Microscopic optical potential from chiral two- and three-nuclear forces

[JWH, N. Kaiser, G. A. Miller and W. Weise, arXiv:1304.3175]

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From Few Nucleon Forces to Many-Nucleon Structure
ECT*, June 13, 2013

Outline

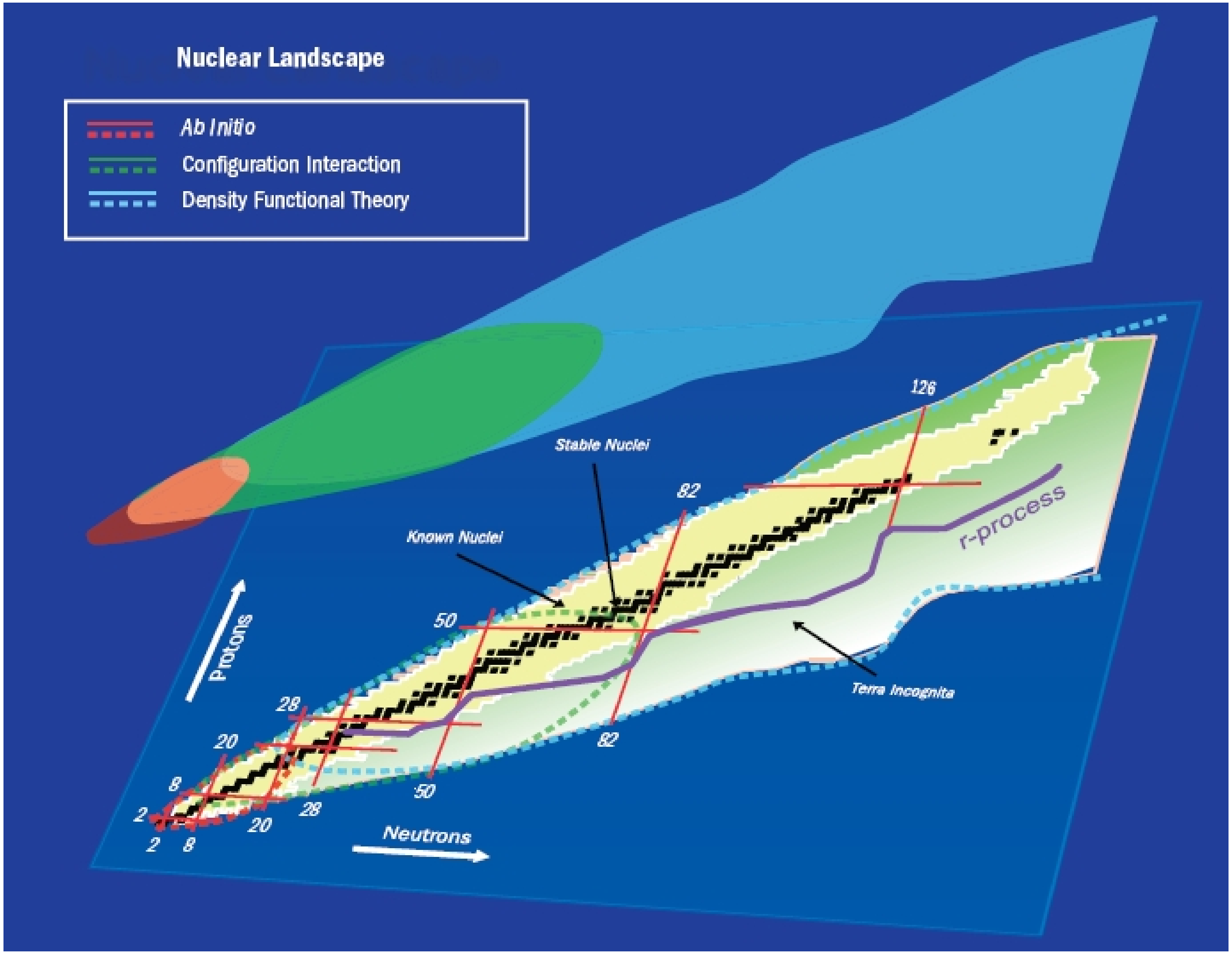
Motivation

- *R-process nucleosynthesis*
- *Neutron capture on exotic isotopes; surrogate reaction (d,p) stripping*
- *Necessary input: nucleon-nucleus optical potential*
- *Reliable extrapolation away from the valley of stability*

Microscopic optical model potentials

- *Perturbative calculation of nucleon self-energy*
- *Three-nucleon forces at first and second order*
- *Benchmarking against empirical optical potentials*

Many-nucleon structure and many-body methods



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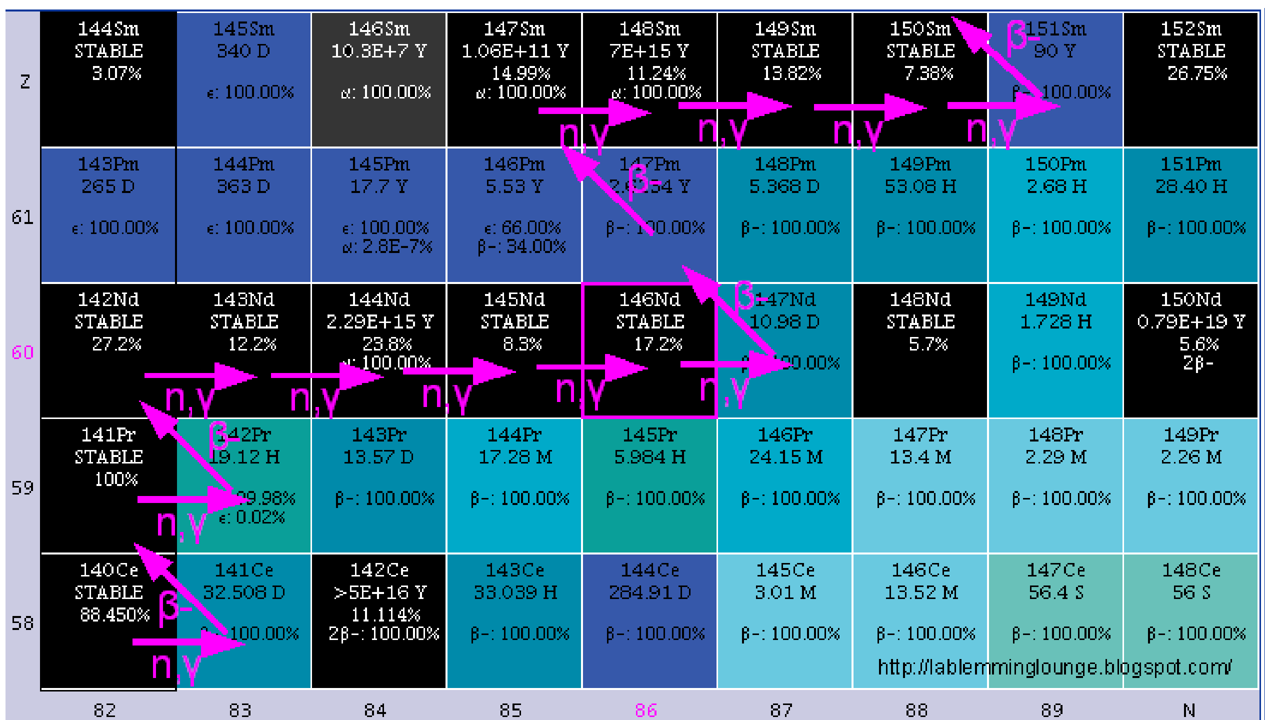
S-process nucleosynthesis

- “Slow" neutron capture (relative to the timescale of nuclear β-decay)
- Relatively low neutron density
- Process occurs in normal stellar burning
- Produces approximately half of the elements heavier than iron



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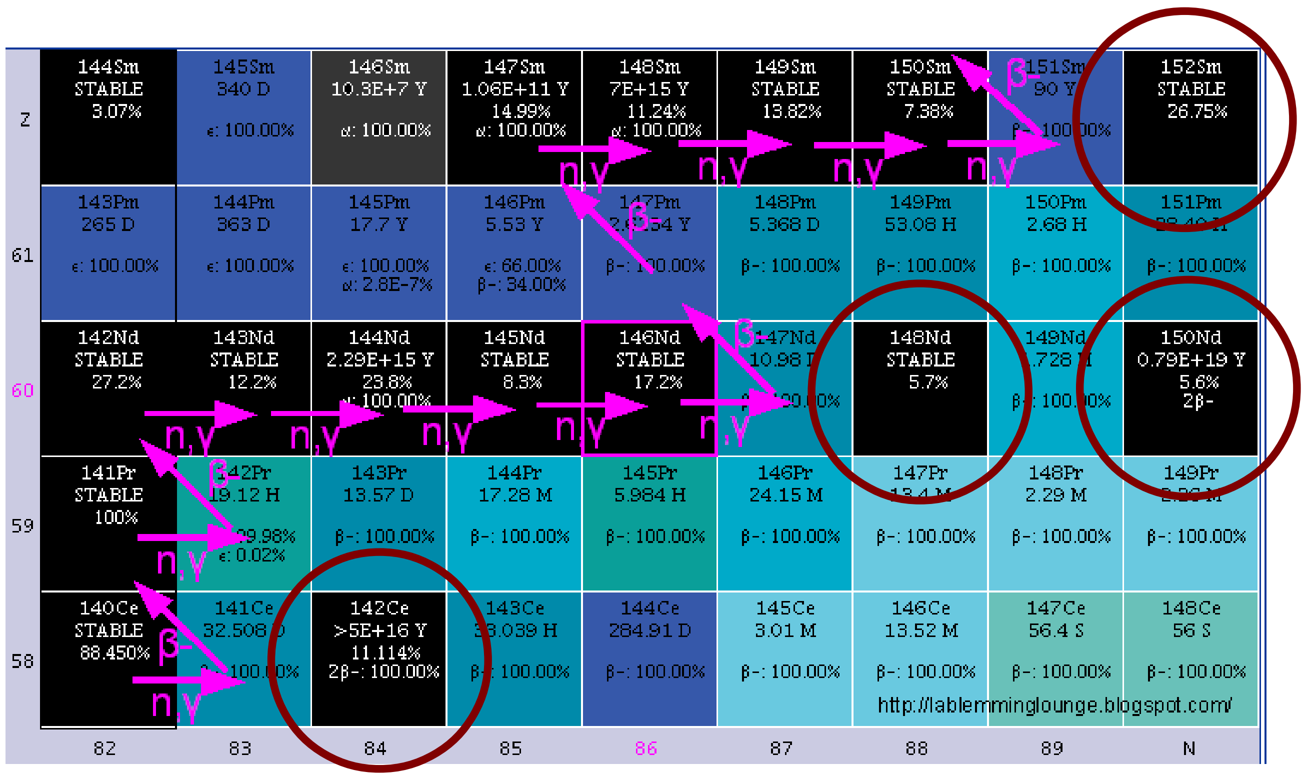
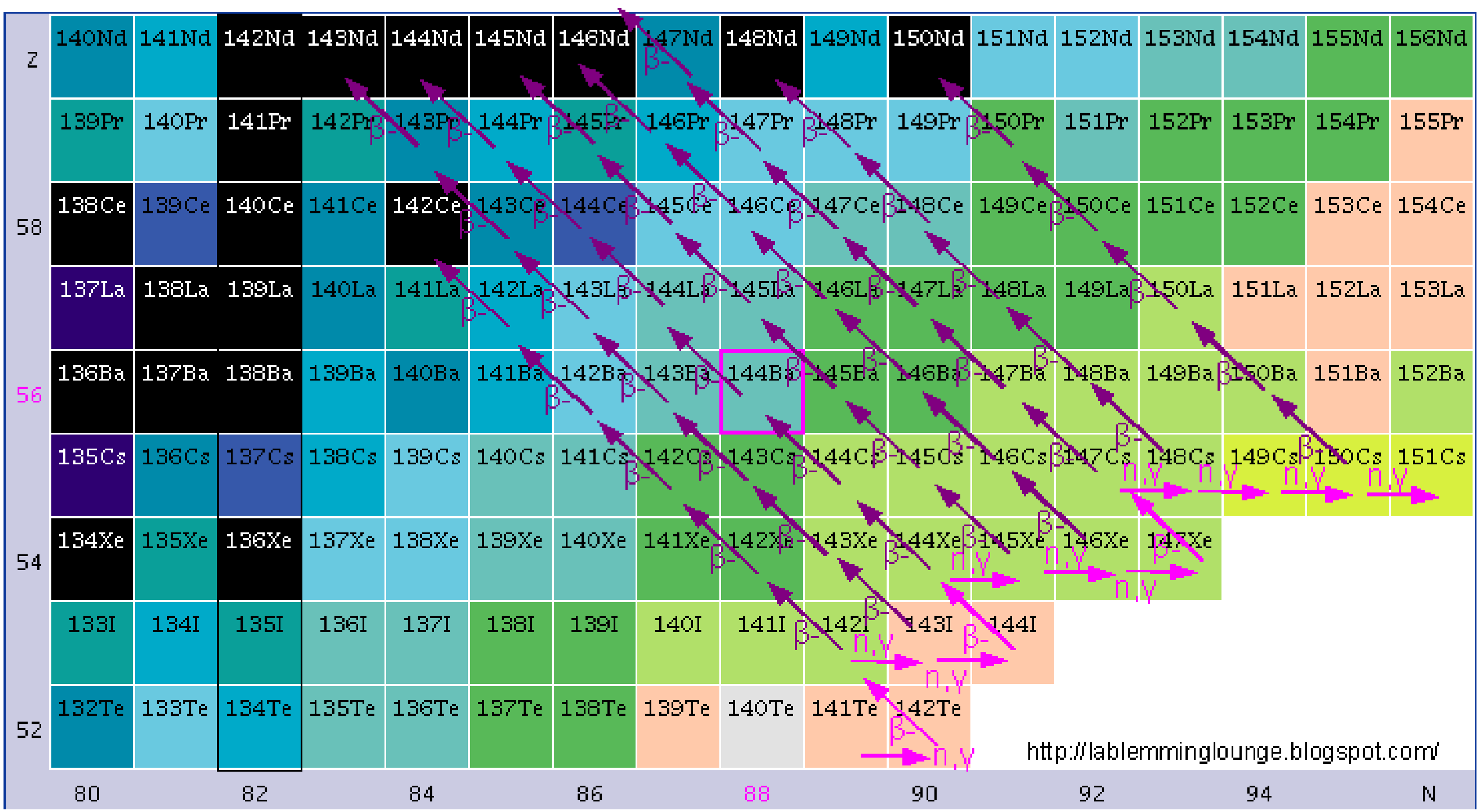


