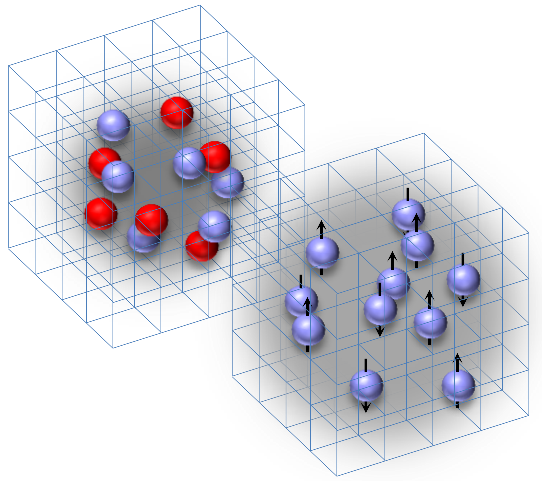
Nuclear structure and excitations from lattice effective field theory



Nuclear Lattice EFT Collaboration

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From Few-Nucleon Forces to Many-Nucleon Structures

ECT* and HIC for FAIR Workshop, Trento
June 11, 2013

Outline

What is lattice effective field theory?

Carbon-12 spectrum and the Hoyle state

Light quark mass dependence of helium burning

Ab initio lattice calculations up to A = 28

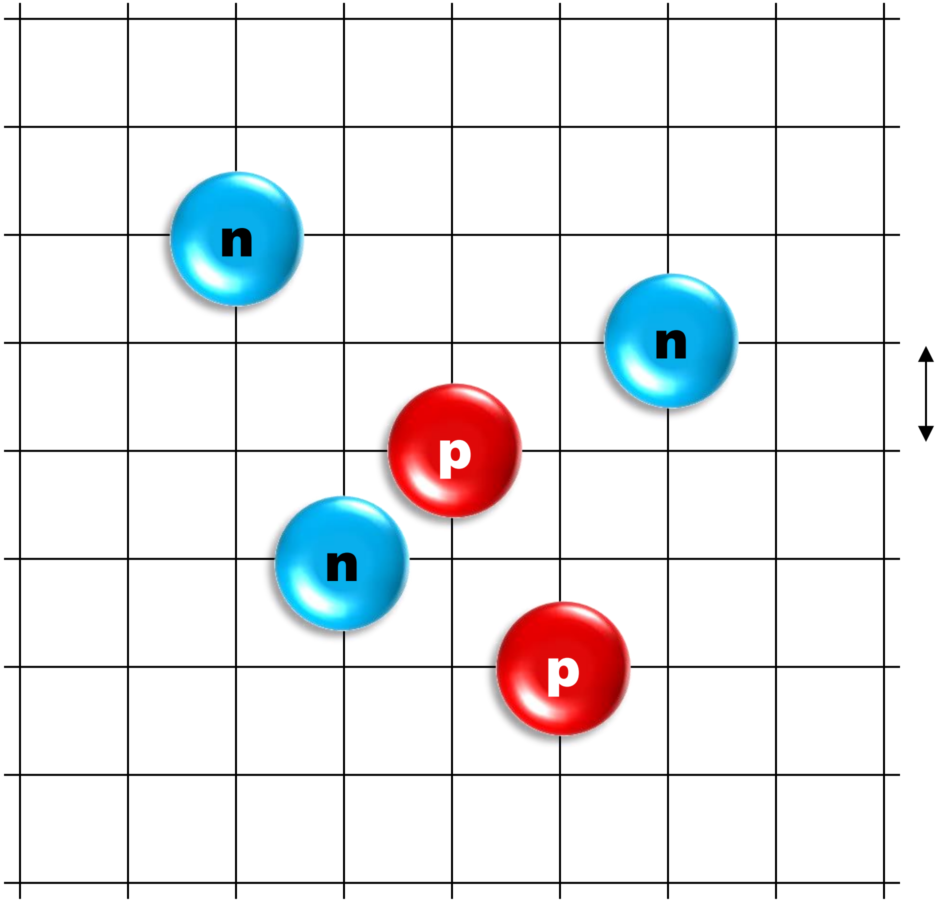
Oxygen-16 structure and spectrum

Properties of neutron matter

Scattering and reactions on the lattice

Summary and future directions

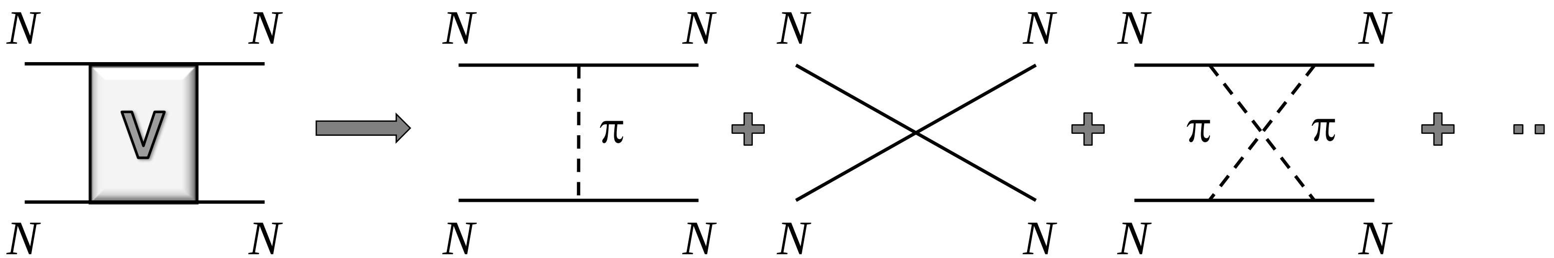
Lattice effective field theory



$a \sim 1 - 5 \text{ fm}$

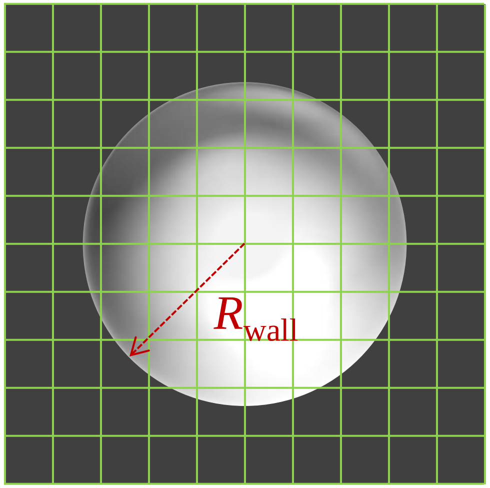
Low energy nucleons: Chiral effective field theory

Construct the effective potential order by order



$$\underbrace{V^{\mathrm{OPEP}} \quad \text{Contact interactions}}_{\text{Leading order (LO)}} \qquad \underbrace{V^{\mathrm{TPEP}}}_{\text{Next-to-leading order (NLO)}}$$

Physical scattering data → Unknown operator coefficients

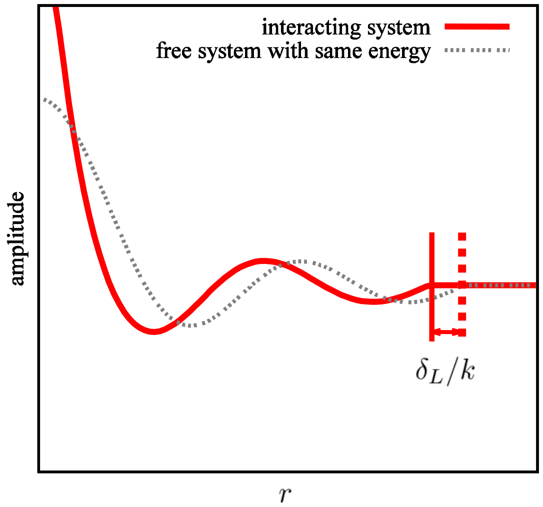


Spherical wall method

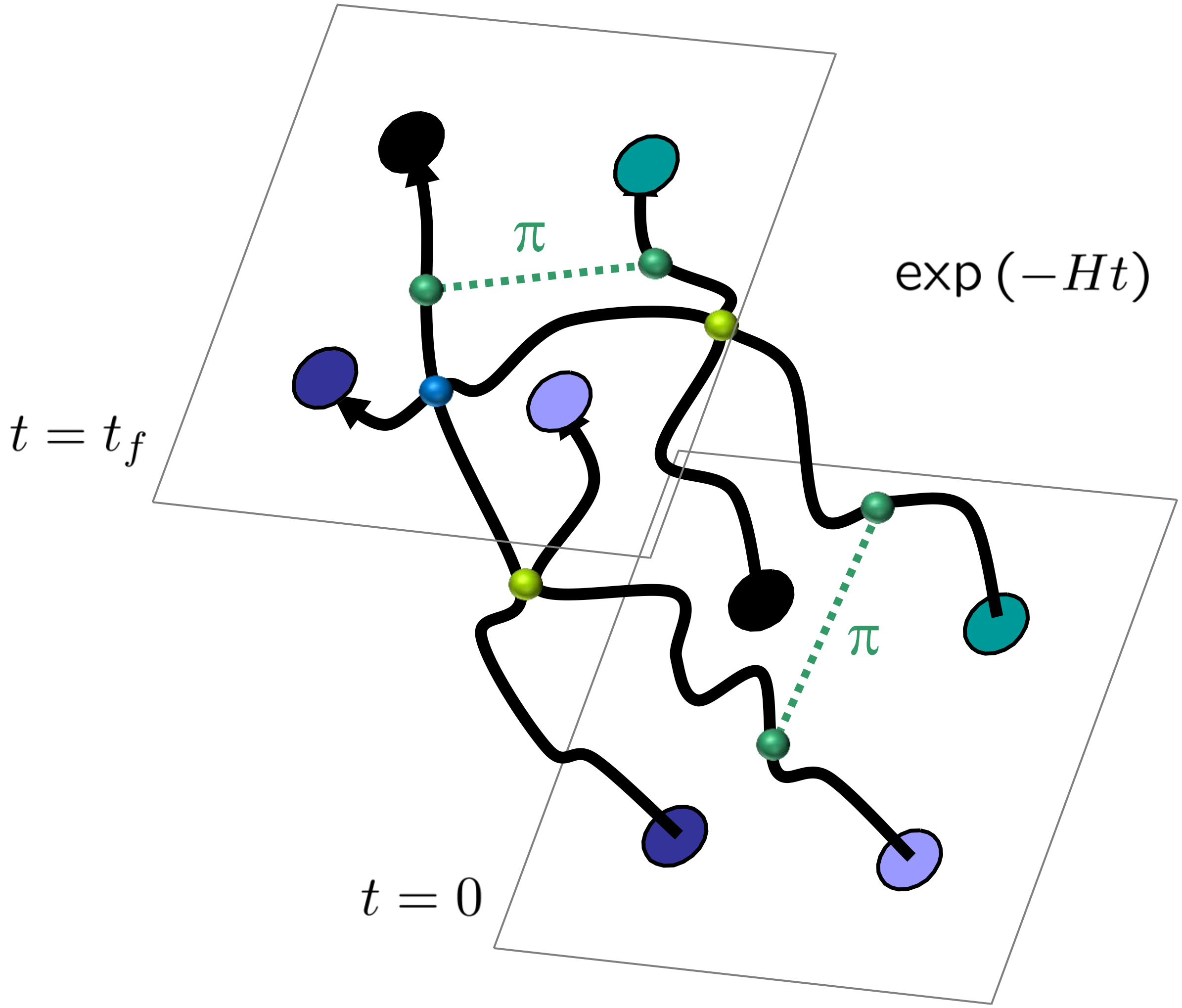
Borasoy, Epelbaum, Krebs, D.L., Meißner,
EPJA 34 (2007) 185

Spherical wall imposed in the center of mass frame

Representation	J_z	Example
A_1	$0 \bmod 4$	$Y_{0,0}$
T_1	$0, 1, 3 \bmod 4$	$\{Y_{1,0}, Y_{1,1}, Y_{1,-1}\}$
E	$0, 2 \bmod 4$	$\left\{Y_{2,0}, \frac{Y_{2,-2}+Y_{2,2}}{\sqrt{2}}\right\}$
T_2	$1, 2, 3 \bmod 4$	$\left\{Y_{2,1}, \frac{Y_{2,-2}-Y_{2,2}}{\sqrt{2}}, Y_{2,-1}\right\}$
A_2	$2 \bmod 4$	$\frac{Y_{3,2}-Y_{3,-2}}{\sqrt{2}}$



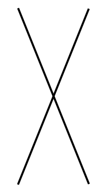
Euclidean time projection



Auxiliary field method

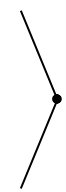
We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^\dagger N)^2\right]$$



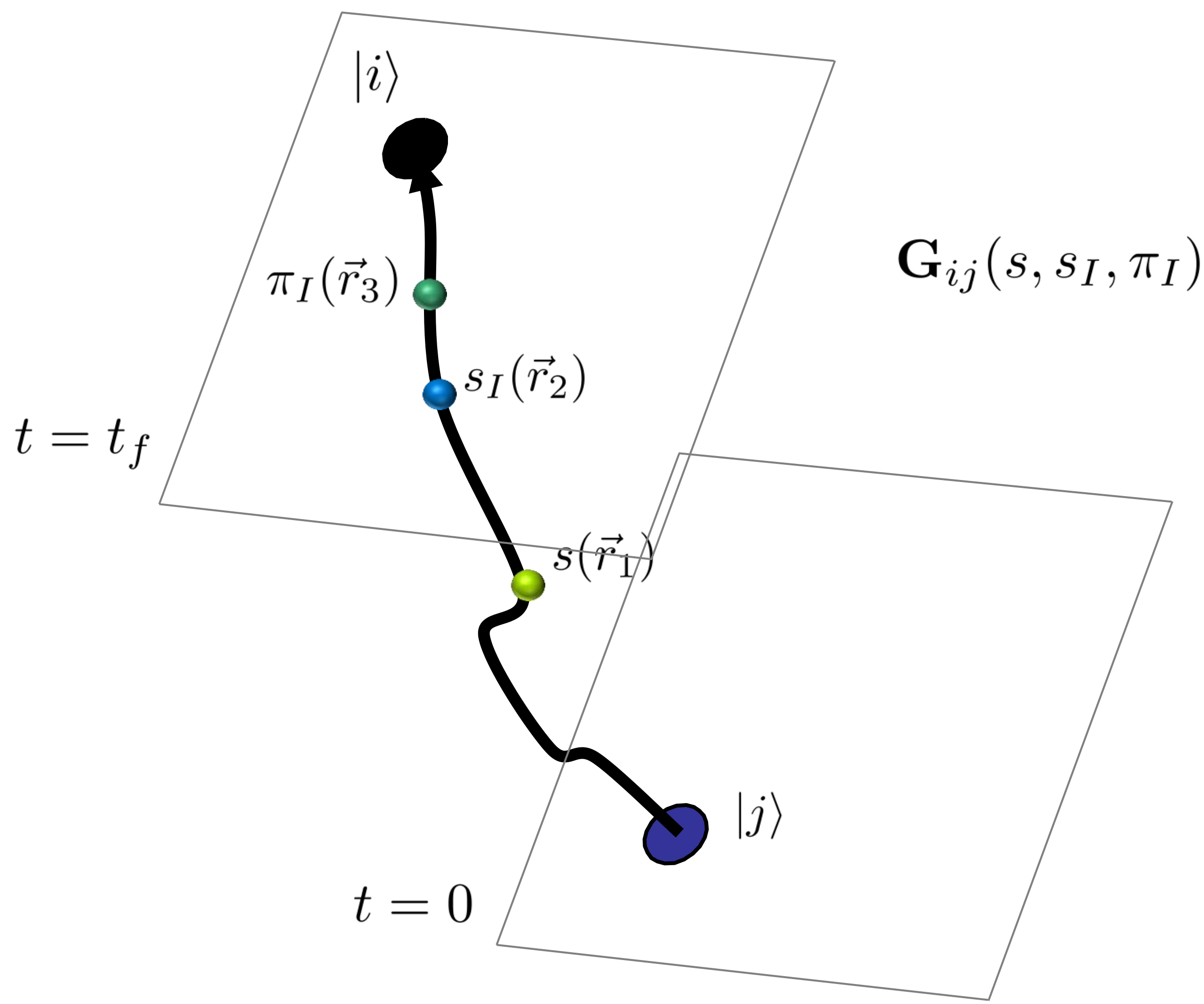
$(N^\dagger N)^2$

$$= \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} ds \exp\left[-\frac{1}{2}s^2 + \sqrt{-C}\, s(N^\dagger N)\right]$$

