The Lorentz Integral Transform and Resonances

- Introduction
- LIT method: controlled resolution
- 0^+ resonance of ^4He
- Case study for two-nucleon case

main point of the LIT :

Schrödinger-like equation with a source

$$\left(H - E_0 - \omega_0 + i\Gamma\right)\tilde{\Psi} = S$$

The $\tilde{\Psi}$ solution is unique and has ***bound state like*** asymptotic behavior

⇒ one can apply ***bound state methods***

Reformulation of the LIT

$$\mathrm{LIT}(\sigma_R,\sigma_I) = -\frac{1}{\sigma_I}\,\mathrm{Im}\left\{\left\langle \Psi_0 \middle| \Theta^\dagger\,(\sigma_R + E_0 - H + i\,\sigma_I)^{-1}\,\Theta\,\middle|\Psi_0\right\rangle\right\}$$

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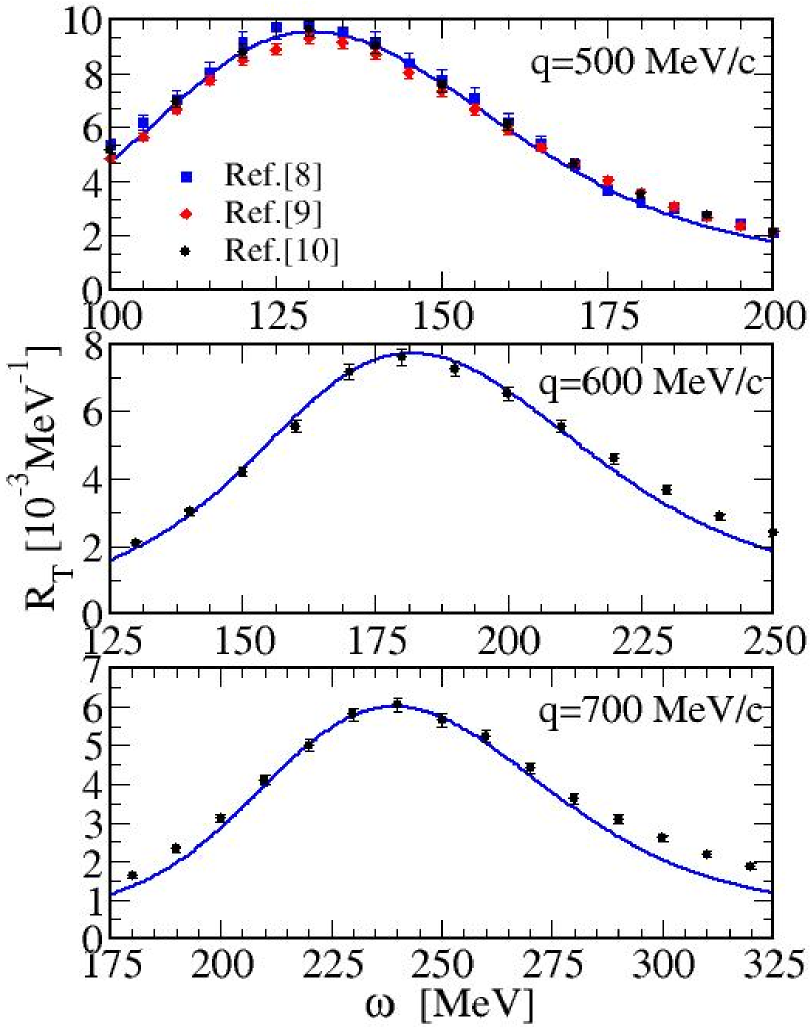
$$R(E=\sigma_R) = -\frac{1}{\pi}\,\mathrm{Im}\left\{\lim_{\sigma_I \to 0}\,\langle\Psi_0|\Theta^\dagger\,(\sigma_R + E_0 - H + i\,\sigma_I)^{-1}\,\Theta\,|\Psi_0\rangle\right\}$$

LIT method allows calculation up into the far continuum!

$R_T(q,\omega)$ of $^3He(e,e')$



NN potential AV18
Three-nucleon force UIX

L. Yuan et al., PLB 706, 90 (2011)

Experimental data:
Bates, Saclay,
world data (J. Carlson et al.)

LIT - Inversion

Standard LIT inversion method

Take the following ansatz for the response function $R(\omega)$ (or $F_{fi}(E,E')$)

$$R(\omega') = \sum_{m=1}^{M_{max}} c_m \, \chi_m(\omega',\alpha_i)$$

with $\omega'=\omega-\omega_{th}$, given set of functions χ_m , and unknown coefficients c_m

Define: $$\tilde{\chi}_m(\sigma_R,\sigma_I,\alpha_i) = \int_0^\infty d\omega' \, \frac{\chi_m(\omega',\alpha_i)}{(\omega'-\sigma_R)^2+\sigma_I^2}$$

Calculate LIT $L(\sigma_R,\sigma_I) = <\tilde{\psi}|\tilde{\psi}>$ for many σ_R and fixed σ_I

and expand in set $\tilde{\chi}_m$: $L(\sigma_R,\sigma_I) = \sum_{m=1}^{M_{max}} c_m \, \tilde{\chi}_m(\omega',\alpha_i)$

Determine c_m via best fit

Increase M_{max} up to the point that stable result is obtained for $R(\omega)$. Even further increase of M_{max} might lead to new structures in $R(\omega)$

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As basis set χ_m we normally use

$$\chi_m(\omega',\alpha_i) = (\omega')^{\alpha} \exp(-\alpha_2\omega'/m) \quad \text{with} \quad m = 1, 2, \ldots, M_{max}$$

LIT method: controlled resolution

LIT - Example

deuteron photodisintegration in unretarded dipole approximation

unretarded dipole approximation $\Rightarrow \Theta = \sum_{i=1}^{A} z_i \frac{1+\tau_{i,z}}{2}$,

z_i, $\tau_{i,z}$: 3rd components of position and isospin coordinates of i-th nucleon

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$\overset{\Theta}{\Rightarrow}$ $\sigma_\gamma(\omega) = 4\pi^2 \alpha\, \omega\, R(\omega)$ with $R(\omega) = \sum\!\!\!\!\!\!\int_f |\langle f|\, \Theta|0\rangle|^2\, \delta(\omega - E_f - E_0)$

with $|0\rangle$ and E_0 bound-state wave function and energy

$|f\rangle$ and E_f final-state wave function and energy

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$|f\rangle$ and E_f final-state wave function and energy

In unretarded dipole approximation $|f\rangle$ contains only $^3P_0$, $^3P_1$, $^3P_2 - {}^3F_2$ NN states

Solution of LIT equation via expansion on HH basis

⇒ Discretization of the continuum

Example: deuteron photodisintegration in unretarded dipole approximation. Expansion of radial part Laguerre polynomials up to order N times an exponential fall-off $\exp(-\rho/b)$ with $\rho \equiv r$

