Towards an Estimation of Nuclear Forces and Nuclear Matrix Elements Uncertainties: Chiral vs Non-Chiral

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From Few-Nucleon Forces to Many-Nucleon Structure
ECT*-Trento, 10 Jun 2013 to 14 Jun 2013

References

- [1] **Coarse graining Nuclear Interactions**
  ERICE Summer School (Sep-2011)
  Prog. Part. Nucl. Phys. **67** (2012) 359 [arXiv:1111.4328 [nucl-th]].
- [2] **Phenomenological High Precision Neutron-Proton Delta-Shell Potential**
  Phys. Lett. B (2013) to appear, arXiv:1202.2689 [nucl-th].
- [3] **Error estimates on Nuclear Binding Energies from Nucleon-Nucleon uncertainties**
  arXiv:1202.6624 [nucl-th].
- [4] **Nuclear Binding Energies and NN uncertainties**
  Quark Nuclear Physics (May-2012)
  PoS QNP **2012** (2012) 145 [arXiv:1206.3508 [nucl-th]].
- [5] **Effective interactions in the delta-shells potential**
  International IUPAP Conference on Few-Body Problems in Physics, Aug-2012
  Few-Body Syst (2013), arXiv:1209.6269 [nucl-th].
- [6] **Nucleon-Nucleon Chiral Two Pion Exchange potential vs Coarse grained interactions**
  Chiral Dynamics Aug-2012. arXiv:1301.6949 [nucl-th].
- [7] **Partial Wave Analysis of Nucleon-Nucleon Scattering below pion production**
  Phys. Rev. C (2013) to appear, arXiv:1304.0895 [nucl-th].

Bottomline

THE PROBLEM

- GOAL: Estimate uncertainties from IGNORANCE of NN,3N,4N interaction
  Reduce computational cost
- Statistical Uncertainties: NN,3N,4N Data
  Data abundance bias
- Systematic Uncertainties: NN,3N,4N potential
  Many forms of potentials possible
- Confidence level of Imperfect theories vs Perfect experiments

OUR APPROACH

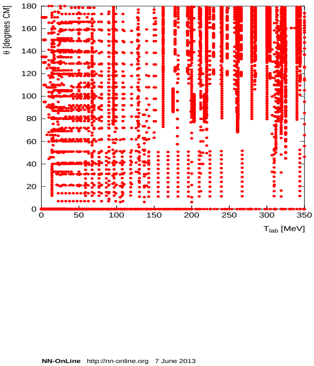
- Start with NN
- Fit data WITH ERRORS with a simple interaction
- Compare different interactions (AV18,CDBonn,N3LO,Nijm,Spec)
- Estimate uncertainties of Effective Interactions and Matrix elements

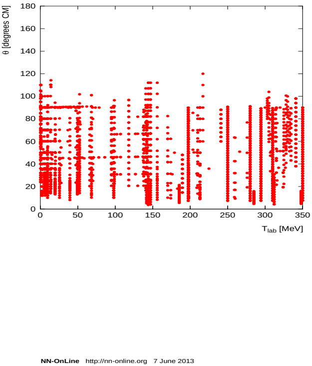
Error Analysis in Nuclear Structure

- Theoretical Predictive Power Flow: From light to heavy nuclei
- Experiment much more precise than theory
- How to estimate theoretical errors based on INPUT data

$$INPUT = NN, 3N, \cdots \rightarrow OUTPUT = 4N, \ldots$$

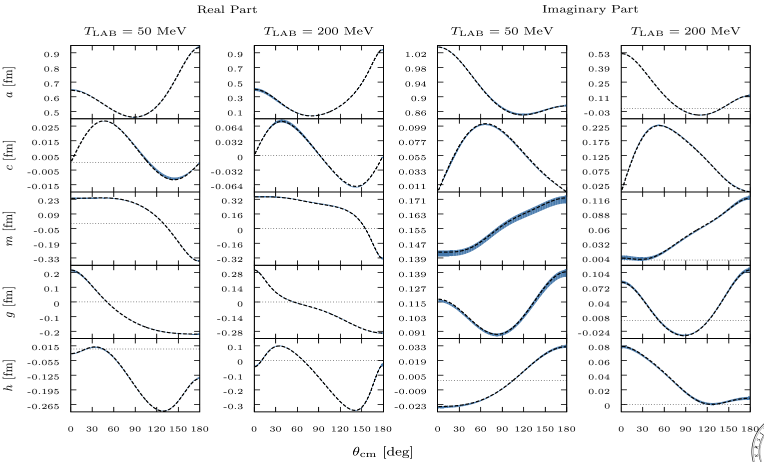
- First Step: INPUT=NN scattering data
- OUTPUT=NN scattering amplitudes