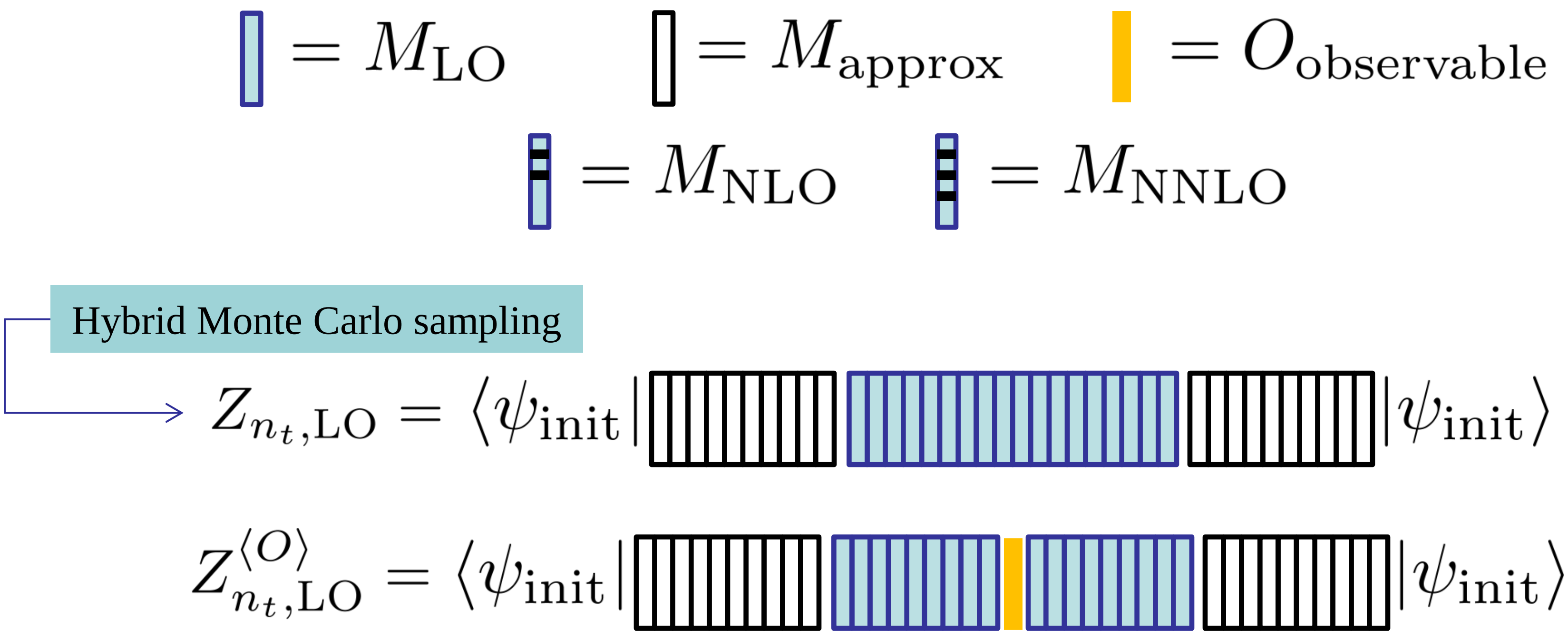


$sN^\dagger N$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.

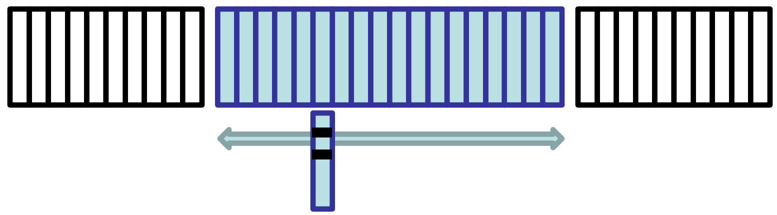

$|i\rangle$
$\mathbf{G}_{ij}(s, s_I, \pi_I)$
$\pi_I(\vec{r}_3)$
$s_I(\vec{r}_2)$
$t = t_f$
$s(\vec{r}_1)$
$|j\rangle$
$t = 0$

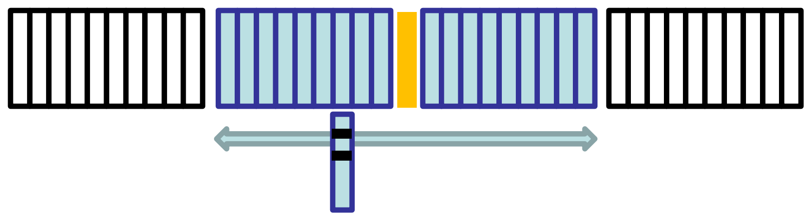
Schematic of lattice Monte Carlo calculation



$$e^{-E_{0,{\rm LO}} a_t} = \lim_{n_t \to \infty} Z_{n_t+1,{\rm LO}} / Z_{n_t,{\rm LO}}$$

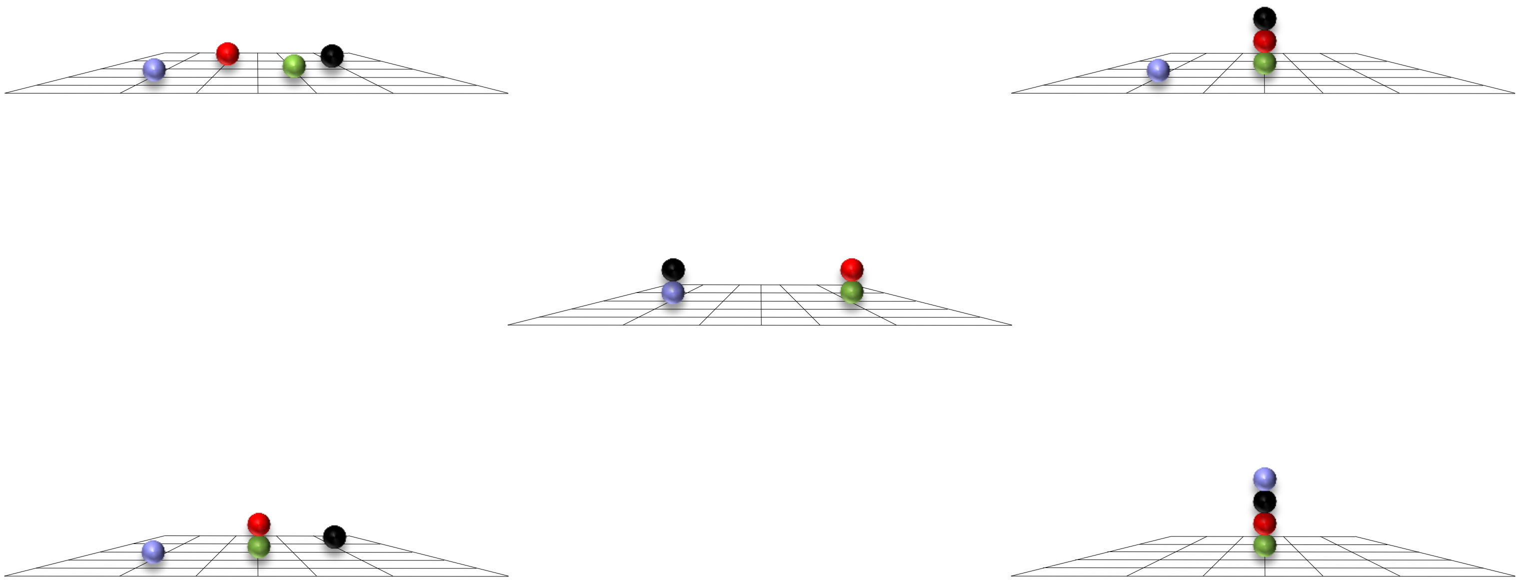
$$\langle O \rangle_{0,{\rm LO}} = \lim_{n_t \to \infty} Z_{n_t,{\rm LO}}^{\langle O \rangle} / Z_{n_t,{\rm LO}}$$

$Z_{n_t,\mathrm{NLO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

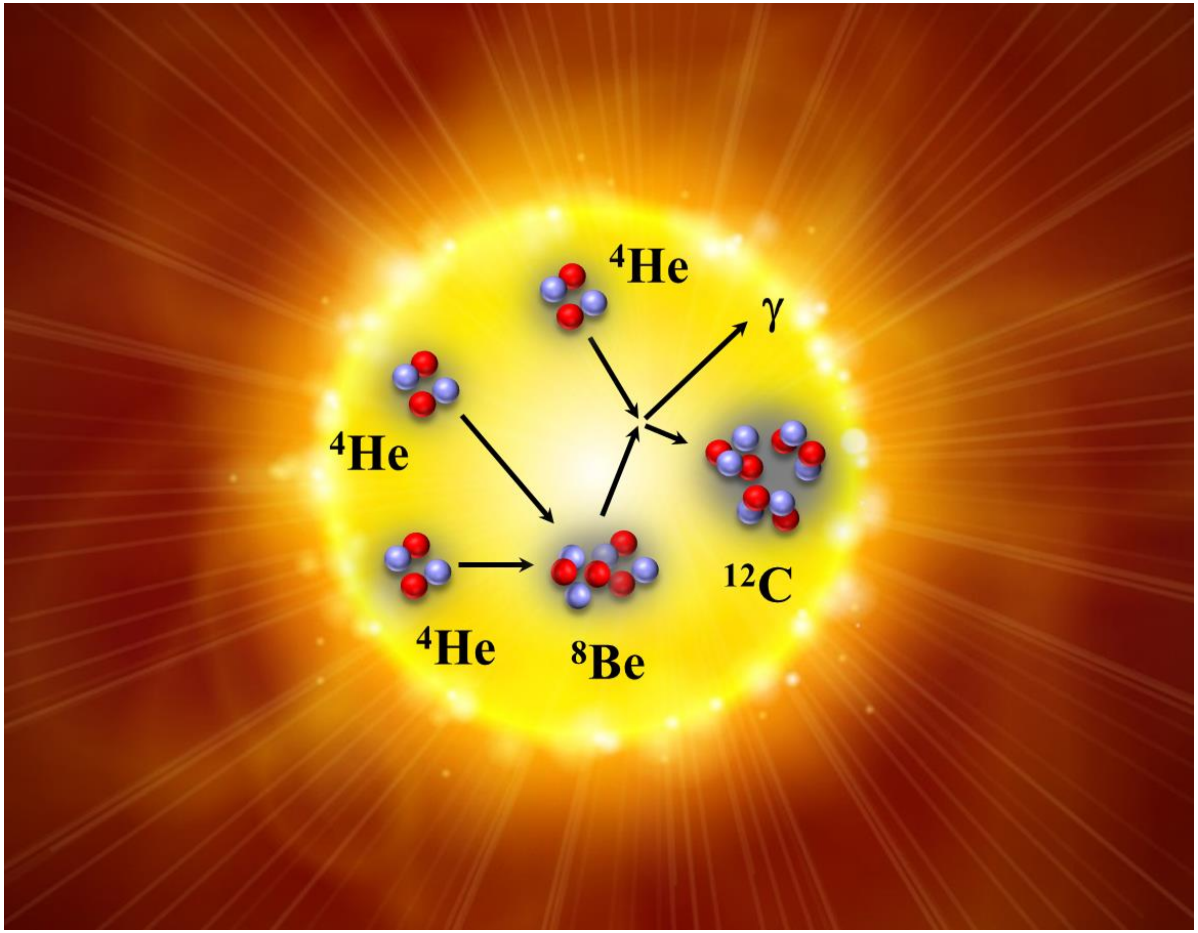
$Z^{\langle O \rangle}_{n_t,\mathrm{NLO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$$\langle O \rangle_{0,\mathrm{NLO}} = \lim_{n_t \to \infty} Z^{\langle O \rangle}_{n_t,\mathrm{NLO}} / Z_{n_t,\mathrm{NLO}}$$

Particle clustering included automatically

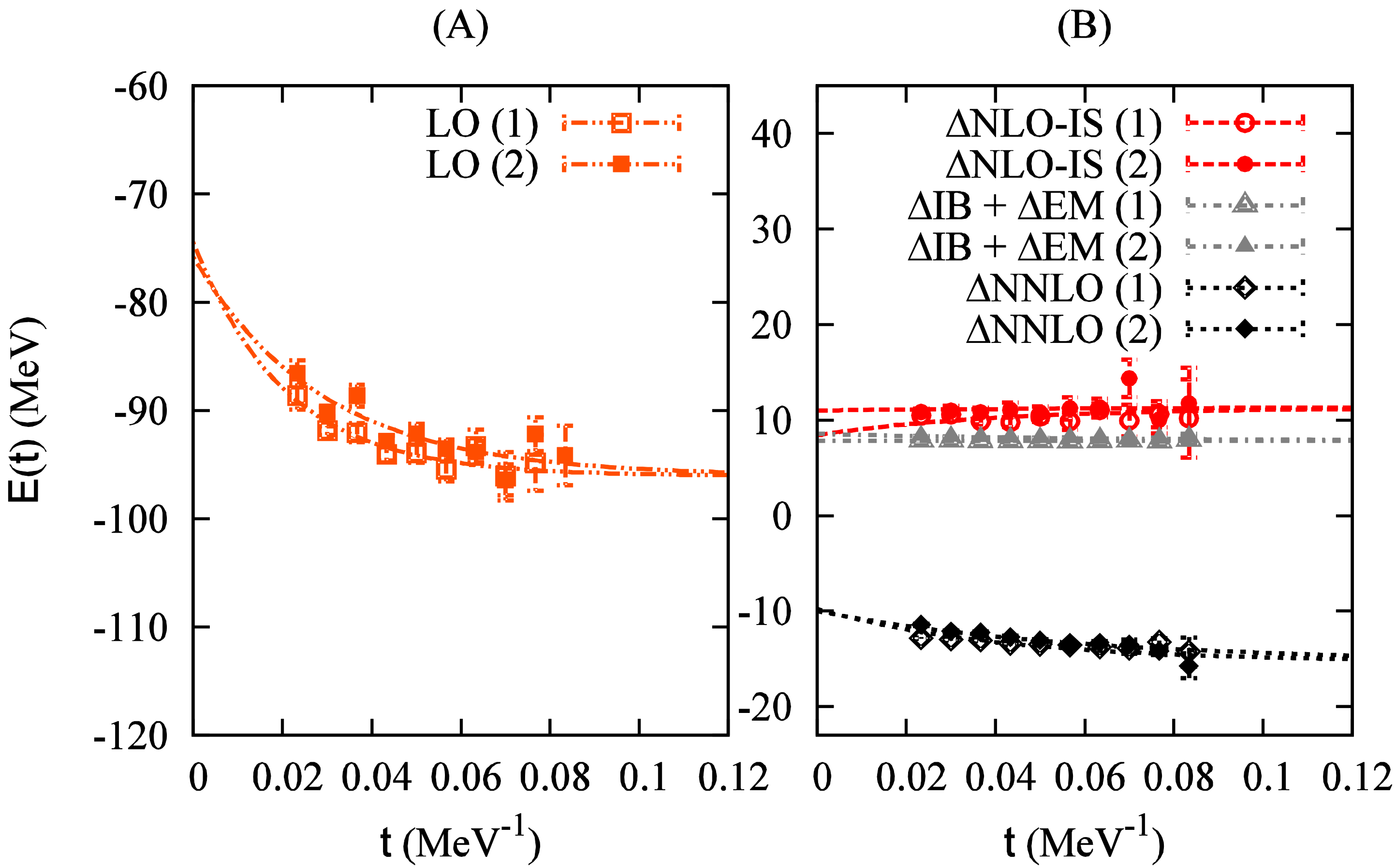


Carbon-12 spectrum and the Hoyle state



Ground state of Carbon-12

$$L = 11.8\,\mathrm{fm}$$



Epelbaum, Krebs, D.L, Meißner, PRL 106 (2011) 192501

Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 (2012) 252501

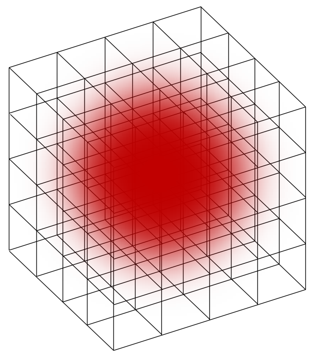
Ground state of Carbon-12

$$L = 11.8\,\mathrm{fm}$$

LO* ($O(Q^0)$)	–96(2) MeV
NLO ($O(Q^2)$)	–77(3) MeV
NNLO ($O(Q^3)$)	–92(3) MeV
Experiment	–92.2 MeV