R-process nucleosynthesis

- Involves "rapid" neutron capture to highly neutron-rich nuclei
- β-decay back to the valley of stability
- Creates about half of the elements above iron

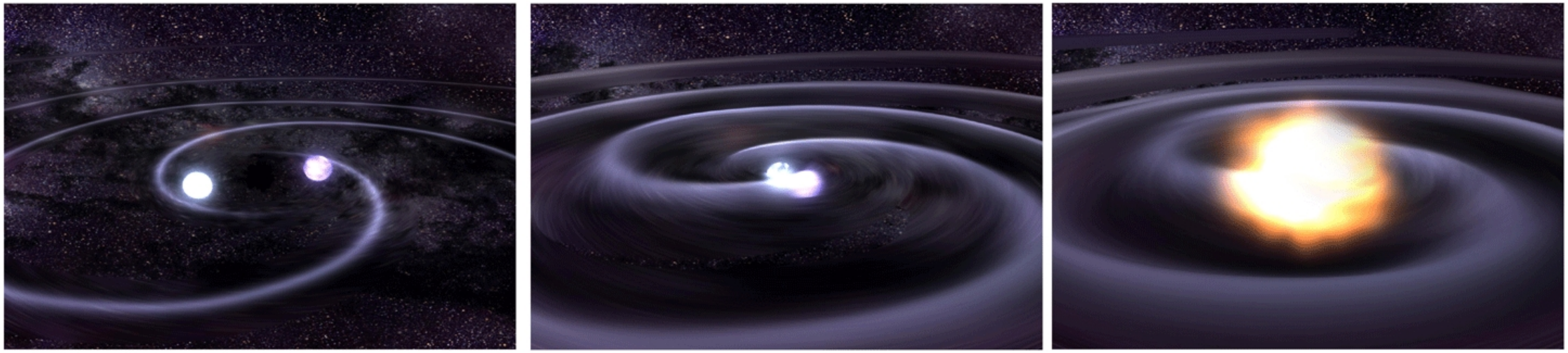


R-process nucleosynthesis sites

- Requires a highly neutron-rich environment

Core collapse supernovae

Binary neutron star mergers



R-process nucleosynthesis sites

- Requires a highly neutron-rich environment

Core collapse supernovae

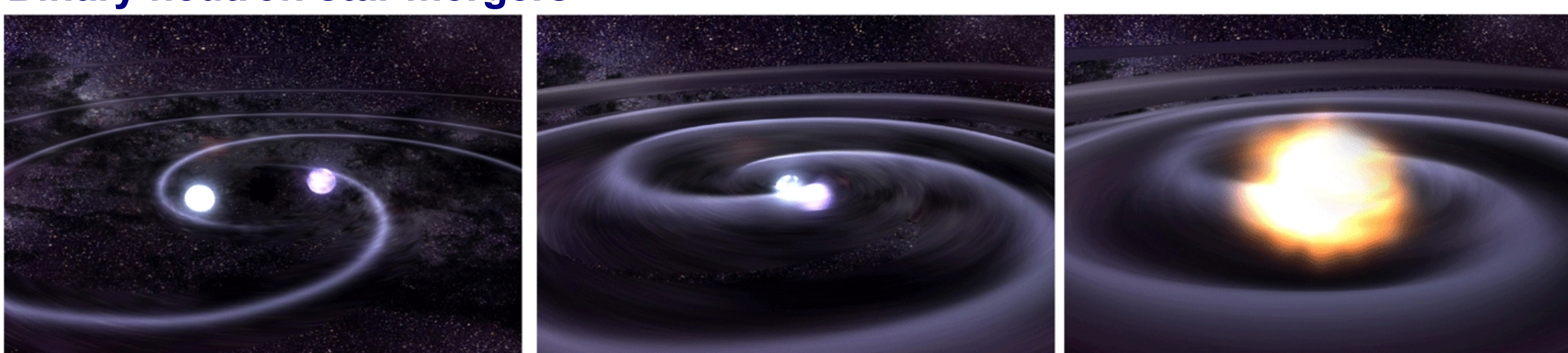
Inputs for numerical simulations:

Isotope masses

Beta-decay lifetimes

Neutron-capture cross sections

Binary neutron star mergers

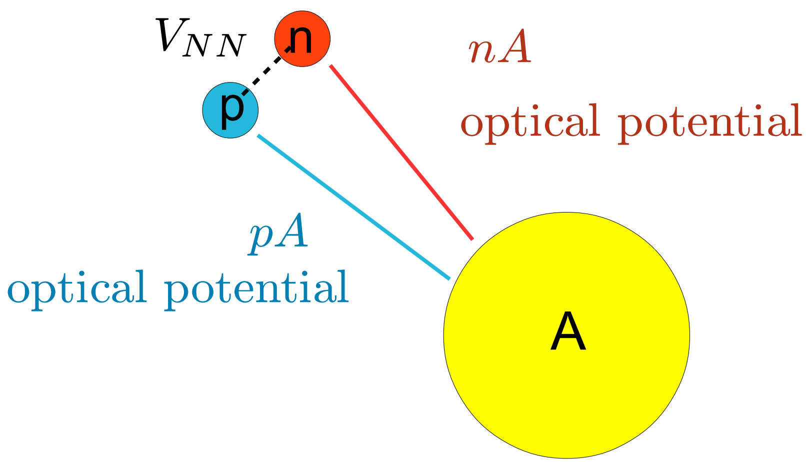


(d,p) stripping reactions

Neutron capture reactions on neutron-rich isotopes are experimentally unfeasible

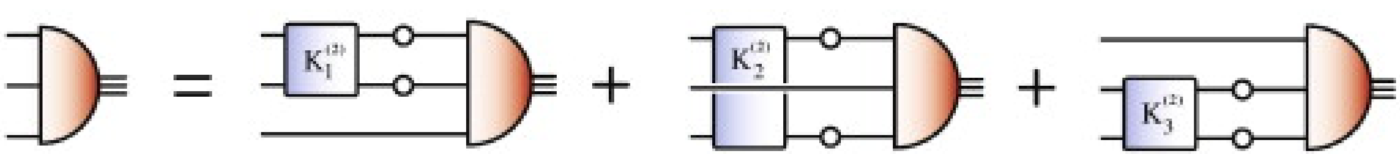
“Surrogate reaction” for neutron capture

(d, p) reaction



$$U(\vec{r}, \vec{r}\,'; E) = V(\vec{r}, \vec{r}\,'; E) + iW(\vec{r}, \vec{r}\,'; E)$$

Effective three-body problem requires $V_{NN}, V_{pA}^{\rm op}, V_{nA}^{\rm op}$



Treats elastic, breakup, and transfer reactions on equal footing

Phenomenological optical potentials

$$\mathcal{U}(r,E) = -\mathcal{V}_V(r,E) - i\mathcal{W}_V(r,E) - i\mathcal{W}_D(r,E) + \mathcal{V}_{SO}(r,E).\mathbf{l}.\sigma + i\mathcal{W}_{SO}(r,E).\mathbf{l}.\sigma + \mathcal{V}_C(r),$$

$$\mathcal{V}_V(r,E) = V_V(E) f(r, R_V, a_V),$$

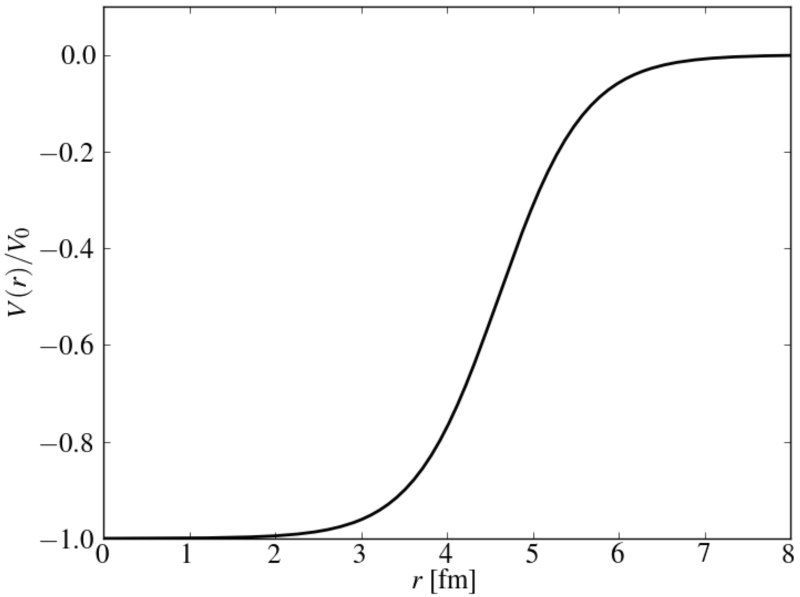
$$\mathcal{W}_V(r,E) = W_V(E) f(r, R_V, a_V),$$

$$\mathcal{W}_D(r,E) = -4a_D W_D(E) \frac{d}{dr} f(r, R_D, a_D),$$

$$\mathcal{V}_{SO}(r,E) = V_{SO}(E) \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr} f(r, R_{SO}, a_{SO}),$$

$$\mathcal{W}_{SO}(r,E) = W_{SO}(E) \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr} f(r, R_{SO}, a_{SO}).$$