Wolfenstein Parameters

- PWA,NimjII,CDBonn,Spec,Reid93,AV18, $\chi^2/\mathrm{dof} \sim 1$



Introduction

- How much do we need to know light nuclei to predict heavy nuclei ?
- Nucleon size $a \sim 1\text{fm}$
- Nuclear Force $\sim 1/m_\pi = 1.4\text{fm}$
- Nuclear matter (interparticle distance)

$$\rho_{nm} = 0.17\text{fm}^{-3} = \frac{1}{(1.8\text{fm})^3}$$

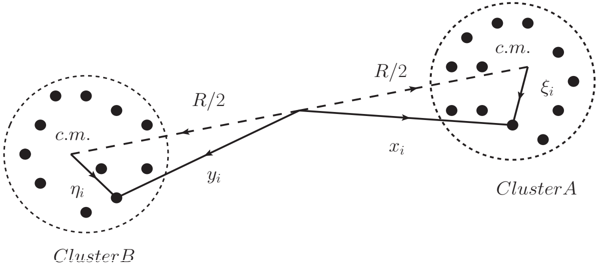
- Fermi Momentum

$$k_F = 270\text{MeV} \qquad \lambda_F = \pi/k_F = 2.3\text{fm} \gg 1/\sqrt{m_\pi M_N} = 0.5\text{fm}$$

Can we ignore explicit core, finite nucleon size and explicit pions ?
What is the confidence level for this scenario ?

Quark Cluster Dynamics (qcd)

- Atomic analogue. Neutral atoms
  - Non-overlapping atoms exchange TWO photons (Van der Waals force)
  - Overlapping atoms are not locally neutral; ONE photon exchange is possible (Chemical bonding)



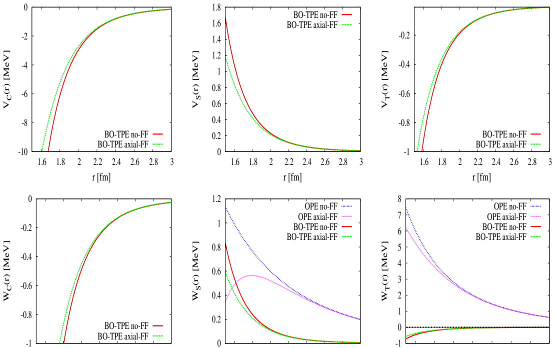
- Overlapping effects (quark exchange) constrain the applicability of Lagrangians

Quark Cluster Dynamics (qcd)

- NN potential in the Born-Oppenheimer approximation — Calle Cordon, RA, '12

$$\bar{V}^{1\pi+2\pi+\cdots}_{NN,NN}(\boldsymbol{r}) = V^{1\pi}_{NN,NN}(\boldsymbol{r}) + 2\,\frac{|V^{1\pi}_{NN,N\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \frac{1}{2}\,\frac{|V^{1\pi}_{NN,\Delta\Delta}(\boldsymbol{r})|^2}{M_N - M_\Delta} + \mathcal{O}(V^3)\,,$$

- Bulk of TWO-Pion Exchange Chiral forces reproduced
- Finite size effects set in at $2\mathrm{fm}$ $\rightarrow$ exchange quark effects become explicit
- High quality potentials confirm these trends.



Anatomy of the unknown NN interaction

- At what distance look nucleons point-like ?

$$r > 2\text{fm}$$

- When is OPE the ONLY contribution ?

$$r_c > 3\text{fm}$$

- What is the minimal resolution where interaction is elastic ?

$$p_{\text{max}} \sim \sqrt{M_N m_\pi} \to \Delta r = 1/p_{\text{max}} = 0.6\text{fm}$$

- How many partial waves must be fitted ?

$$l_{\text{max}} = p_{\text{max}} r_c r_c / \Delta r = 5$$

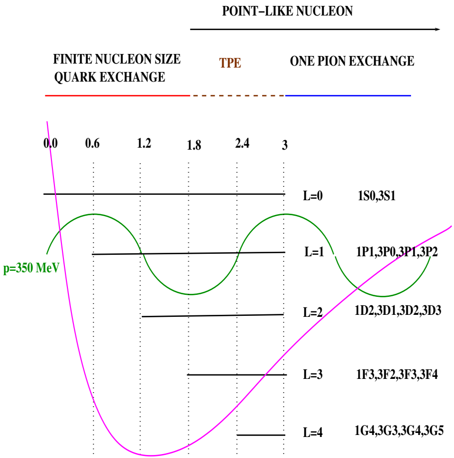
- Minimal distance where centrifugal barrier dominates

$$\frac{l(l+1)}{r_{\text{min}}^2} \leq p^2$$

- How many parameters ?
  $({}^1S_0, {}^3S_1)$, $({}^1P_1, {}^3P_0, {}^3P_1, {}^3P_2)$, $({}^1D_2, {}^3D_1, {}^3D_2, {}^3D_3)$, $({}^1F_3, {}^3F_2, {}^3F_3, {}^3F_4)$

$$2 \times 5 + 4 \times 4 + 4 \times 3 + 4 \times 2 + 4 \times 1 = 50$$

Anatomy of the unknown NN interaction

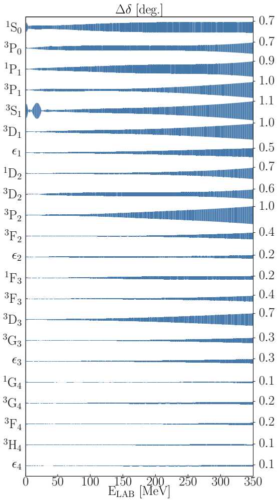


Motivation

- Study of the NN interaction for over 60 years
- More than 7800 experimental scattering data from 1950 to 2013
- Several partial wave analyses (PWA) and potentials since the 1950's
  - Hamada Johnston, Yale, Paris, Bonn, Nijmegen, Argonne, ...
- $\chi^2/\mathrm{d.o.f.} \sim 1$ possible by 1993