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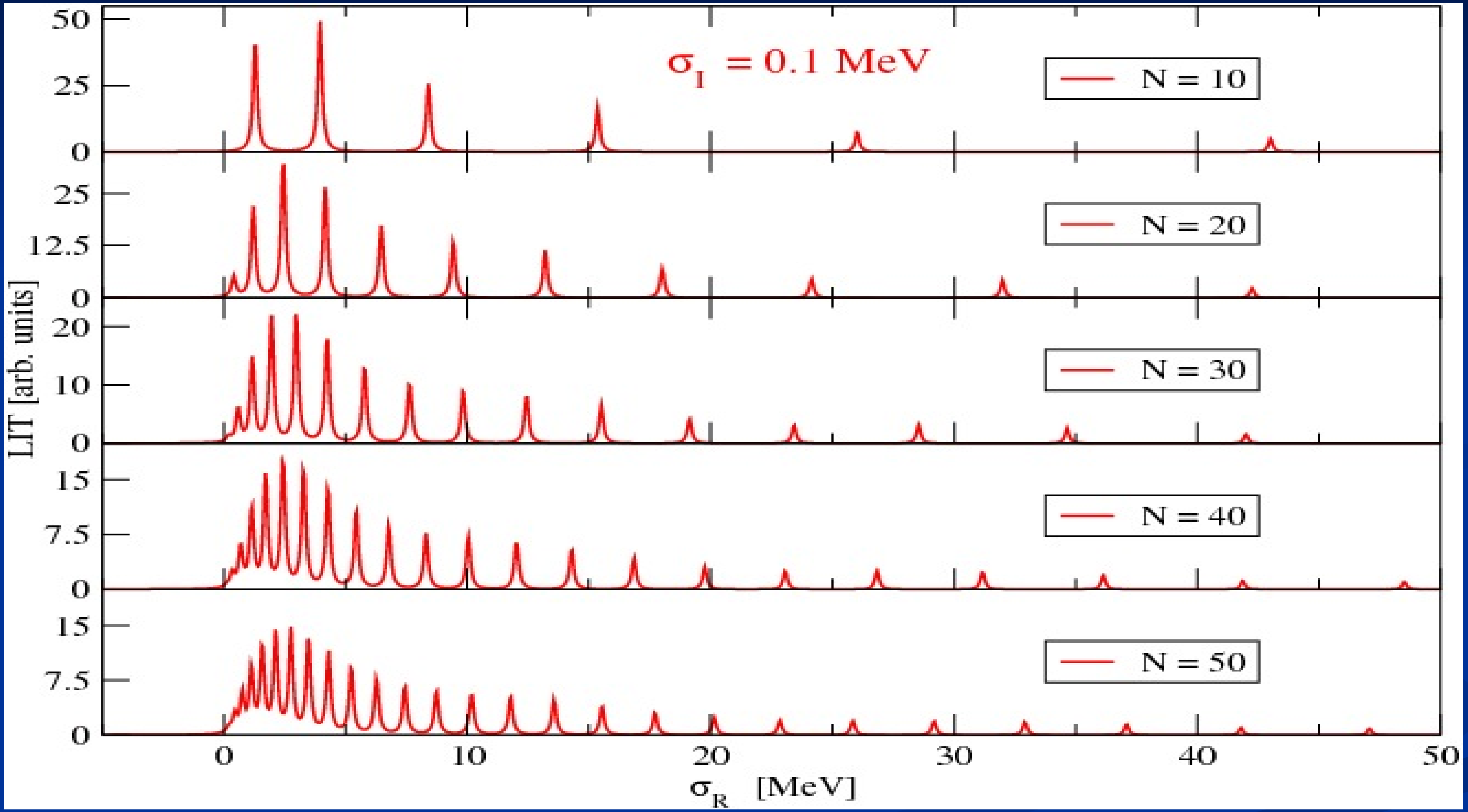
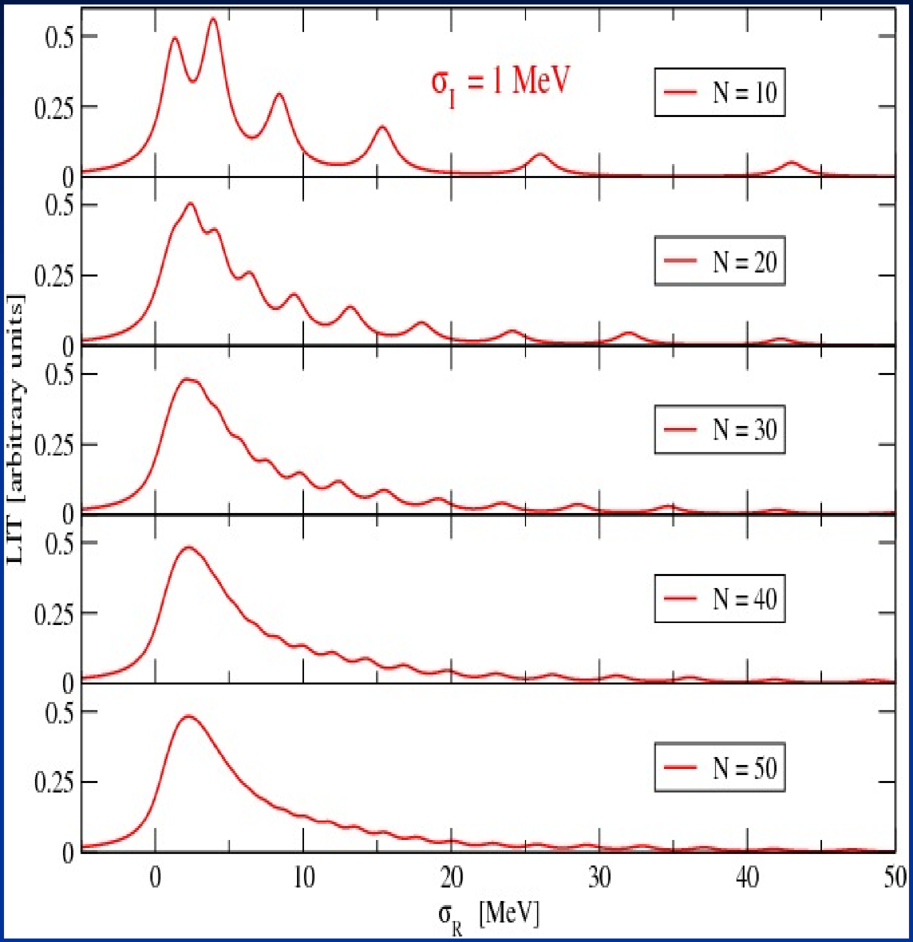
LIT results with various resolutions σ_I

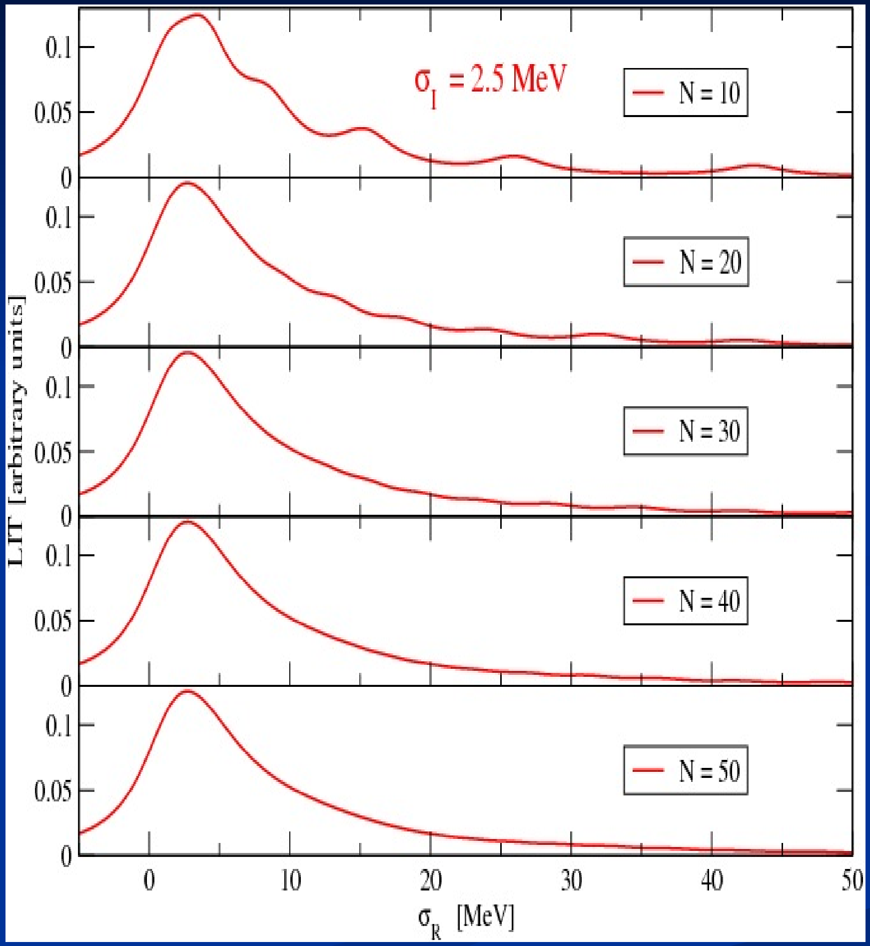
(only transitions to $^3P_1$)

This leads to the following LITs with Laguerre polynomials up to order N with exponential fall-off (b=0.5 fm):



Laguerre polynomials up to order N (exponential fall-off)