$$f(r, R_i, a_i) = \left(1 + \exp\left[(r - R_i)/a_i\right]\right)^{-1}$$



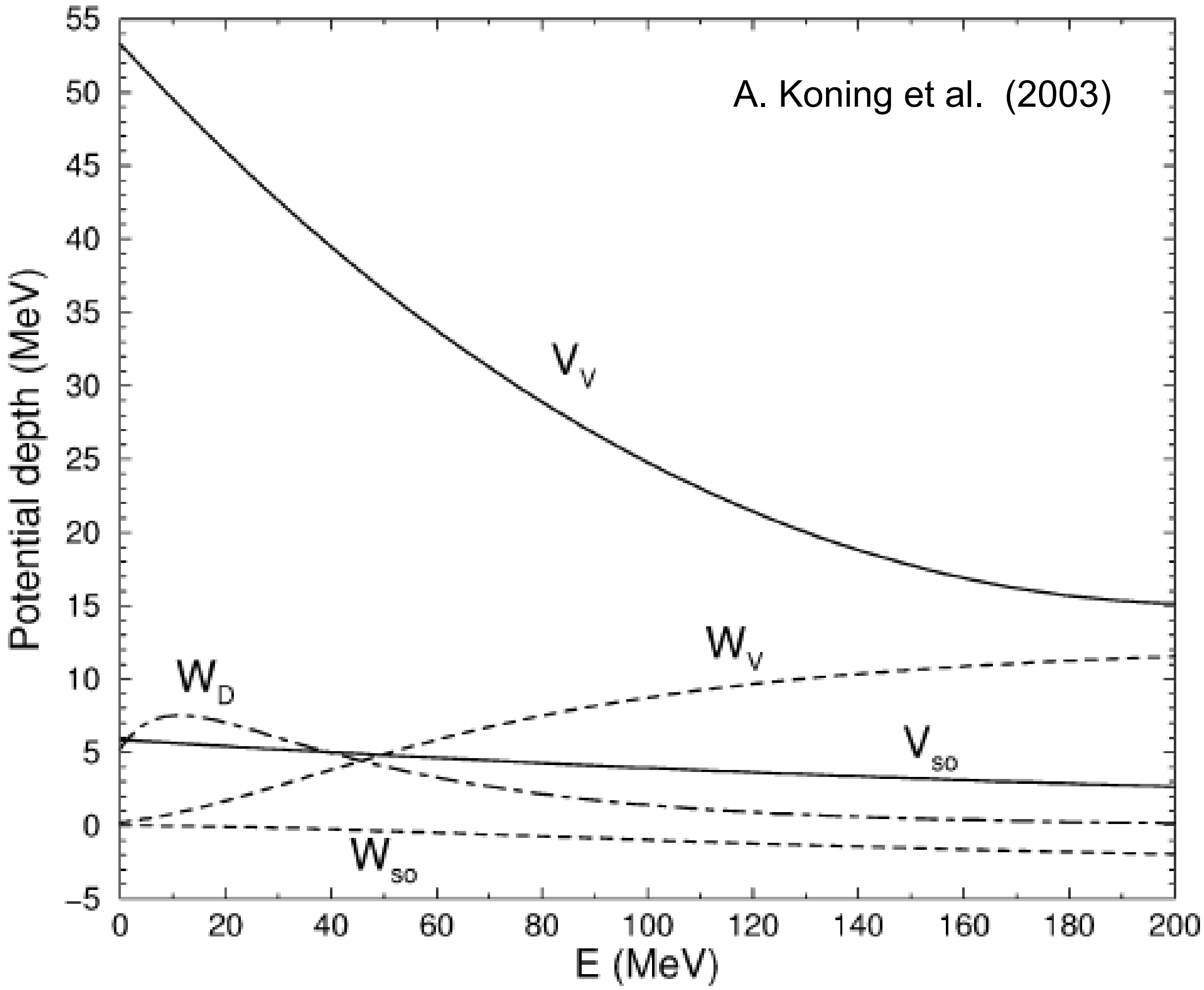
$$V_V(E) = v_1\left[1 - v_2(E - E_f) + v_3(E - E_f)^2 - v_4(E - E_f)^3\right],$$

$$W_V(E) = w_1 \frac{(E - E_f)^2}{(E - E_f)^2 + (w_2)^2},$$

$$r_V = \text{constant},$$

$$a_V = \text{constant},$$

Phenomenological optical potentials



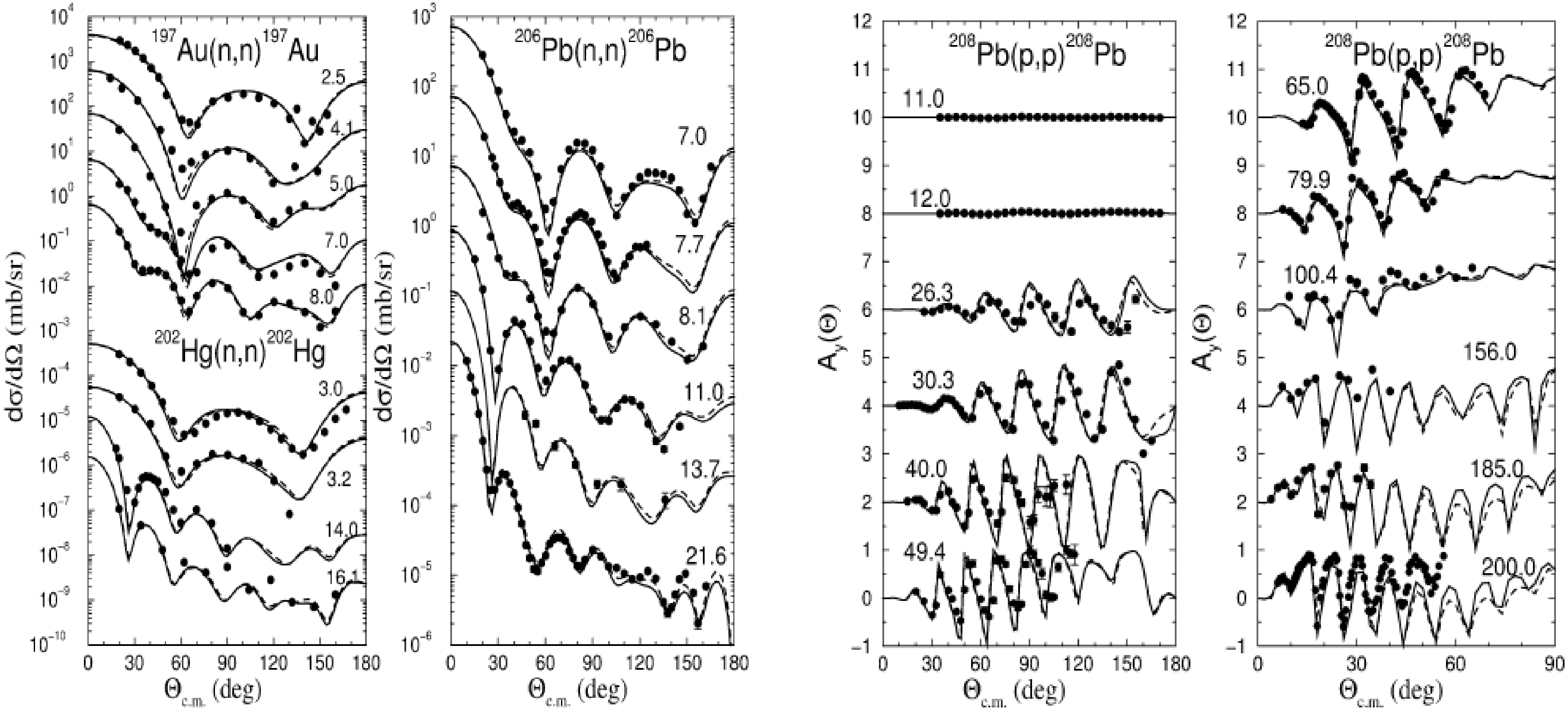
Empirical optical potentials fit to scattering data close to the valley of stability

Extrapolations to reactions on neutron-rich nuclei not well constrained

Microscopic optical potentials have controlled uncertainties

Comparison to scattering observables

Successful description of total cross sections, elastic scattering angular distributions, and analyzing powers

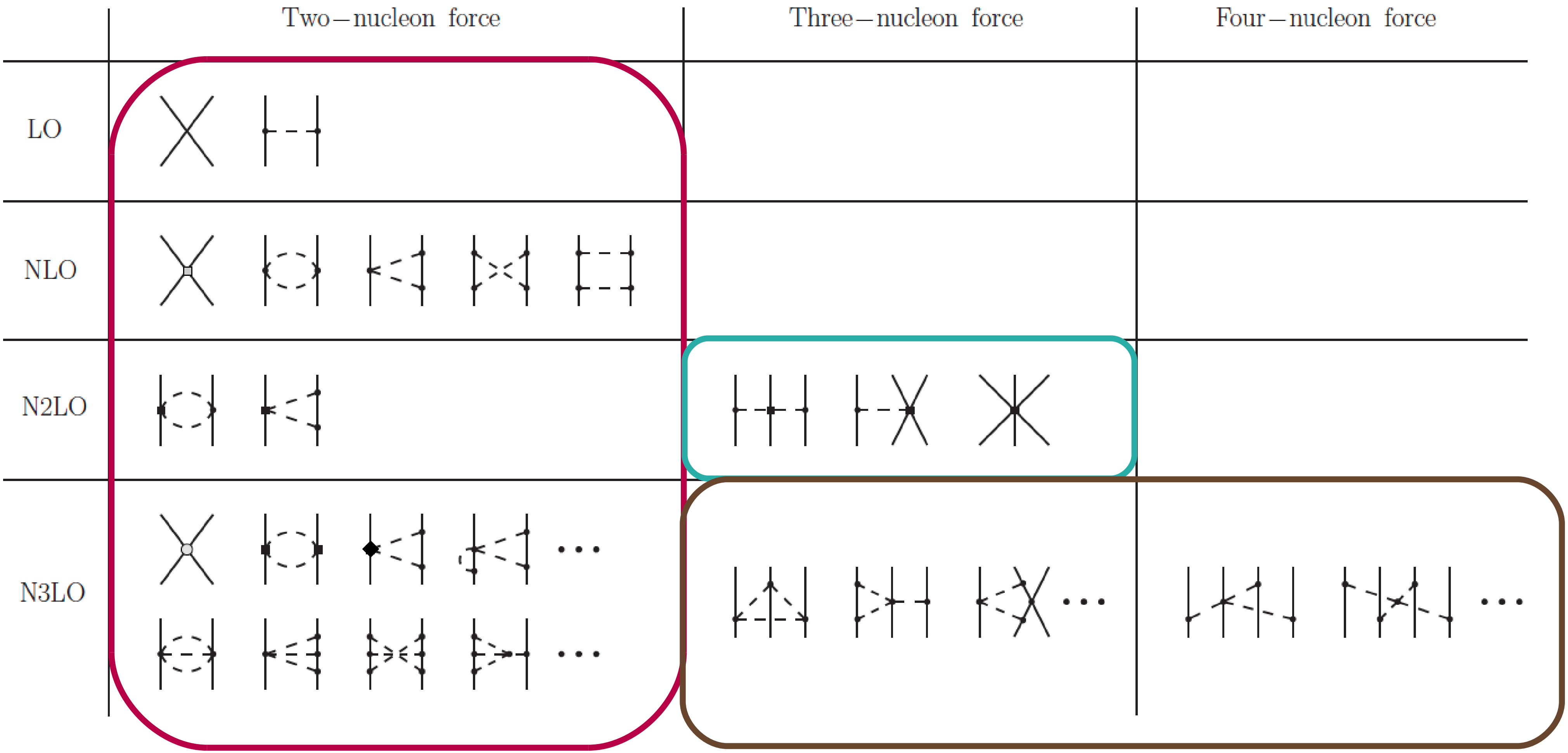


A. Koning et al. (2003)

Microscopic approach: nuclear forces from chiral EFT

SYSTEMATIC EXPANSION in powers of Q/Λ_χ : $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{NN}^{(0)} + \cdots$

$Q = p, m_\pi$



Microscopic optical potentials

Want to reproduce qualitative features of emirical optical potentials

- Depth of real and imaginary parts
- Energy dependence

Optical potential identified with the nucleon self energy $\Sigma(\vec{r}_1, \vec{r}_2, \omega)$ [J. Bell (1959)]