  [Stoks et al, Phys. Rev. C 48 (1993), 792]
- Chiral potentials appear in the mid 1990's

Motivation



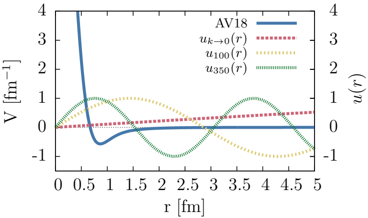
- No unique determination of the NN interaction
- Different phenomenological potentials
    - Fitted to experimental scattering data
    - High accuracy $\chi^2/\text{d.o.f.} \sim 1$
    - Dispersion in Phaseshifts
    - OPE as the long range interaction
    - ~ 40 parameters for the short and intermediate range
    - Repulsive core for most of them
        - Short range correlations
- Nuclear structure calculations become complicated
- No statistical uncertainties estimates

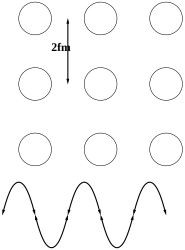
Motivation

- Effective coarse graining
  - Oscillator Shell Model
  - Euclidean Lattice EFT
  - $V_{\rm lowk}$ interaction
- Characteristic distance $\sim 0.5 - 1.0$ fm
- Nyquist Theorem
  - Optimal sampling
  - Finite Bandwidth

$$\Delta r \Delta k \sim 1$$

  - de Broglie wavelength of the most energetic particle





COARSE GRAINED INTERACTION

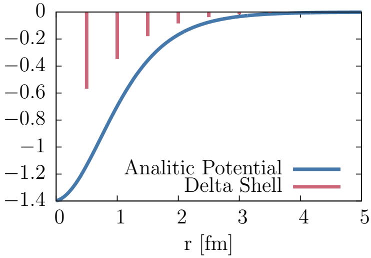
Delta Shell Potential

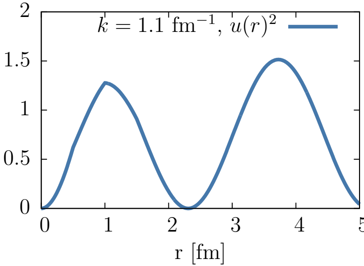
- A sum of delta functions

$$V(r) = \sum_i \frac{\lambda_i}{2\mu}\delta(r - r_i)$$

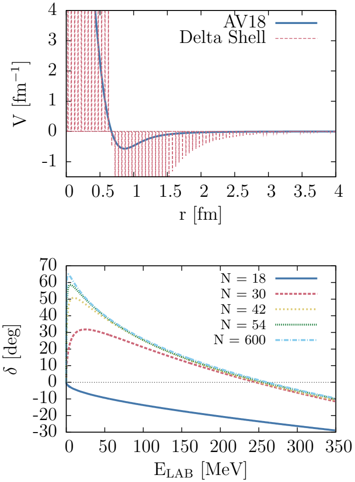
[Aviles, Phys.Rev. C6 (1972) 1467]

- Optimal and minimal sampling of the nuclear interaction
- Pion production threshold $\Delta k \sim 2$ fm^{-1}
- Optimal sampling, $\Delta r \sim 0.5$fm



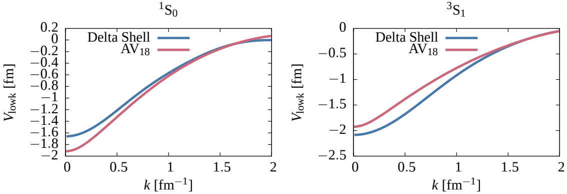


Coarse Graining the AV18 potential



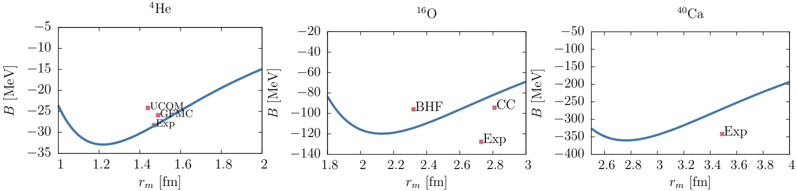
Delta Shell Potential

- Comparison with $V_{\rm lowk}$



- Nuclear structure calculations

[Prog.Part.Nucl.Phys. 67 (2012) 359]

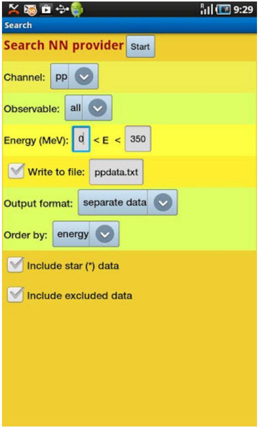


Delta Shell Potential

- 3 well defined regions
- Innermost region $r \leq 0.5$ fm
  - Short range interaction
  - No delta shell (No repulsive core)
- Intermediate region $0.5 \leq r \leq 3.0$ fm
  - Unknown interaction
  - λ_i parameters fitted to scattering data
- Outermost region $r \geq 3.0$ fm
  - Long range interaction
  - Described by OPE and EM effects
    - Coulomb interaction V_{C1} and relativistic correction V_{C2} (pp)
    - Vacuum polarization V_{VP} (pp)
    - Magnetic moment V_{MM} (pp and np)

np AND pp PARTIAL WAVE ANALYSIS

Fitting NN observables



- Database of NN scattering data obtained till 2013
  - http://nn-online.org/
  - http://gwdac.phys.gwu.edu/
  - NN provider for Android
    - Google Play Store