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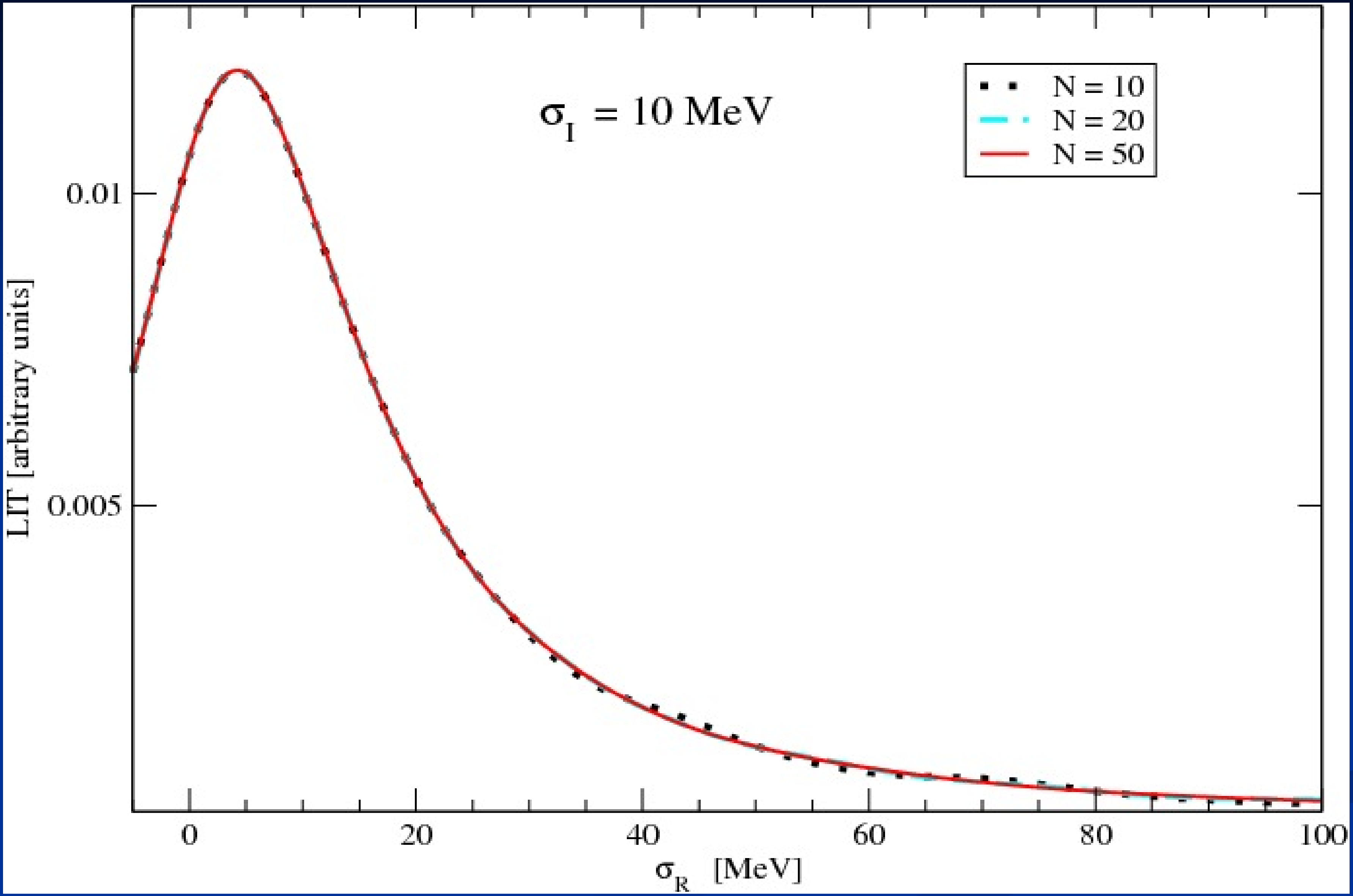

LIT method is an approach with a controlled resolution!

Strength for a given discrete state of energy E is not the actual strength for this energy, but can only be interpreted correctly within an integral transform approach.

The correct distribution of strength is obtained via the inversion of the integral transform.

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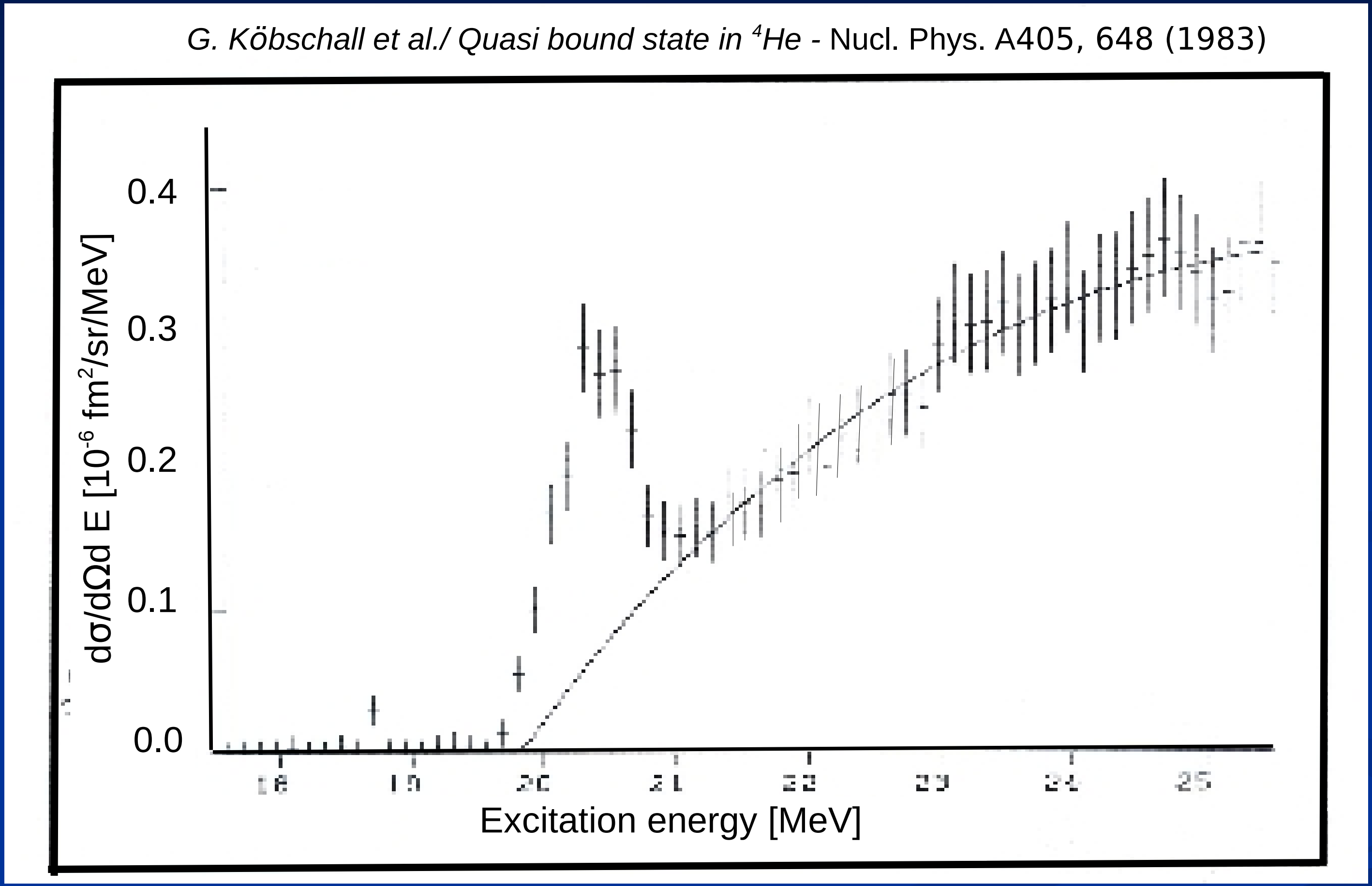
Correct procedure: Ensure convergence of expansion and search for the smallest possible resolution

LIT method and resonances

O^+ resonance in longitudinal response function R_L in $^4He(e,e')$

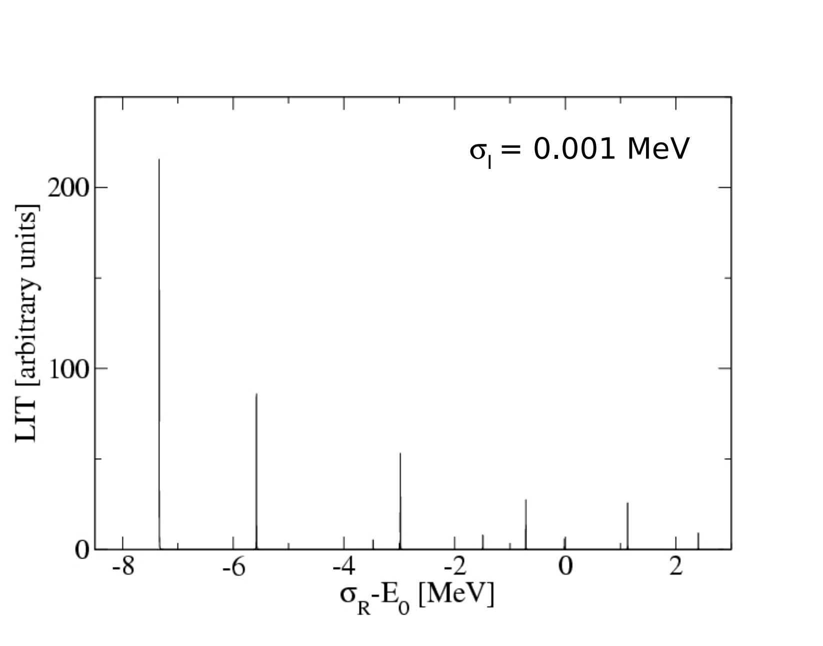
S. Bacca, N. Barnea, WL, G. Orlandini, PRL 110, 042503 (2013)