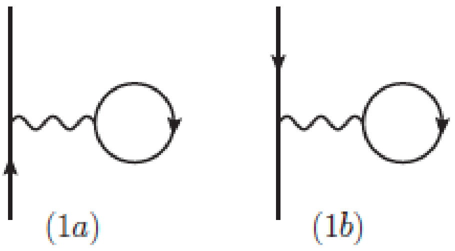
- In general, nonlocal and energy dependent

Translationally-invariant systems $\Sigma(\vec{r}_1, \vec{r}_2, \omega) \to \Sigma(q, \omega)$

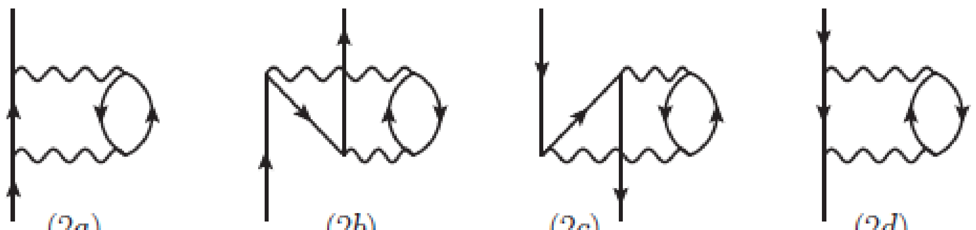
On-shell approximation $U(q, k_f) = \Sigma(q, \omega = q^2/2M; k_f)$

Hartree-Fock contribution is real, energy independent, nonlocal

$$\Sigma_{2N}^{(1)}(q; k_f) = \sum_1 \langle \vec{q}\, \vec{h}_1 s s_1 t t_1 | \bar{V}_{2N} | \vec{q}\, \vec{h}_1 s s_1 t t_1 \rangle n_1$$



Second-order perturbative contributions



$$\Sigma_{2N}^{(2a)}(q,\omega;k_f) = \frac{1}{2}\sum_{123} \frac{|\langle \vec{p}_1 \vec{p}_3 s_1 s_3 t_1 t_3 | \bar{V} | \vec{q}\, \vec{h}_2 s s_2 t t_2 \rangle|^2}{\omega + \epsilon_2 - \epsilon_1 - \epsilon_3 + i\eta} \bar{n}_1 n_2 \bar{n}_3 (2\pi)^3 \delta(\vec{p}_1 + \vec{p}_3 - \vec{q} - \vec{h}_2)$$

$$\Sigma_{2N}^{(2b)}(q,\omega;k_f) = \frac{1}{2}\sum_{123} \frac{|\langle \vec{h}_1 \vec{h}_3 s_1 s_3 t_1 t_3 | \bar{V} | \vec{q}\, \vec{p}_2 s s_2 t t_2 \rangle|^2}{\omega + \epsilon_2 - \epsilon_1 + \epsilon_3 - i\eta} n_1 \bar{n}_2 n_3 (2\pi)^3 \delta(\vec{h}_1 + \vec{h}_3 - \vec{q} - \vec{p}_2)$$

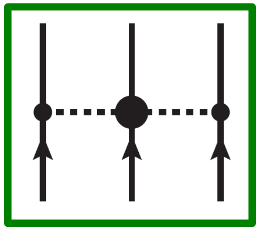
Expressions identical for $\Sigma_{2N}^{(2a)}(q,\omega;k_f)$ and $\Sigma_{2N}^{(2c)}(q,\omega;k_f)$

$\Sigma_{2N}^{(2b)}(q,\omega;k_f)$ and $\Sigma_{2N}^{(2d)}(q,\omega;k_f)$

Second-order contribution is complex, non-local, and energy-dependent

Only $\Sigma_{2N}^{(2a)}(q,\omega;k_f)$ and $\Sigma_{2N}^{(2d)}(q,\omega;k_f)$ are complex

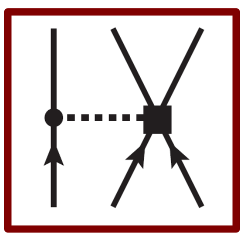
Leading-order chiral three-nucleon force



$$V_{3N}^{(2\pi)} = \sum_{i\neq j\neq k} \frac{g_A^2}{8f_\pi^4} \frac{\vec{\sigma}_i\cdot\vec{q}_i\,\vec{\sigma}_j\cdot\vec{q}_j}{(\vec{q}_i^{\,2}+m_\pi^2)(\vec{q}_j^{\,2}+m_\pi^2)} F_{ijk}^{\alpha\beta}\tau_i^\alpha\tau_j^\beta$$

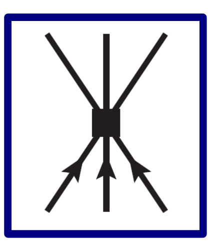
$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}\left(-4c_1 m_\pi^2 + 2c_3\vec{q}_i\cdot\vec{q}_j\right) + c_4\epsilon^{\alpha\beta\gamma}\tau_k^\gamma\vec{\sigma}_k\cdot(\vec{q}_i\times\vec{q}_j)$$

$$c_1 = -0.81,\ c_3 = -3.2,\ c_4 = 5.4\ [\mathrm{GeV}^{-1}]$$



$$V_{3N}^{(1\pi)} = -\sum_{i\neq j\neq k} \frac{g_A c_D}{8f_\pi^4\Lambda_\chi} \frac{\vec{\sigma}_j\cdot\vec{q}_j}{\vec{q}_j^{\,2}+m_\pi^2}\vec{\sigma}_i\cdot\vec{q}_j\,\vec{\tau}_i\cdot\vec{\tau}_j$$

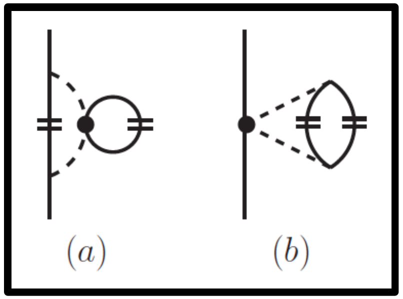
$$c_D(2.5\,\mathrm{fm}^{-1}) = -0.2$$



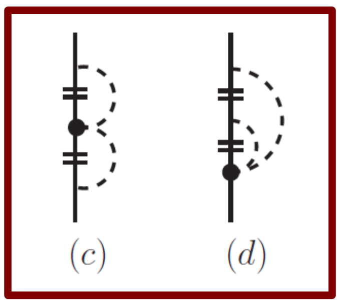
$$V_{3N}^{(\mathrm{ct})} = \sum_{i\neq j\neq k} \frac{c_E}{2f_\pi^4\Lambda_\chi}\vec{\tau}_i\cdot\vec{\tau}_j$$

$$c_E(2.5\,\mathrm{fm}^{-1}) = -0.205$$

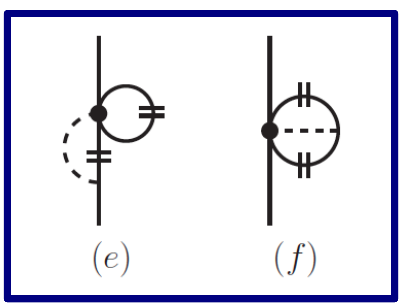
Hartree-Fock contribution from three-body forces



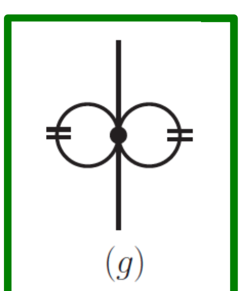
$$U(q,k_f) = \frac{g_A^2 m_\pi^6}{(2\pi f_\pi)^4}\Big\{14(c_3-c_1)u^4 + (3c_1-2c_3)u^2 - 4c_3u^6 + (12c_1-10c_3)u^3 \times \Big[\arctan 2u + \arctan(u+x) + \arctan(u-x)\Big] + \Big[\frac{c_3}{2}(1+9u^2) - \frac{3c_1}{4}(1+8u^2)\Big] \times \ln(1+4u^2) + \frac{u^3}{x}\Big[3c_3 - 4c_1 + 2(c_1-c_3)(x^2-u^2)\Big]\ln\frac{1+(u+x)^2}{1+(u-x)^2}\Big\}$$



$$U(q,k_f) = \frac{g_A^2 m_\pi^6}{(4\pi f_\pi)^4 x^2}\Big\{3c_1 H^2(x,u) + \Big(\frac{c_3}{2}-c_4\Big)G_S^2(x,u) + (c_3+c_4)G_T^2(x,u) + \int_0^u d\xi\Big[6c_1 H(\xi,u)\frac{\partial H(\xi,x)}{\partial x} + (c_3-2c_4)G_S(\xi,u)\frac{\partial G_S(\xi,x)}{\partial x} + 2(c_3+c_4)G_T(\xi,u)\frac{\partial G_T(\xi,x)}{\partial x}\Big]\Big\}$$

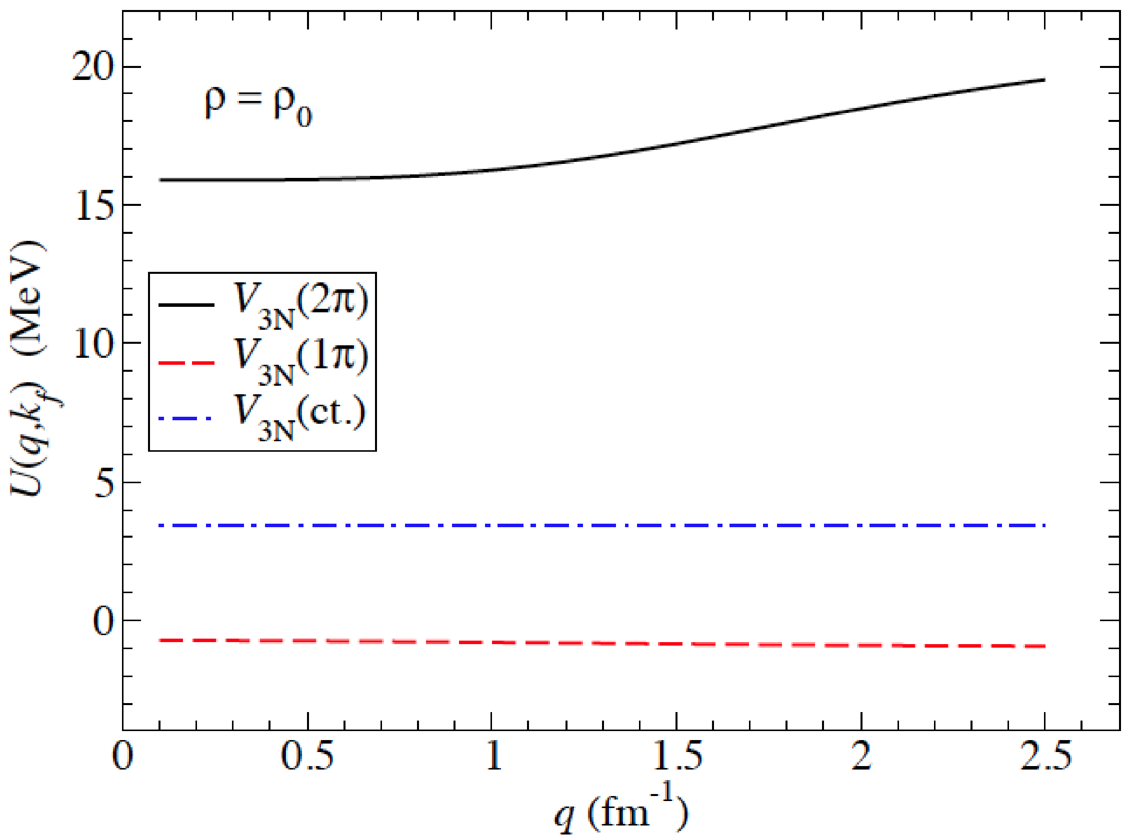


$$U(q,k_f) = \frac{g_A c_D m_\pi^6}{(2\pi f_\pi)^4 \Lambda_\chi}\Big\{u^6 - \frac{7u^4}{4} + \frac{u^2}{8} - \frac{1+12u^2}{32}\ln(1+4u^2) + u^3\Big[\arctan 2u + \arctan(u+x) + \arctan(u-x)\Big] + \frac{u^3}{4x}(x^2-u^2-1)\ln\frac{1+(u+x)^2}{1+(u-x)^2}\Big\}$$