[J.E. Amaro, R. Navarro-Perez, and E. Ruiz-Arriola]

- 2868 pp data and 4991 np data
- 3σ criterion by Nijmegen to remove possible outliers

Fitting NN observables

- Delta shell potential in every partial wave

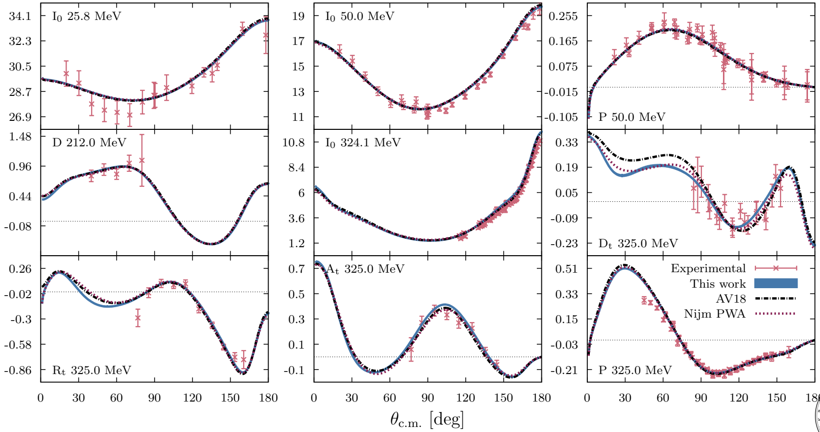
$$V^{JS}_{l,l'}(r) = \frac{1}{2\mu_{\alpha\beta}} \sum_{n=1}^{N} (\lambda_n)^{JS}_{l,l'} \delta(r - r_n) \qquad r \leq r_c = 3.0\text{fm}$$

- Strength coefficients λ_n as fit parameters
- Fixed and equidistant concentration radii $\Delta r = 0.6$ fm
- EM interaction is crucial for pp scattering amplitude

$$\begin{aligned} V_{C1}(r) &= \frac{\alpha'}{r} \,, \\ V_{C2}(r) &\approx -\frac{\alpha\alpha'}{M_p r^2} \,, \\ V_{VP}(r) &= \frac{2\alpha\alpha'}{3\pi r} \int_1^\infty dx\, e^{-2m_e r x} \left[1 + \frac{1}{2x^2}\right] \frac{(x^2-1)^{1/2}}{x^2} \,, \\ V_{MM}(r) &= -\frac{\alpha}{4M_p^2 r^3} \left[\mu_p^2 S_{ij} + 2(4\mu_p - 1)\mathbf{L}\cdot\mathbf{S}\right] \end{aligned}$$

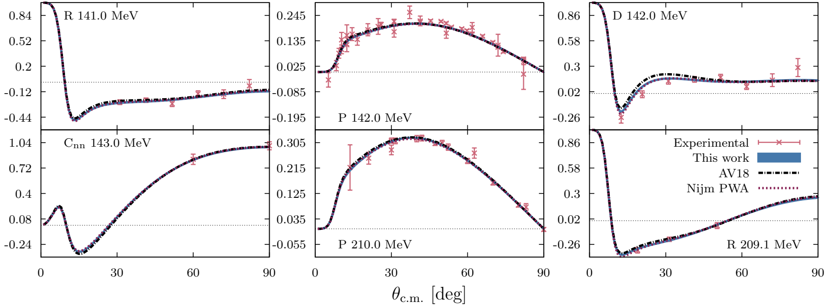
Scattering Observables

- Comparing with Potentials and Experimental data
- np data



Scattering Observables

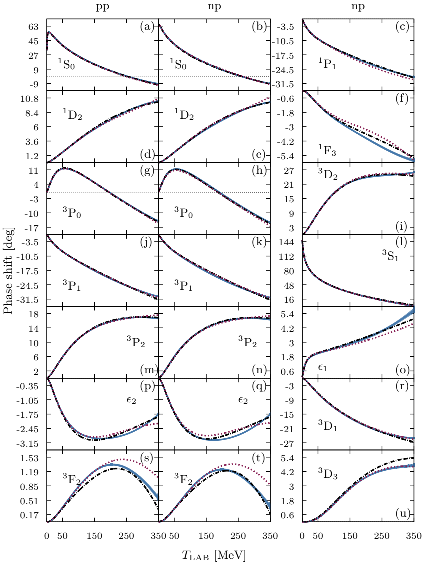
- Comparing with Potentials and Experimental data
- pp data