0^+ Resonance in the ^{4}He compound system

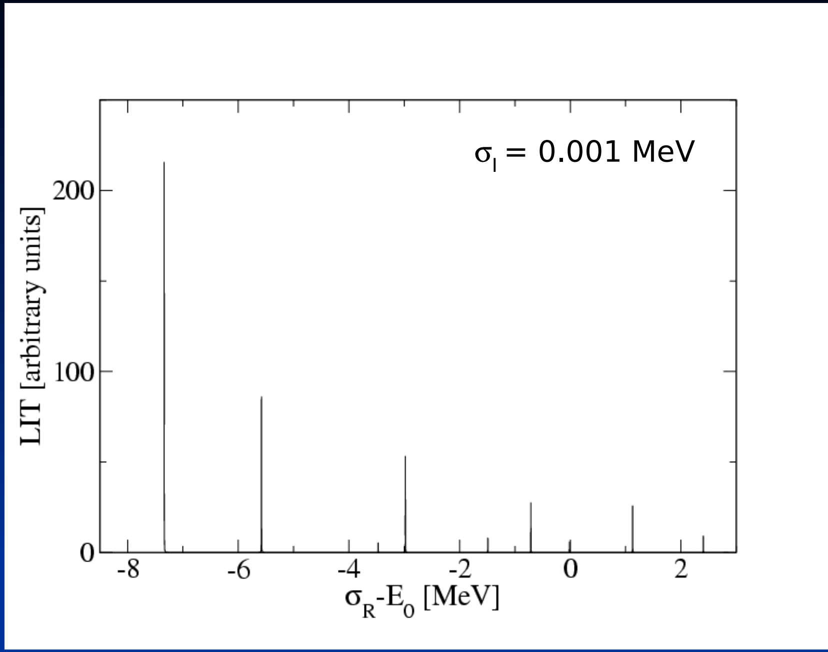


Resonance at E_R = -8.2 MeV, i.e. above the ^{3}H-p threshold. Strong evidence in electron scattering off ^{4}He, Γ = 270±70 keV

Results of our LIT calculation



σI = 0.001 MeV
LIT [arbitrary units]
200
100
0
-8
-6
-4
-2
0
2
σR-E0 [MeV]


σI = 0.001 MeV
LIT [arbitrary units]
200
100
0
-8
-6
-4
-2
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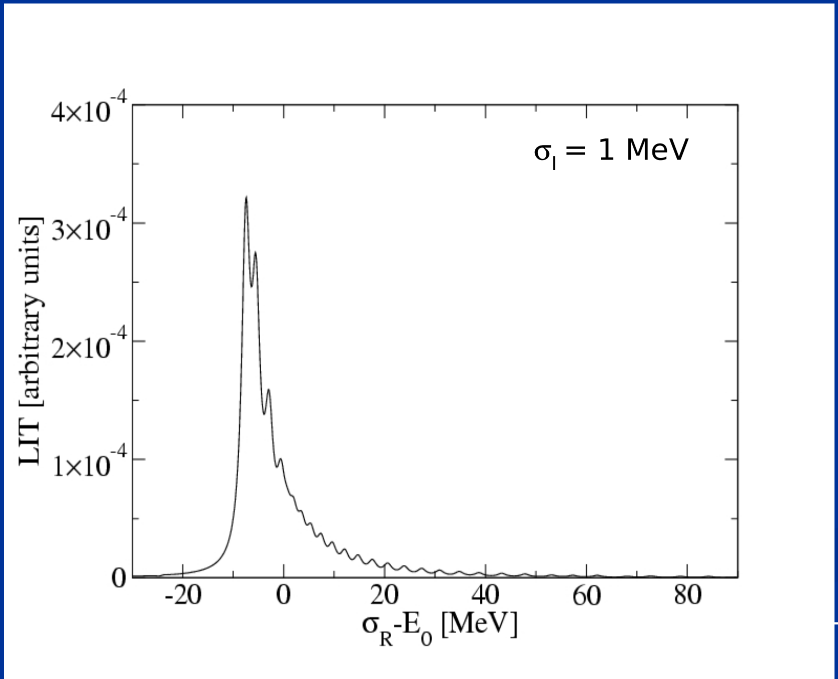
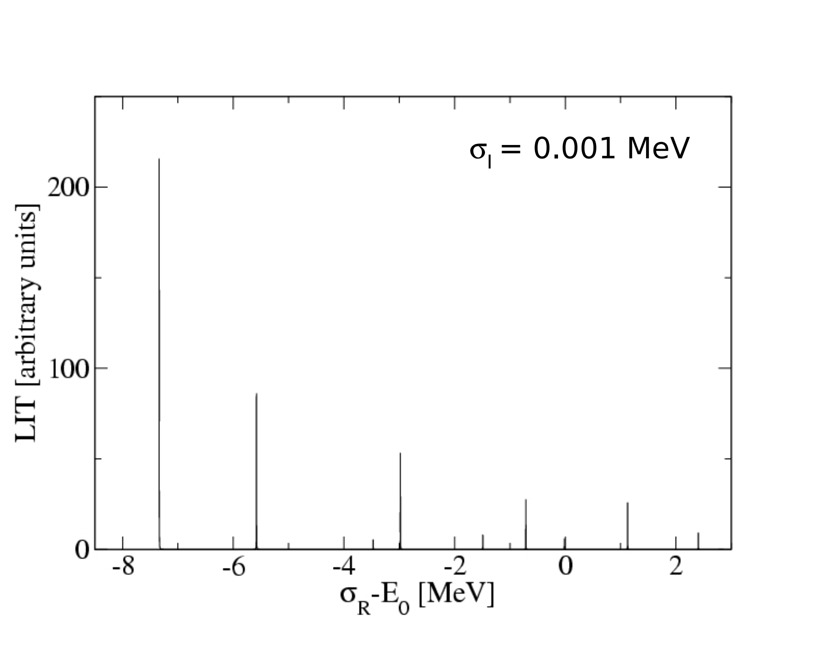
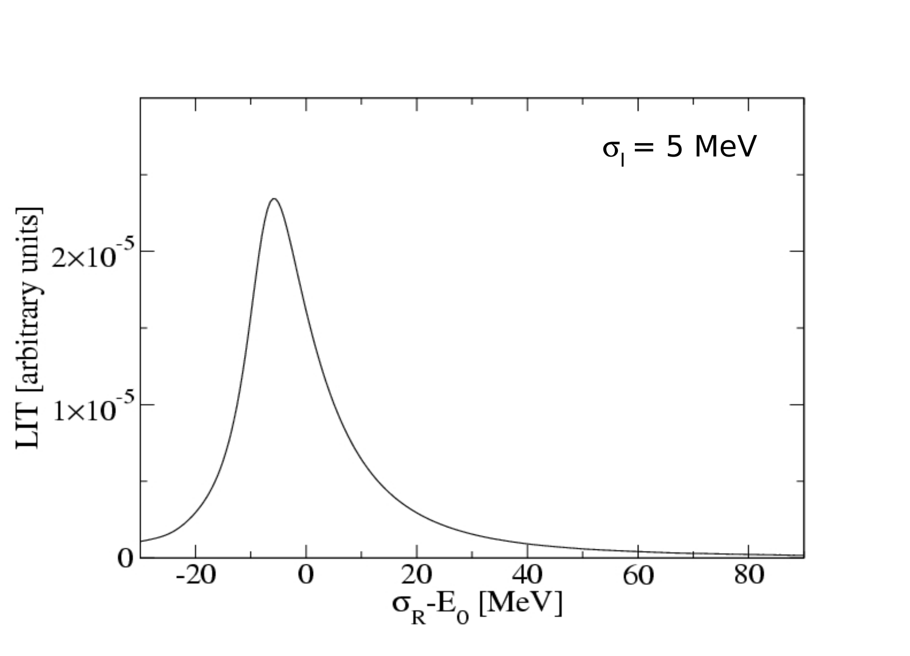
2
σR-E0 [MeV]

