Selfconsistent mean field calculations of the nuclear response using a realistic nucleon-nucleon interaction with a density dependent corrective term

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Workshop “From Few-Nucleon Forces to Many-Nucleon Structure“

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Basic ingredients: HFB, TDA, RPA methods, realistic NN interaction, density-dependent (DD) phenomenological interaction as simulation of the 3-body terms, spin-orbit (s.o.) splitting, TDA phonons, Equation of Motion, Hilbert space divided to 0,1,2,3,...,n – phonon subspaces, exact treating of Pauli principle (no any quasiboson approximation)

Motivation of study: using the self-consistent methods like HFB, TDA, RPA in the context of the realistic NN potentials, comparison of TDA & RPA, dependence of the s.p. and TDA spectrum on the phenomenological DD and s.o. terms, TDA phonons as a basis for calculations beyond one-phonon approaches, fragmentation of giant resonance, pygmy resonances and possibly other types of transitions with strong anharmonic effects

Formalism & DD interaction

$$H = T_{int} + h_{s.o.} + V + V^{DD}[\rho]$$

2-body density-dependent int.

2-body “realistic“ NN interaction

$$T_{int} = \left(1 - \frac{1}{A}\right)\sum_i \frac{p_i^2}{2m} - \frac{1}{mA}\sum_{i<j} p_i . p_j$$

intrinsic kinetic term

$$h_{s.o.} = C l.s$$

phenom. spin-orbit term

$$V^{DD}[\rho] = \frac{C_{3N}}{6}(1 + P_\sigma)\rho\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right)\delta^{(3)}(\mathbf{r}_1 - \mathbf{r}_2)$$

2-body density-dependent int.
contact in S=1, T=0 channel

$$V^{3N} = C_{3N}\delta^{(3)}(\mathbf{r}_1 - \mathbf{r}_2)\delta^{(3)}(\mathbf{r}_2 - \mathbf{r}_3)$$

the same contribution to E_{HF} energy as pure contact 3-body interaction

Formalism & DD interaction

$$H = T_{int} + h_{s.o.} + V + V^{DD}[\rho]$$

$$T_{int} = \left(1 - \frac{1}{A}\right) \sum_i \frac{p_i^2}{2m} - \frac{1}{mA} \sum_{i<j} p_i . p_j$$

intrinsic kinetic term

$$h_{s.o.} = C l.s$$

phenom. spin-orbit term

2-body “realistic“ NN interaction

2-body density-dependent int.

$$V^{DD}[\rho] = \frac{C_0}{6}(1+P_\sigma)\,\rho\left(\frac{\mathrm{r}_1+\mathrm{r}_2}{2}\right)\delta^{(3)}(\mathrm{r}_1-\mathrm{r}_2)+\frac{C_1}{6}(1-P_\sigma)\,\rho\left(\frac{\mathrm{r}_1+\mathrm{r}_2}{2}\right)\delta^{(3)}(\mathrm{r}_1-\mathrm{r}_2)$$

(S=1, T=0) term

(S=0, T=1) term

The DD interaction can be introduced phenomenologically without any relation to some 3-body term.

Formalism & DD interaction

The interaction matrix elements for DD force – like-wise particles & proton-neutron

$$< l_i j_i, J|\hat{V}^{DD}|l_k j_k, J >_{pp}= \frac{V_1}{4\pi}\frac{1}{12(2J+1)}\frac{1}{\sqrt{(1+\delta_{ij})(1+\delta_{kl})}}\sqrt{(2j_i+1)(2j_j+1)(2J_k+1)(2j_l+1)}I_{rad}$$
$$(-1)^{l_j+l_l+j_j+j_l}(1+(-1)^{l_i+l_j+J})(1+(-1)^{l_k+l_l+J})C^{J0}_{j_i 1/2 j_j -1/2}C^{J0}_{j_k 1/2 j_l -1/2}$$

$$< l_i j_i, J|\hat{V}^{DD}|l_k j_k, J >_{pn}=$$
$$\frac{V_0}{4\pi}\frac{1}{24(2J+1)}\sqrt{(2j_i+1)(2j_j+1)(2J_k+1)(2j_l+1)}I_{rad}[2(-1)^{j_i+j_j+j_k+j_l}(1+(-1)^{l_i+l_j+l_k+l_l})$$
$$C^{J1}_{j_i 1/2 j_j 1/2}C^{J1}_{j_k 1/2 j_l 1/2}+(-1)^{l_i+l_k+j_i+j_k+1}(1+(-1)^{l_i+l_j+J+1})(1+(-1)^{l_k+l_l+J+1})C^{J0}_{j_i 1/2 j_j -1/2}C^{J0}_{j_k 1/2 j_l -1/2}]$$
$$+\frac{V_1}{4\pi}\frac{1}{24(2J+1)}\sqrt{(2j_i+1)(2j_j+1)(2J_k+1)(2j_l+1)}I_{rad}$$
$$(-1)^{l_j+l_l+j_j+j_l}(1+(-1)^{l_i+l_j+J})(1+(-1)^{l_k+l_l+J})C^{J0}_{j_i 1/2 j_j -1/2}C^{J0}_{j_k 1/2 j_l -1/2}$$

radial integral

$$I_{rad}=\sum_{n_m l_m j_m} V^2_{n_m l_m j_m}\frac{2j_m+1}{4\pi}\int \mathrm{d}r r^2 R_{n_i l_i}(r)R_{n_j l_j}(r)R_{n_k l_k}(r)R_{n_l l_l}(r)R_{n_m l_m}(r)R_{n_m l_m}(r)$$

Formalism & DD interaction

relation DD 2-body & 3-body interaction

$$V^{DD,pp}_{ijkl} = \sum_{m<E_F} V^{ppp}_{ijmklm} + \sum_{m<E_F} V^{ppn}_{ijmklm}$$

$$V^{DD,nn}_{ijkl} = \sum_{m<E_F} V^{nnn}_{ijmklm} + \sum_{m<E_F} V^{pnn}_{mijmkl}$$

$$V^{DD,pn}_{ijkl} = \sum_{m<E_F} V^{ppn}_{mijmkl} + \sum_{m<E_F} V^{pnn}_{ijmklm}$$

Due to Wick theorem:

- contribution to HF energy

$$E_{HF} = \frac{1}{3}\left(\frac{1}{2}\sum_{ij<E_F} V^{DD,pp}_{ijij} + \frac{1}{2}\sum_{ij<E_F} V^{DD,nn}_{ijij} + \sum_{ij<E_F} V^{DD,pn}_{ijij}\right)$$

- contribution to s.p. energies:

$$e^p_i = \frac{1}{2}\sum_{k<E_F}\left(V^{DD,pp}_{ikik} + V^{DD,pn}_{ikik}\right)$$

- contribution to 2-body part of Hamiltonian:

$$\begin{aligned}\hat{H}^{(2)} &= \frac{1}{4}\sum_{ijkl}\left(V^{pp}_{ijkl} + V^{DD,pp}_{ijkl}\right) a^\dagger_i a^\dagger_j a_k a_l \\ &+\frac{1}{4}\sum_{ijkl}\left(V^{nn}_{ijkl} + V^{DD,nn}_{ijkl}\right) a^\dagger_i a^\dagger_j a_k a_l \\ &+\sum_{ijkl}\left(V^{pn}_{ijkl} + V^{DD,pn}_{ijkl}\right) a^\dagger_i a^\dagger_j a_k a_l\end{aligned}$$

Formalism & DD interaction

relation DD 2-body & 3-body interaction

$$V_{ijkl}^{DD,pp} = \sum_{m<E_F} V_{ijmklm}^{ppp} + \sum_{m<E_F} V_{ijmklm}^{ppn}$$

$$V_{ijkl}^{DD,nn} = \sum_{m<E_F} V_{ijmklm}^{nnn} + \sum_{m<E_F} V_{mijmkl}^{pnn}$$

$$V_{ijkl}^{DD,pn} = \sum_{m<E_F} V_{mijmkl}^{ppn} + \sum_{m<E_F} V_{ijmklm}^{pnn}$$

Due to Wick theorem:

- contribution to HF energy

$$E_{HF} = \frac{1}{3}\left(\frac{1}{2}\sum_{ij<E_F} V_{ijij}^{DD,pp} + \frac{1}{2}\sum_{ij<E_F} V_{ijij}^{DD,nn} + \sum_{ij<E_F} V_{ijij}^{DD,pn}\right)$$

- contribution to s.p. energies:

$$e_i^p = \frac{1}{2}\sum_{k<E_F}\left(V_{ikik}^{DD,pp} + V_{ikik}^{DD,pn}\right)$$

- contribution to 2-body part of Hamiltonian:

$$\hat{H}^{(2)} = \frac{1}{4}\sum_{ijkl}\left(V_{ijkl}^{pp} + V_{ijkl}^{DD,pp}\right) a_i^\dagger a_j^\dagger a_k a_l + \frac{1}{4}\sum_{ijkl}\left(V_{ijkl}^{nn} + V_{ijkl}^{DD,nn}\right) a_i^\dagger a_j^\dagger a_k a_l + \sum_{ijkl}\left(V_{ijkl}^{pn} + V_{ijkl}^{DD,pn}\right) a_i^\dagger a_j^\dagger a_k a_l$$

Formalism & EMPM model

Basic building blocks - TDA phonons

$$O^\dagger_\lambda = \sum_{ph} c^\lambda_{ph} a^\dagger_p a_{\bar{h}}$$

Construction of n-phon states

as 1phon excitation on (n-1)-phon

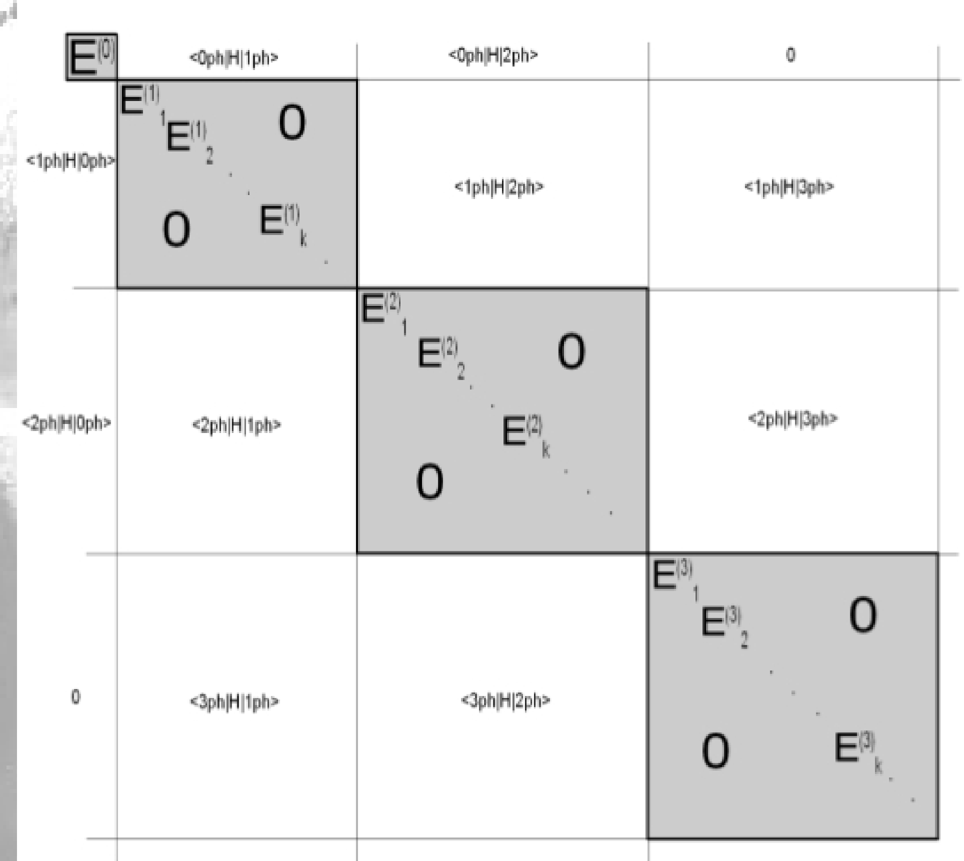
$$|n;\beta\rangle = \sum_{\lambda\alpha} C^{(\beta)}_{\lambda\alpha}\, O^\dagger_\lambda\; |n-1;\;\alpha\rangle$$