$$U(q,k_f) = -\frac{c_E k_f^6}{4\pi^4 f_\pi^4 \Lambda_\chi}$$

Momentum dependence of 3NF contributions

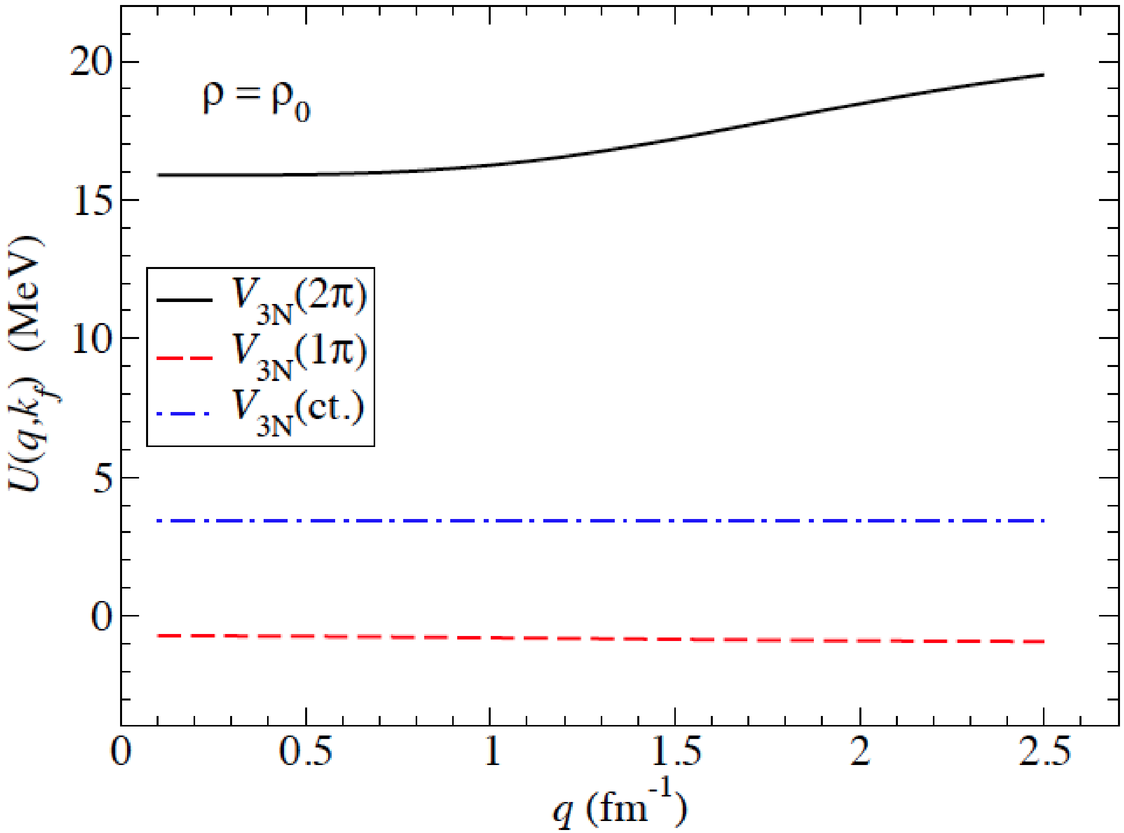


Most of the repulsive strength arises from the two-pion exchange 3NF

Term proportional to c_D nearly independent of the momentum

Strong correlation between c_D and c_E

Momentum dependence of 3NF contributions



Equivalent 3NF mean field

$$c_E = \alpha \cdot c_D + \text{const.}$$

$$\alpha = 0.21 \pm 0.02$$

Most of the repulsive strength arises from the two-pion exchange 3NF

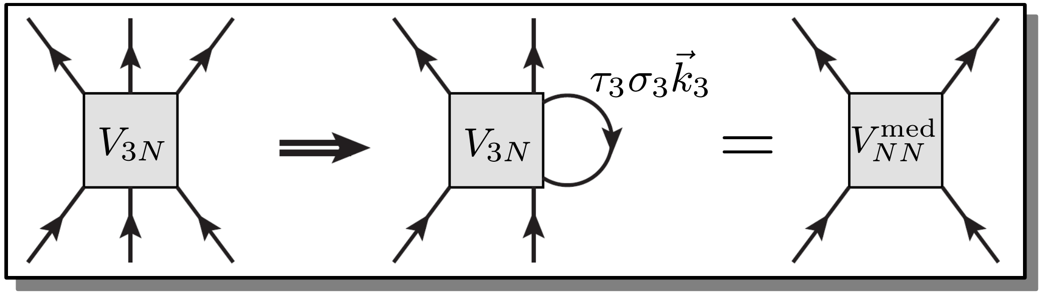
Term proportional to c_D nearly independent of the momentum

Strong correlation between c_D and c_E

Higher-order perturbative contributions: in-medium NN interactions

Free fermi gas reference state $\rho = 2k_f^3/3\pi^2$

Sum over occupied states in the Fermi sea



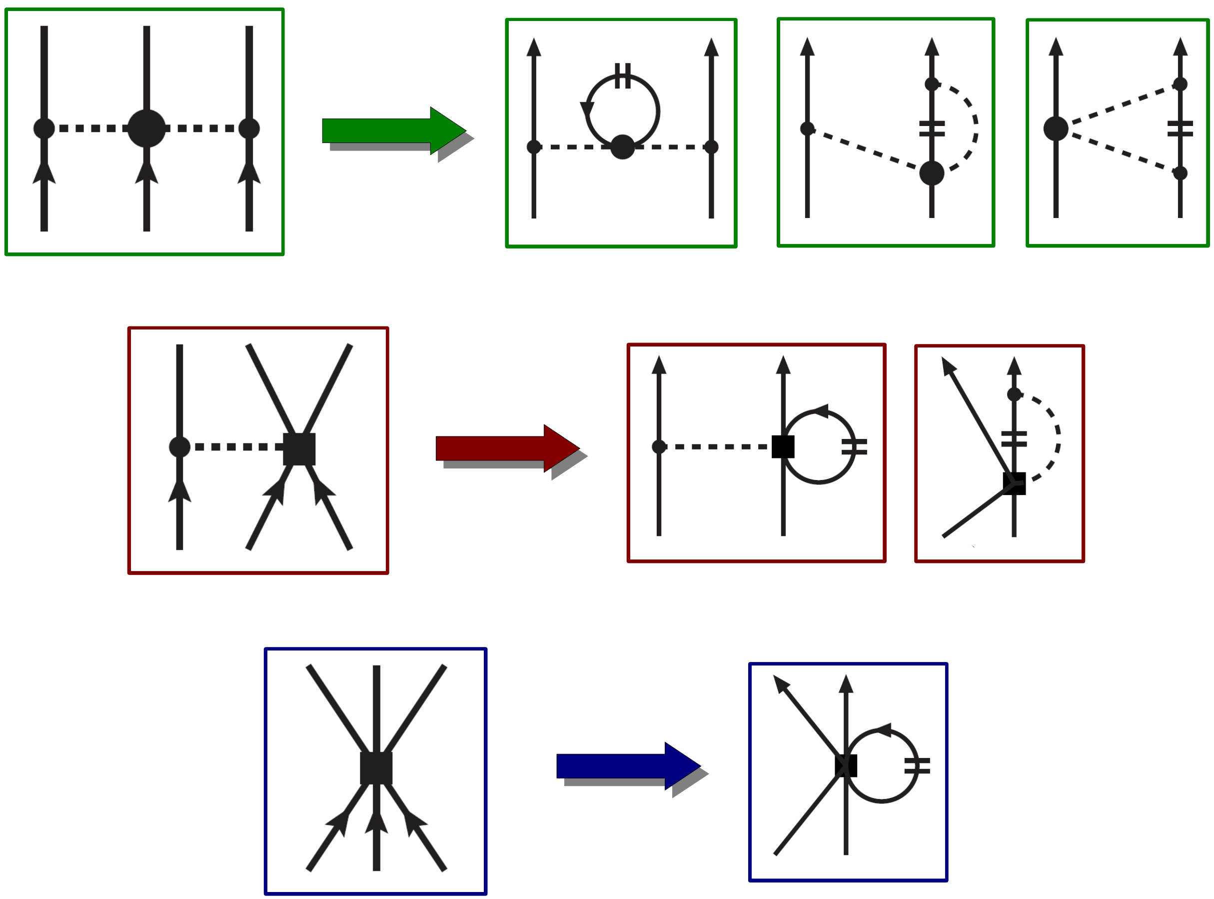
Approximation: On-shell scattering in CM frame $N(\vec{p}) + N(-\vec{p}) \to N(\vec{p}+\vec{q}) + N(-\vec{p}-\vec{q})$

$$V(\vec{p},\vec{q}) = V_C + \vec{\tau}_1 \cdot \vec{\tau}_2 \, W_C + [V_S + \vec{\tau}_1 \cdot \vec{\tau}_2 \, W_S] \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 + [V_T + \vec{\tau}_1 \cdot \vec{\tau}_2 \, W_T] \, \vec{\sigma}_1 \cdot \vec{q} \, \vec{\sigma}_2 \cdot \vec{q}$$
$$+ [V_{SO} + \vec{\tau}_1 \cdot \vec{\tau}_2 \, W_{SO}] \; i(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{q} \times \vec{p}) + [V_Q + \vec{\tau}_1 \cdot \vec{\tau}_2 \, W_Q] \; \vec{\sigma}_1 \cdot (\vec{q} \times \vec{p}) \, \vec{\sigma}_2 \cdot (\vec{q} \times \vec{p})$$

form is same as free-space NN interaction

Density-dependent NN interactions

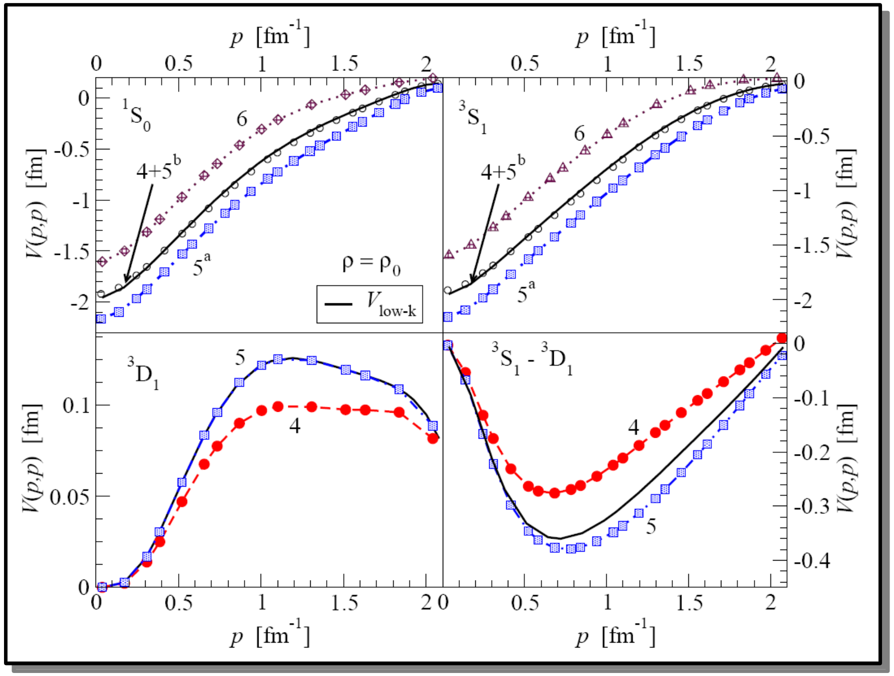
Holt, Kaiser, Weise (2009)