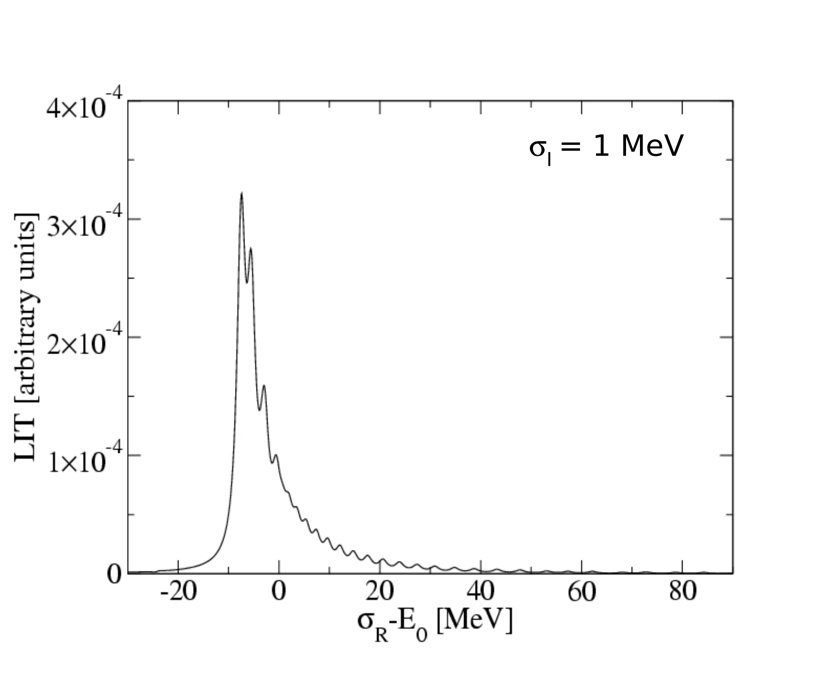

σI = 1 MeV
LIT [arbitrary units]
4×10-4
3×10-4
2×10-4
1×10-4
0
-20
0
20
40
60
80
σR-E0 [MeV]



σ_I = 0.001 MeV
LIT [arbitrary units]
200
100
0
-8
-6
-4
-2
0
2
σ_R-E_0 [MeV]



σ_I = 5 MeV
LIT [arbitrary units]
2×10^{-5}
1×10^{-5}
0
-20
0
20
40
60
80
σ_R-E_0 [MeV]



σ_I = 1 MeV
LIT [arbitrary units]
4×10^{-4}
3×10^{-4}
2×10^{-4}
1×10^{-4}
0
-20
0
20
40
60
80
σ_R-E_0 [MeV]

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Of course not by taking the strength to the discretized state, but by rearranging the inversion in a suitable way:

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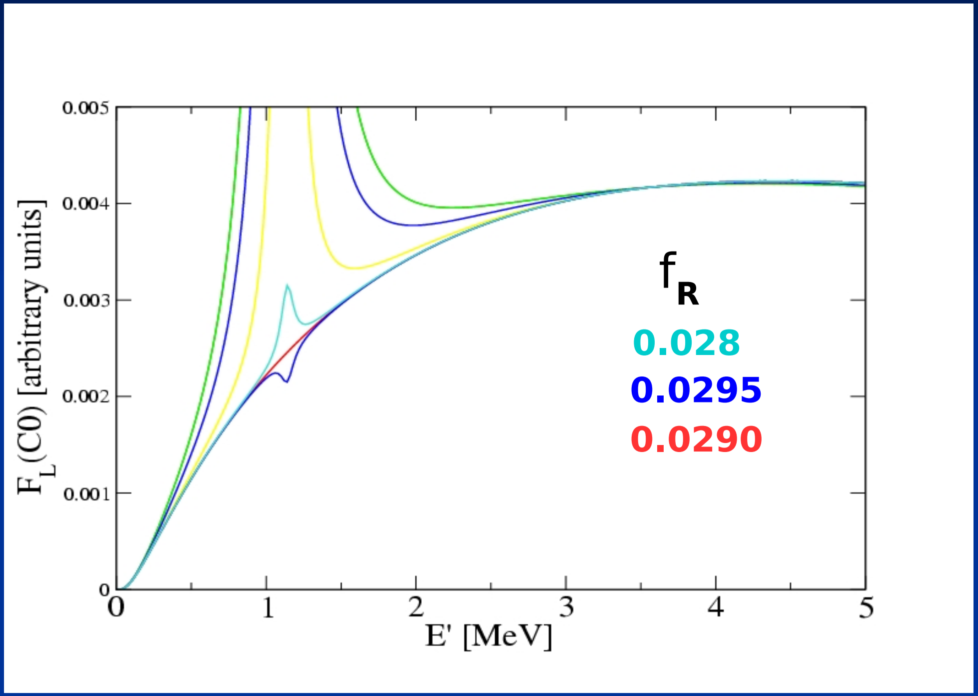
However, the strength of the resonance can be determined!

Of course not by taking the strength to the discretized state, but by rearranging the inversion in a suitable way:

Reduce strength of the first state up to the point that the inversion does not show any resonant structure at the resonance energy $E_{\mathbf{R}}$:

$$\mathrm{LIT}(\sigma_R, \sigma_I) \rightarrow \mathrm{LIT}(\sigma_R,\sigma_I) - f_R \,/\, [(E_{\mathbf{R}} - \sigma_R)^2 + \sigma_I^2\,] \equiv \mathrm{LIT}(\sigma_R, \sigma_I, f_R)$$

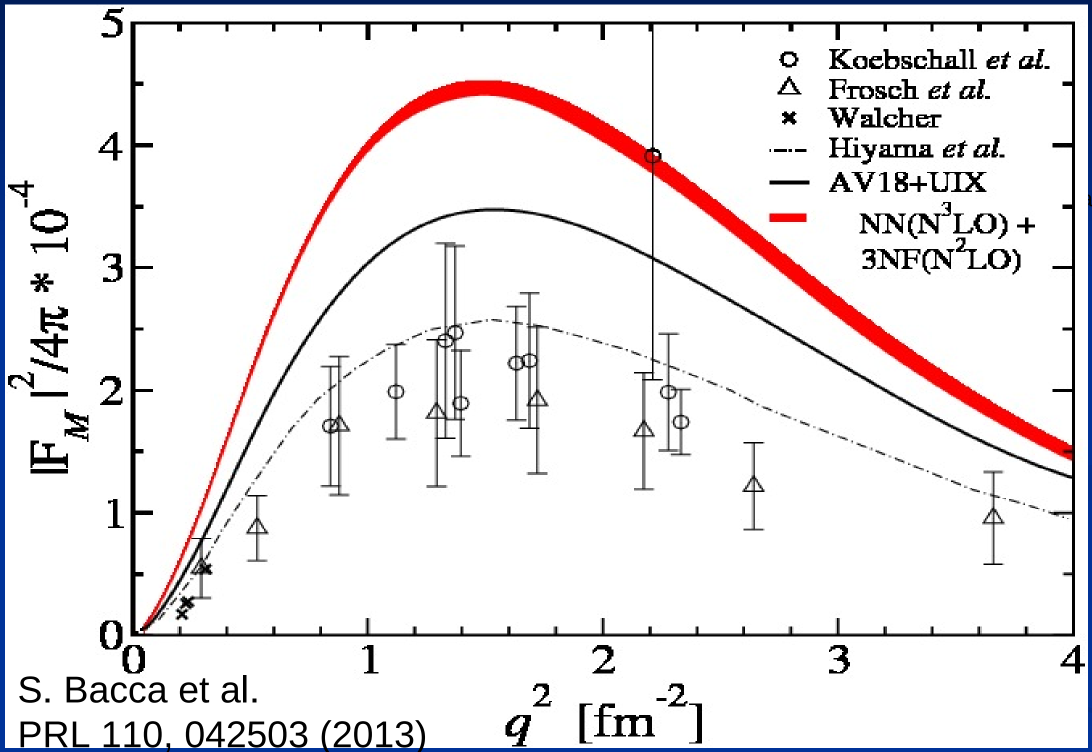
with resonance strength f_R



Inversion results with different f_R values

AV18+UIX, q=300 MeV/c

Comparison to experimental results



S. Bacca et al.
PRL 110, 042503 (2013)

LIT/EIHH Calculation for AV18+UIX and Idaho-N3LO+N2LO

Dotted: AV8' + central 3NF (Hiyama et al.)