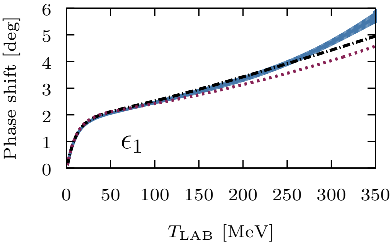
- $\chi^2/\mathrm{d.o.f.} = 1.06$ with $N = 2747|_{\mathrm{pp}} + 3691|_{\mathrm{np}}$

[arXiv:1304.0895]

Phase shifts



- Phase shifts for every partial
- Statistical uncertainty propagated directly from covariance matrix



Wolfenstein Parameters

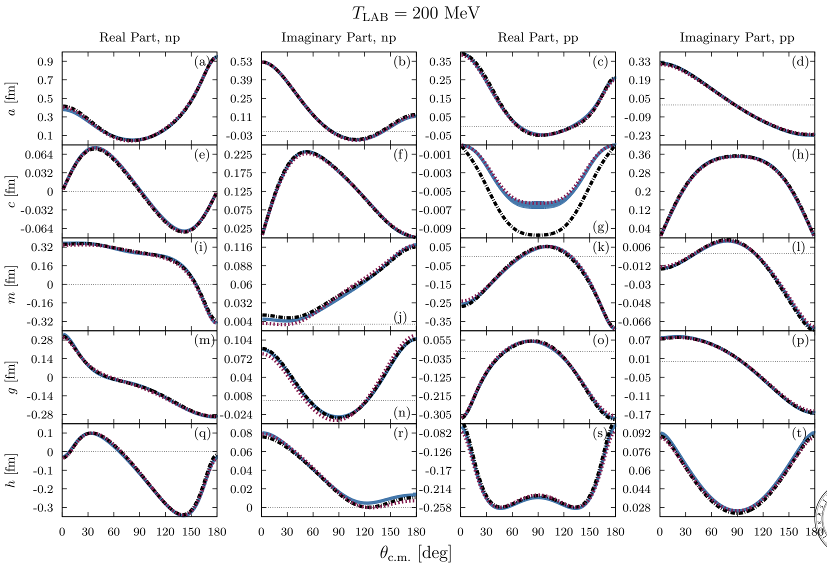
- A complete parametrization of the on-shell scattering amplitudes
- Five independent complex quantities
- Function of Energy and Angle

$$M(\mathbf{k}_f, \mathbf{k}_i) = a + m(\sigma_1, \mathbf{n})(\sigma_2, \mathbf{n}) + (g - h)(\sigma_1, \mathbf{m})(\sigma_2, \mathbf{m}) + (g + h)(\sigma_1, \mathbf{l})(\sigma_2, \mathbf{l}) + c(\sigma_1 + \sigma_2, \mathbf{n})$$

- Scattering observables can be calculated from M

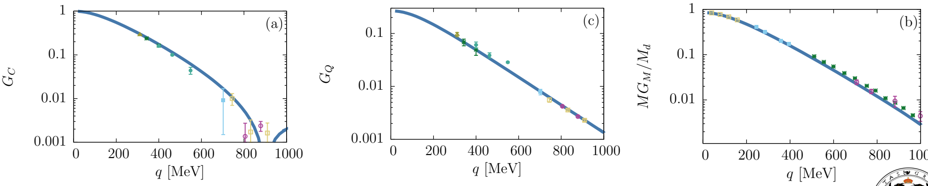
[Bystricky, J. et al, Jour. de Phys. 39.1 (1978) 1]

Wolfenstein Parameters



Deuteron Properties

	Delta Shell	Empirical	Nijm I	Nijm II	Reid93	AV18	CD-Bonn
E_d(MeV)	Input	2.224575(9)	Input	Input	Input	Input	Input
η	0.02493(8)	0.0256(5)	0.0253	0.0252	0.0251	0.0250	0.0256
$A_S(\mathrm{fm}^{1/2})$	0.8829(4)	0.8781(44)	0.8841	0.8845	0.8853	0.8850	0.8846
$r_m(\mathrm{fm})$	1.9645(9)	1.953(3)	1.9666	1.9675	1.9686	1.967	1.966
$Q_D(\mathrm{fm}^2)$	0.2679(9)	0.2859(3)	0.2719	0.2707	0.2703	0.270	0.270
P_D	5.62(5)	5.67(4)	5.664	5.635	5.699	5.76	4.85
$\langle r^{-1}\rangle(\mathrm{fm}^{-1})$	0.4540(5)			0.4502	0.4515		



Including Chiral Two Pion Exchange

- Inclusion of χTPE interactions at long and intermediate ranges
- pp PWA by the Nijmegen group

  [Rentmeester et al, Phys. Rev. Lett. 82 (1999), 4992]

  - Improvement in the χ^2 value compared to OPE only
  - Reduction of the number of parameters
  - Determination of chiral constants c_1, c_3, c_4
- Preliminary test using the δ-shell potential
  - OPE, TPE(l.o.) and TPE(s.o.)
  - Different cut radius, $r_c = 3.0,\ 2.4,\ 1.8$fm

Comparing OPE and χTPE

- Fitting all NN data

r_c [fm]	1.8		2.4		3.0	
	N_p	χ^2/ν	N_p	χ^2/ν	N_p	χ^2/ν
OPE	31	1.80	39	1.56	46	1.54
TPE(l.o.)	31	1.72	38	1.56	46	1.52
TPE(s.o.)	30+3	1.60	38+3	1.56	46+3	1.52