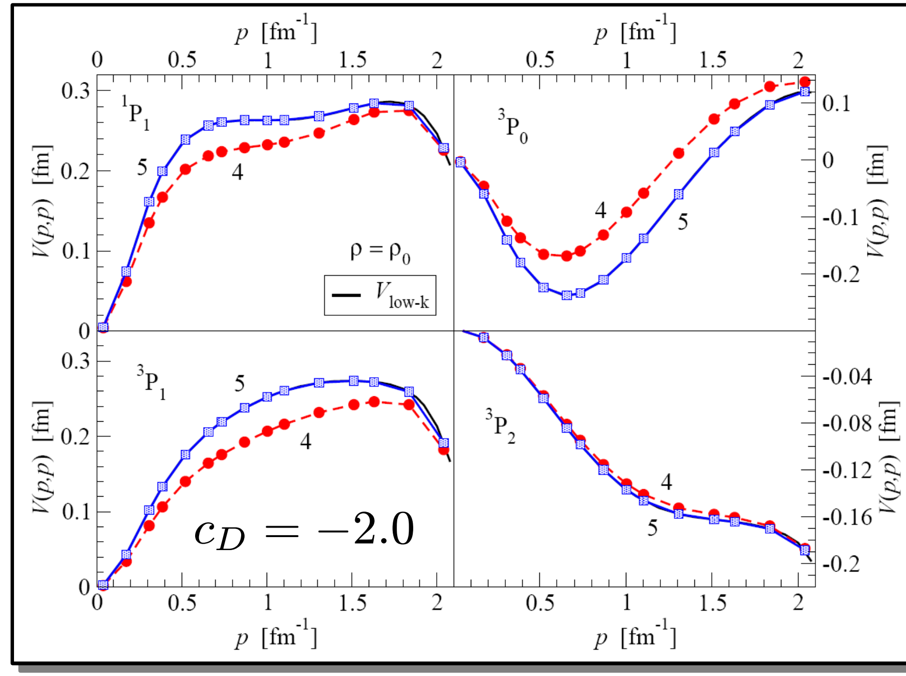


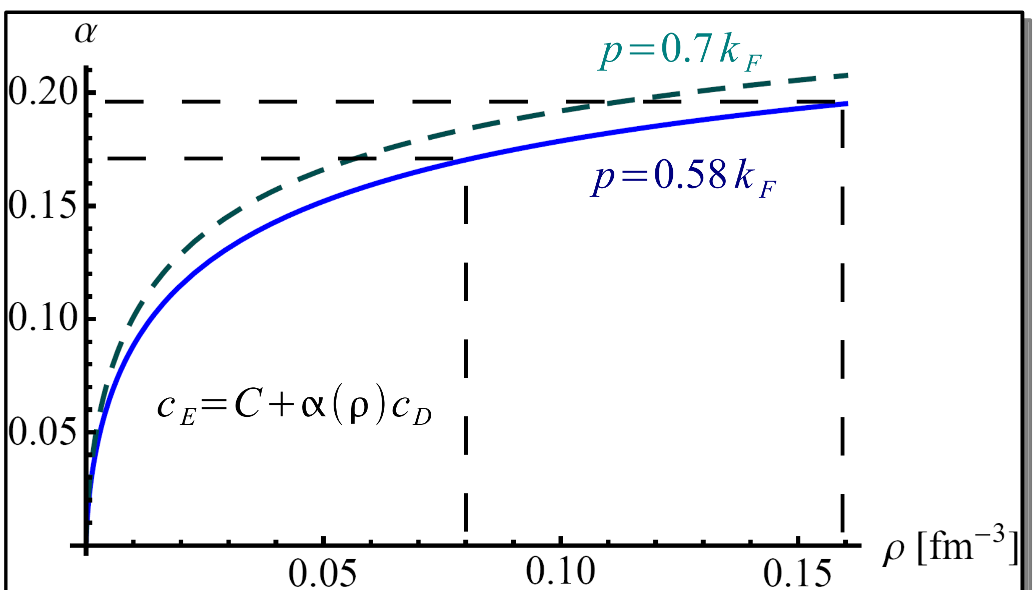
Approximates the exact Hartree-Fock results to within ~ 5%

Evaluate in second-order diagrams with two-body forces

Correlation between c_D and c_E







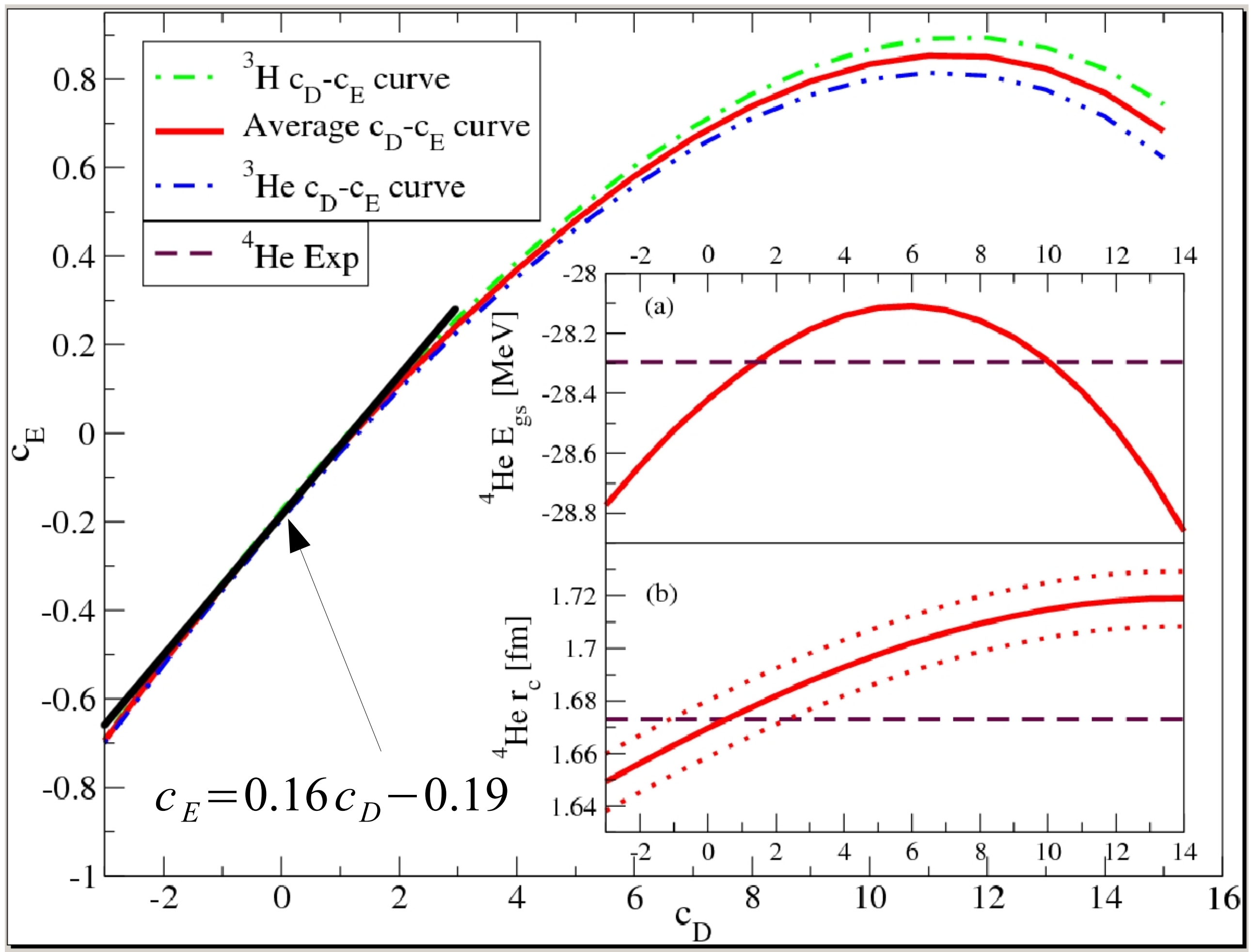
$$\alpha(p, \rho) = \frac{g_a}{8}\left(2 - \frac{2m_\pi^2\Gamma_0 + \Gamma_2}{\pi^2\rho}\right)$$

Weak dependence on $\boldsymbol{k_F}$ and $\boldsymbol{p}$

$$\frac{\rho_0}{2} \leq \rho \leq \rho_0 \longrightarrow 0.17 \leq \alpha \leq 0.20$$

Correlation between c_D and c_E

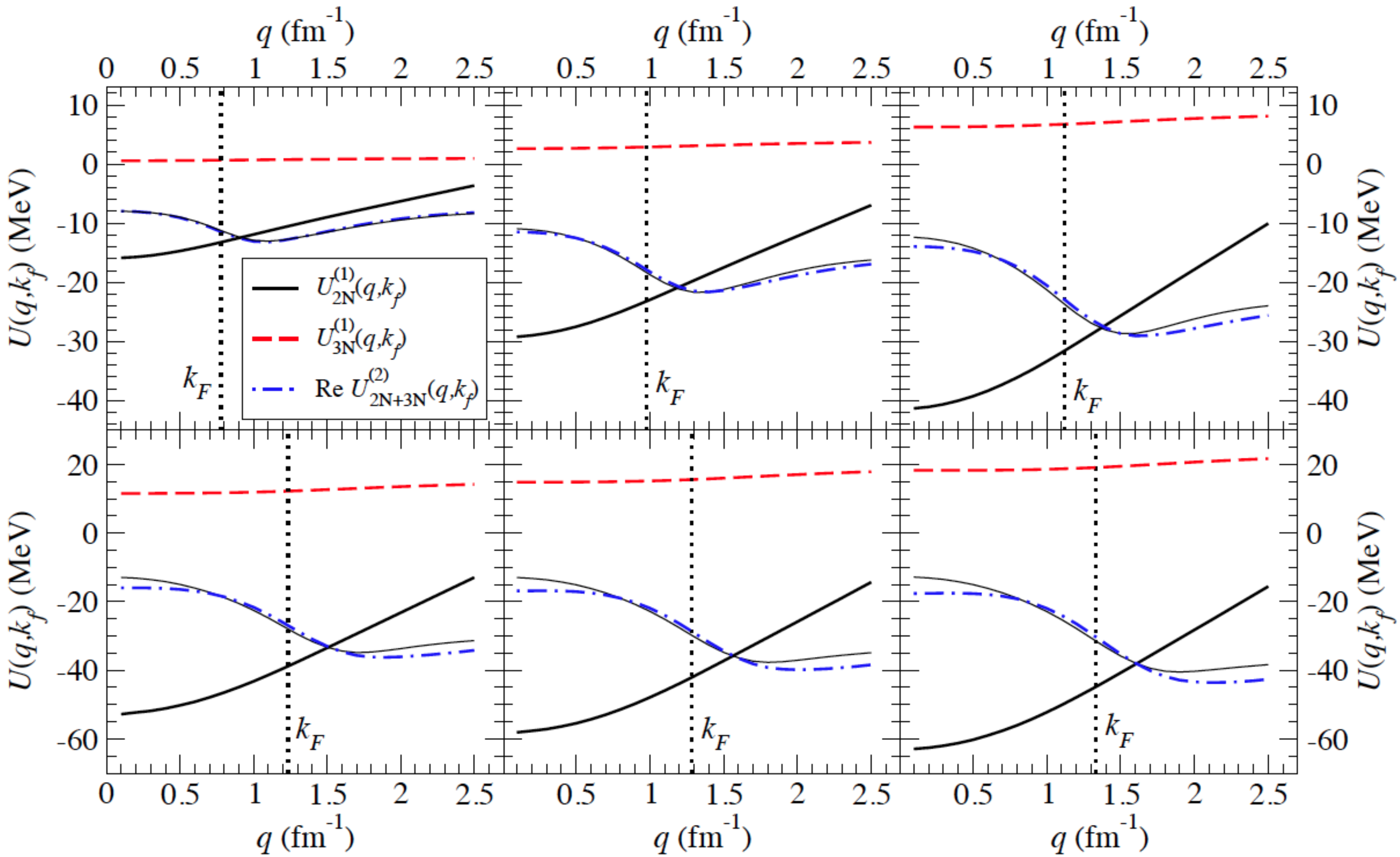
P. Navrátil et al., PRL (2007)