Study of Resonance for the deuteron case

NN potential with fictitious resonance in $^3P_1$ partial wave

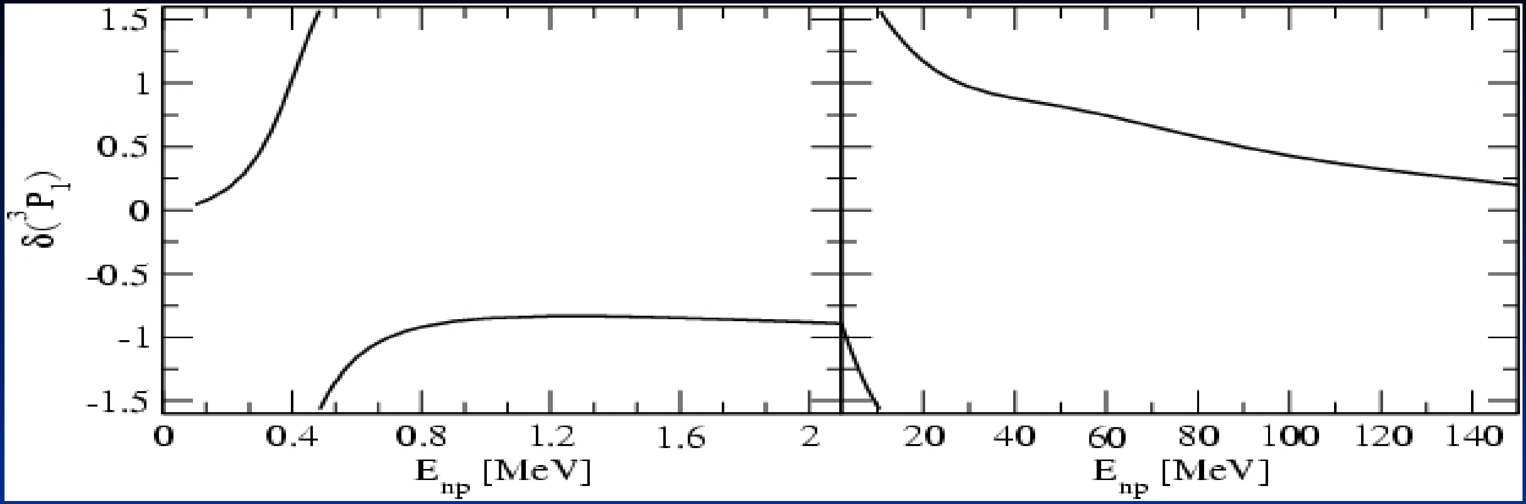
$$V(^3P_1) \longrightarrow V(^3P_1) + V_{add}$$

With $$V_{add} = -\frac{57.6\ \mathrm{MeV}}{r}\left(1-\exp(-2r^2)\right)\left(1+\exp\left(\frac{r-5}{0.2}\right)\right)^{-1}$$

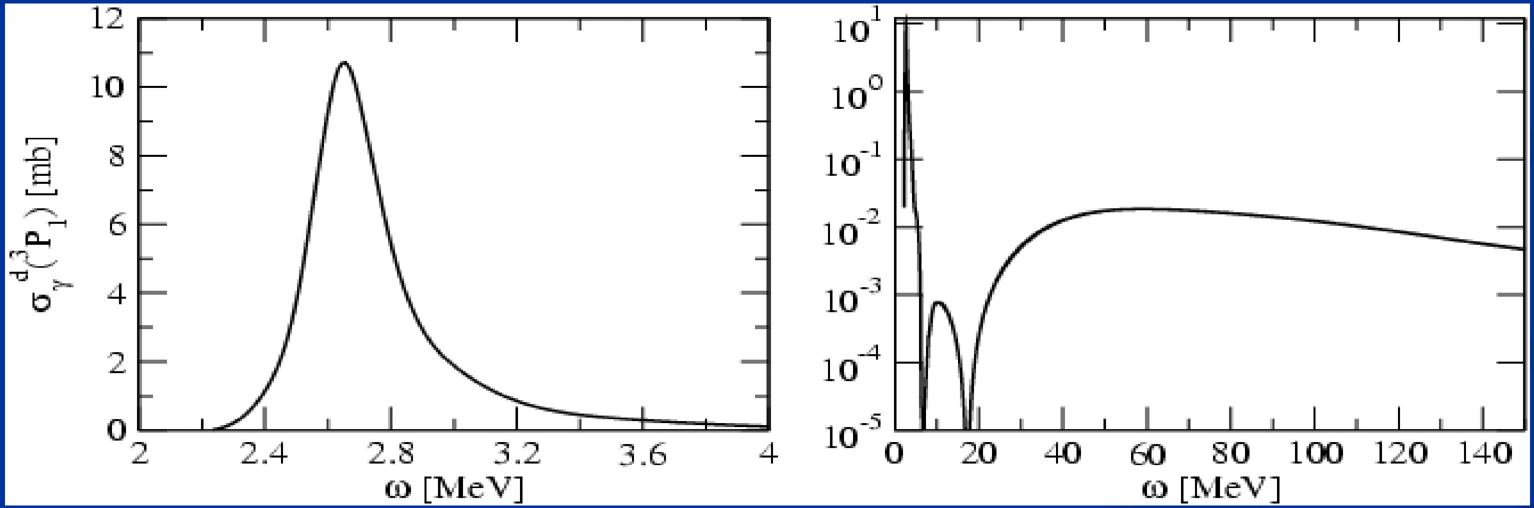
and relative coordinate r in units of fm

Why such a potential?

To understand this better let us have a look on corresponding phaseshift $^3P_1$ and deuteron photoabsorption cross section in $^3P_1$ partial wave



Phase shifts shows two resonances one at $E_{\mathbf{np}}$ = 0.48, 10.5 MeV

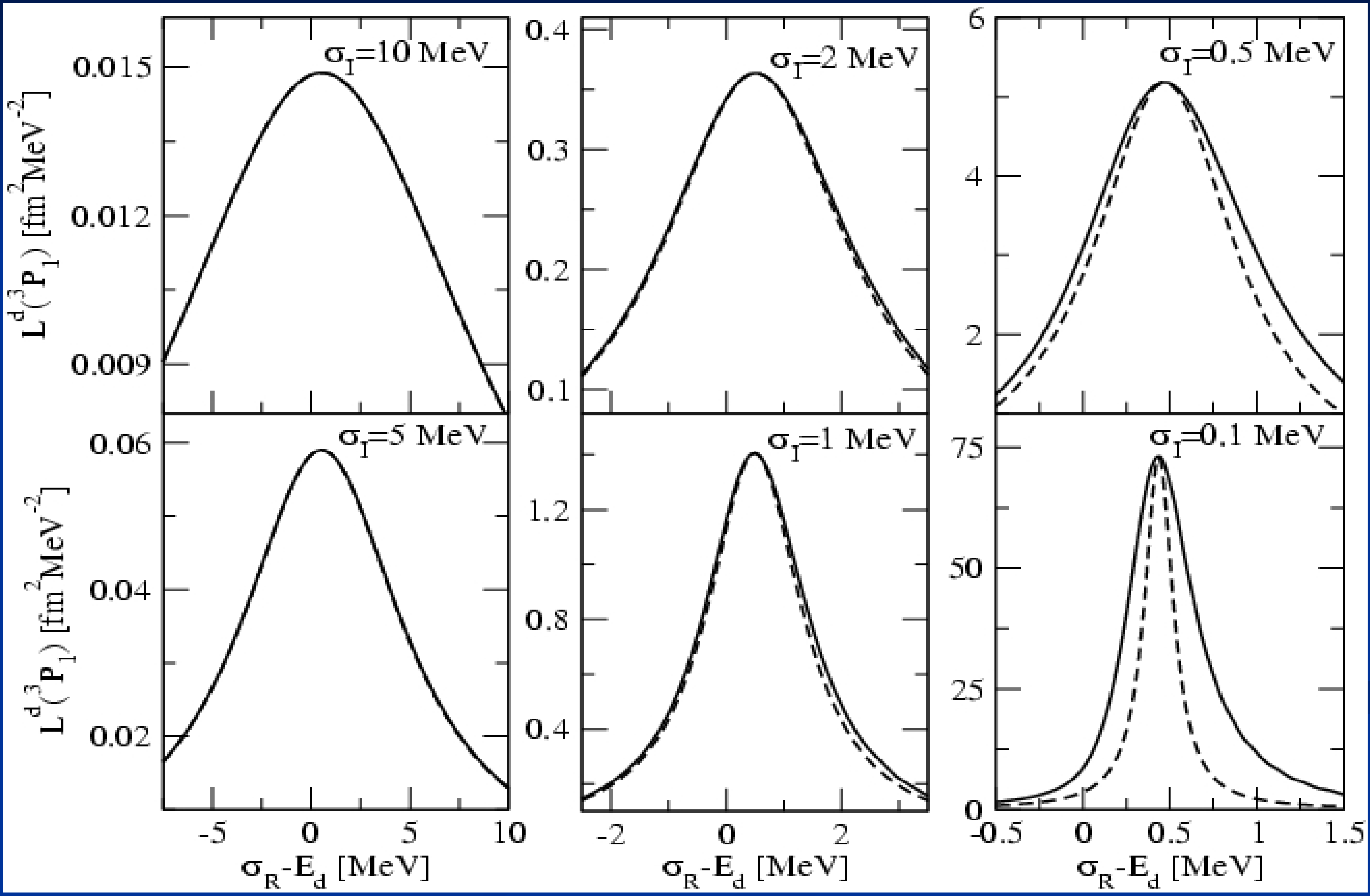


$\sigma_\gamma(^3P_1)$ shows two corresponding resonances: low-energy resonance very pronounced with small width Γ=270 KeV, the other one is much weaker and has a larger width