EoM: generate n-phon energies

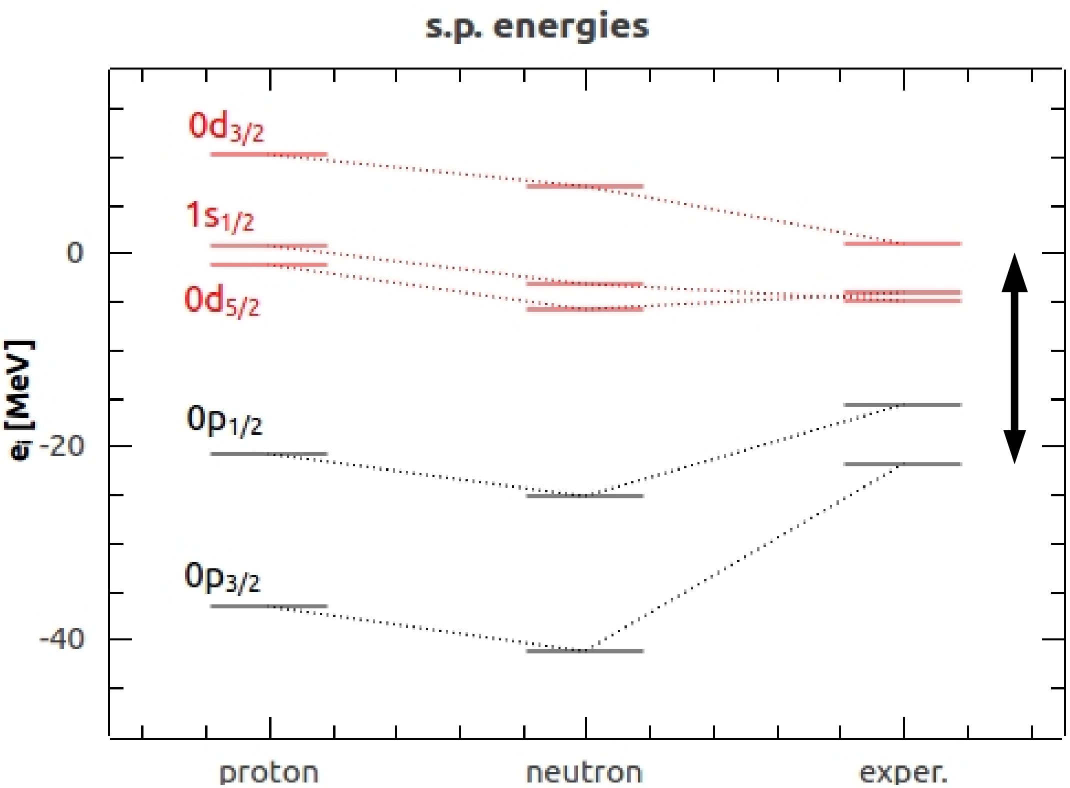
$$\langle n, \beta\| [H, O^\dagger_\lambda] \| n-1, \alpha\rangle = (E_\beta - E_\alpha) X^{(\beta)}_{\lambda\alpha}$$

Transitions from one correlated state to another correlated state:

$$\begin{aligned}\mathcal{M}_\lambda(i \to f) &= \langle \Psi_{\nu_f} | \mathcal{M}_\lambda | \Psi_{\nu_i}\rangle \\ &= \sum_{\beta_i \beta_f} C^{(\nu_i)}_{\beta_i} C^{(\nu_f)}_{\beta_f} \langle n_f, \beta_f | \mathcal{M}_\lambda | n_i, \alpha_i\rangle\end{aligned}$$

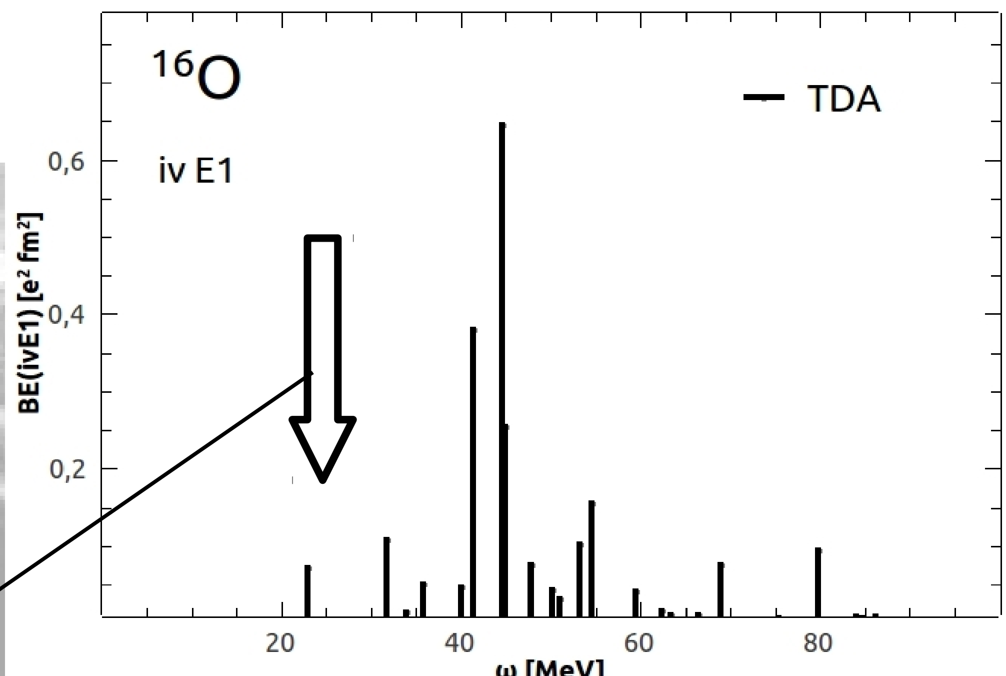


Spectra without DD interaction



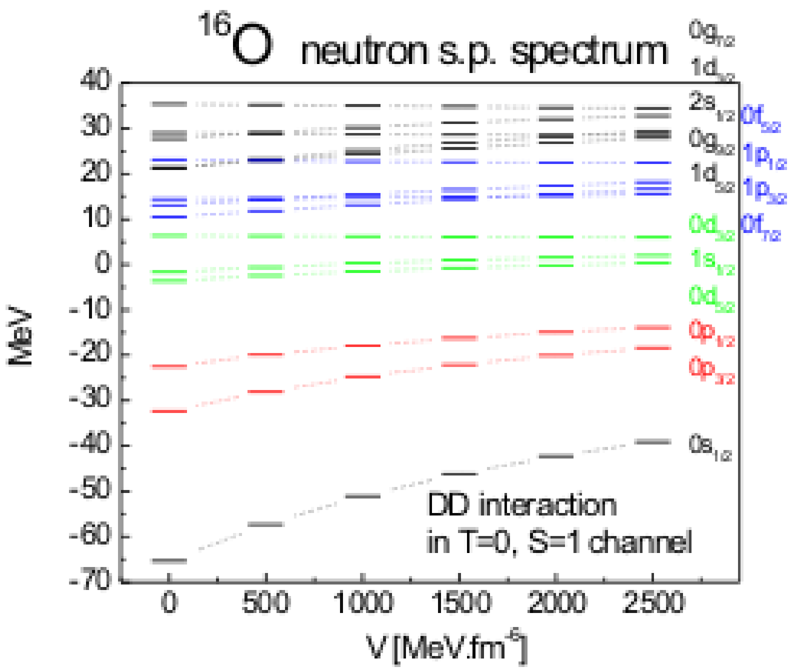
NN interaction: CD-Bonn + V_{lowk}
with $k_{cut\text{-}off} = 1.9\ fm^{-1}$

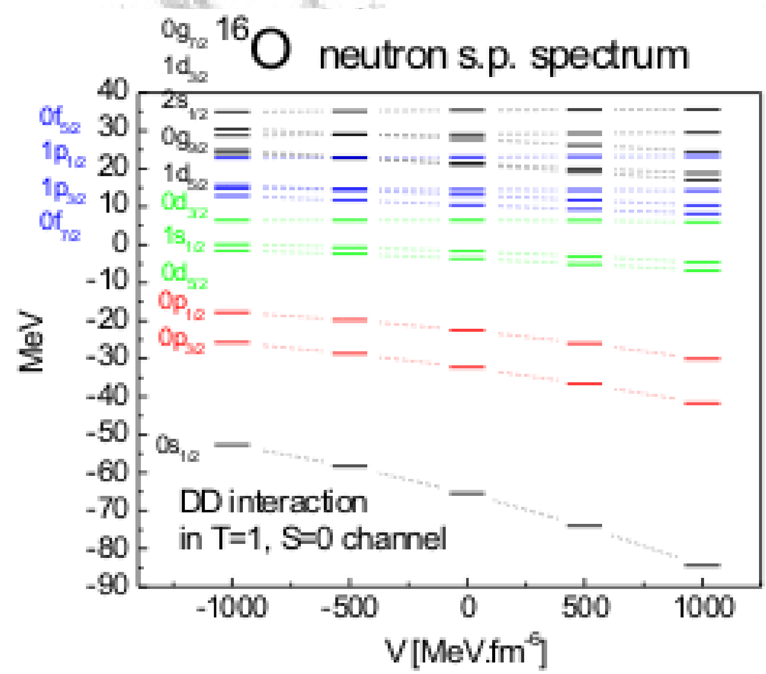
"experimental gap" between p & (sd) shells much smaller than in HF calculation with realistic NN force



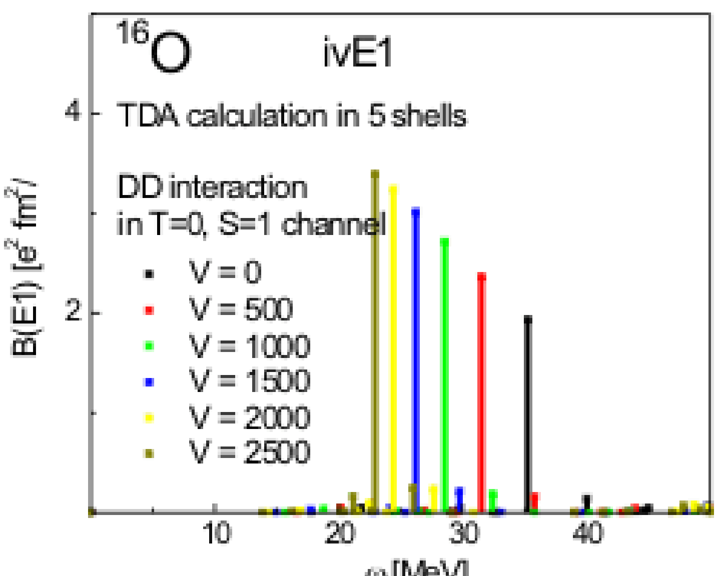
In experiment the E1 resonance occurs at ~ 25 MeV