$$c_E = C + \alpha(\rho) c_D$$

$$0.17 \leq \alpha \leq 0.20$$

Contributions to real part of nucleon self-energy

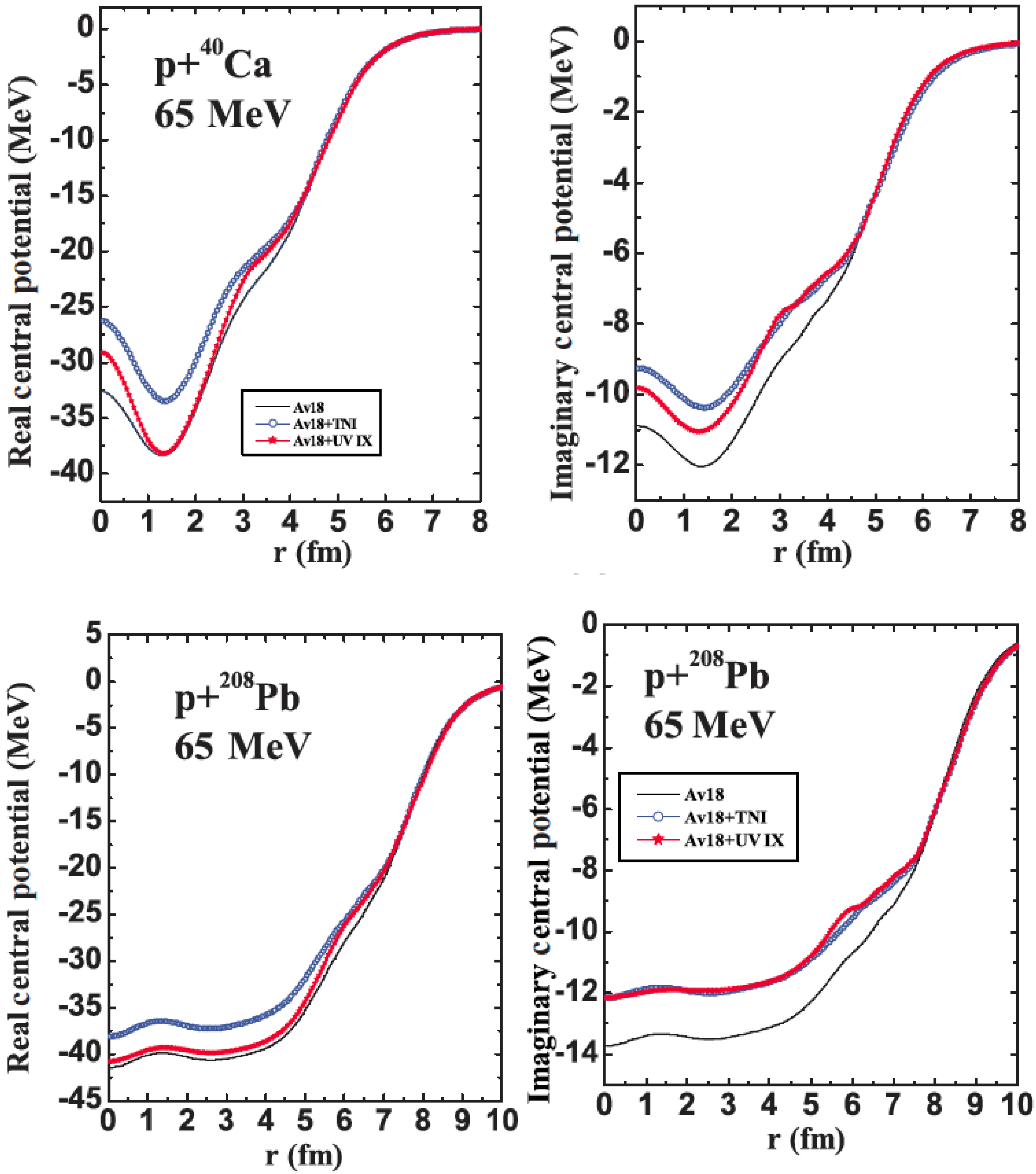


Contributions from 3NF increase almost linearly with the nuclear density

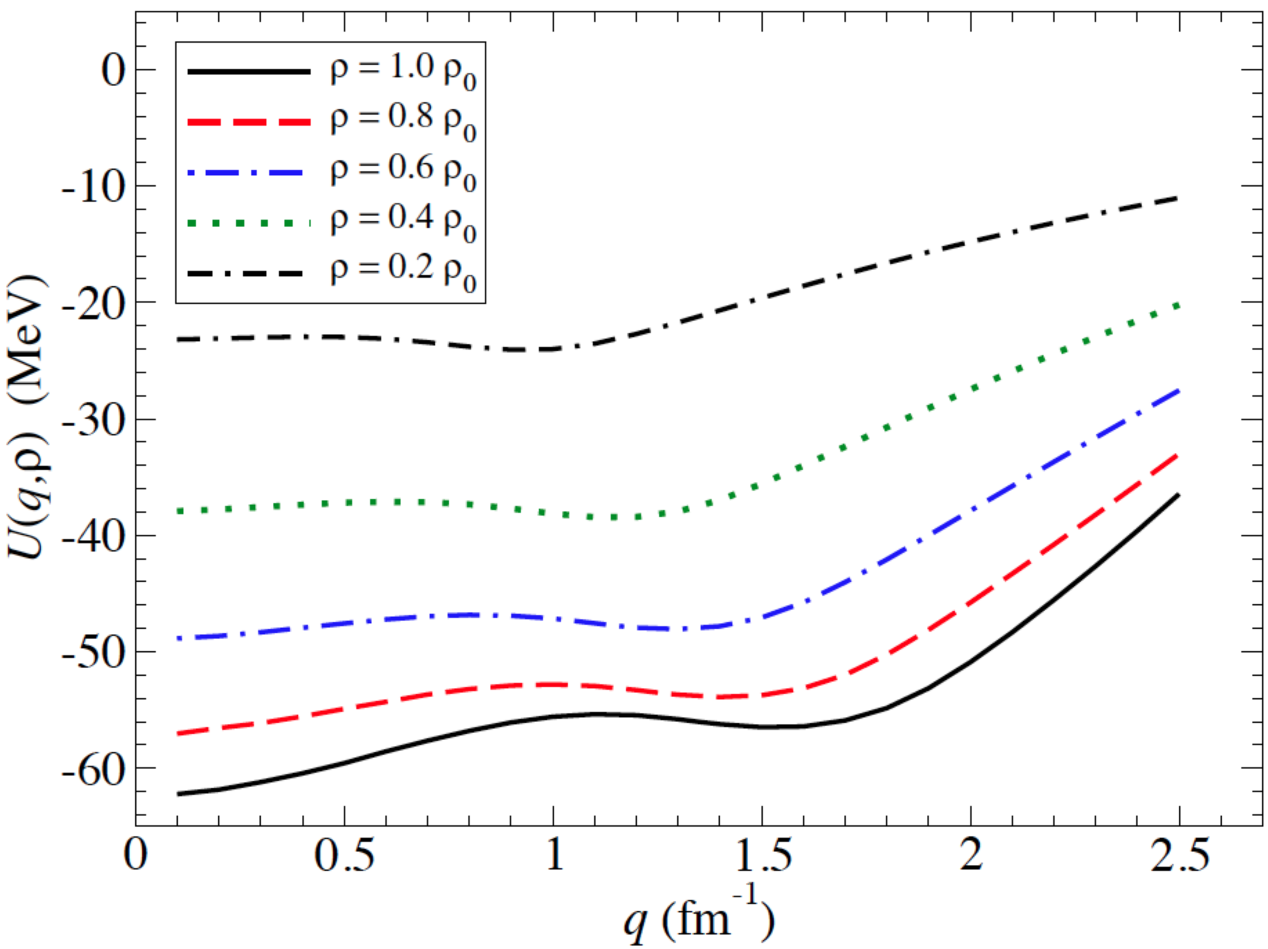
Three-body forces at second order in perturbation theory weakly attractive

Comparison to previous literature

Rafi et al. (2013)



Total real part of optical potential

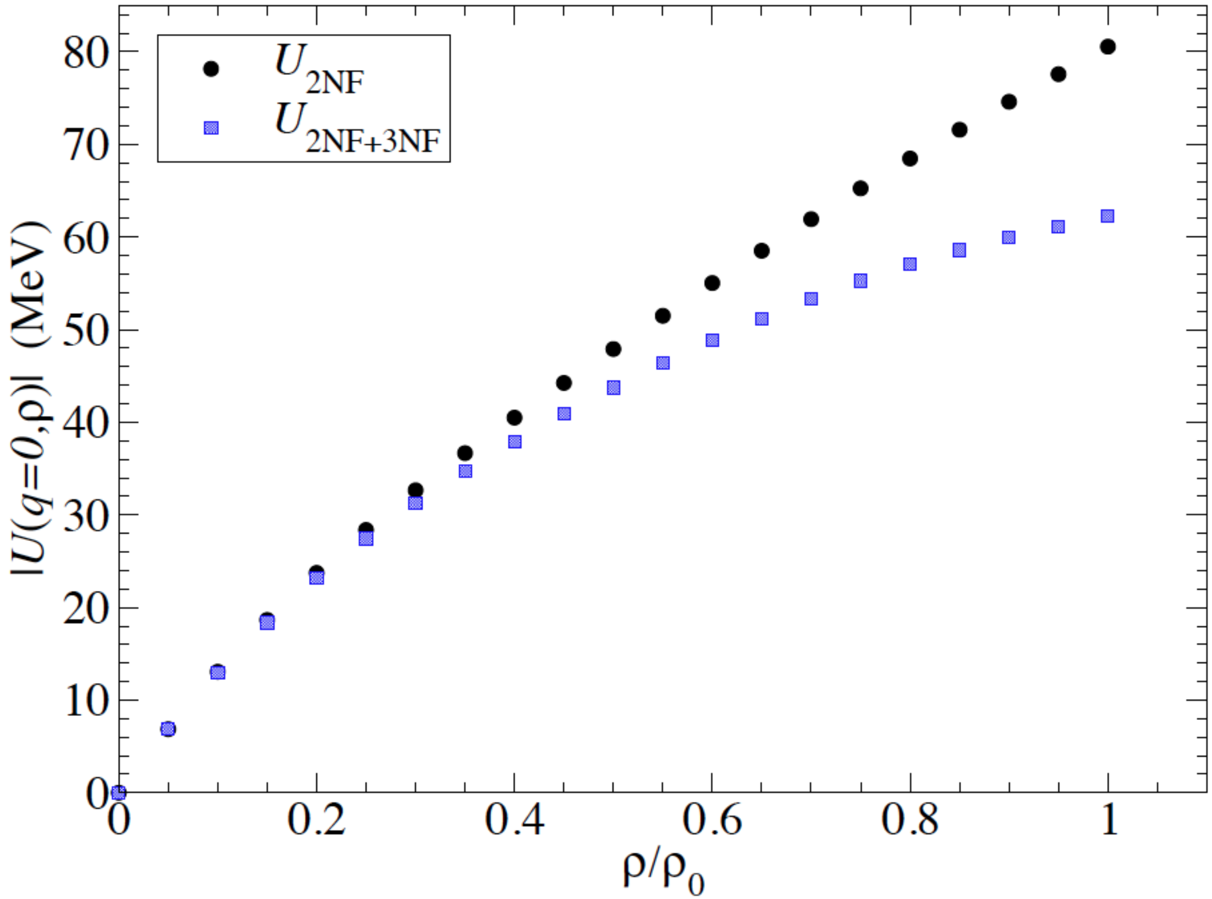


For momenta $q < k_f$ the nuclear mean field is essentially constant (small densities)