*contains some interactions promoted from NLO

Simulations using general initial/final state wavefunctions



$$\bigwedge_{j=1,\cdots,A} |\psi_j(\vec{n})\rangle$$

Construct states with well-defined momentum using all possible translations.

$$L^{-3/2}\sum_{\vec{m}} e^{i\vec{P}\cdot\vec{m}} \bigwedge_{j=1,\cdots,A} |\psi_j(\vec{n}-\vec{m})\rangle$$

Shell model wavefunctions

$$\psi_j(\vec{n}) = \exp(-c\vec{n}^2)$$
$$\psi_j'(\vec{n}) = n_x \exp(-c\vec{n}^2)$$
$$\psi_j''(\vec{n}) = n_y \exp(-c\vec{n}^2)$$
$$\psi_j'''(\vec{n}) = n_z \exp(-c\vec{n}^2)$$
$$\vdots$$

Alpha cluster wavefunctions

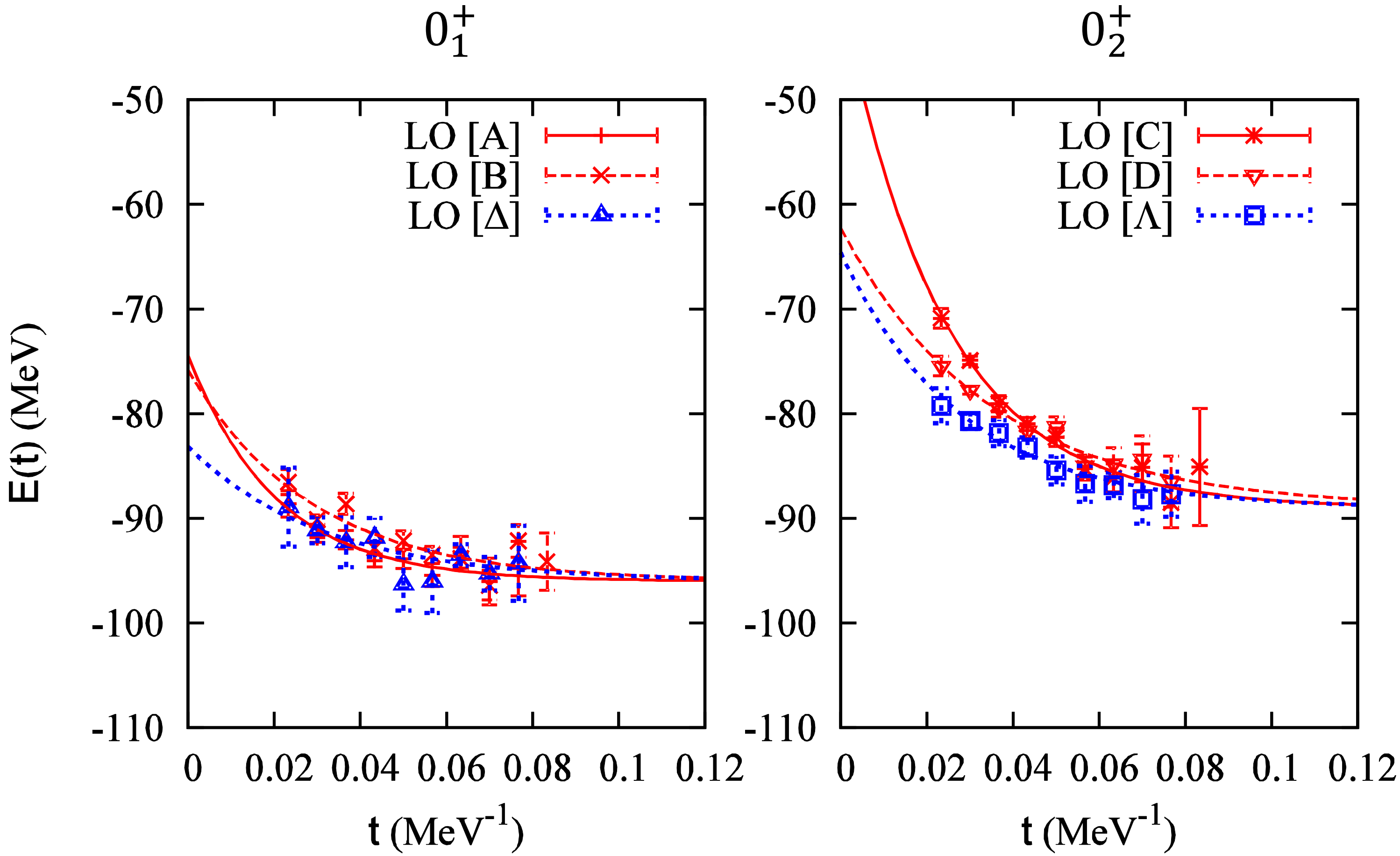
$$\psi_j(\vec{n}) = \exp[-c(\vec{n} - \vec{m})^2]$$
$$\psi_j'(\vec{n}) = \exp[-c(\vec{n} - \vec{m}')^2]$$
$$\psi_j''(\vec{n}) = \exp[-c(\vec{n} - \vec{m}'')^2]$$
$$\vdots$$

Shell model wavefunctions by themselves do not have enough local four nucleon correlations,

$$< (N^\dagger N)^4 >$$

Needs to develop the four nucleon correlations via Euclidean time projection.

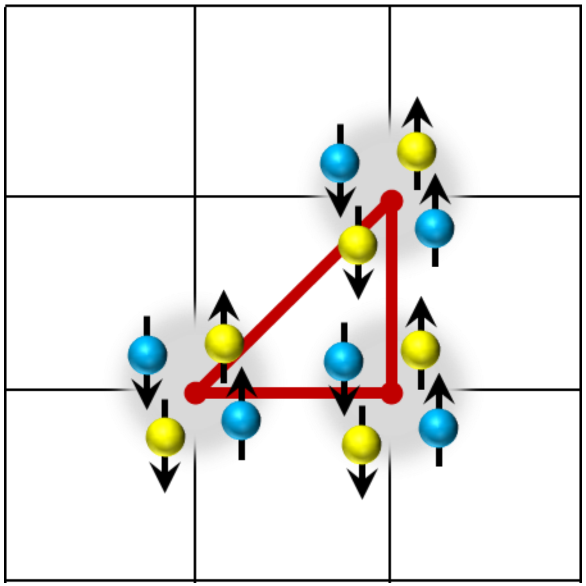
But can reproduce same results starting directly from alpha cluster wavefunctions [Δ and Λ in plots on next slide].



Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 252501 (2012)

Structure of ground state and first 2+

Strong overlap with compact triangle configuration

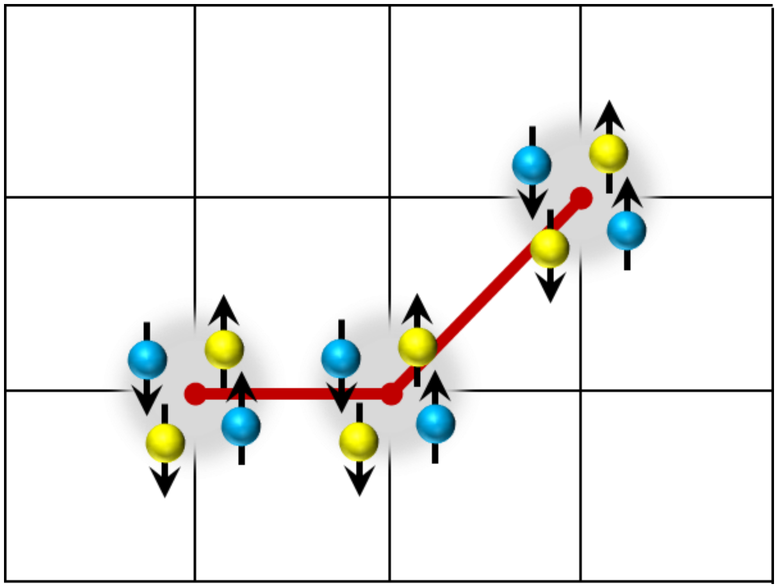


12 rotational orientations

$$b = 1.97 \text{ fm}$$

Structure of Hoyle state and second 2+

Strong overlap with bent arm configuration



24 rotational orientations

$$b = 1.97 \text{ fm}$$

Excited state spectrum of carbon-12 (even parity)

	2_1^+	0_2^+	2_2^+
LO* ($O(Q^0)$)	–94(2) MeV	–89(2) MeV	–88(2) MeV
NLO ($O(Q^2)$)	–74(3) MeV	–72(3) MeV	–70(3) MeV
NNLO ($O(Q^3)$)	–89(3) MeV	–85(3) MeV	–83(3) MeV
Experiment	–87.72 MeV	–84.51 MeV	–82.6(1) MeV (*A*,*B*) –81.1(3) MeV (*C*) –82.13(11) MeV (*D*)

*contains some interactions promoted from NLO

A – Freer et al., PRC 80 (2009) 041303

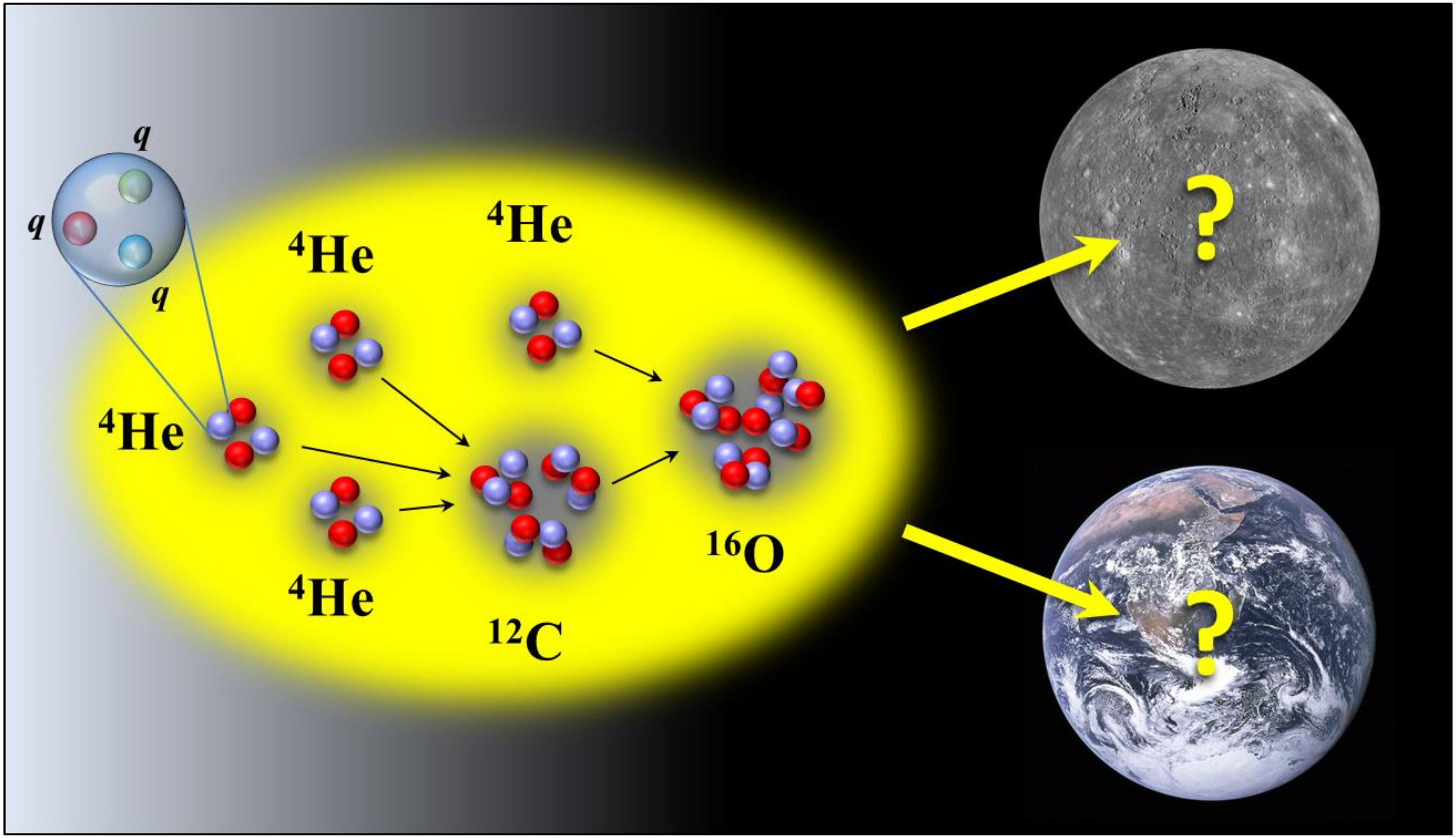
B – Zimmerman et al., PRC 84 (2011) 027304