Test of the DD term

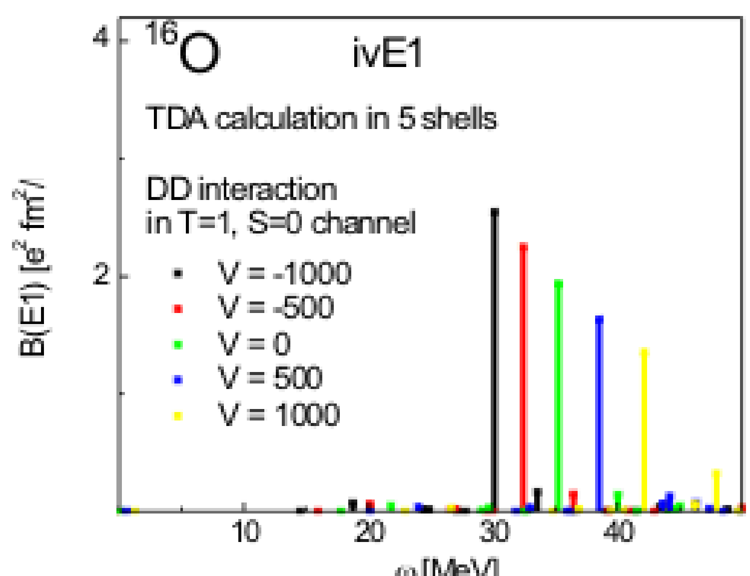




The spectrum becomes more realistic but …



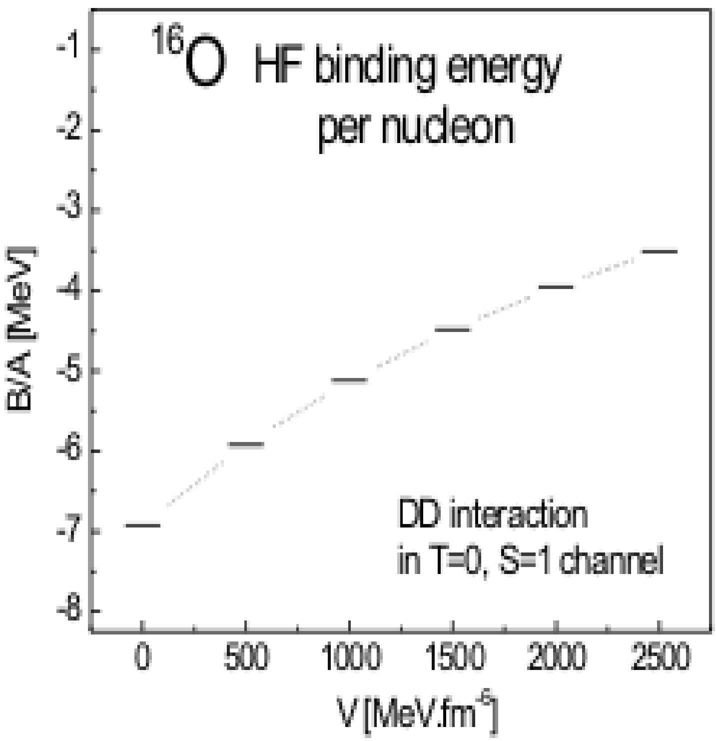
we pay a price for it (see next page)...

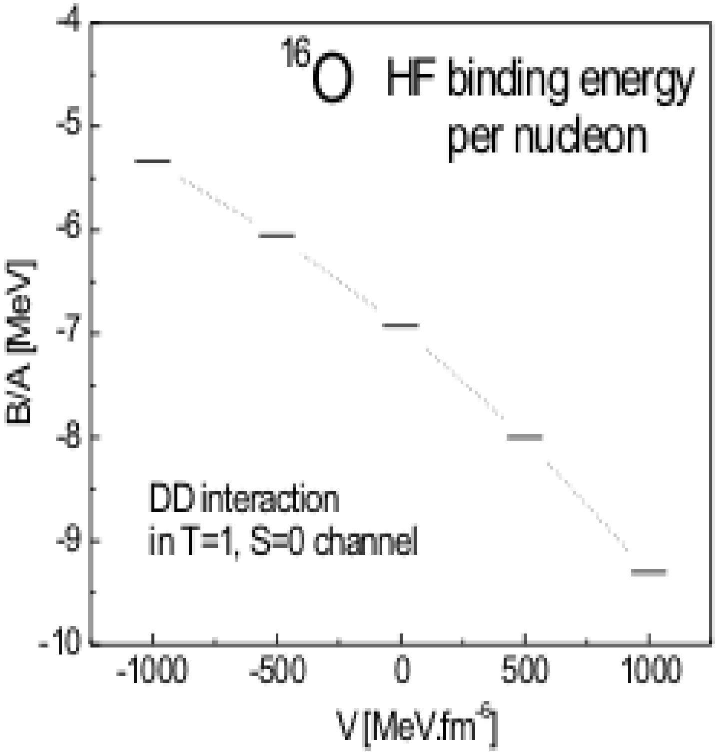


Test of the DD term

the better description of spectra the worse (i.e. less) of the binding energy (HF ground state energy per nucleon)

For ^{16}O $(B/A)_{exp.}$ = 7.98 MeV



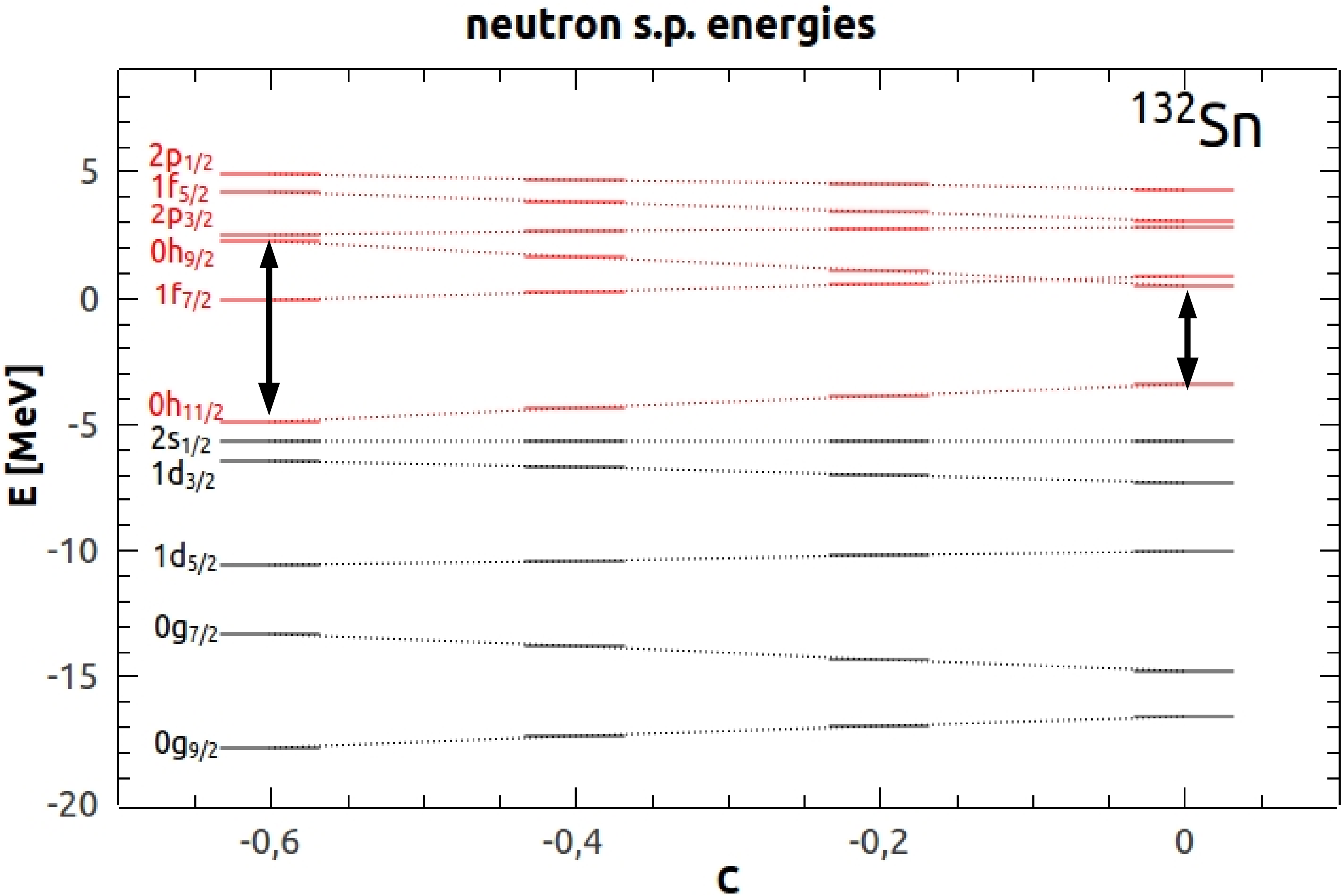


Test of the spin-orbit term

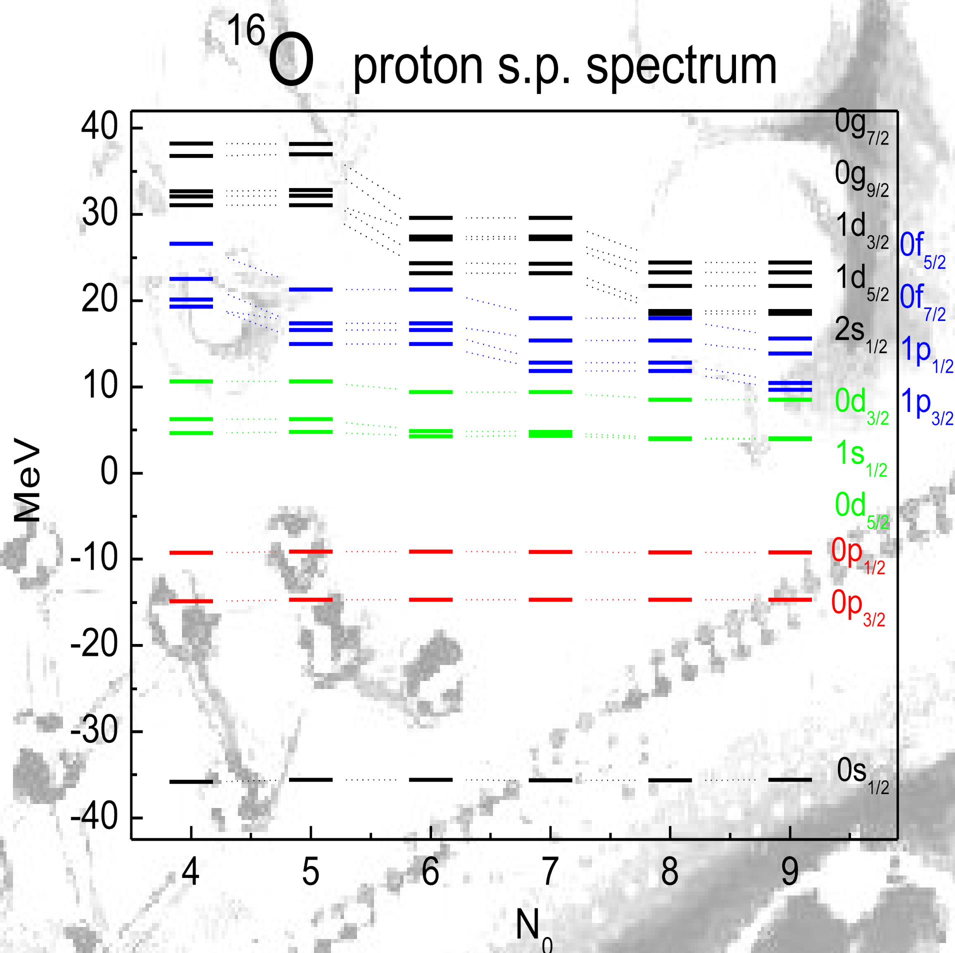
Example of spin-orbit splitting:

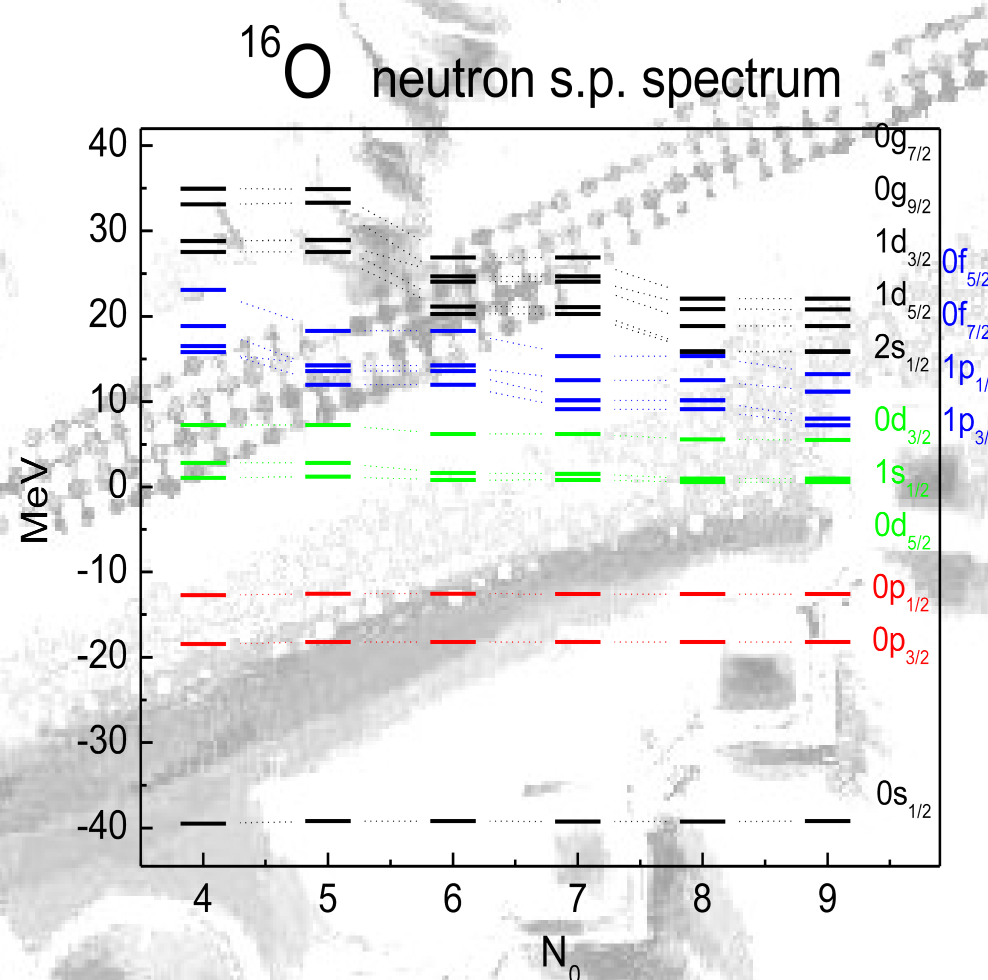
neutron 0h in ^{132}Sn

exper.= 6.53 MeV



Effect of the configuration space



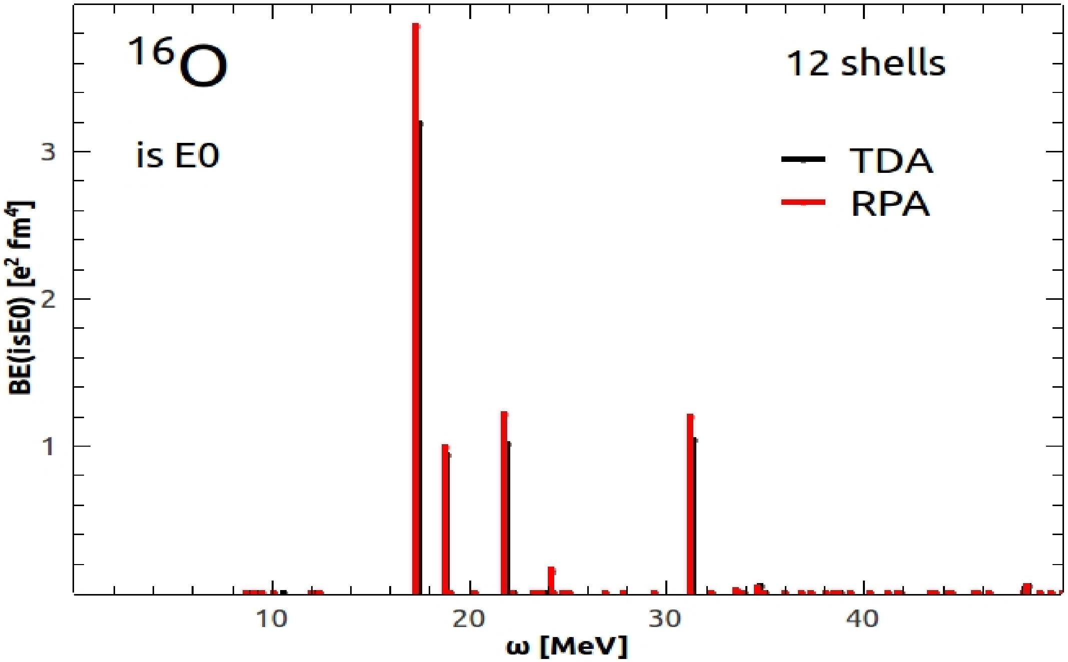


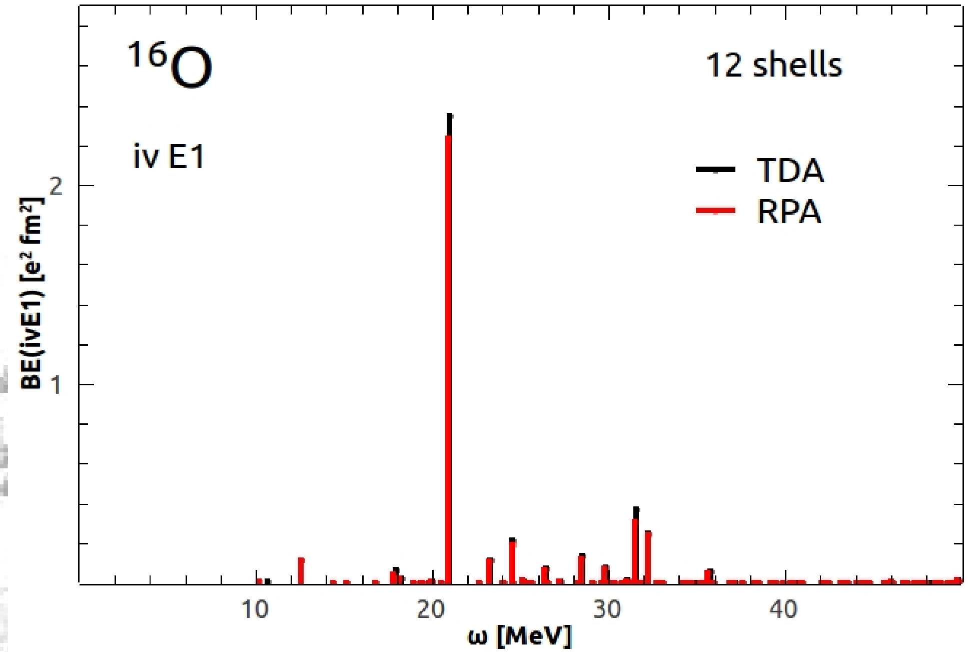
The s.p. spectrum is comprised with enlarging the configuration space.
(dissappearance of the shell structure above the Fermi level)

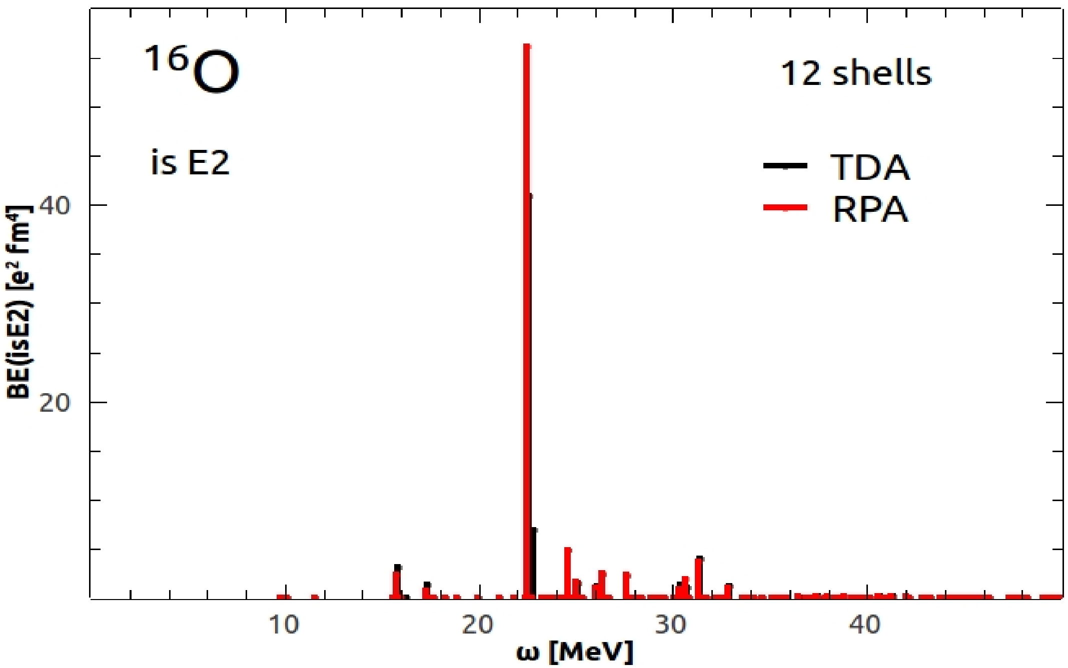
DD int. :
V(S=1,T=0)=1300 MeV.fm^6
V(S=0,T=1)=-1300 MeV.fm^6