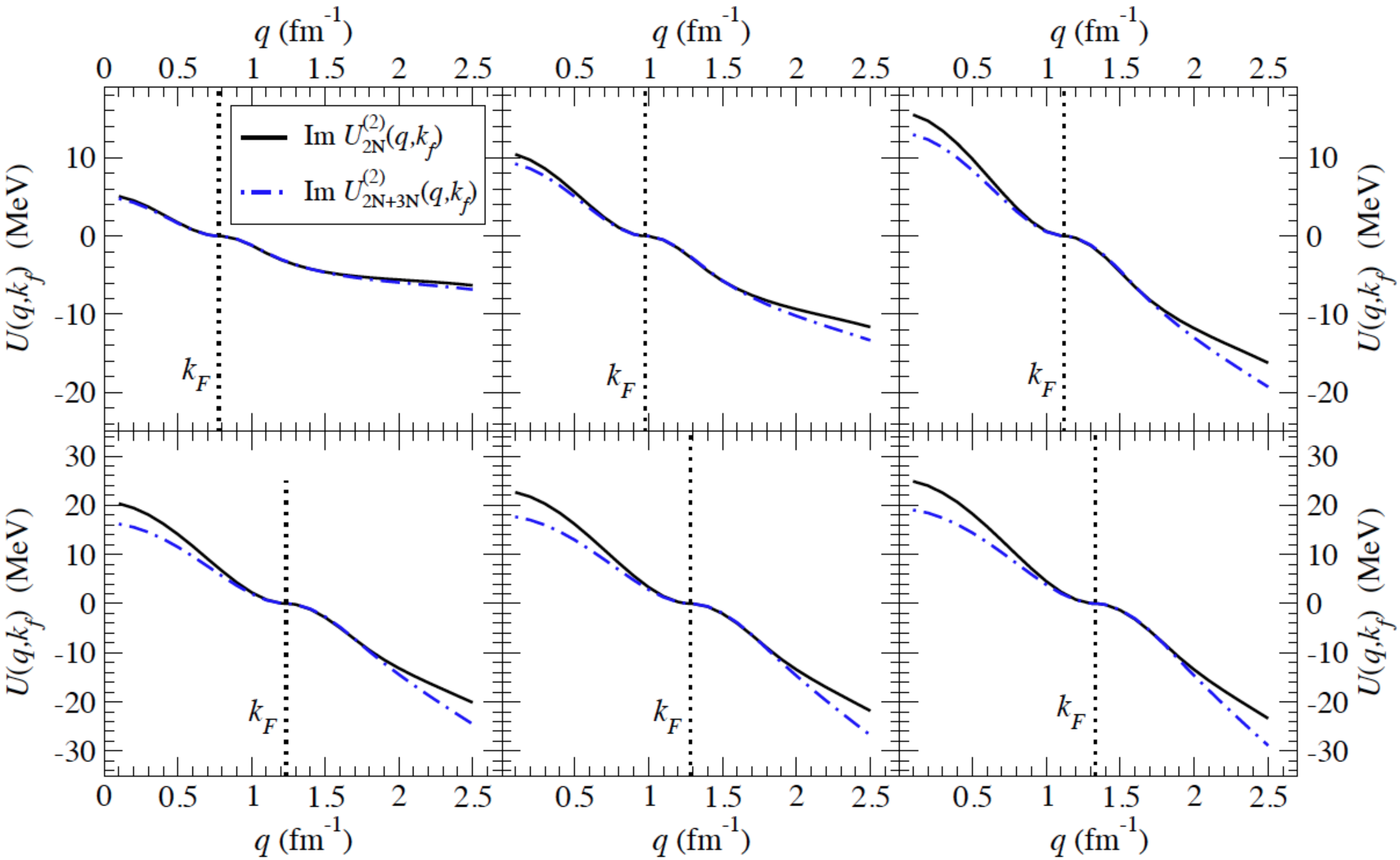
Hugenholtz—Van-Hove theorem: $(E/A)_{\rm sat} \simeq -20\,{\rm MeV}$ (nuclear matter overbound)

Attractive potential well too deep $U_0^{mc} = 57\,{\rm MeV}$ vs. $U_0^{ph} = 52\,{\rm MeV}$

Density dependence of 3NF contribution



Imaginary part of optical potential



Neary symmetric about k_f (feature assumed in dispersion optical model treatments)

Absorptive potential well too deep $W_0^{mc} = 29\,\mathrm{MeV}$ vs. $W_0^{ph} = 10 - 12\,\mathrm{MeV}$

Future extensions

Finite nuclei through local density approximation:

- requires point nucleon densities

$$U_{LDA}(r,E) = U_{NM}(q,k_f(r))$$

Self-consistent single-particle energies:

$$\epsilon_q = \frac{q^2}{2M_N} + \mathrm{Re}\,\Sigma(q,\epsilon_q)$$

- reduces both the real and imaginary parts of the optical potential
- Imaginary part receives additional reduction factor [J. Negele (1981)]

$$W_{ph}(q,E) = \left(1 + \frac{M_N}{q}\frac{\partial U}{\partial q}\right)^{-1} W_{mc}(q,E)$$

Isospin asymmetric nuclear matter k_f^p, k_f^n

Future extensions

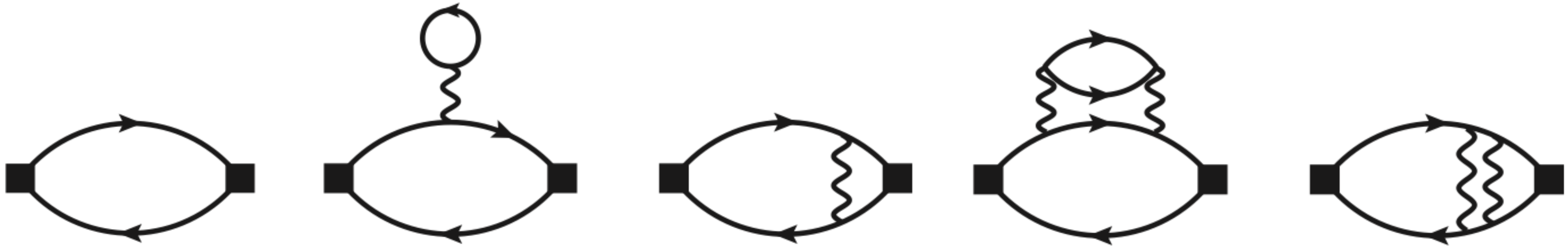
Dynamics of stellar core collapse and evolution of residual neutron star

- sensitive to neutrino processes (scattering, absorption and production)

Neutrino mean free path:

$$\frac{1}{\lambda(\vec{k}_i, T)} = \frac{G_F^2}{32\pi^3} \int d^3k_f [(1+\cos\theta) S^{(0)}(\omega, \vec{q}, T) + g_A^2 (3-\cos\theta) S^{(1)}(\omega, \vec{q}, T)]$$

Structure factors $S^{(S)}(\omega, \vec{q}, T)$ given by the imaginary part of response function



E. Rrapaj, JWH, and S. Reddy, in prog.

Summary/Outlook

Understand heavy element formation in ***r*-process nucleosynthesis**

Connection to current and future rare-isotope reactions: (*d,p*)

Microscopic optical model potential from chiral two- and three-nucleon forces

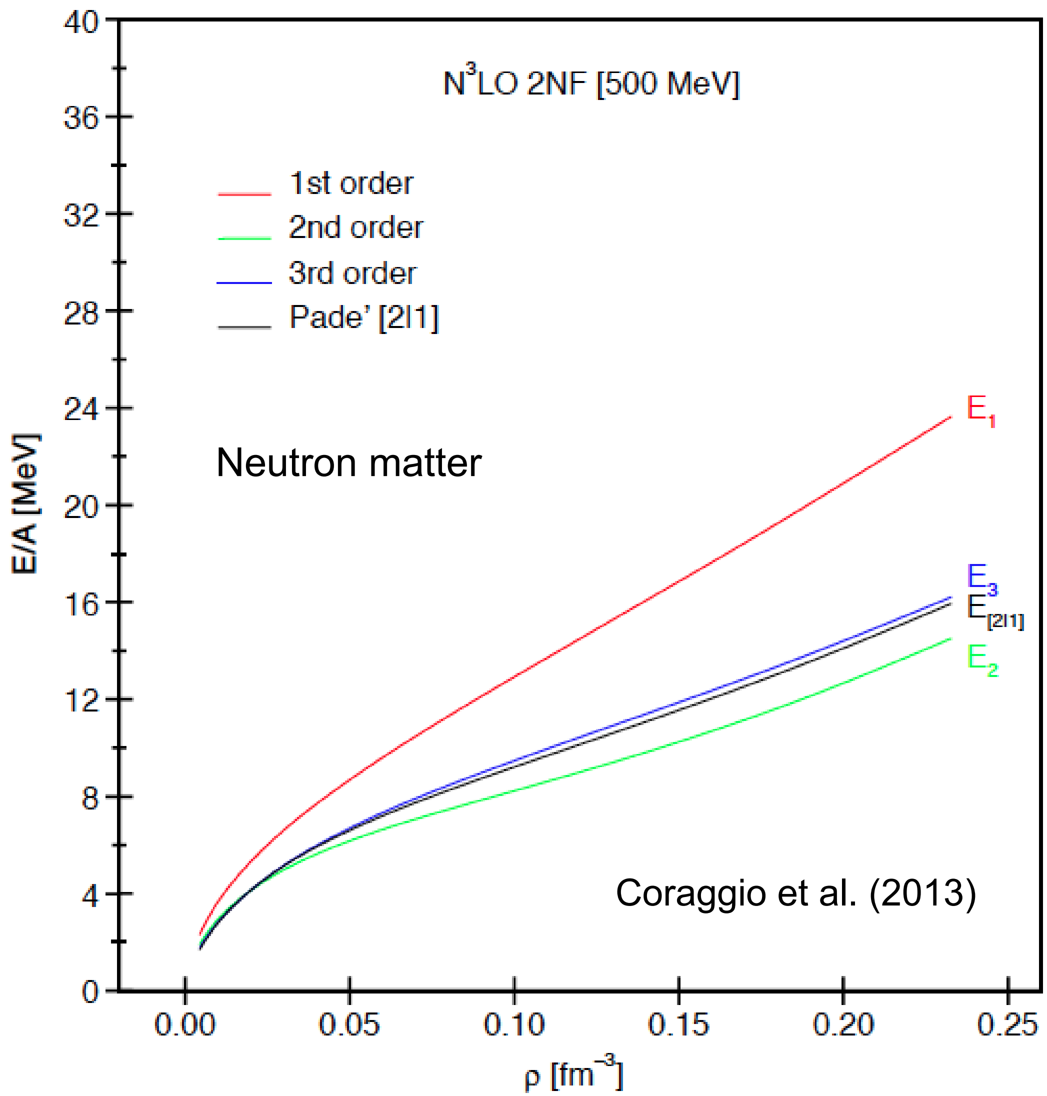
FUTURE PROJECTS

Include corrections due to **isospin asymmetry**

Extend to **finite nuclei**

Neutron matter response functions

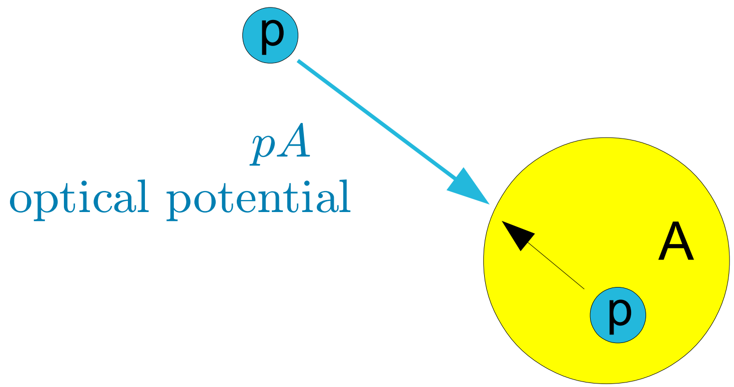
Convergence in many-body perturbation theory



- Good convergence properties of chiral nuclear interactions with Λ ≤ 500 MeV

Range of validity

- Chiral nuclear interactions with $\Lambda \leq 500$ MeV are perturbative
- Constrains the allowed projectile energies that can be described



- Maximum momentum of nucleons inside nucleus

$$k_f \simeq 250\,\mathrm{MeV}$$

- Chiral potential can describe nucleon-nucleus scattering up to

$$E_{\mathrm{max}} \simeq 250\,\mathrm{MeV}$$