C – Hyldegaard et al., PRC 81 (2010) 024303

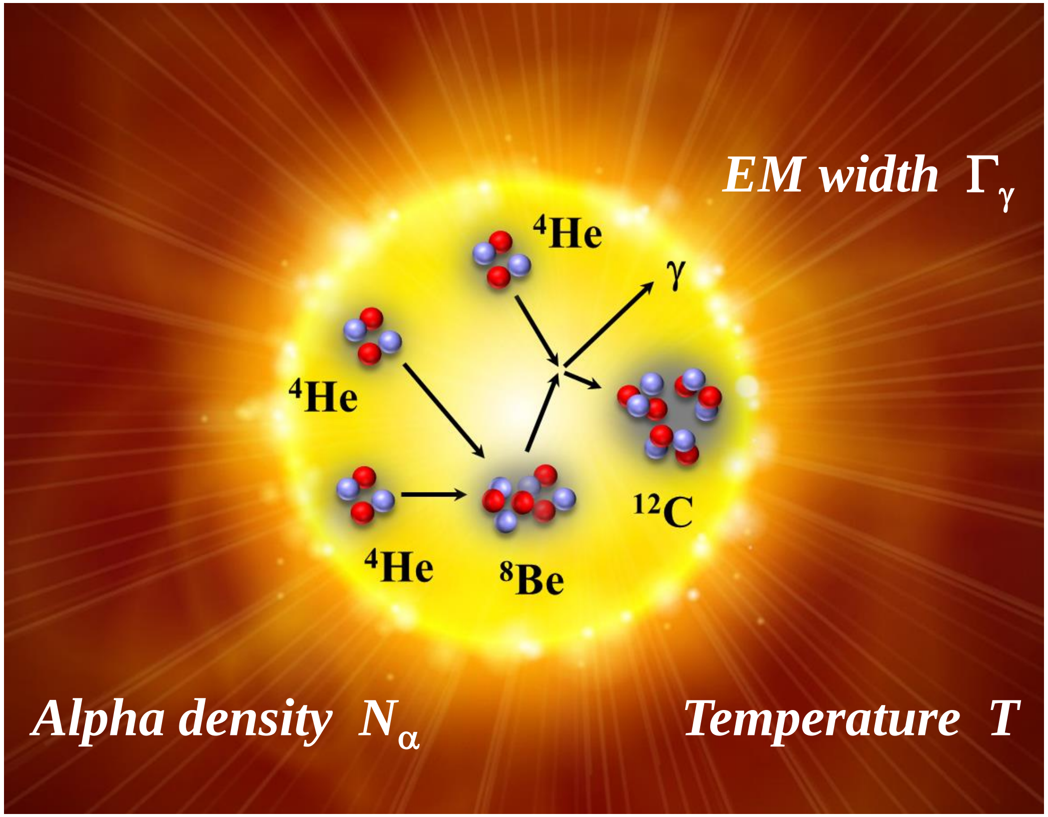
D – Itoh et al., PRC 84 (2011) 054308

Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 252501 (2012)

Light quark mass dependence of helium burning



Triple alpha reaction rate



$$r_{3\alpha} \propto \Gamma_\gamma \, (N_\alpha / k_B T)^3 \times \exp(-\varepsilon / k_B T)$$

$\varepsilon = E_h - 3E_\alpha$ Hoyle relative to triple-alpha

Is nature fine-tuned?

$$\varepsilon = E_h - 3E_\alpha \approx 380\,\mathrm{keV}$$

$$\varepsilon > 480\,\mathrm{keV}$$

Less resonance enhancement.
Rate of carbon production smaller by several orders of magnitude.
Low carbon abundance is unfavorable for carbon-based life.

$$\varepsilon < 280\,\mathrm{keV}$$

Carbon production occurs at lower stellar temperatures and oxygen production greatly reduced.
Low oxygen abundance is unfavorable for carbon-based life.

Schlattl et al., Astrophys. Space Sci., 291, 27–56 (2004)

We investigate the dependence on the fundamental parameters of the standard model such as the light quark masses. Can be parameterized by the pion mass.

See talk by
Evgeny at
this workshop

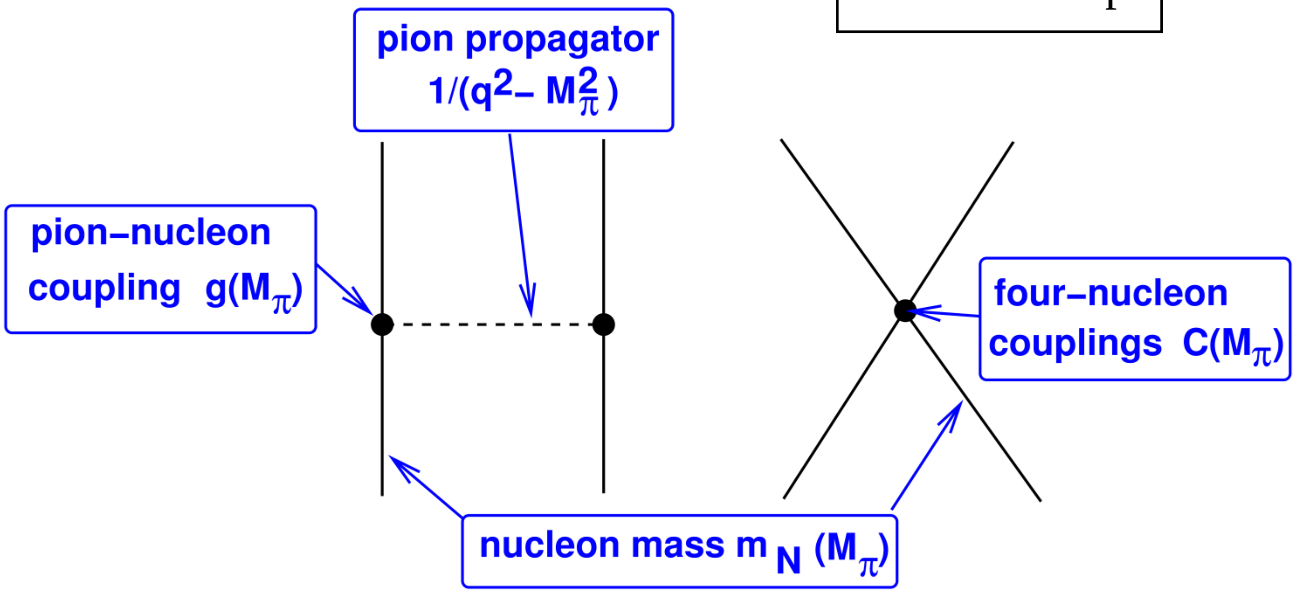
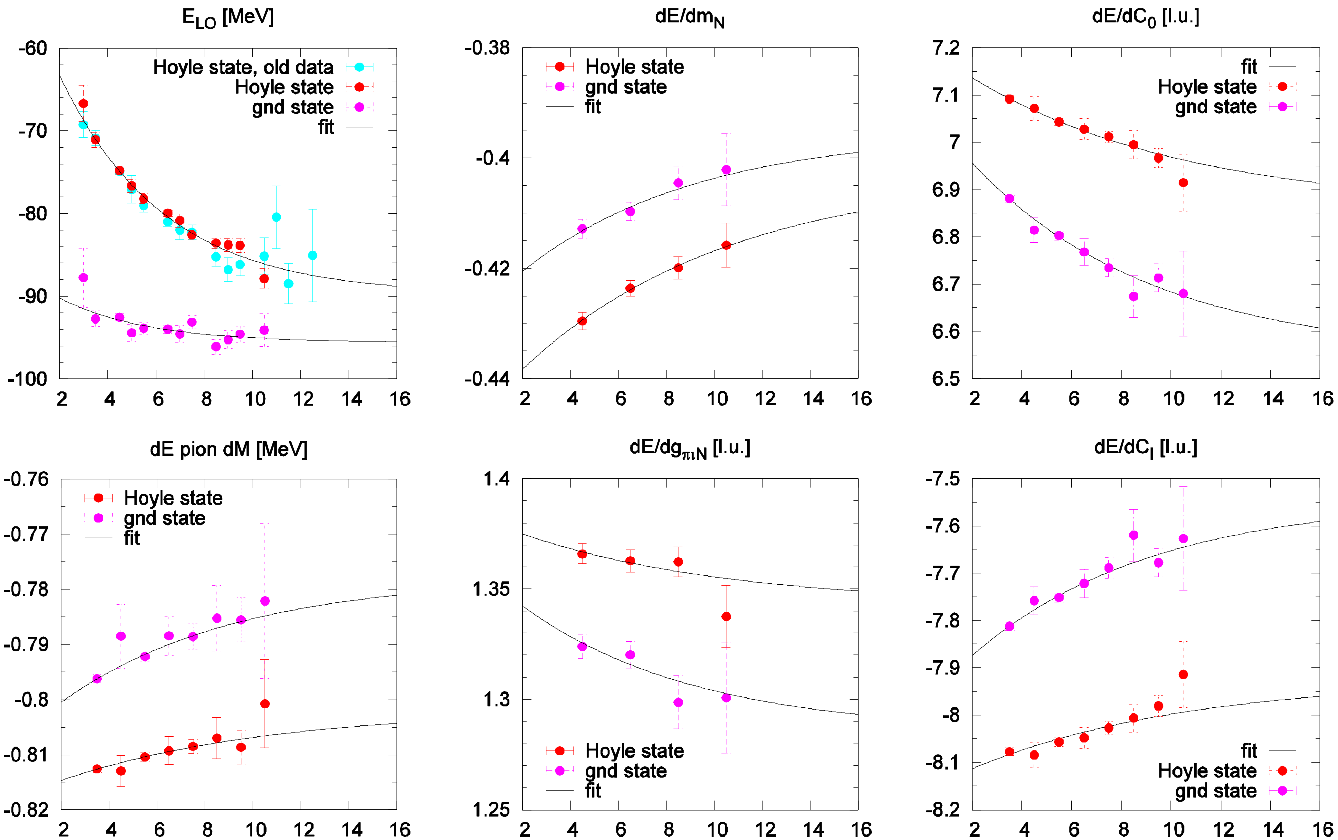


Figure courtesy of U.-G. Meißner

Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; ibid., arXiv:1303.4856
Berengut et al., Phys. Rev. D 87 (2013) 085018

Lattice results for pion mass dependence



$$\Delta E_h = E_h - E_b - E_\alpha$$ Hoyle relative to Be-8-alpha

$$\Delta E_b = E_b - 2E_\alpha$$ Be-8 relative to alpha-alpha

$$\varepsilon = E_h - 3E_\alpha$$ Hoyle relative to triple-alpha

$$\left.\frac{\partial \Delta E_h}{\partial M_\pi}\right|_{M_\pi^{\rm ph}} = -0.455(35)\bar{A}_s - 0.744(24)\bar{A}_t + 0.051(19)$$

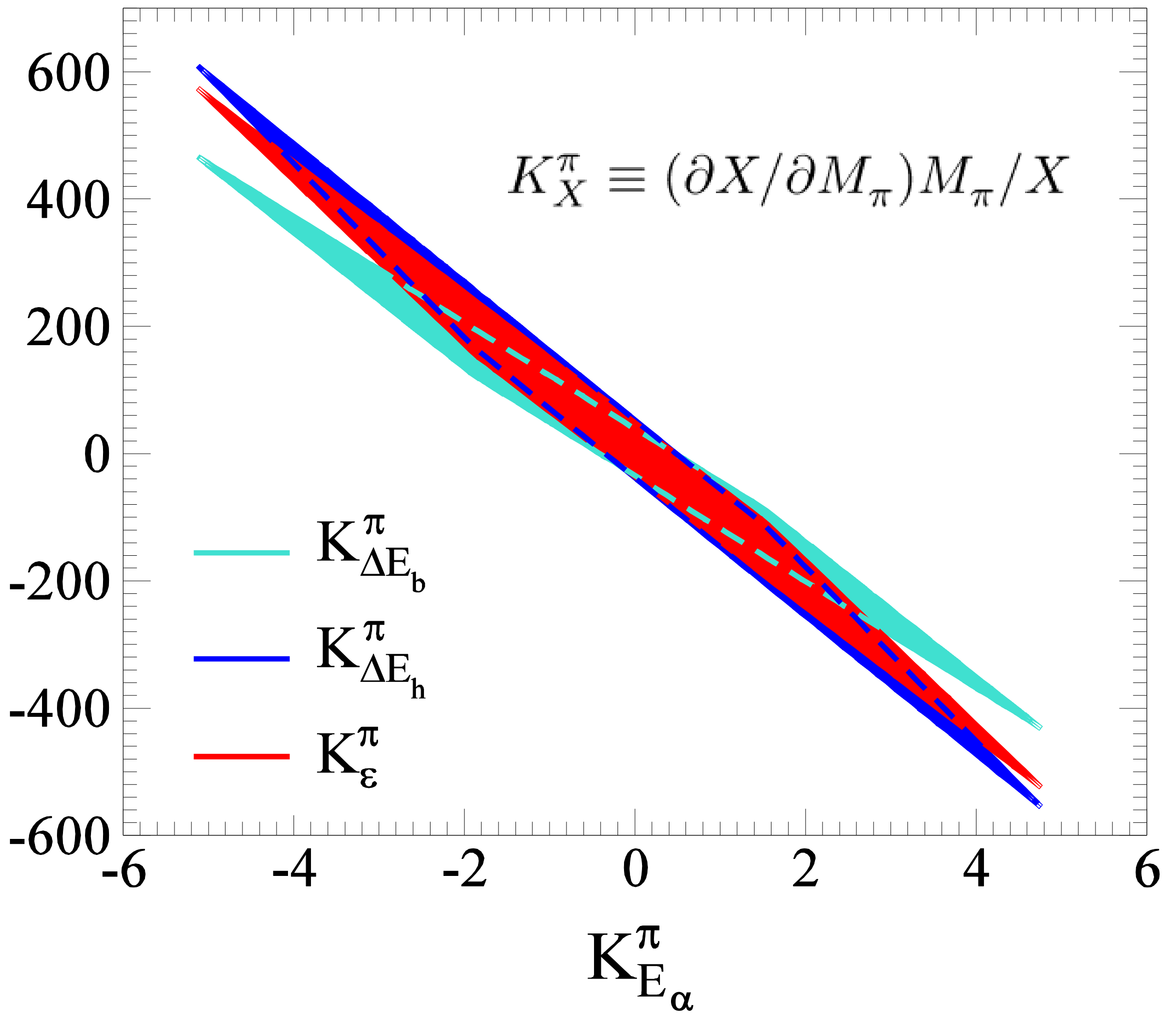
$$\left.\frac{\partial \Delta E_b}{\partial M_\pi}\right|_{M_\pi^{\rm ph}} = -0.117(34)\bar{A}_s - 0.189(24)\bar{A}_t + 0.013(12)$$

$$\left.\frac{\partial \varepsilon}{\partial M_\pi}\right|_{M_\pi^{\rm ph}} = -0.572(19)\bar{A}_s - 0.933(15)\bar{A}_t + 0.064(16)$$

$$\bar{A}_s \equiv \partial a_s^{-1}/\partial M_\pi\big|_{M_\pi^{\rm ph}} \qquad \bar{A}_t \equiv \partial a_t^{-1}/\partial M_\pi\big|_{M_\pi^{\rm ph}}$$