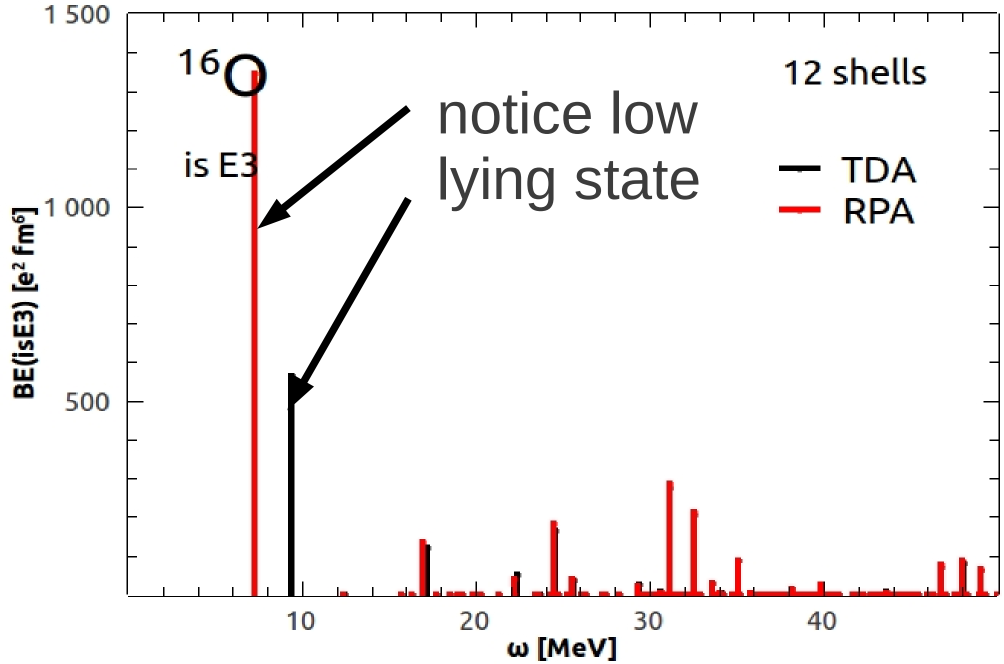
TDA & RPA methods

s.o. term :
C = -0.9 MeV



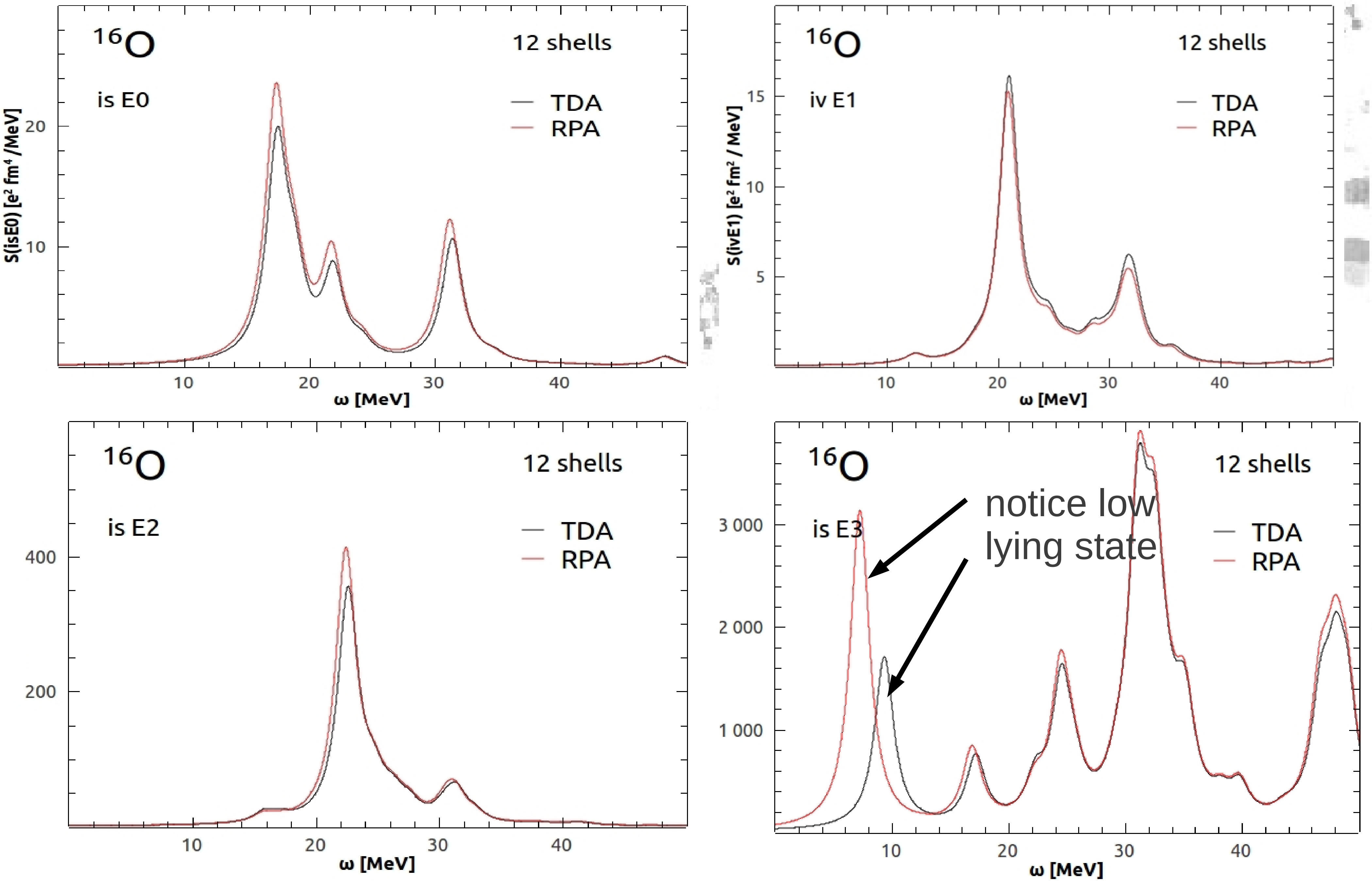




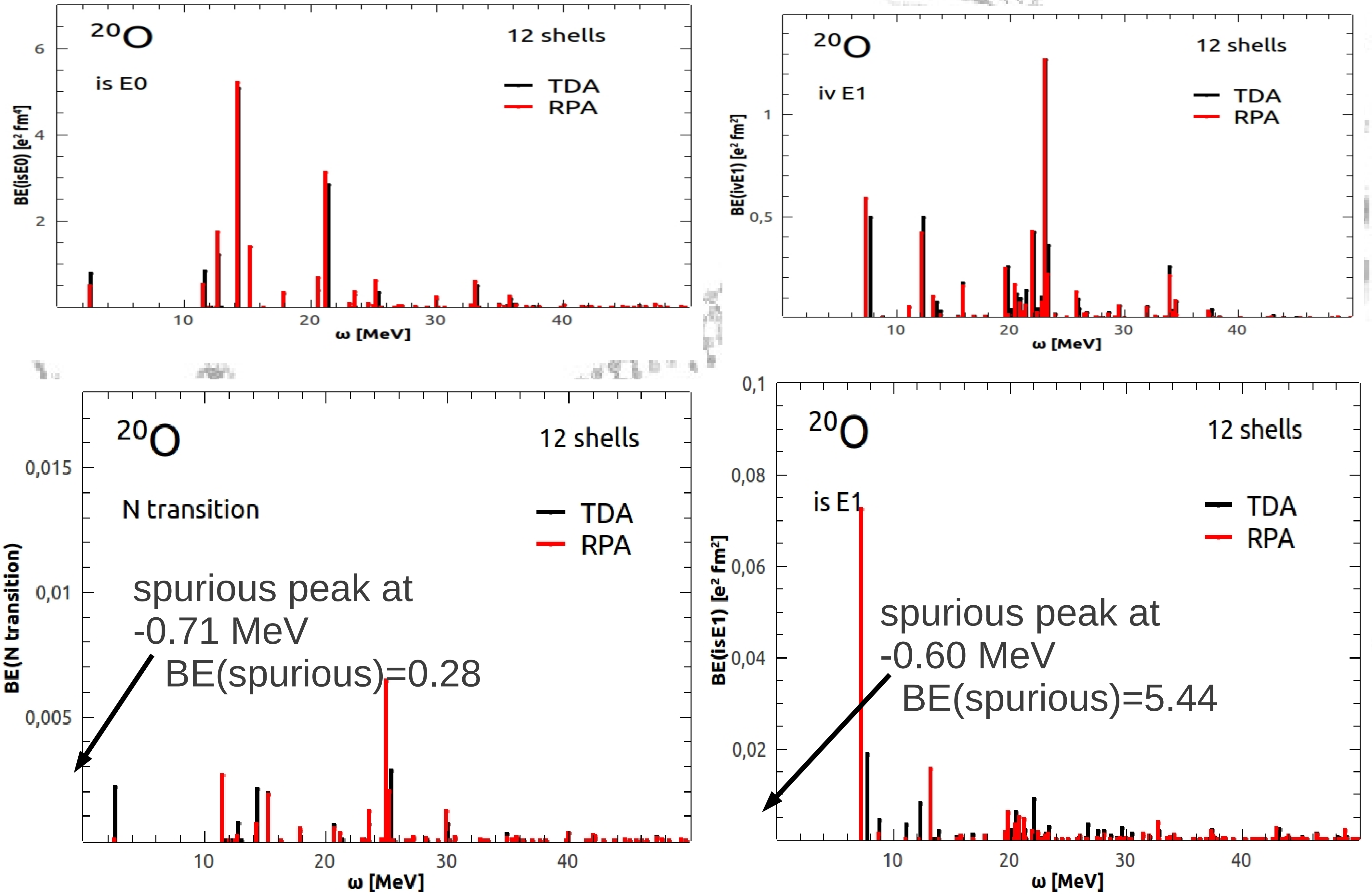


TDA & RPA methods

(the same with Lorentz averaging)



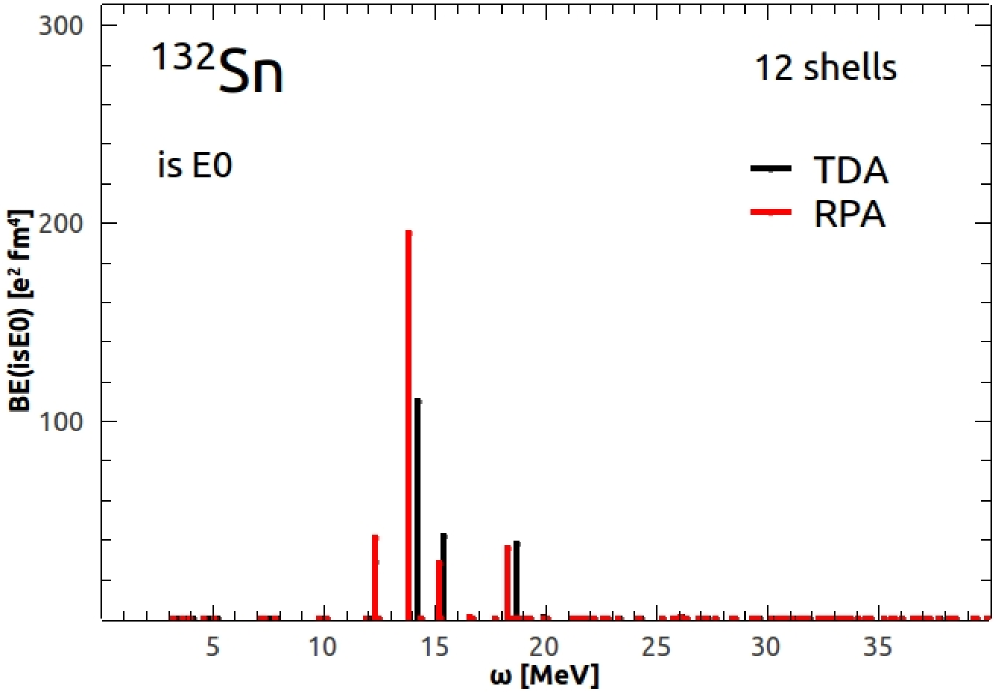
TDA & RPA methods

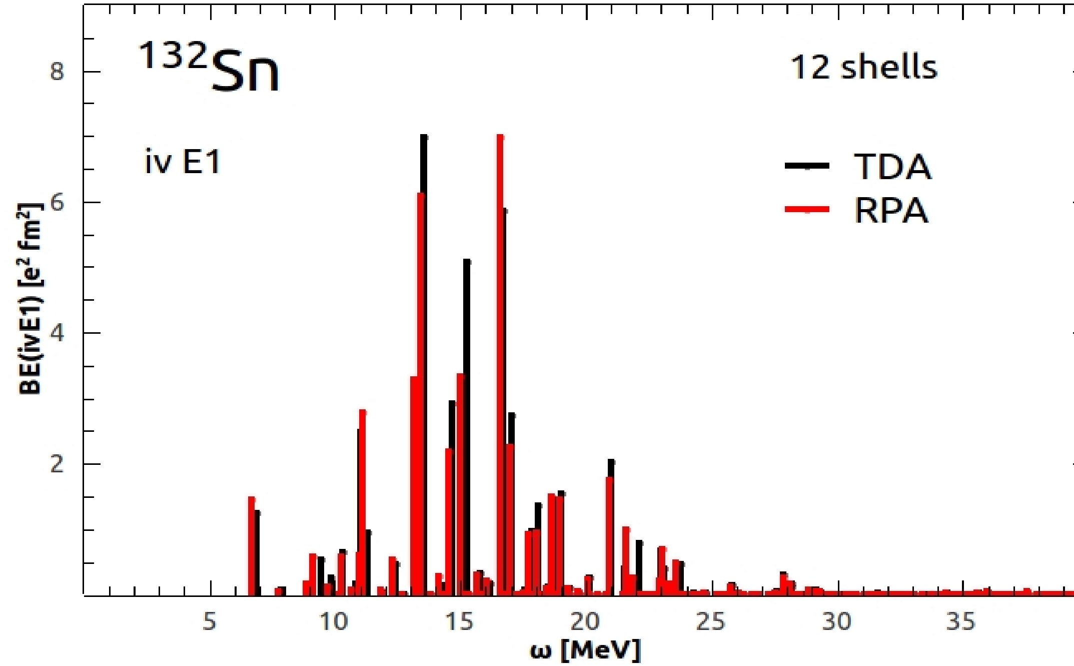


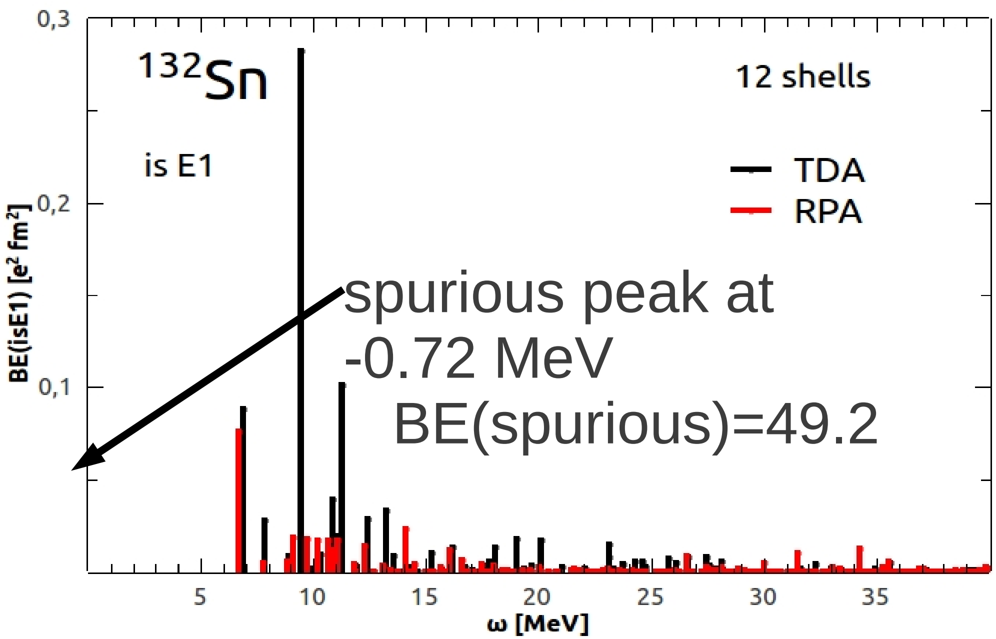
DD int. :
V(S=1,T=0)=2400 MeV.fm^6
V(S=0,T=1)=-1800 MeV.fm^6

