Exercises for Quantum Field Theory II, WS 2015/16

H.-W. Hammer

Exercise Sheet 3   27.11.2015

A.3: Derivation of effective field theories: Integrating out a heavy field

Consider a renormalizable theory where a "heavy" scalar field H is coupled to a fermion field ψ:

$$\mathcal{L} = -\frac{1}{2} H(\partial_\mu \partial^\mu + m_H^2) H + \bar{\psi}(\mathrm{i}\not{\partial} - m_\psi)\psi - g\bar{\psi}\psi H \, .$$

(a) Using the path-integral formalism, write down the generating functional W for this theory. Introduce appropriate sources and express the interaction part of the Lagrangian in terms of functional derivatives with respect to the sources.

(b) Now write down the same generating functional without coupling the field H to an external source. Using an appropriate shift, you can carry out the path integral over H explicitly. What is the *effective interaction Lagrangian* generated by this procedure?

<u>Hint</u>:

> *To simplify the calculation, it is useful to introduce a short-hand notation for the differential operators,*
>
> $$\hat{D}_H = \partial^2 + m_H^2 \qquad , \quad \hat{D}_\psi = i\not{\partial} - m_\psi \, ,$$
>
> *and to write the combination $g\bar{\psi}\psi$ as a single function.*

(c) The effective Lagrangian derived in part (b) is non-local. Sketch the behavior of the heavy-field Feynman propagator as a function of the distance in order to identify the dominant contributions. Taylor-expand the fermion density $(\bar{\psi}\psi)$ and keep only the leading term to obtain a local effective Lagrangian.

(d) Draw appropriate diagrams and write down the interaction Feynman rules for

- i) the original Lagrangian,
- ii) the non-local Lagrangian obtained in part (b), and
- iii) the final result from part (c).

(e) Physically interpret the procedure carried out above.