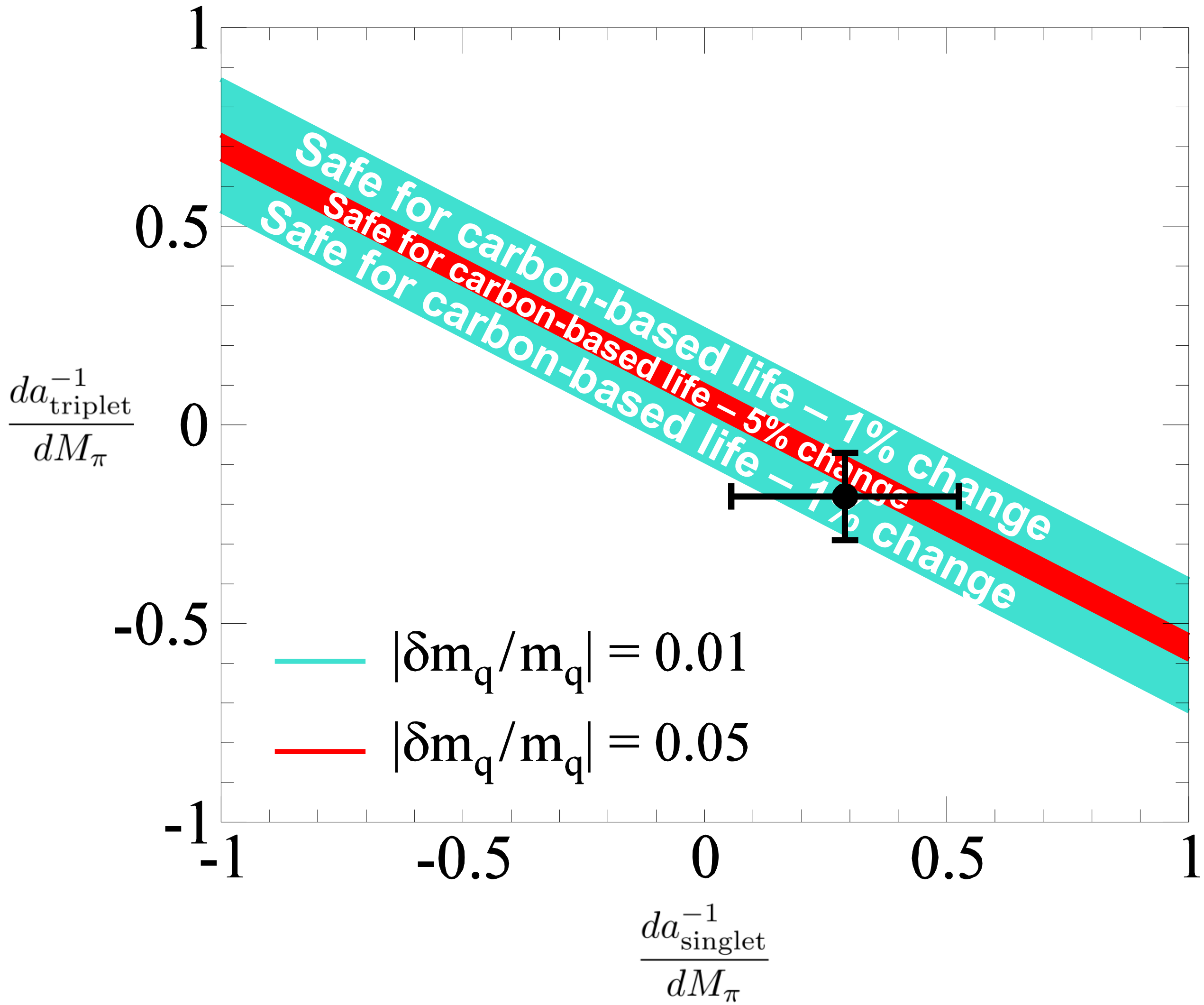
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; ibid., arXiv:1303.4856
Berengut et al., Phys. Rev. D 87 (2013) 085018

Evidence for correlation with alpha binding energy



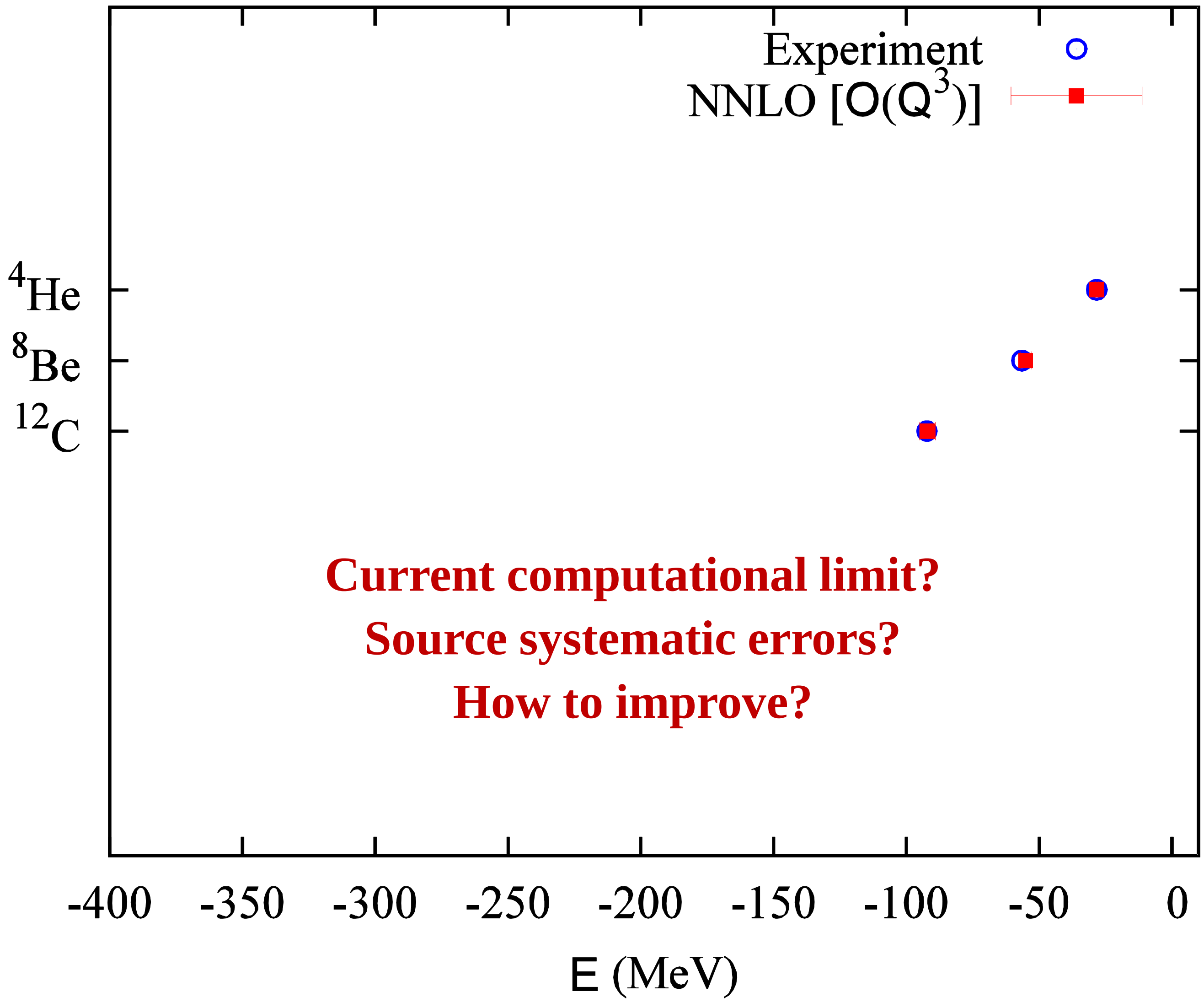
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; ibid., arXiv:1303.4856

“End of the world” plot

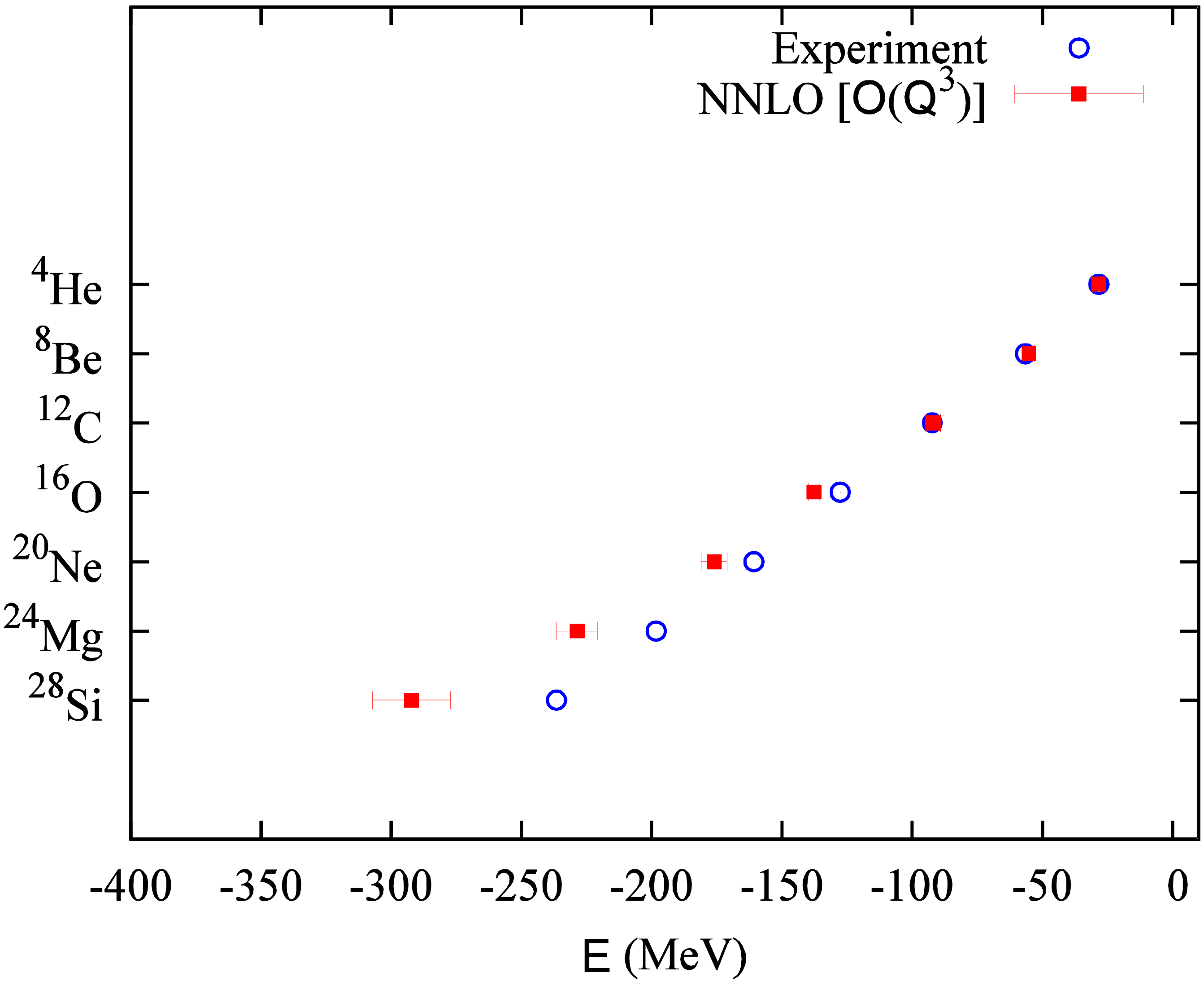


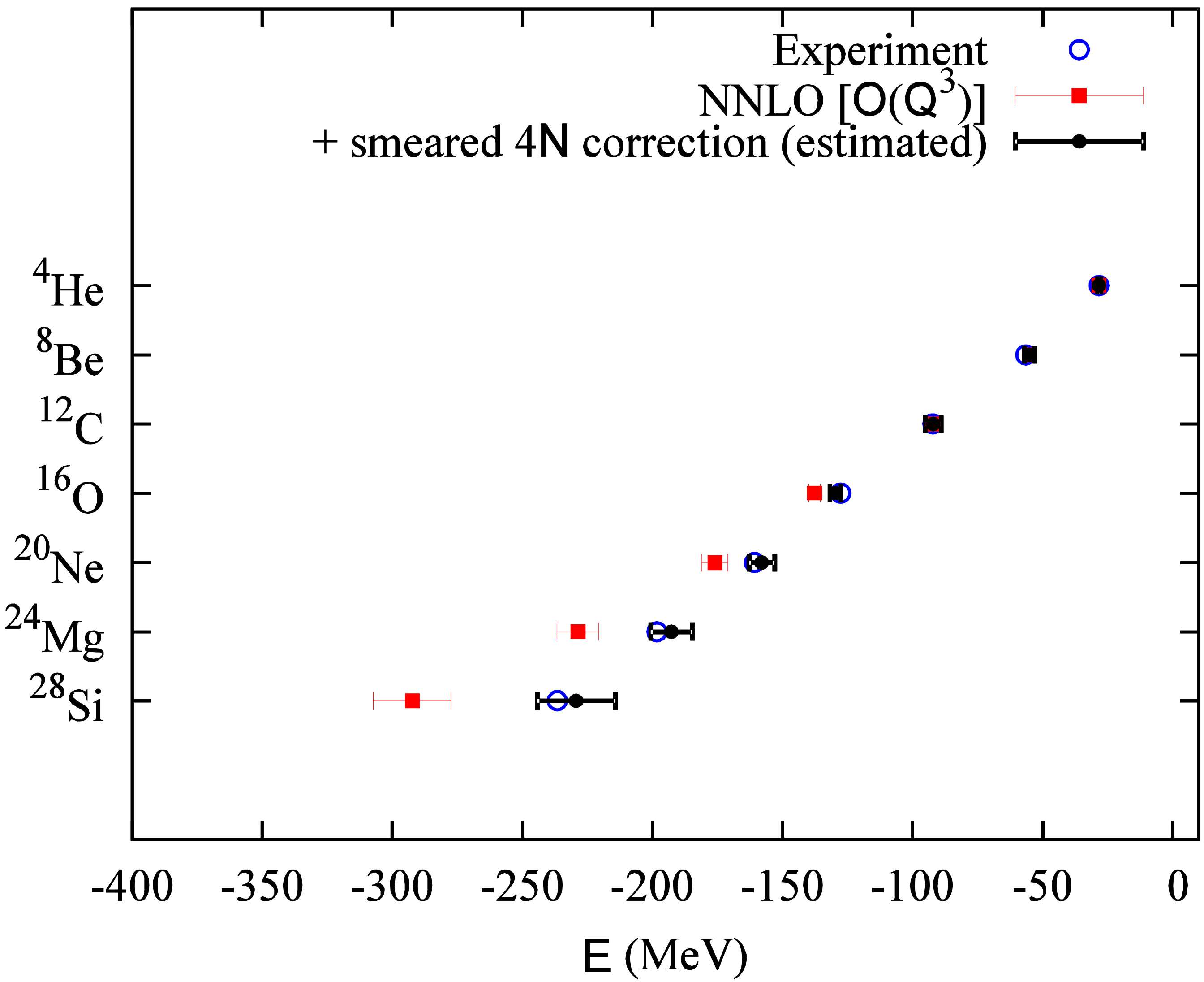
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; ibid., arXiv:1303.4856