Results with modified $^3P_1$ potential

First LIT in the region of the low-energy resonance

LITs in the resonance region with various σ_I (full curves);
comparison with single Lorentzians of corresponding σ_I (dashed curves)

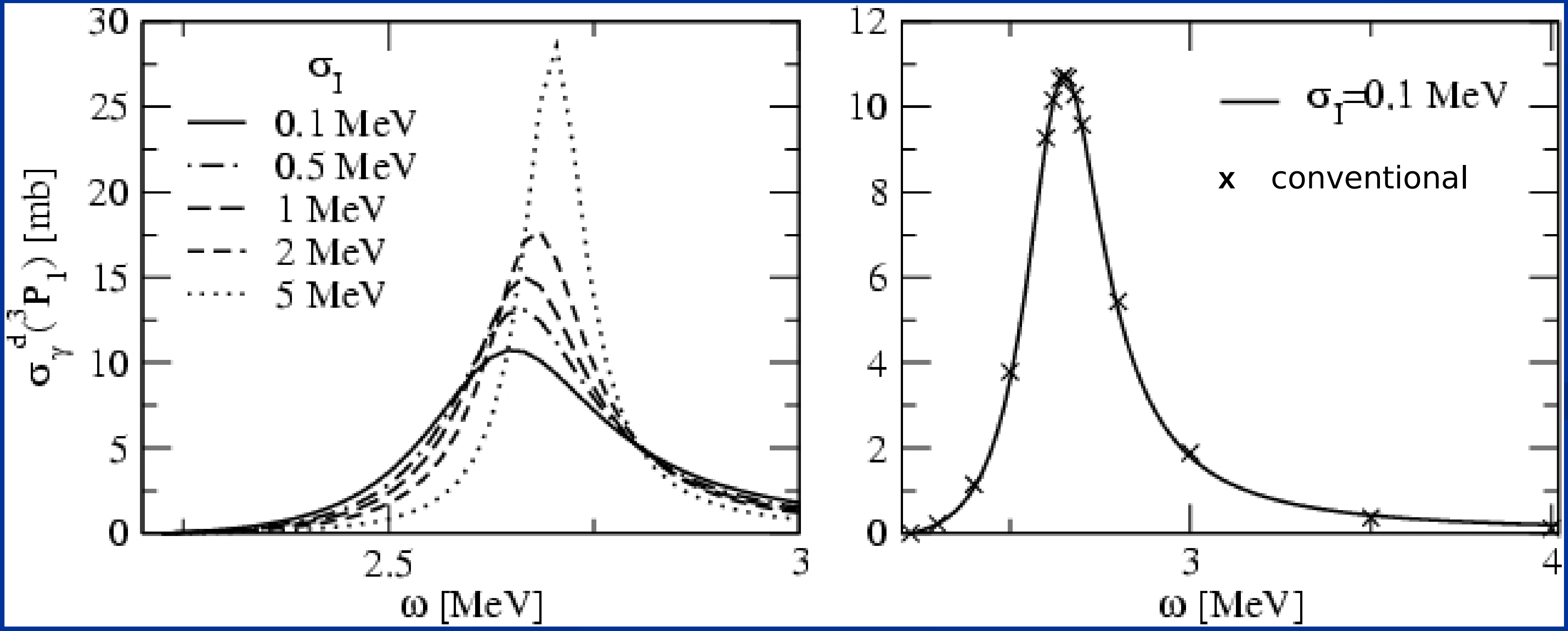


Incomplete Inversion

Instead of using set χ_m defined previously we take M_{max}=1 and take

$$\chi_1^{res} = \frac{1}{(E_{np} - E_{res})^2 + (\Gamma/2)^2}\left(\frac{1}{1+\exp(-1)} - \frac{1}{1+\exp((E_{np} - \alpha_3)/\alpha_3)}\right)$$