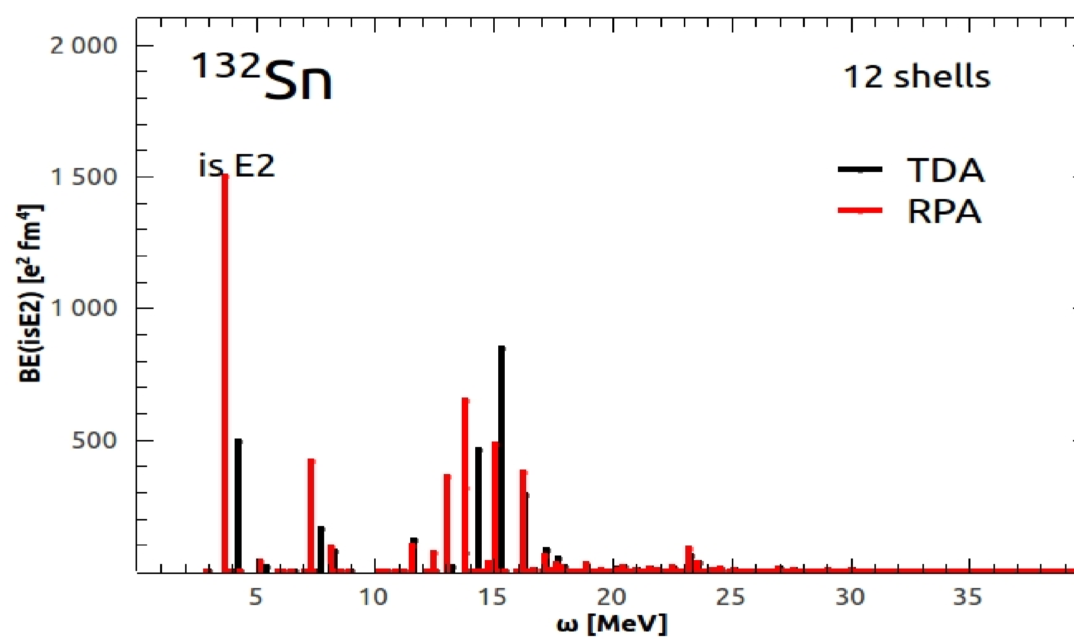
TDA & RPA methods

s.o. term :
C = -0.6 MeV







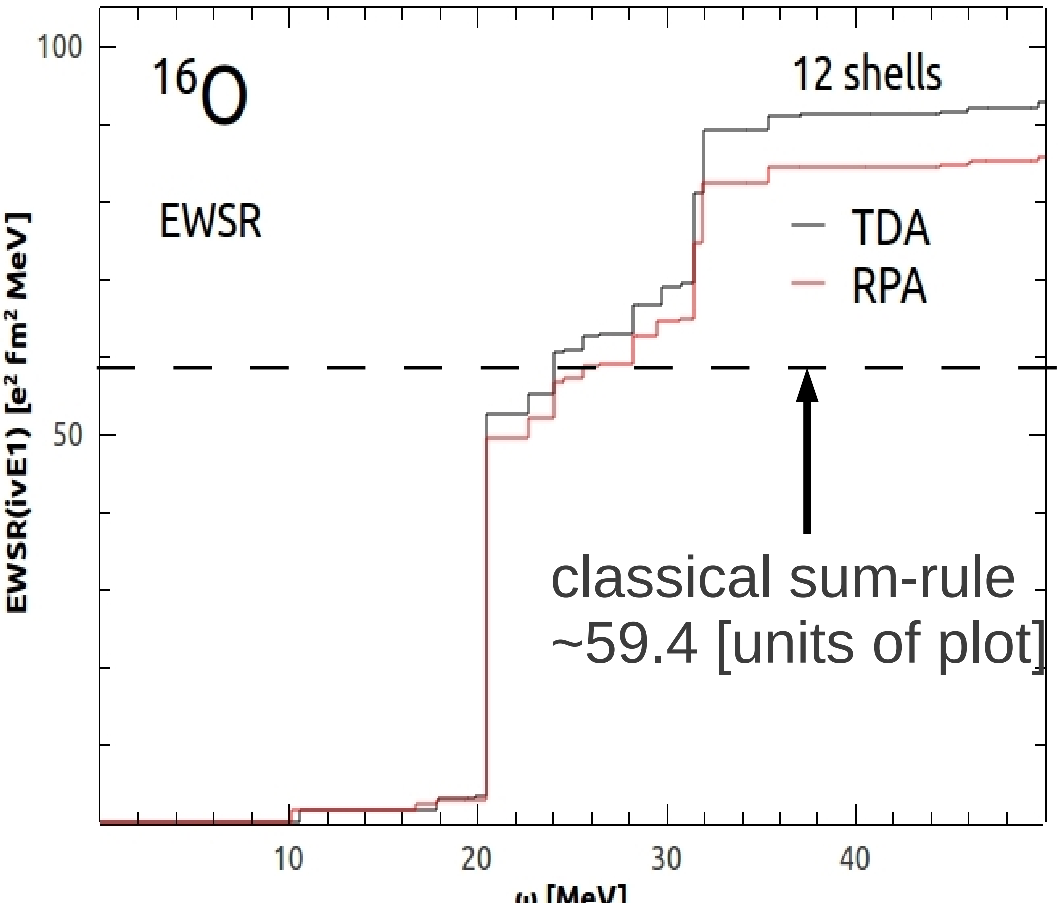


TDA & RPA methods

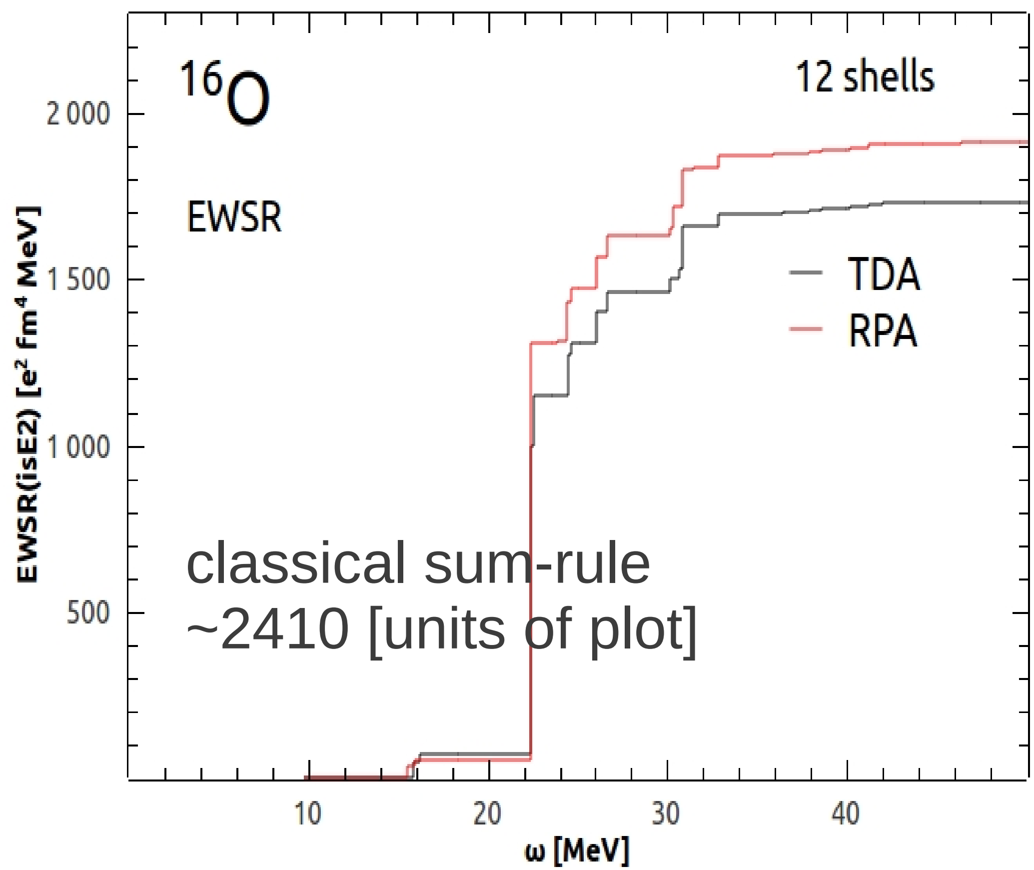
running energy-weighted sum rule (EWSR) for ivE1 and isE2

ivE1 … overestimate
isE2 … underestimate

Should EWSR be precisely reached?
EWSR derived for unperturped g.s.
EWSR in shell model?

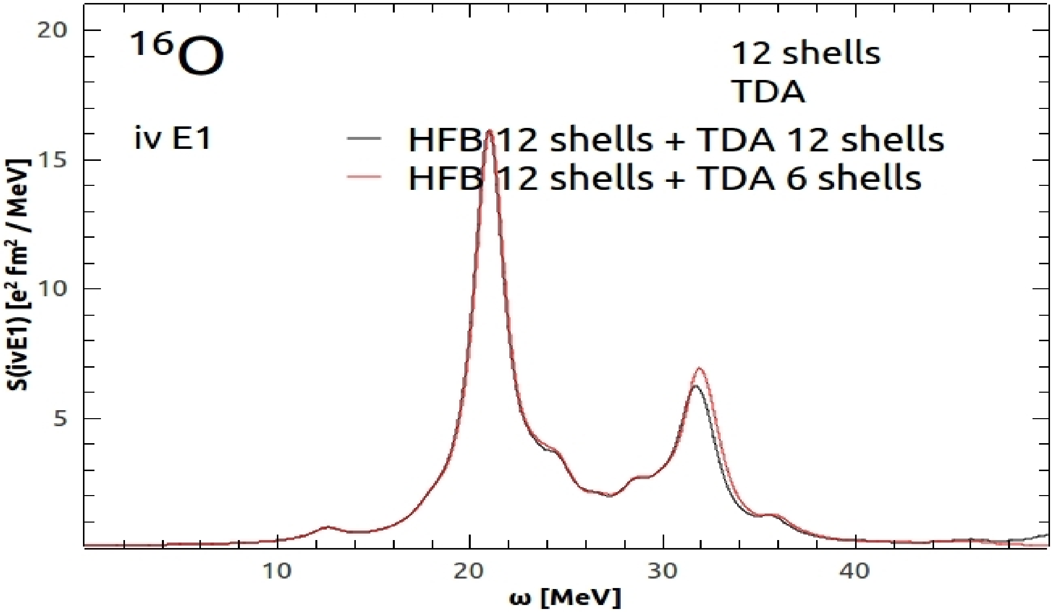


classical sum-rule
~59.4 [units of plot]