Preliminary: *Ab initio* lattice calculations up to A = 28

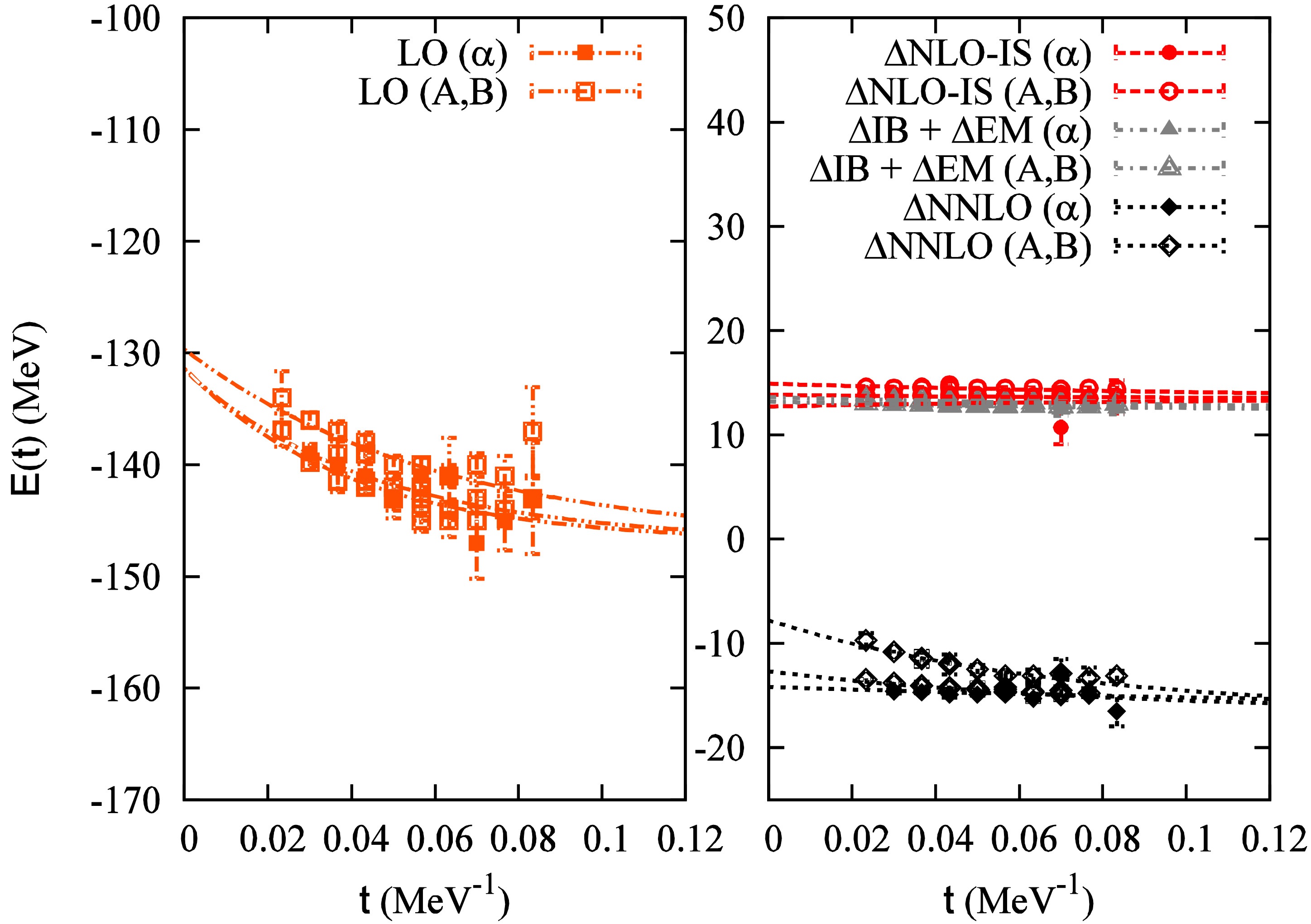


Preliminary: *Ab initio* lattice calculations up to A = 28

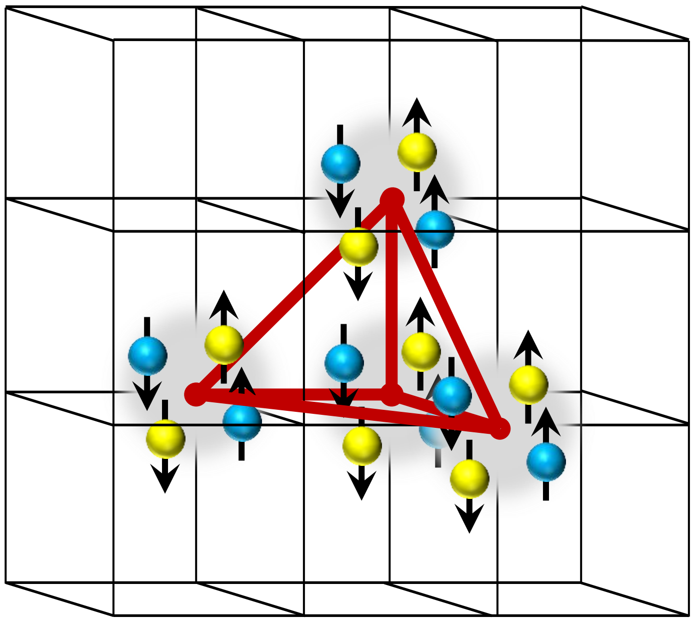



Experiment
NNLO [O(Q^3)]
+ smeared 4N correction (estimated)
4He
8Be
12C
16O
20Ne
24Mg
28Si
-400
-350
-300
-250
-200
-150
-100
-50
0
E (MeV)

Preliminary: Oxygen-16 structure and spectrum



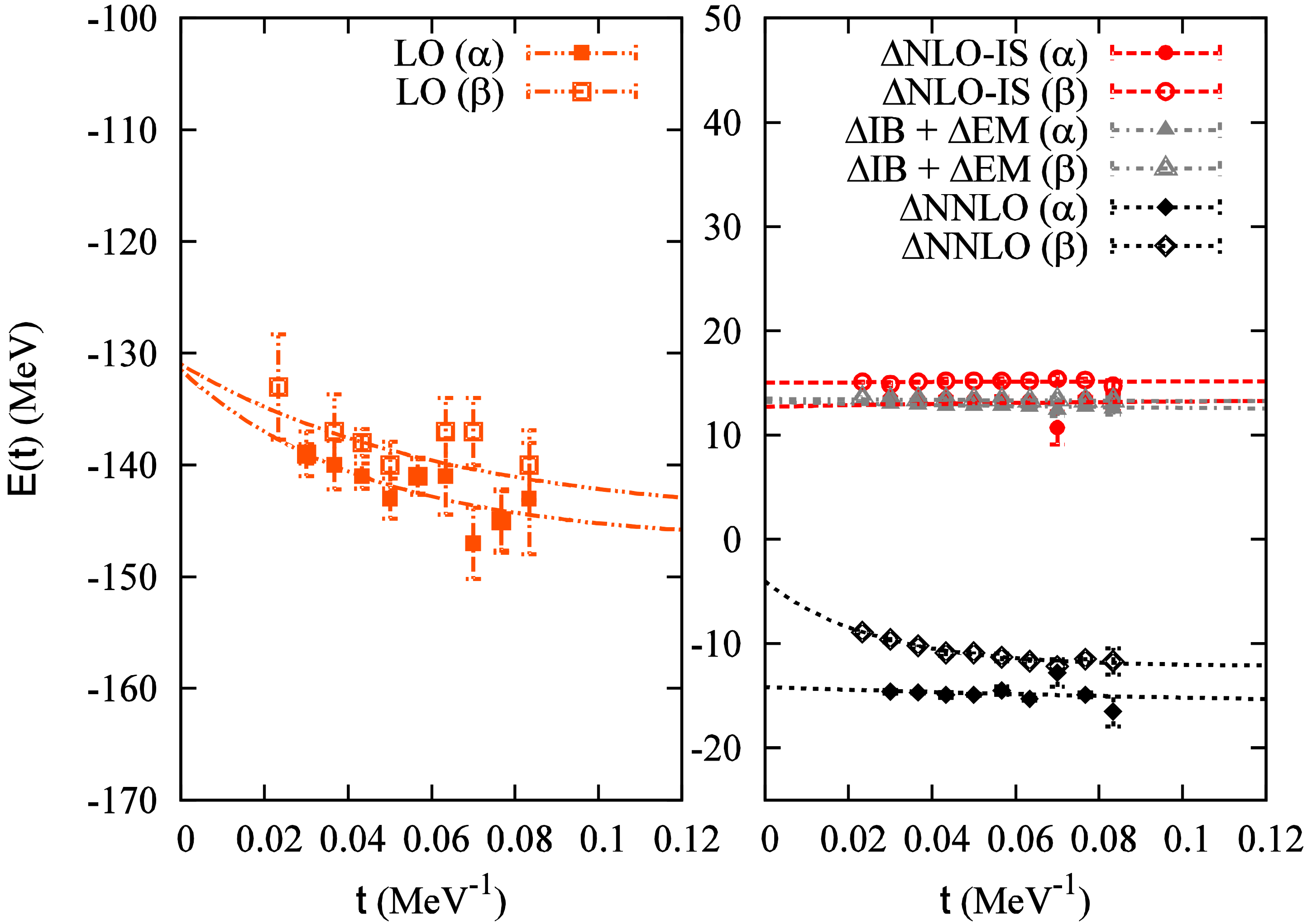
Tetrahedral cluster structure of first 0^+ and 3^-



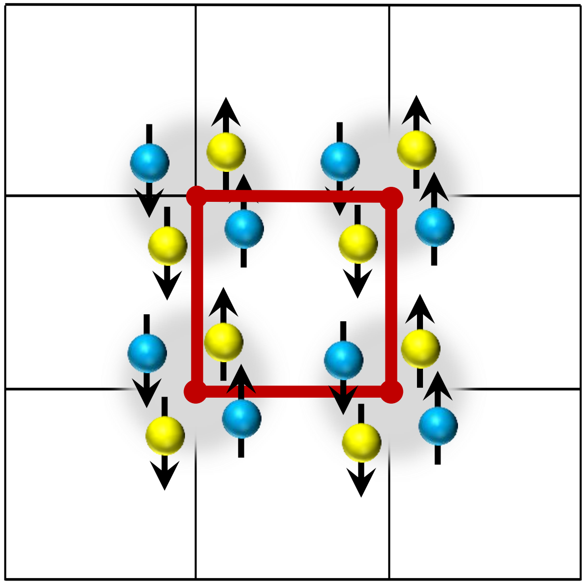
8 rotational orientations

$$b = 1.97 \text{ fm}$$

<u>Finding the second 0^+ state</u>



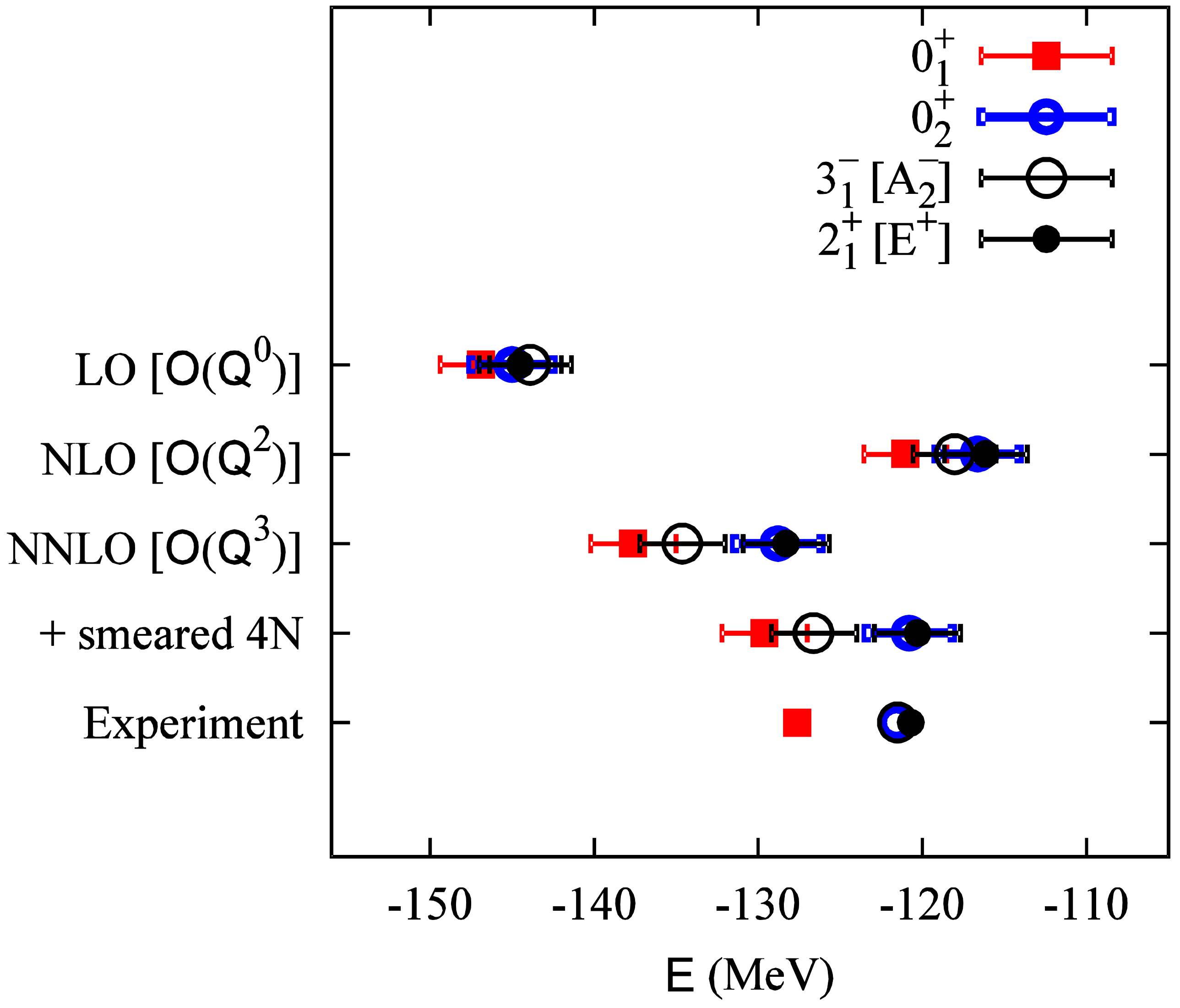
Cluster structure of second 0^+ state and first 2^+ state



