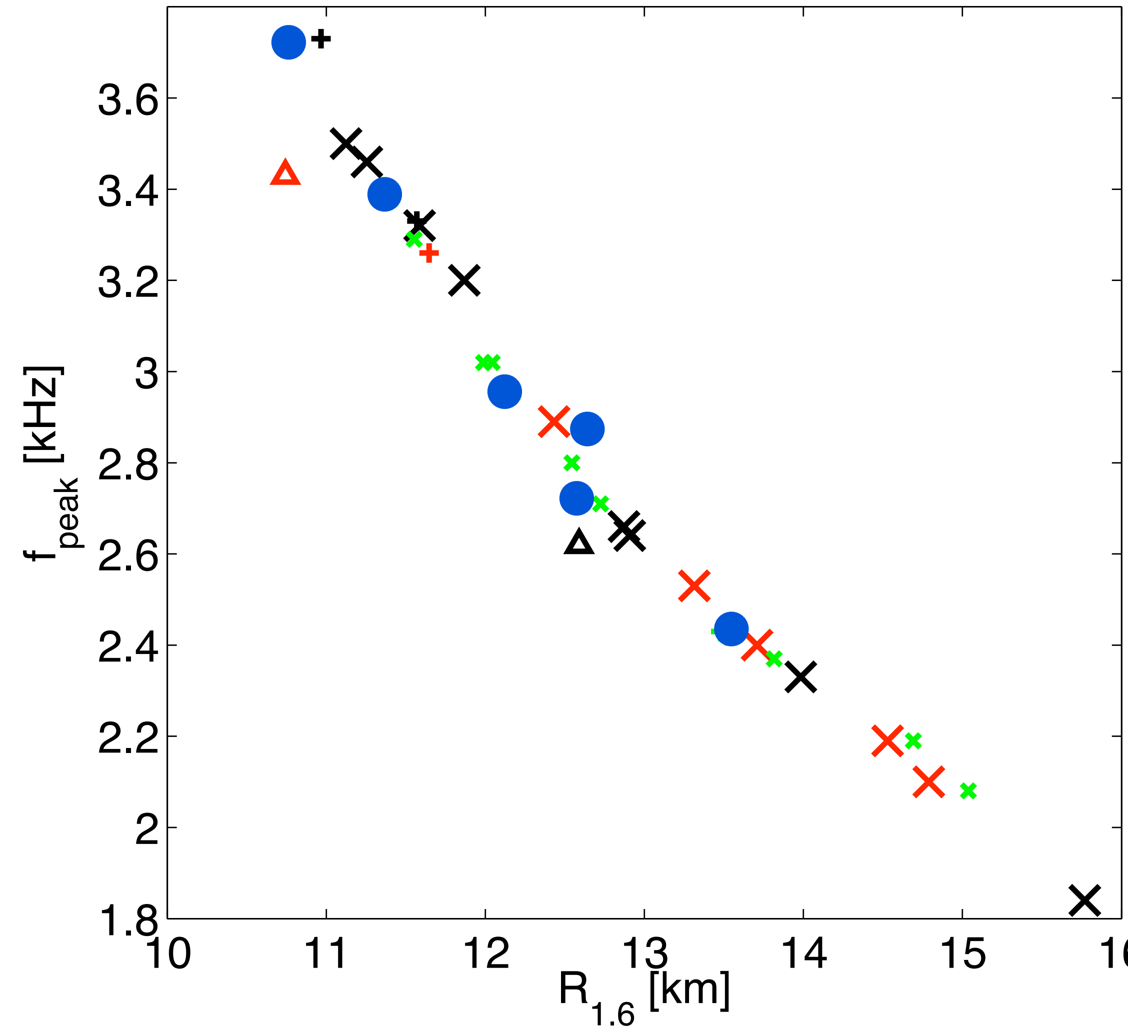
KH, Lattimer, Pethick, Schwenk, arXiv:1303.4662
see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- radius constraint for typical $1.4\,M_\odot$ neutron star: $9.8 - 13.4\,\mathrm{km}$

Gravitational wave signals from neutron star binary mergers







Bauswein and Janka PRL 108, 011101 (2012),
Bauswein, Janka, KH, Schwenk arXiv: PRD 86, 063001 (2012)

- high-density part of nuclear EOS only loosely constrained
- simulations of NS binary mergers show strong correlation between between $f_{\rm peak}$ of the GW spectrum and the radius of a NS
- measuring $f_{\rm peak}$ is key step for constraining EOS systematically at large ρ