- Fitting 3σ compatible NN data

	N_{Data}	N_p	χ^2/ν	N_{Data}	N_p	χ^2/ν	N_{Data}	N_p	χ^2/ν
OPE	5766	31	1.10	6363	39	1.09	6438	46	1.06
TPE(l.o.)	5841	31	1.10	6432	38	1.10	6423	46	1.06
TPE(s.o.)	6220	30+3	1.07	6439	38+3	1.10	6422	46+3	1.06

- OPE only at 3.0fm describes the data
- 1.8 $\leq r \leq$ 3.0fm OPE + something else
- χTPE most of that something else

EFFECTIVE INTERACTIONS

Motivation

- Effective Interaction [Skyrme, Moshinsky]
- Useful simplifications in many body calculations [Brink, Vaughterin]
- Power expansion in CM momenta

$$
\begin{aligned}
V(\mathbf{p}', \mathbf{p}) &= \int d^3x e^{-i\mathbf{x}\cdot(\mathbf{p}'-\mathbf{p})} \hat{V}(\mathbf{x}) \\
&= t_0(1 + x_0 P_\sigma) + \frac{t_1}{2}(1 + x_1 P_\sigma)(\mathbf{p}'^2 + \mathbf{p}^2) \\
&+ t_2(1 + x_2 P_\sigma)\mathbf{p}' \cdot \mathbf{p} + 2i t_V \mathbf{S} \cdot (\mathbf{p}' \wedge \mathbf{p}) \\
&+ \frac{t_T}{2}\left[\sigma_1 \cdot \mathbf{p}\sigma_2 \cdot \mathbf{p} + \sigma_1 \cdot \mathbf{p}'\sigma_2 \cdot \mathbf{p}' - \frac{1}{3}\sigma_1\sigma_2(\mathbf{p}'^2 + \mathbf{p}^2)\right] \\
&+ \frac{t_U}{2}\left[\sigma_1 \cdot \mathbf{p}\sigma_2 \cdot \mathbf{p}' + \sigma_1 \cdot \mathbf{p}'\sigma_2 \cdot \mathbf{p} - \frac{2}{3}\sigma_1\sigma_2\mathbf{p}' \cdot \mathbf{p}\right] \\
&+ \mathcal{O}(p^4)
\end{aligned}
$$

Skyrme parameters

- Skyrme parameters in terms of paratial waves
- Partial Wave potential in momentum space

$$V^{JS}_{l'l'}(p',p) = \frac{(4\pi)^2}{M} \int_0^\infty dr r^2 j_{l'}(p'r) j_l(pr) V^{JS}_{l'l}(r)$$

- Using the Bessel function expansion

$$j_l(x) = \frac{x^l}{(2l+1)!!} \left[1 - \frac{x^2}{2(2l+3)} + \cdots \right]$$

Skyrme parameters

- Comparing similar terms

$$
\begin{aligned}
(t_0, x_0 t_0) &= \frac{1}{2}\int d^3x \left[V_{3_{S_1}}(r) \pm V_{1_{S_0}}(r) \right] \\
(t_1, x_1 t_1) &= -\frac{1}{12}\int d^3x r^2 \left[V_{3_{S_1}}(r) \pm V_{1_{S_0}}(r) \right] \\
(t_2, x_2 t_2) &= \frac{1}{54}\int d^3x r^2 \left[V_{3_{P_0}}(r) + 3V_{3_{P_1}}(r) + 5V_{3_{P_2}}(r) \pm 9V_{1_{P_1}}(r) \right] \\
t_V &= \frac{1}{72}\int d^3x r^2 \left[2V_{3_{P_0}}(r) + 3V_{3_{P_1}}(r) - 5V_{3_{P_2}}(r) \right] \\
t_U &= \frac{1}{36}\int d^3x r^2 \left[-2V_{3_{P_0}}(r) + 3V_{3_{P_1}}(r) - V_{3_{P_2}}(r) \right] \\
t_T &= \frac{1}{5\sqrt{2}}\int d^3x r^2 V_{\epsilon_1}(r)
\end{aligned}
$$