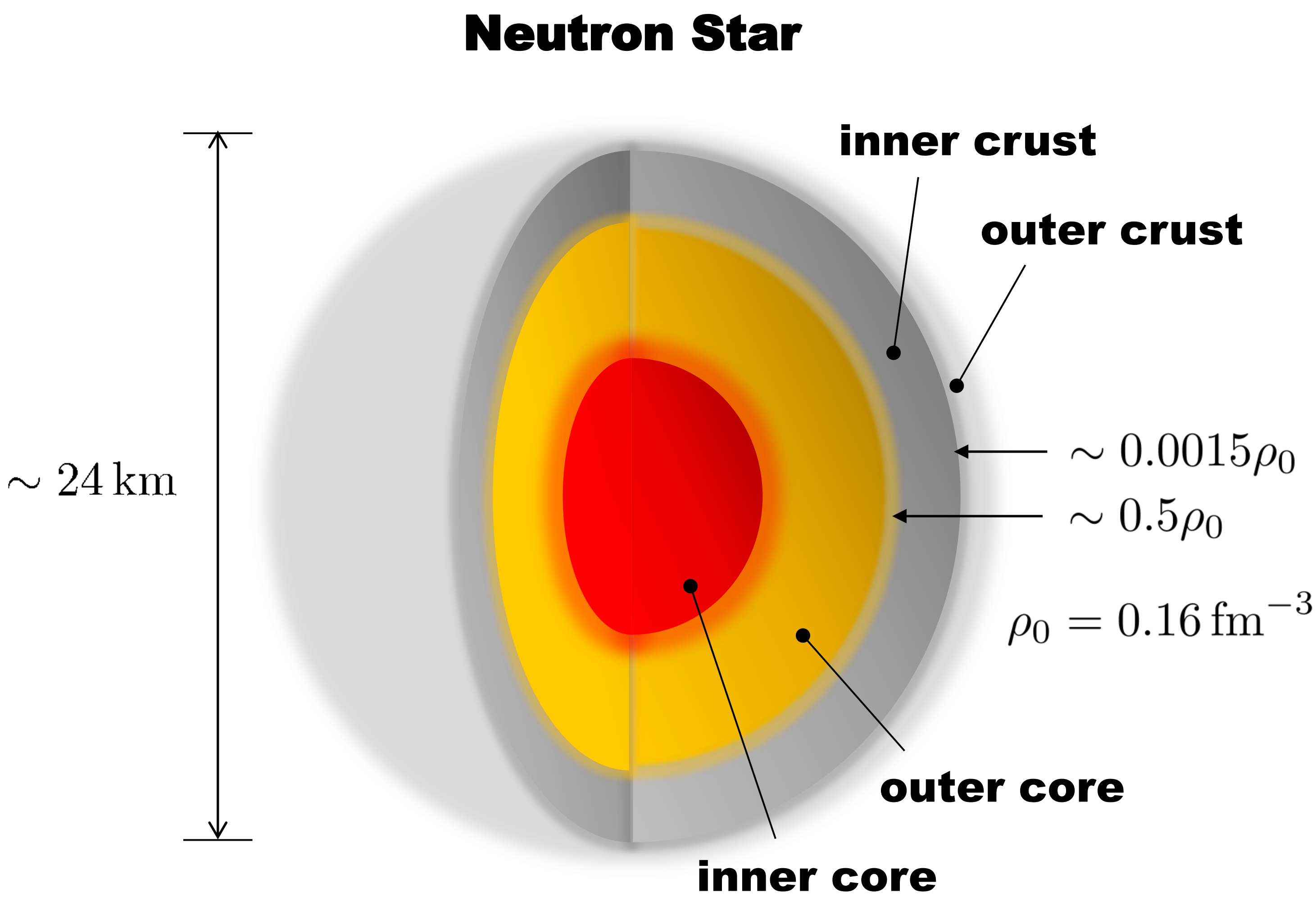
$b = 1.97$ fm

6 rotational orientations

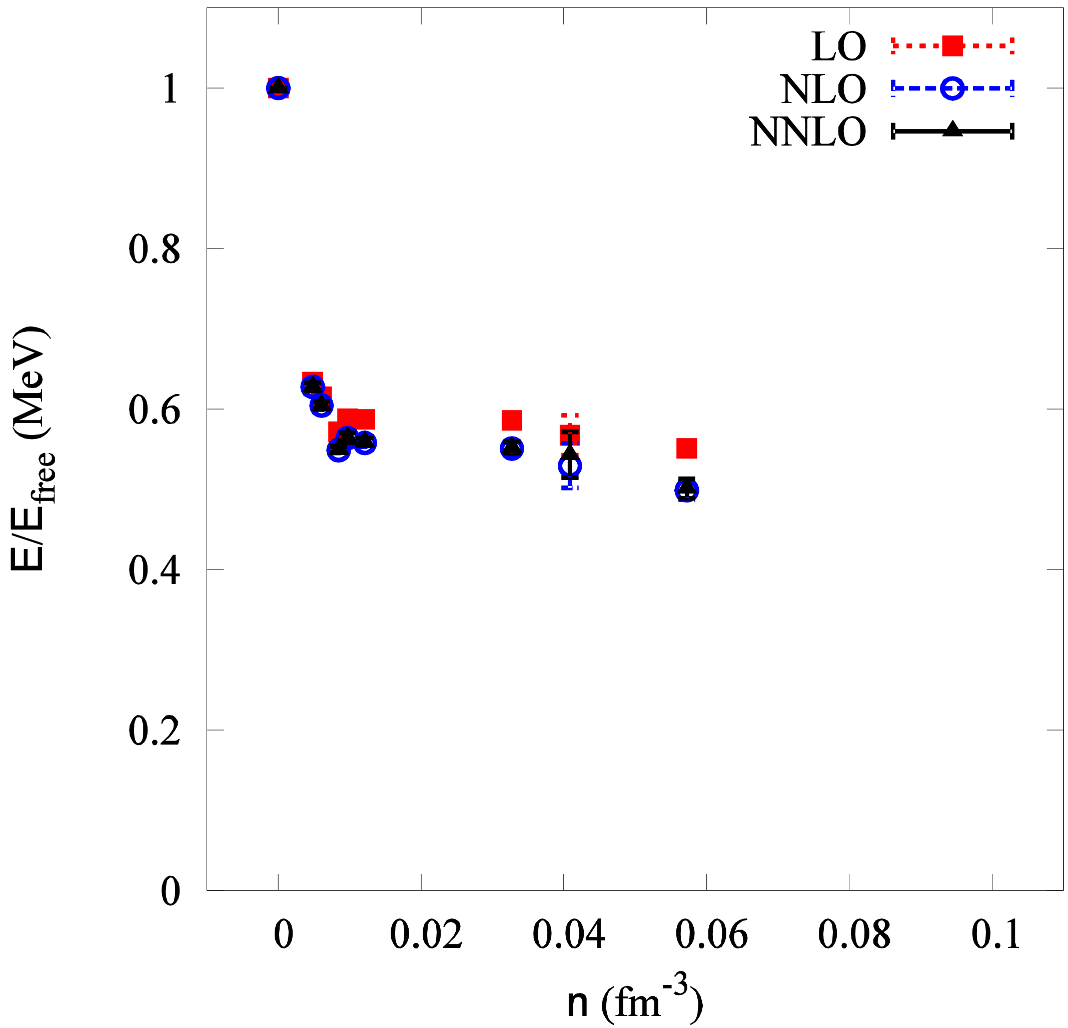
Low-lying spectrum of Oxygen-16



Preliminary: Properties of neutron matter



Energy of the ground state as fraction of free Fermi gas



See talks by Achim and Kai at this workshop

Energy per neutron in the ground state

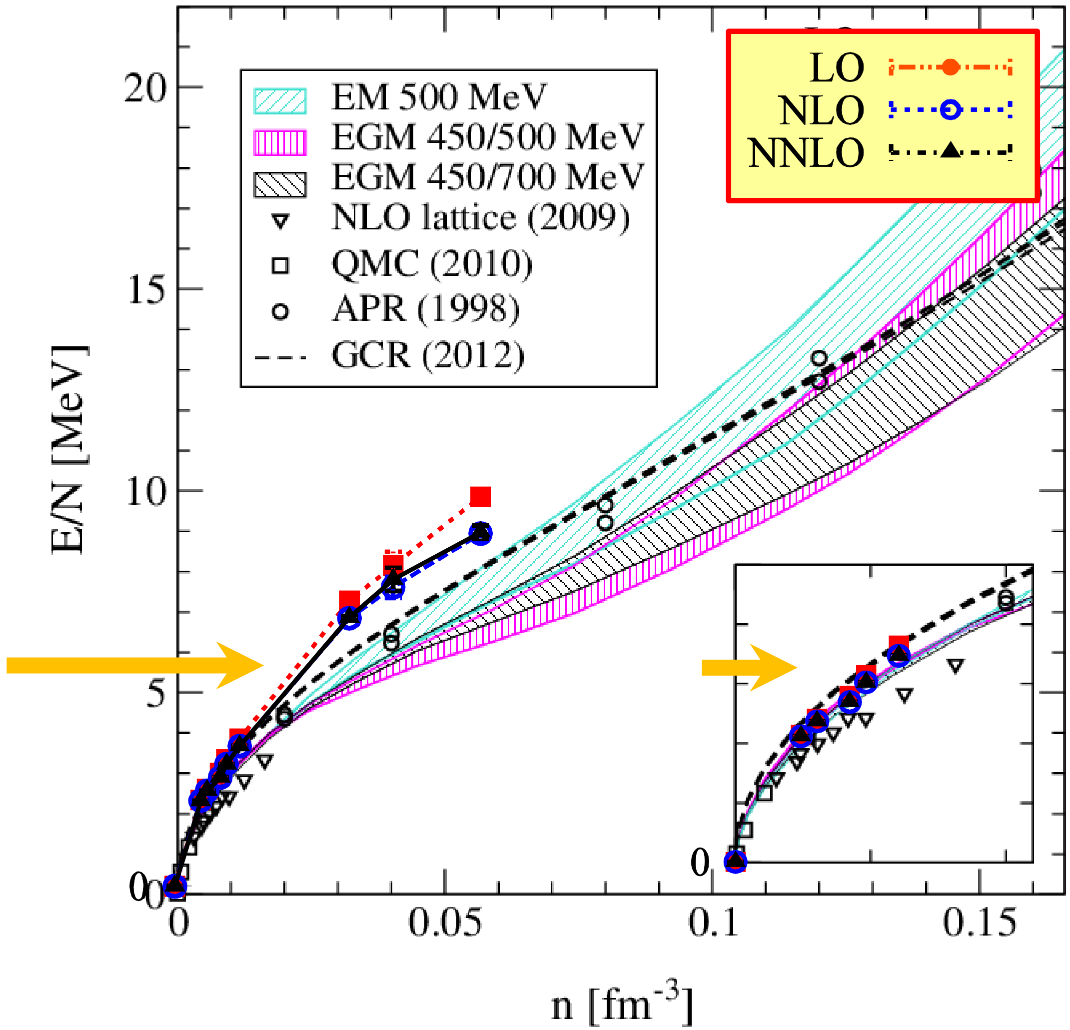


Figure adapted from Tews, et al., PRL 110 (2013) 032504

<u>Energy per neutron in the ground state</u>

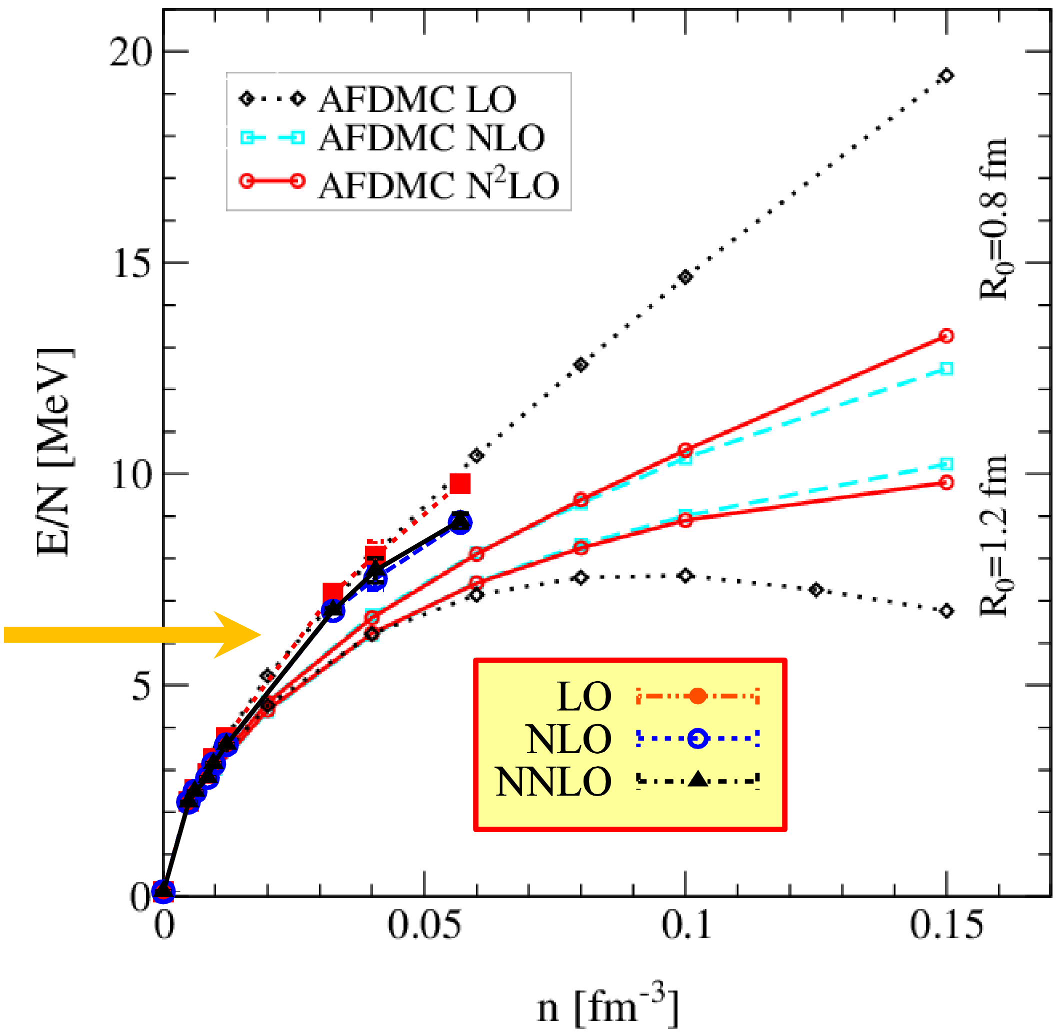
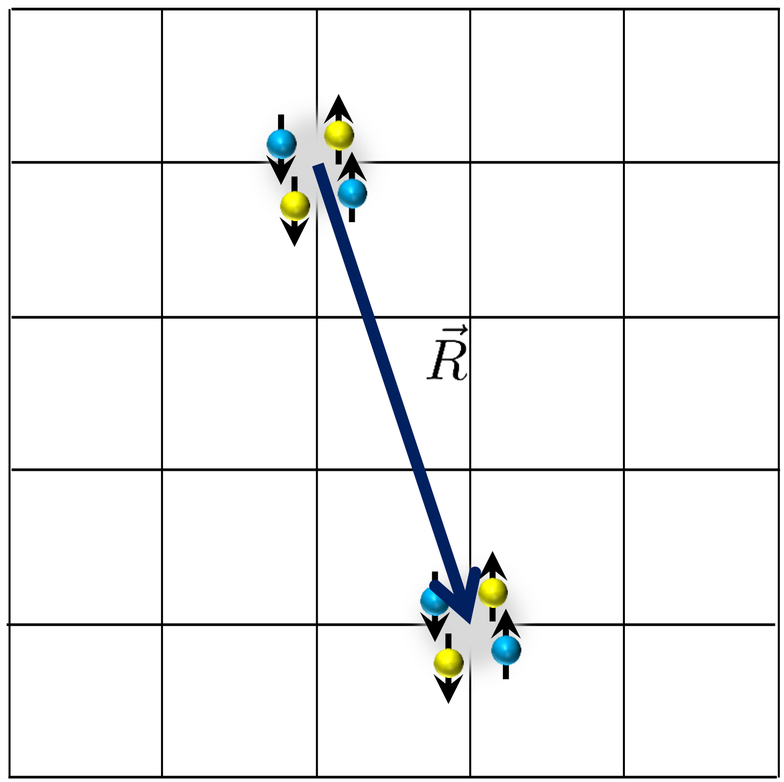