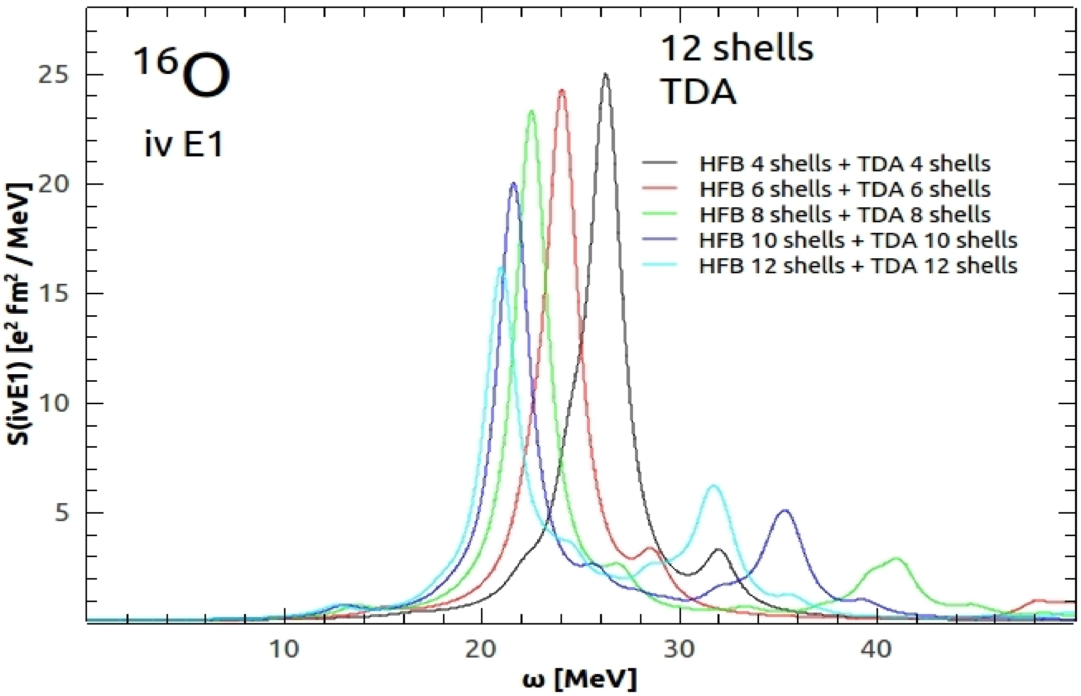
classical sum-rule
~2410 [units of plot]

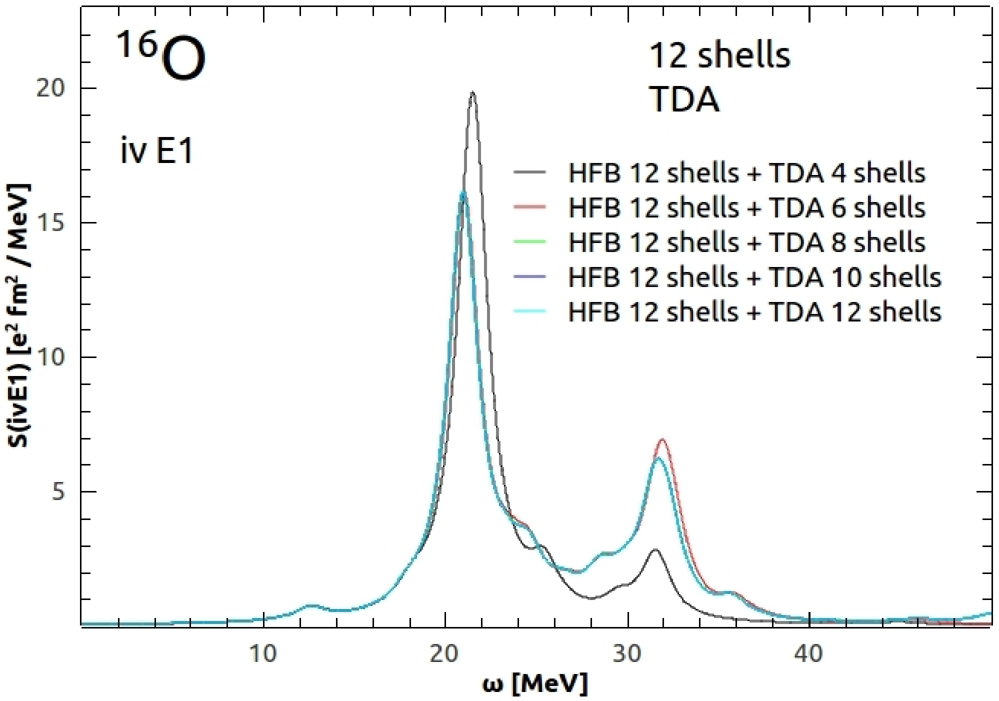
TDA – effect of configuration space



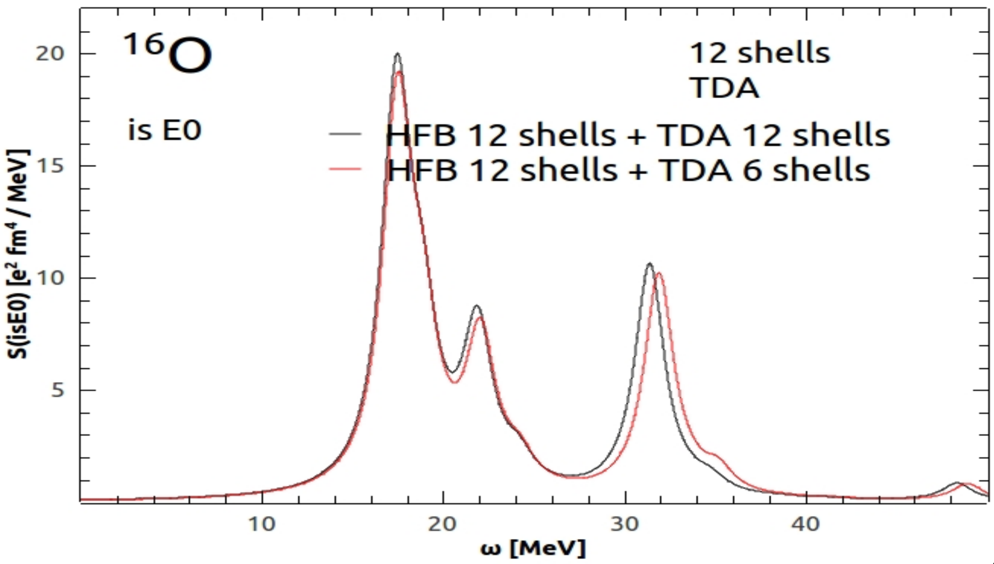
“natural cut“ of TDA configuration space
HFB calculated in maximal space
cut-off only at the level of TDA