Skyrme parameters

- Straightforward for δ-shell potential

$$t \propto \sum \tilde{\lambda}_i r_i^n$$

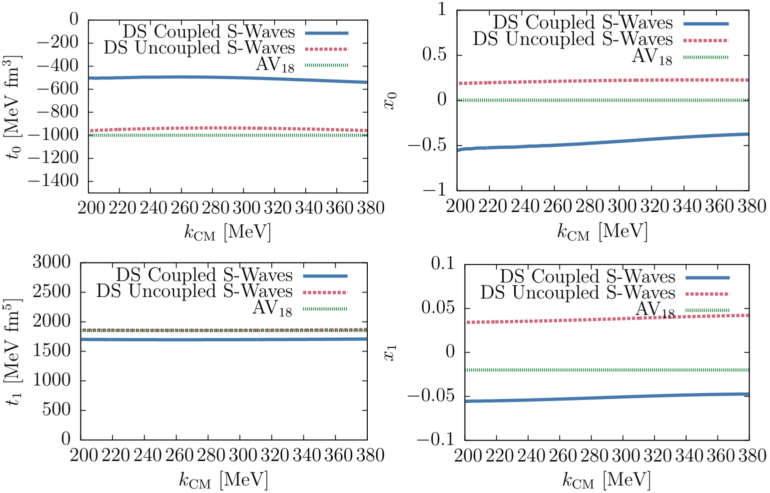
- Integrable for OPE starting at r_c

$$t \propto \frac{f_{\pi NN}^2}{m_\pi^2} \Gamma(n, m_\pi r_c)$$

  - Where $f_{\pi NN}^2/(4\pi) \sim 0.08$

Skyrme parameters

- Skyrme parameters fitting at different energy ranges



Skyrme parameters

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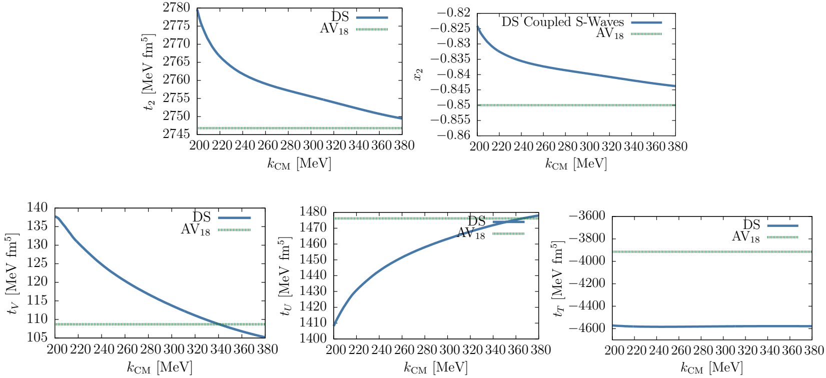
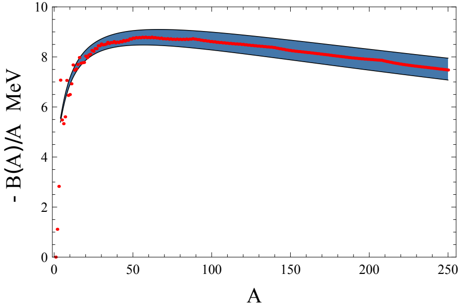


Skyrme Parameters

- Fermi type shape density

$$\rho(x) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

  - $R = r_0 A^{1/3}$, $r_0 = 1.1$fm and $a = 0.7$fm
- Error band for stable nuclei binding energy

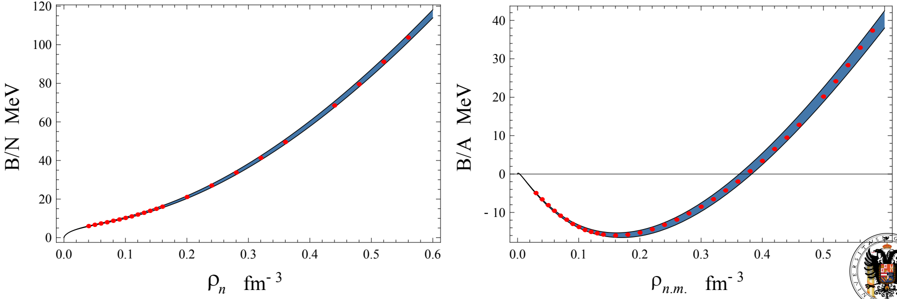


$$\frac{\Delta B}{A} = \frac{3}{8A} \Delta t_0 \int d^3x \, \rho(x)^2$$

Skyrme Parameters

- Nuclear and Neutron matter
    - Error grows linearly with the density

$$\frac{\Delta B_{n.m.}}{A} = \frac{3}{8}\Delta t_0 \rho \sim 3.75\rho$$
$$\frac{\Delta B_n}{A} = \frac{1}{4}\Delta[t_0(1-x_0)]\rho_n \sim 3.5\rho_n$$



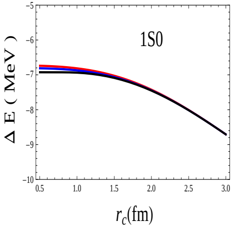
SHELL MODEL MATRIX ELEMENTS

Renormalization of Nuclear Matrix elements

- Harmonic oscillator shell model

$$V_{\rm HO}(r) = \frac{r^2}{2Mb^4} \to \epsilon_{nl} = \frac{1}{2Mb^2}(4n+2l-1)$$

- Distortion due to OPE and TPE $\to$ Energy shift $\Delta\epsilon_{nl}$

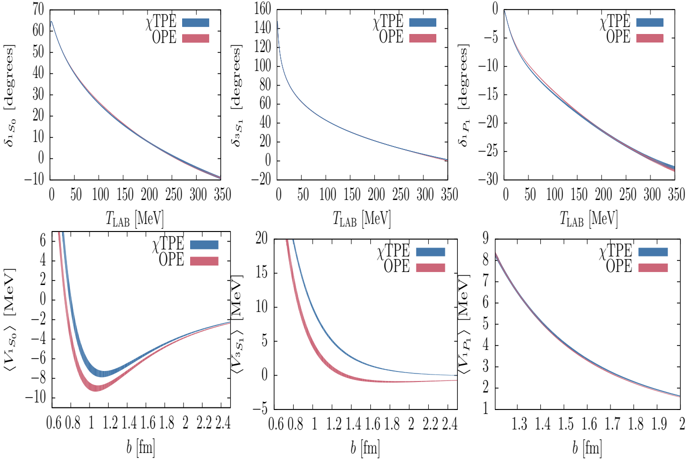


$$\Delta\epsilon_{nl} = \langle\varphi_{nl}|K(\epsilon_{nl}+\Delta\epsilon_{nl})|\varphi_{nl}\rangle$$

- In order to see the differences we need to look into short distances.

