Exercises for Quantum Field Theory II, WS 2015/16

H.-W. Hammer

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A.1: Renormalization of ϕ^3-theory in 6d

Consider the following theory of a real scalar field in **6 spacetime dimensions**:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - c\phi - \frac{m^2}{2}\phi^2 - \frac{g}{3!}\phi^3 \,.$$

Since the potential ϕ^3 is not bounded from below, this theory does not have a proper vacuum state. For small c, however, there exists a metastable vacuum in the vicinity of the origin. This metastable state is sufficient to treat the theory in perturbation theory.

(a) Show that the coupling constant g is dimensionless. Work now in $6 - 2\epsilon$ spacetime dimensions. What is the exponent x in the substitution $g \to \mu^x g$ (μ is a mass scale) that allows g to remain dimensionless?

(b) Draw all 1-particle-irreducible (1PI) one-loop diagrams of this theory.

(c) In order to renormalize the theory in the MS scheme (Minimal Subtration) you need those parts of the diagrams in (b) that diverge when they are Taylor-expanded in the external momenta. Calculate these terms using dimensional regularization. Make sure to use the correct symmetry factors! You need to calculate four integrals, one for each operator in the Lagrangian. For each case, identify the coefficient of the $1/\epsilon$ pole.

<u>Hint</u>:

It holds that

$$\int d^n k \, \frac{1}{(k^2 + a^2)^r} = \pi^{n/2} \, (a^2)^{n/2 - r} \, \frac{\Gamma(r - n/2)}{\Gamma(r)} \,,$$

for any integer $r \geq 1$.

(d) In the MS scheme, the counterterms are chosen to exactly cancel the $1/\epsilon$ parts of the one-loop graphs. What are the four counterterms needed in this scheme in order to renormalize the one-loop diagrams of the theory we are considering here?