much quicker convergence due to cut-off only for TDA



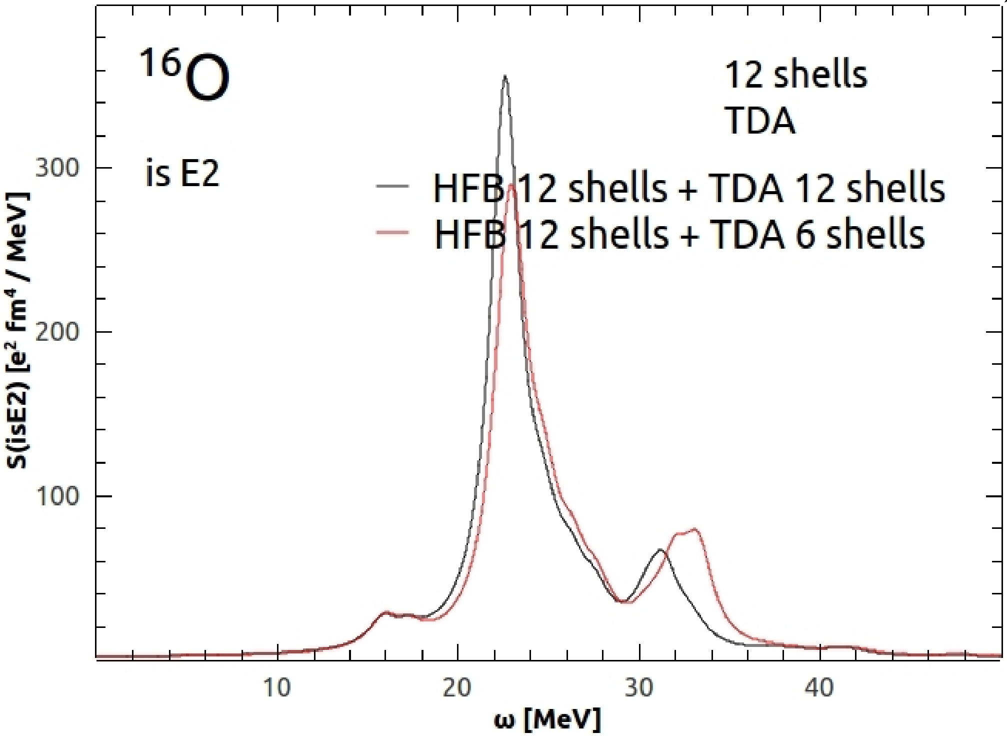


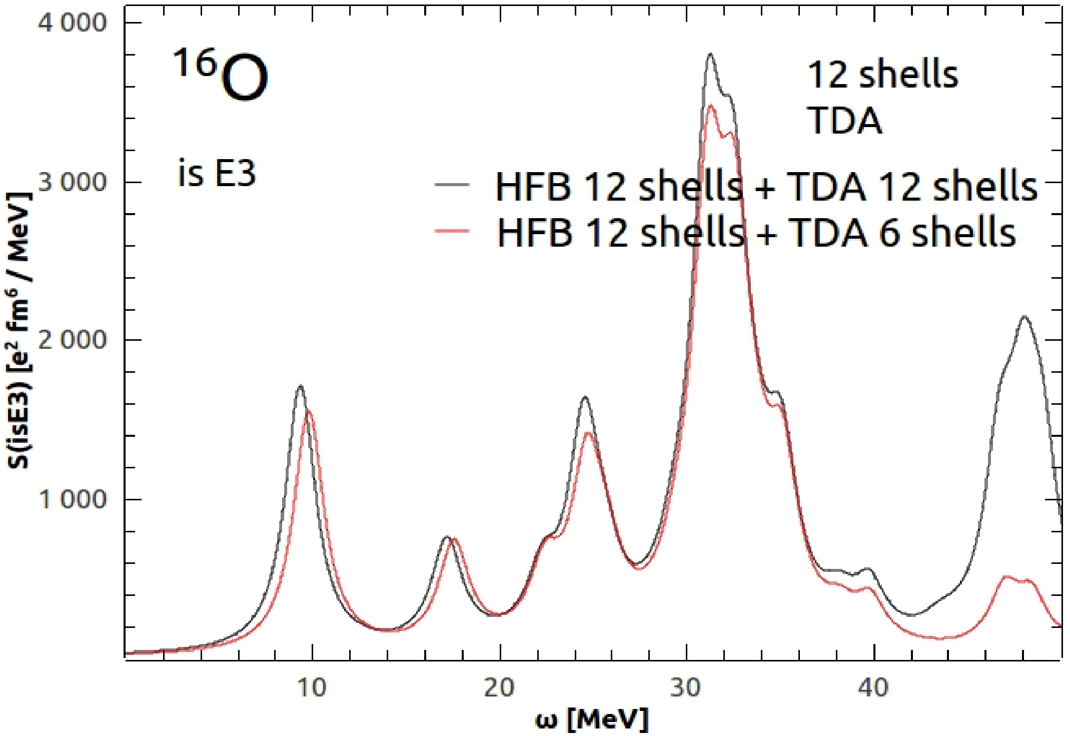
TDA – effect of configuration space



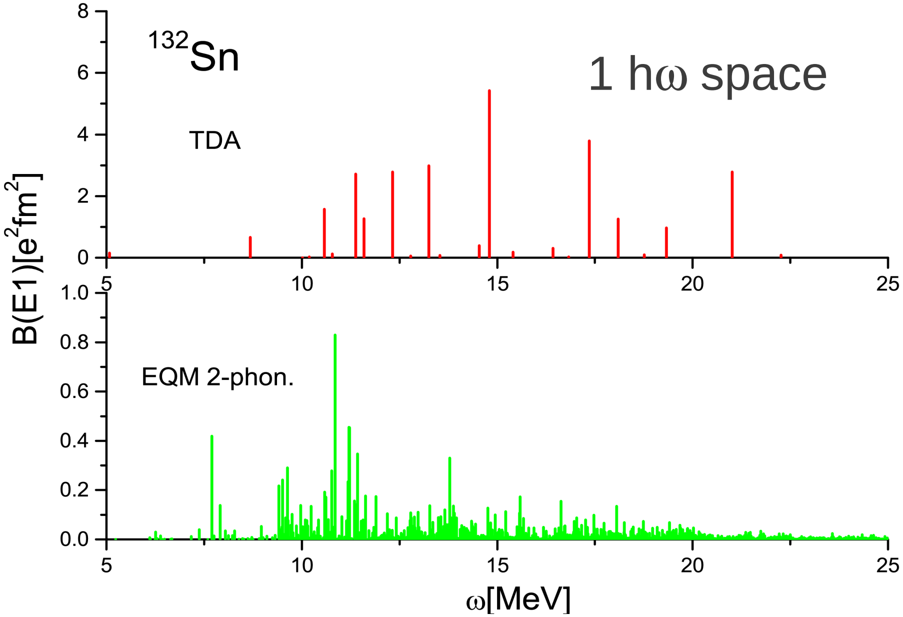
the effect on other multipolarities

E0, E2, E3





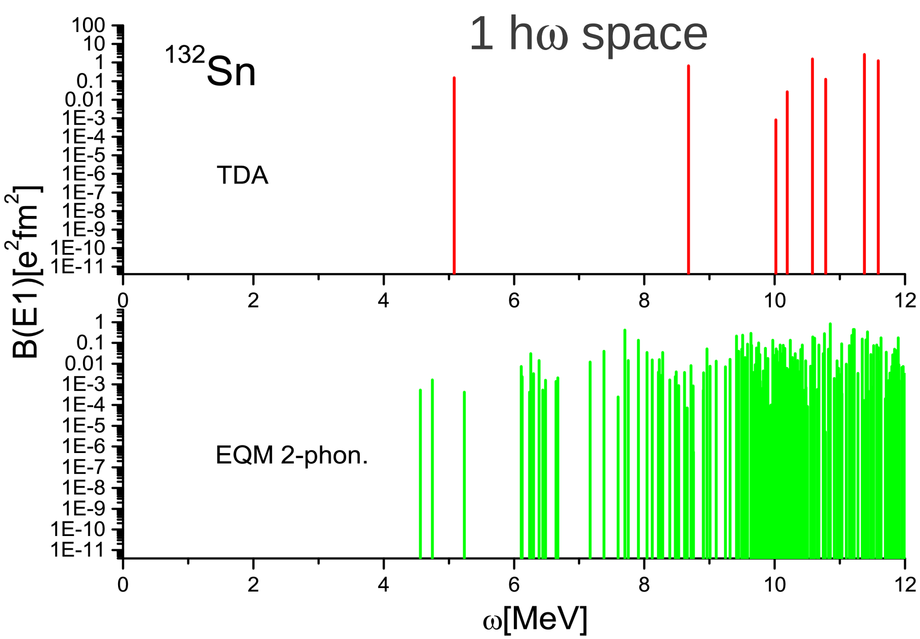
EMPM calculations in HF basis

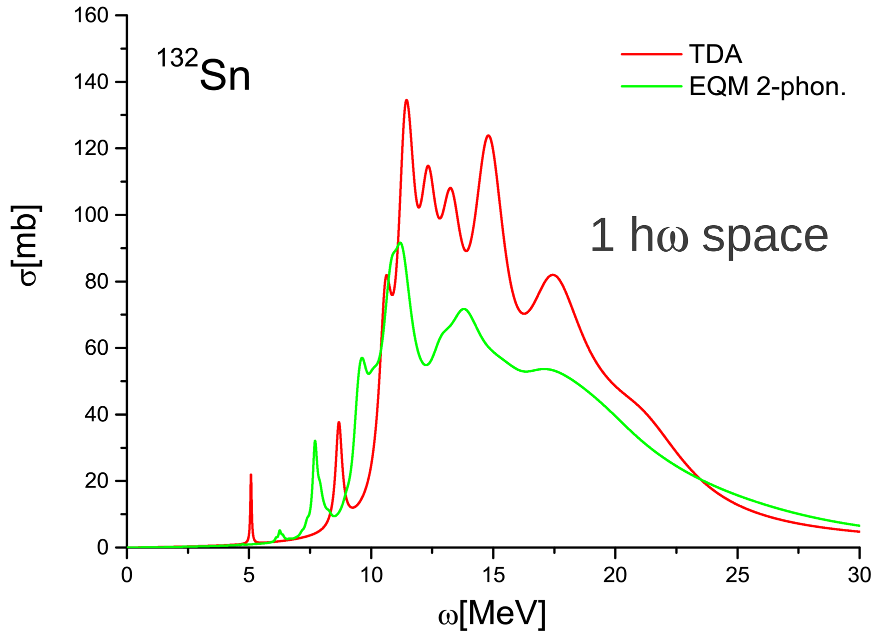


E1 calculation up to 2-phonons in ^{132}Sn...
Excitation energies meassured from HF vacuum energy.

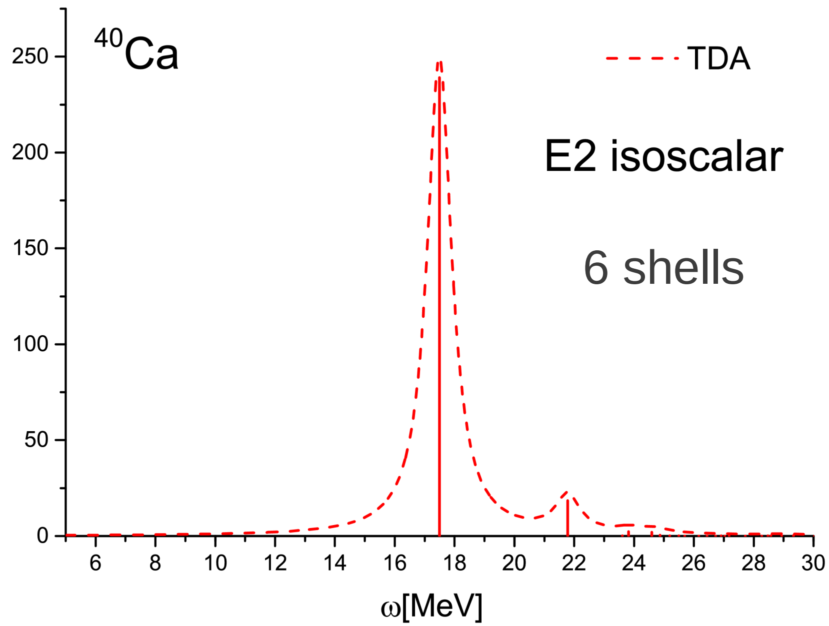
$$E_{corr.} = -5.8 \text{ MeV}$$

Systematical downshift of spectra expected at the 3-phon and 4-phon level
Visible multifragmentation of the spectrum...





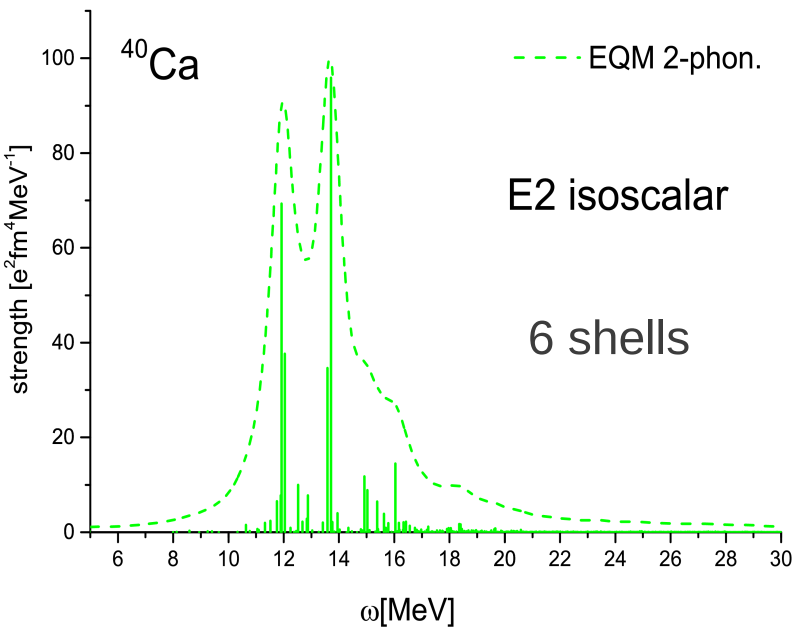
EMPM calculations in HF basis

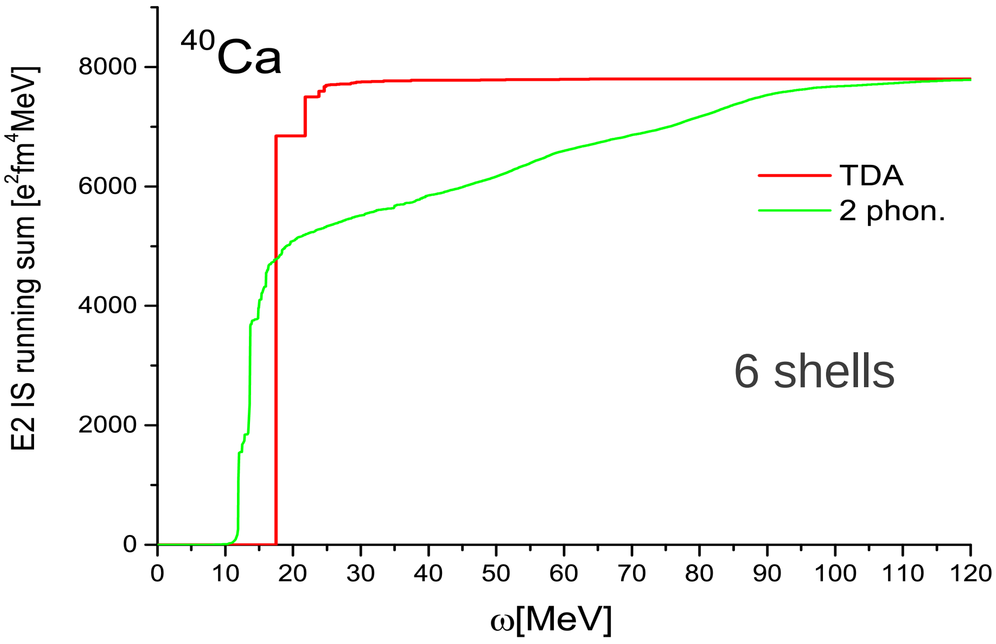


E2 calculation up to 2-phonons in ^{40}Ca...
Excitation energies meassured from HF vacuum energy.

$$E_{corr.} = -43.9 \text{ MeV}$$

Multifragmentation effect – visible also on the running sum





Plans for the near future:

- EMPM calculations in self-consistent basis for isotopes of calcium, tin, lead on a way
- Quasiparticle formulation of the EMPM (application of the model to semi-magic and neutron rich nuclei)
- Study of pygmy resonances, M1 transitions (quenching effect, occurence of spin-M1 for spin-saturated nucleus ^{40}Ca), multifragmentation of the giant resonances
- The study of particle-phonon or particle-multiphonon coupling (description of the odd systems)