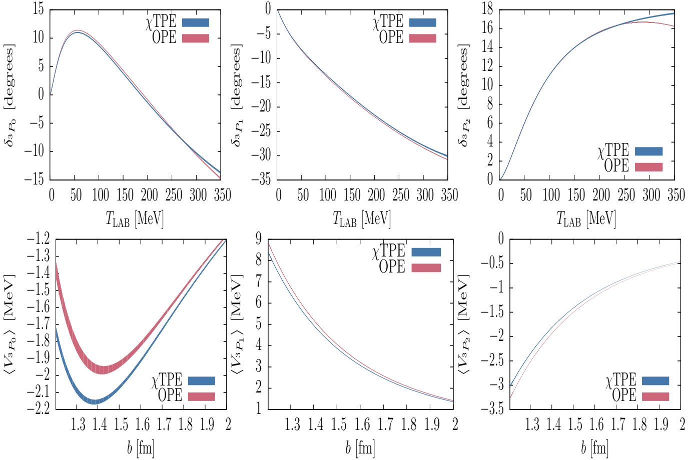
Errors in Nuclear Matrix elements

$$(T_{\rm LAB} \le 350{\rm MeV}, r_c|_{\rm OPE} = 3{\rm fm},\ r_c|_{\rm TPE} = 1.8{\rm fm}\ )$$



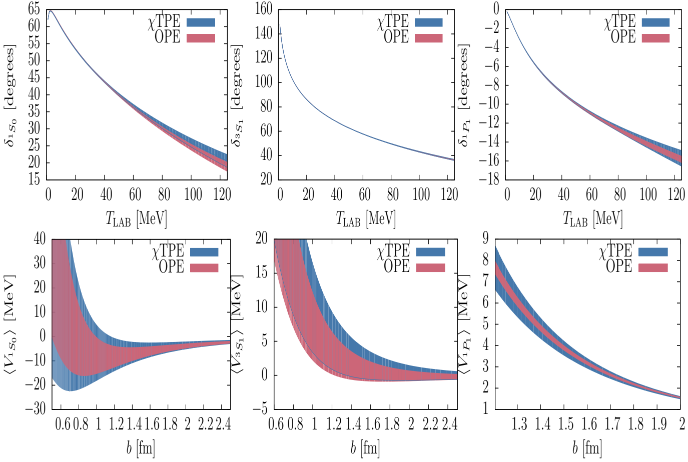
Errors in Nuclear Matrix elements

$(T_{\rm LAB} \le 350{\rm MeV}, r_c|_{\rm OPE} = 3{\rm fm},\ r_c|_{\rm TPE} = 1.8{\rm fm}\ )$



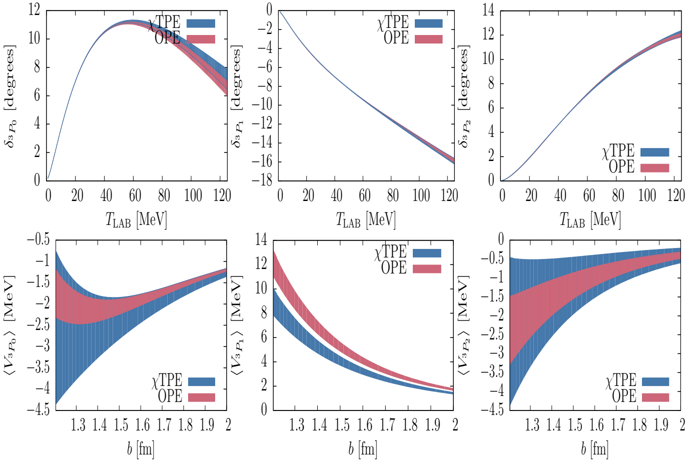
Errors in Nuclear Matrix elements

$(T_{\rm LAB} \le 125{\rm MeV}, r_c|_{\rm OPE} = 1.8{\rm fm},\ r_c|_{\rm TPE} = 1.8{\rm fm}\ )$



Errors in Nuclear Matrix elements

$(T_{\rm LAB} \le 125{\rm MeV}, r_c|_{\rm OPE} = 1.8{\rm fm},\ r_c|_{\rm TPE} = 1.8{\rm fm}\ )$



CONCLUSIONS

Summary

- Sampling of the NN interaction by a delta shell potential

$$1/\sqrt{m_\pi M} \lesssim \Delta r \lesssim 1/m_\pi$$

- Quantitative comparison of OPE and Chiral TPE → Reduccion of Parameters
- Statistical uncertainty propagation possible
- δ-shell representation allows straightforward calculations
- Comparing OPE and χTPE matrix elements with errors

- TAKE AWAY: Before cranking the machine accuracy make sure it does not exceed theoretical uncertainty