Figure adapted from Gezerlis, et al., arXiv: 1303.6243

Preliminary: Scattering and reactions on the lattice

Projected adiabatic matrix method

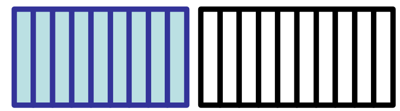


Using cluster wavefunctions for initial continuum scattering states

$$|\vec{R}>$$

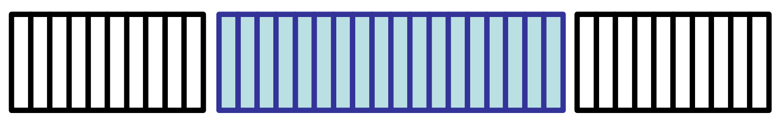
Use projection Monte Carlo to propagate cluster wavefunctions in Euclidean time

$$|\vec{R}>_t = e^{-Ht}|\vec{R}>$$

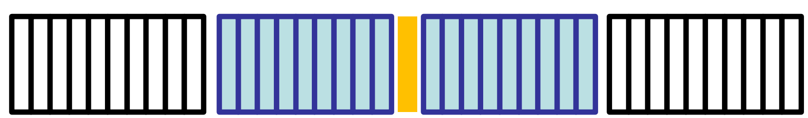
$|\vec{R}>_t =$  $|\vec{R}>$

Construct a norm matrix and matrix of expectation values

$$<N>_t = {}_t<\vec{R}'|\vec{R}>_t =$$

$<\vec{R}'|$  $|\vec{R}>$

$$<O>_t = {}_t<\vec{R}'|O|\vec{R}>_t =$$

$<\vec{R}'|$  $|\vec{R}>$

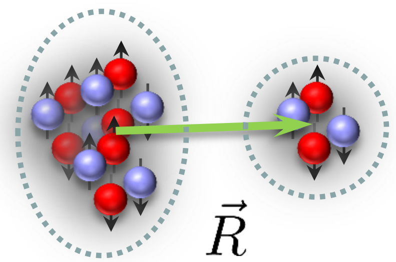
Compute the projected adiabatic matrix

$$< O >_{\text{adiab}} =< N >_t^{-1/2} < O >_t < N >_t^{-1/2}$$

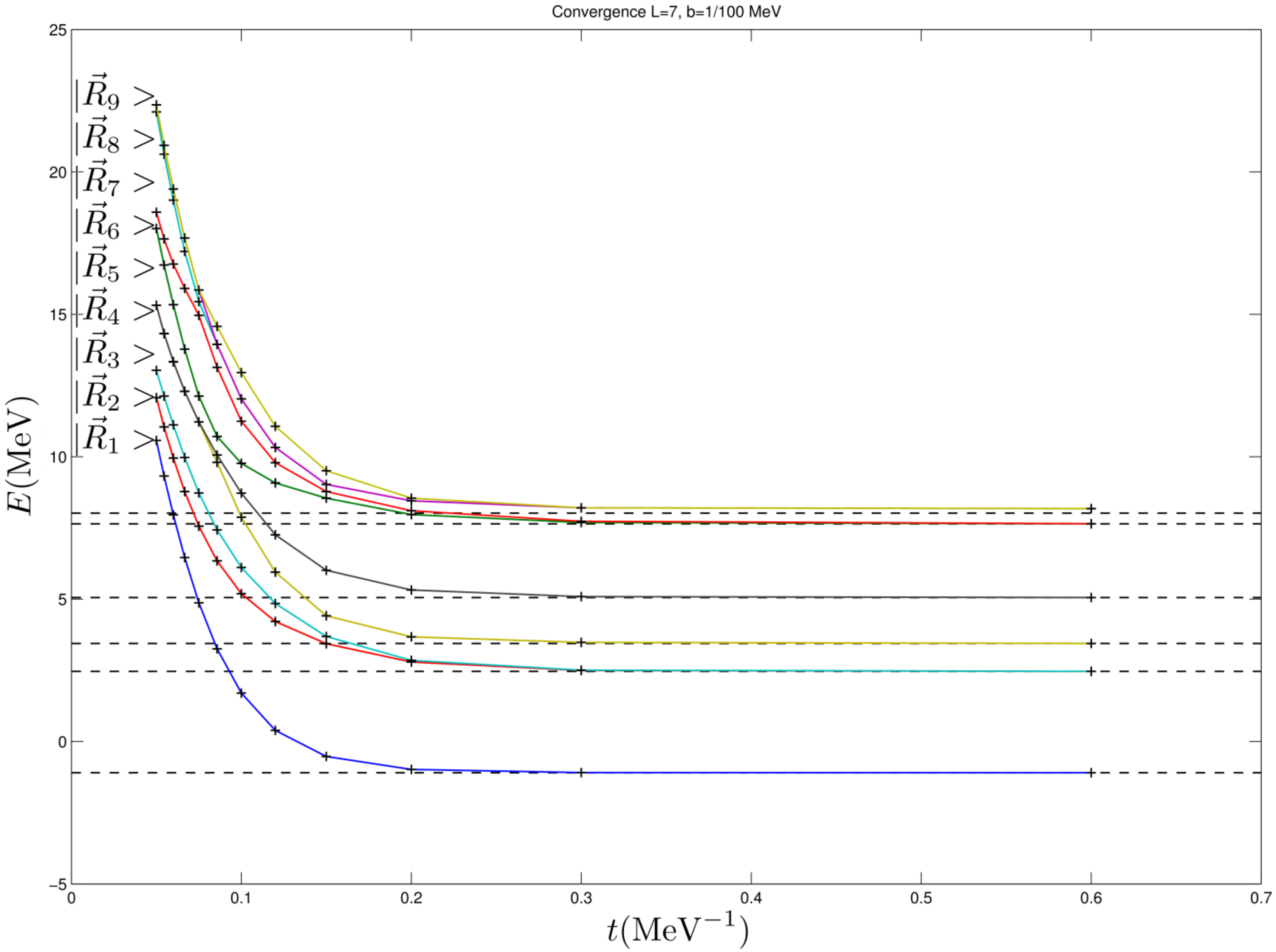
Projected adiabatic Hamiltonian is now an effective two-body Hamiltonian. Similar in spirit to no-core shell model with resonating group method.

See talk by Petr
at this workshop

But some differences. Distortion of the nucleus wavefunctions is automatic due to projection in Euclidean time.

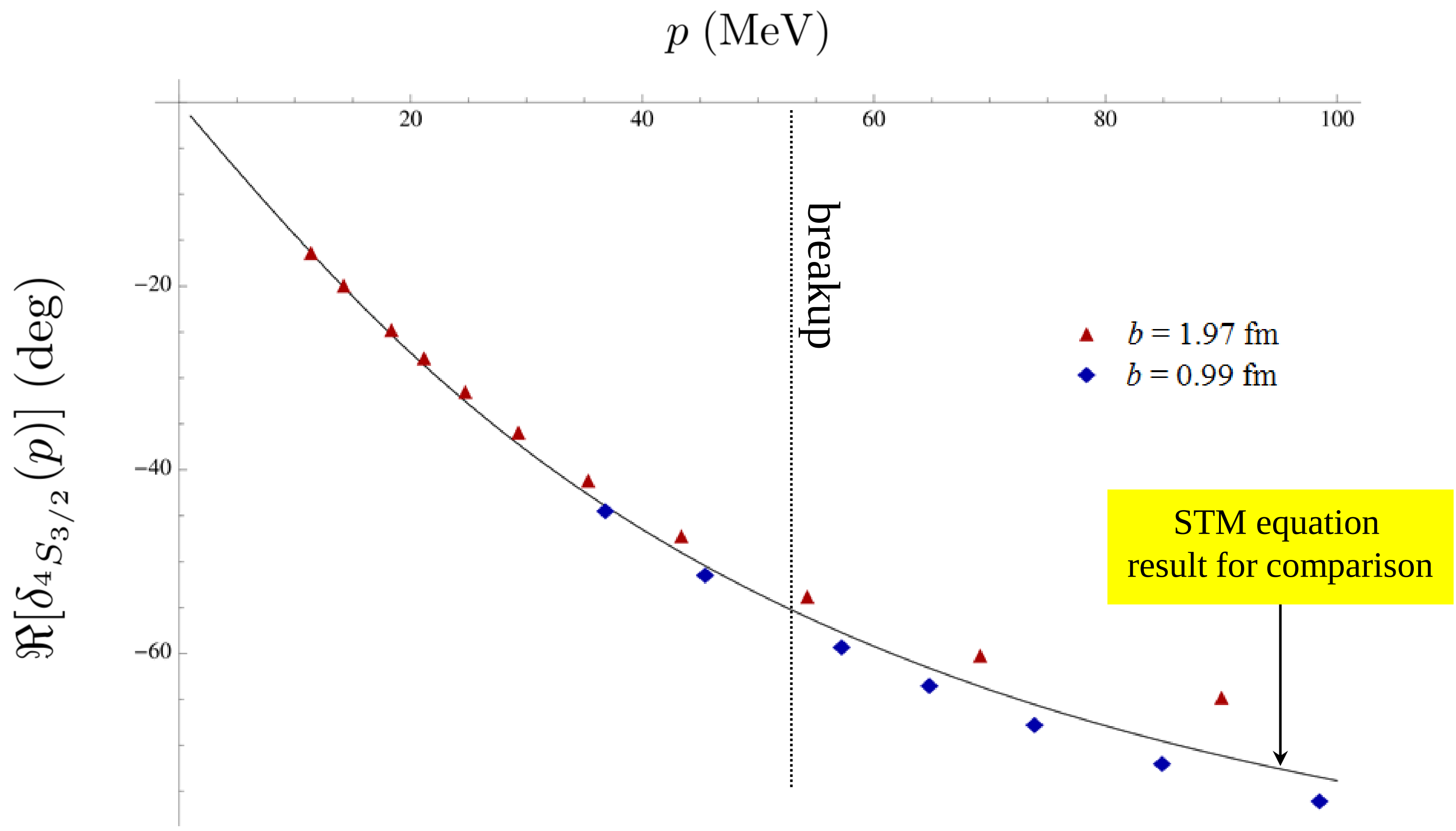


Example: Quartet neutron-deuteron scattering



Pine, D.L., Rupak, work in progress

Quartet neutron-deuteron scattering (pionless EFT at LO)

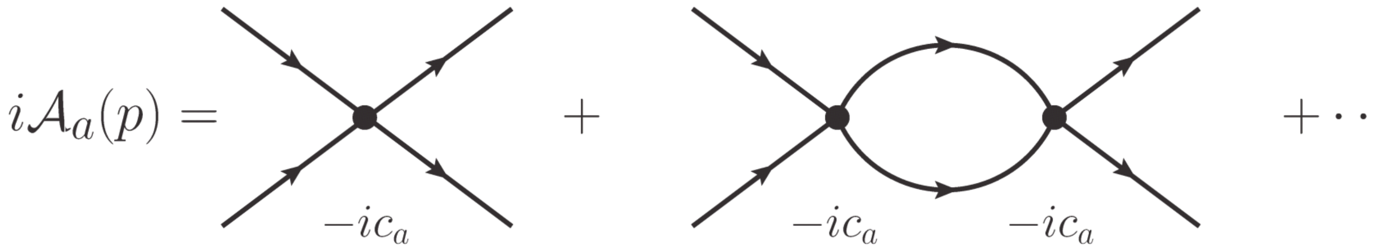


Pine, D.L., Rupak, work in progress

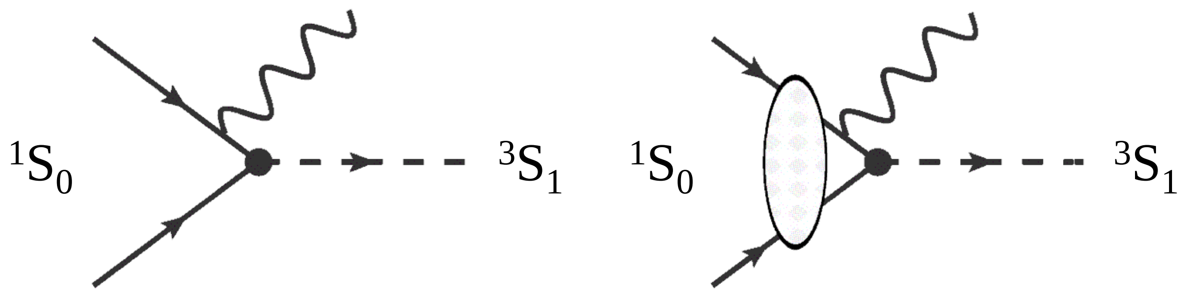
Use coupled channels for capture reactions and break up processes.

Lattice Green's function methods for radiative capture tested for $n + p \rightarrow d + \gamma$ in pionless effective field theory at leading order.

Elastic scattering amplitude ($^1S_0$ and $^3S_1$)

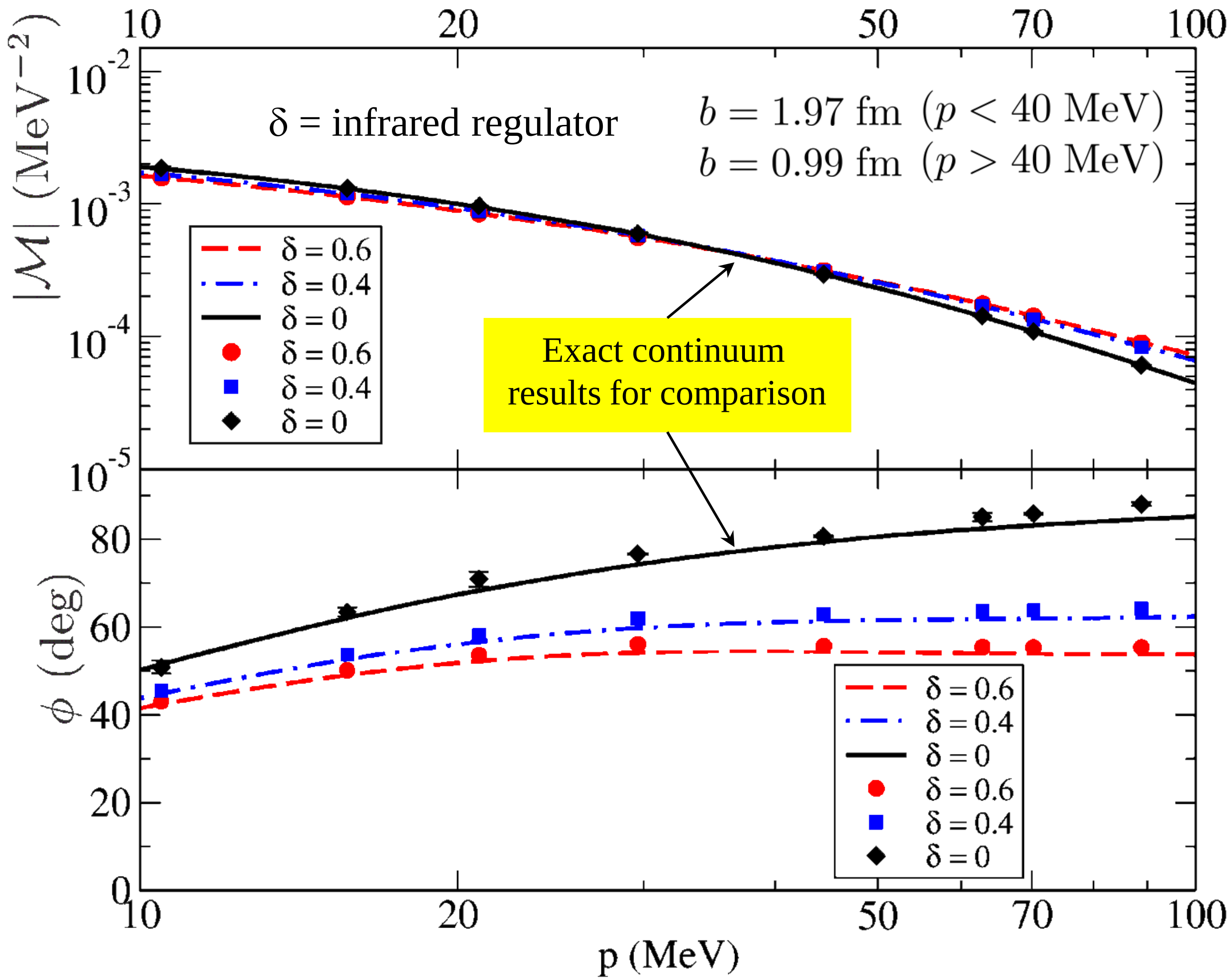


M1 radiative capture amplitude



Rupak, D.L., arXiv:1302.4158 [nucl-th]

M1 transition amplitude $n + p \to d + \gamma$



Rupak, D.L., arXiv:1302.4158 [nucl-th]

Summary

A golden age for nuclear theory from first principles. Big science discoveries being made and many more around the corner.

Lattice effective field theory is a relatively new and promising tool that combines the framework of effective field theory and computational lattice methods. May play a significant role in the future of *ab initio* nuclear theory.

Additional topics to be addressed in the near future…

Different lattice spacings, $N \neq Z$ nuclei, transition from S-wave to P-wave pairing in superfluid neutron matter, etc.