E_{res} , Γ, and α_3 are fit parameters

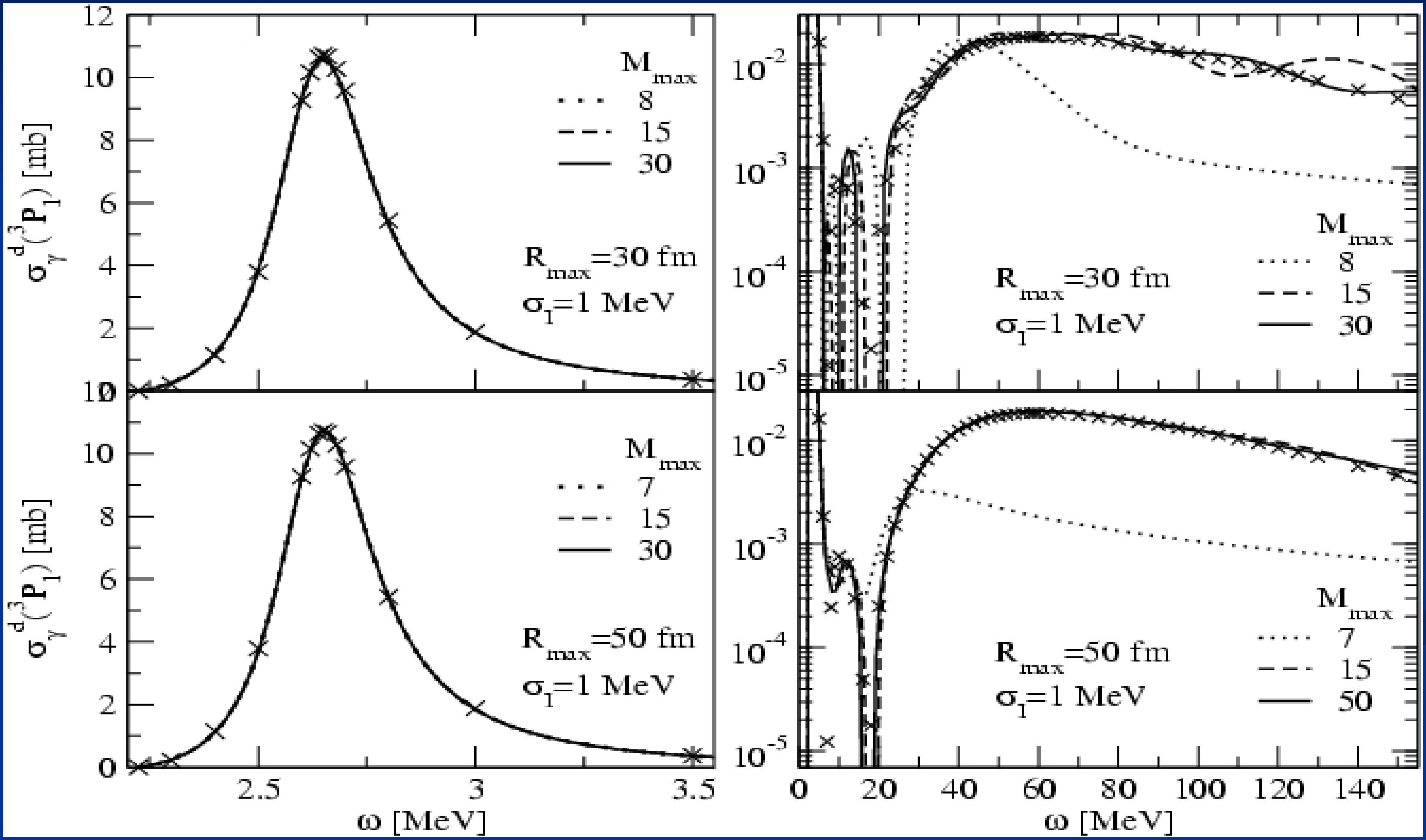


Results with modified $^3P_1$ potential

Now to the LIT results beyond low-energy resonance

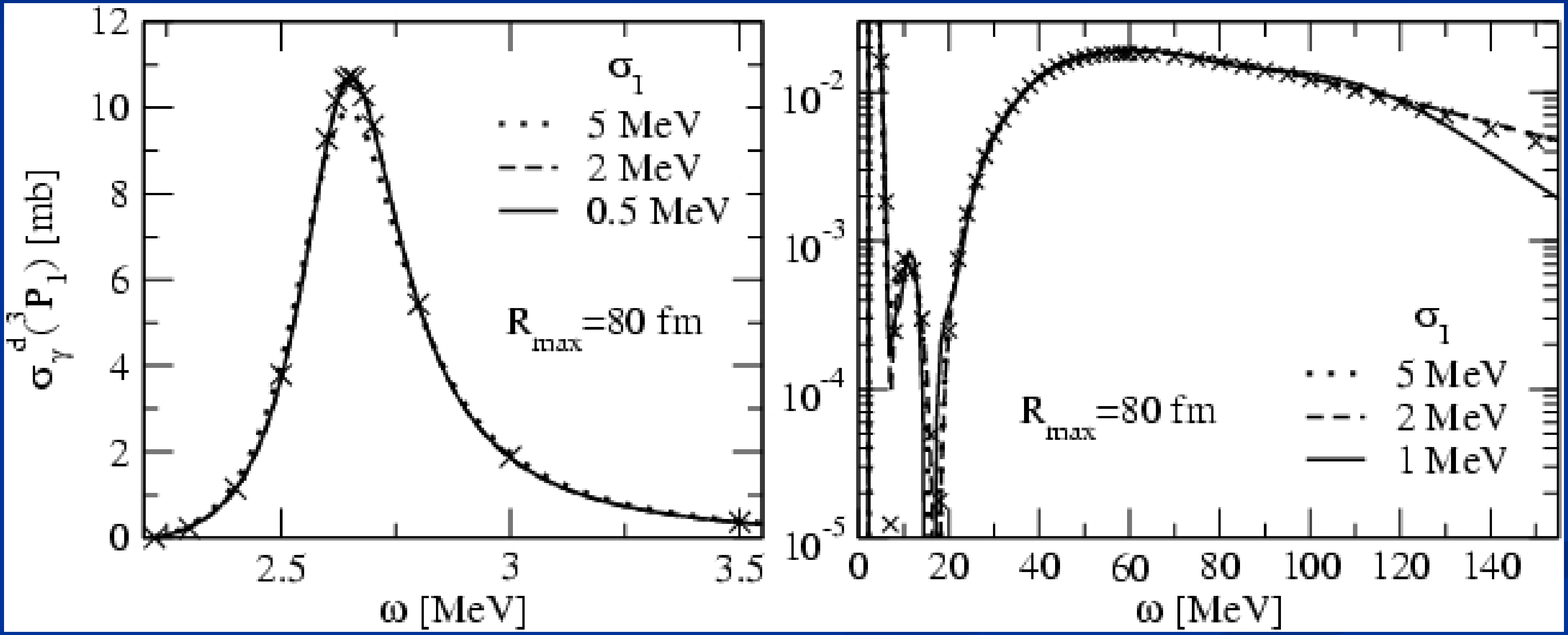
Complete inversion with set χ_m defined previously using in addition as new first basis function χ_1^{res}

σ_I = 1 MeV, R_{max} = 30 and 50 fm, various M_{max}



Complete inversion with set χ_m defined previously using in addition as new first basis function χ_1^{res}

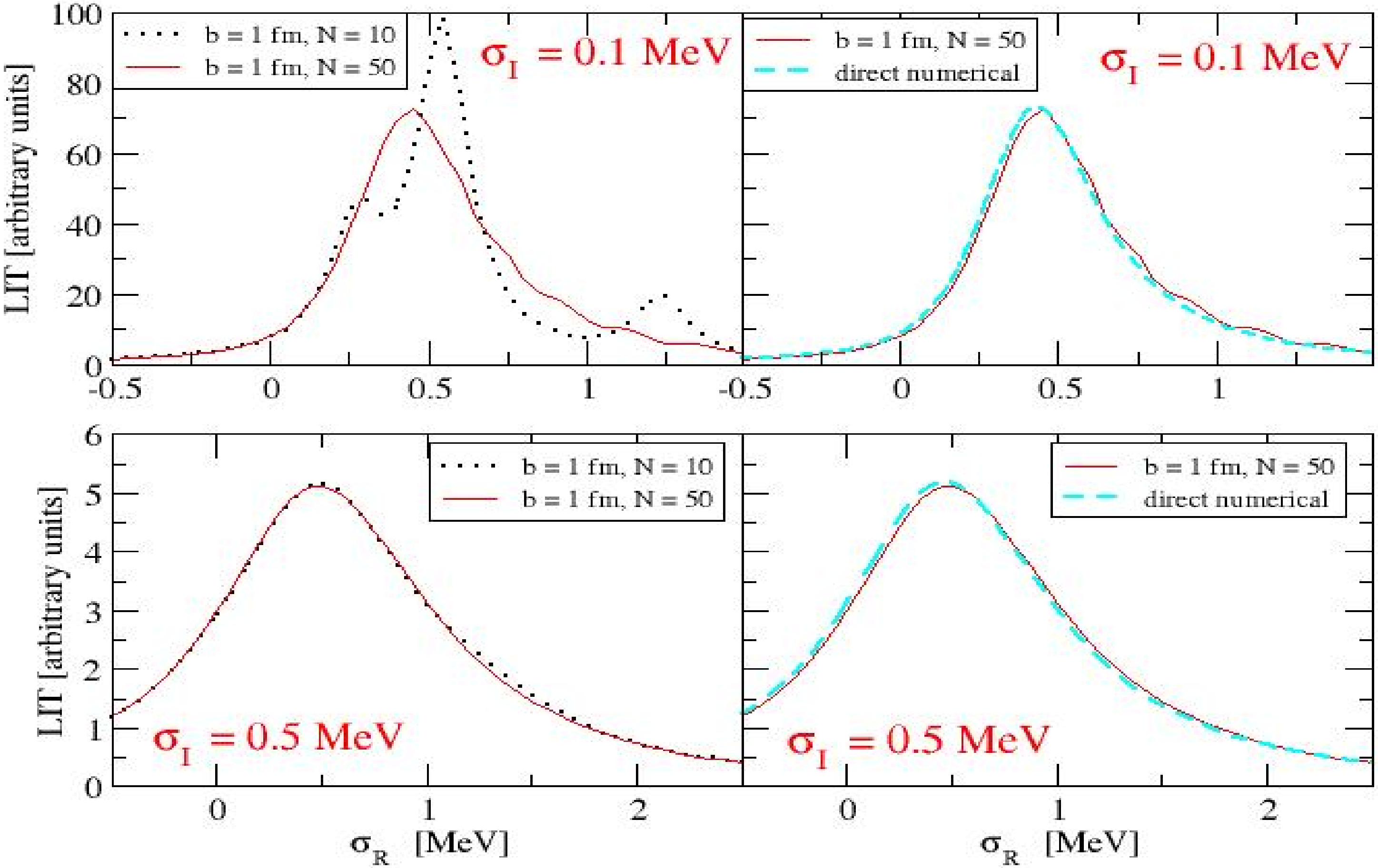
various σ_I , R_{max} = 80 fm, M_{max} =30



Up to now direct numerical solutions of Schrödinger equation for bound state and LIT equation for $\widetilde{\Psi}$

For A > 2 it is more convenient to use expansions in complete sets using expansions in HH or HO functions

Expansion of radial part of $\widetilde{\Psi}$ in Laguerre polynomials up to order N times an exponential fall-off exp(-ρ/b) with ρ≡r



b = 1 fm, N = 10
b = 1 fm, N = 50
$\sigma_I = 0.1$ MeV
direct numerical
$\sigma_I = 0.5$ MeV
LIT [arbitrary units]
σ_R [MeV]

Conclusions

- the LIT metod opens up the possibility to carry out ab-initio calculations of reactions into the A-body continuum for A > 2
- only **bound states** techniques are needed
- the LIT is a method with controlled resolution
- Exact determination of resonant shapes might be difficult, but is in principle possible