Oscillating Magnetic Field near a Feshbach Resonance

(with Langmack, Smith, Mohapatra)

and

Range Corrections for Efimov Features

(with Ji, Phillips, Platter)

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support: National Science Foundation
Simons Foundation

Range Corrections for Efimov Features

previous papers by Chen Ji, Lucas Platter, Daniel R. Phillips

Beyond universality in three-body recombination: an Effective Field Theory treatment
Europhys. Lett. 92:13003,2010 [arXiv:1005.1990]

The three-boson system at next-to-leading order in an effective field theory for systems with a large scattering length
Annals Phys. 327, 1803 (2012) [arXiv:1106.3837]

- use EFT to expand range corrections in powers of range
- calculate 1st-order range corrections in terms of the effective range and a 2nd 3-body parameter

recent paper by Ji, Braaten, Platter, Phillips

Universal Relations for Range Corrections to Efimov Features
arXiv:1506.02334

- reveals simple pattern in 1st-order range corrections
- explains pattern in terms of “running Efimov parameter”

Ken Wilson's Last Physics Paper

Precise numerical results for limit cycles in the quantum three-body problem
R.F. Mohr, R.J. Furnstahl, R.J. Perry, K.G. Wilson, H.-W. Hammer
Annals Phys. 321, 225-259 (2006) [nucl-th/0509076]

numerical solutions of the 3-body problem for identical bosons with 12 digits of accuracy

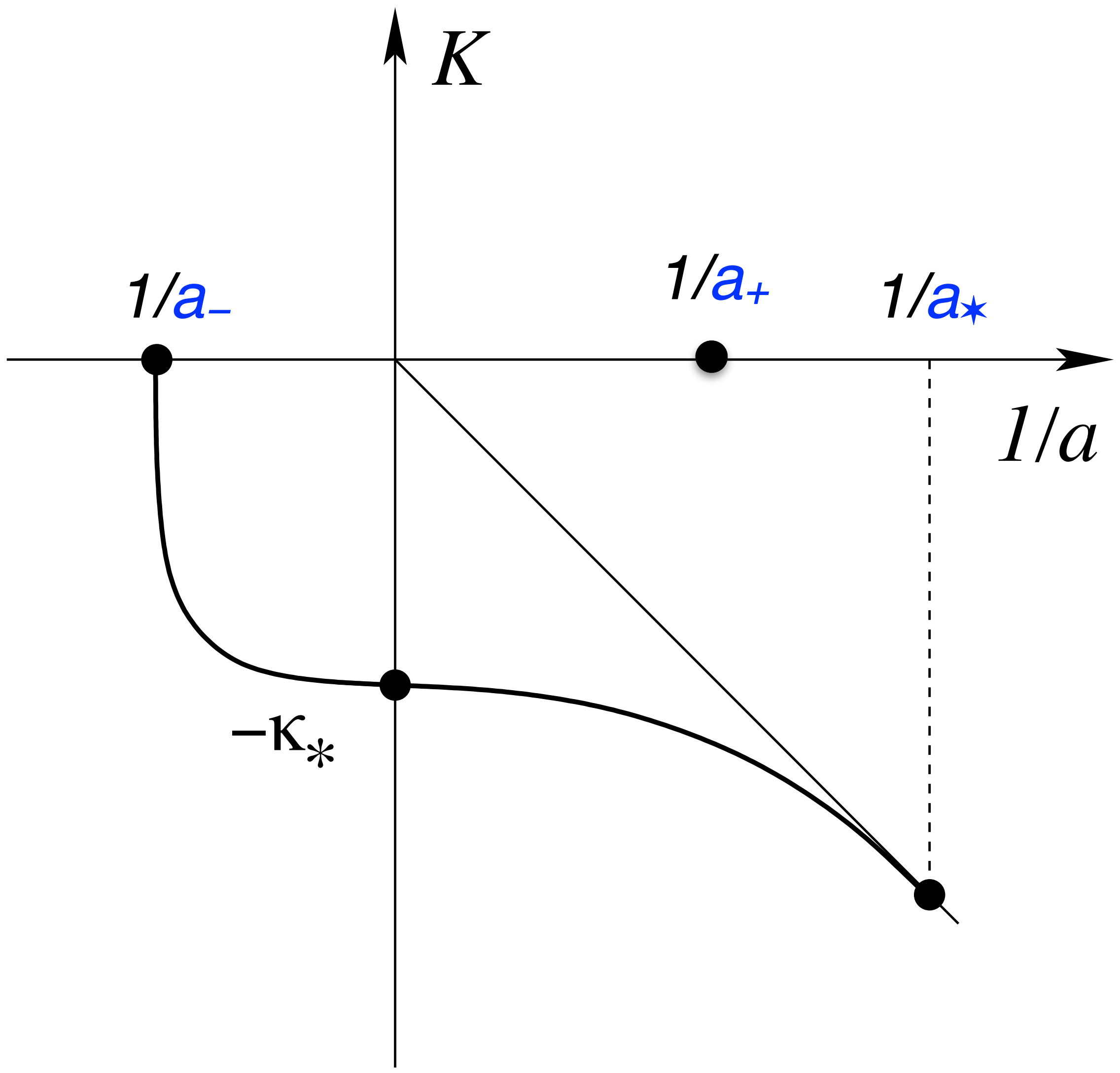
Binding energies of Efimov states 1 and 2 when Efimov state 0 is at the atom-dimer threshold

Λ	B_3 #1	B_3 #2
100000.00000000	6.750290150257678	1406.13039320296
738905.60989306	6.750290150257678	1406.13039320593
2008553.6923187	6.750290150255419	1406.13039320345
14841315.910257	6.750290150268966	1406.13039320345
298095798.70417	6.750290150257678	1406.13039320593
5987414171.5197	6.750290150259935	1406.13039320345
44241339200.892	6.750290150257678	1406.13039320296

Q. Why did Wilson want 12 digits of accuracy?

A. He knew there were logarithmic renormalization effects in the range corrections.

Efimov Features



Features associated with one branch of Efimov trimers

κ_*: binding momentum of Efimov trimer at unitarity

$a = a_-$: Efimov trimer at 3-atom threshold

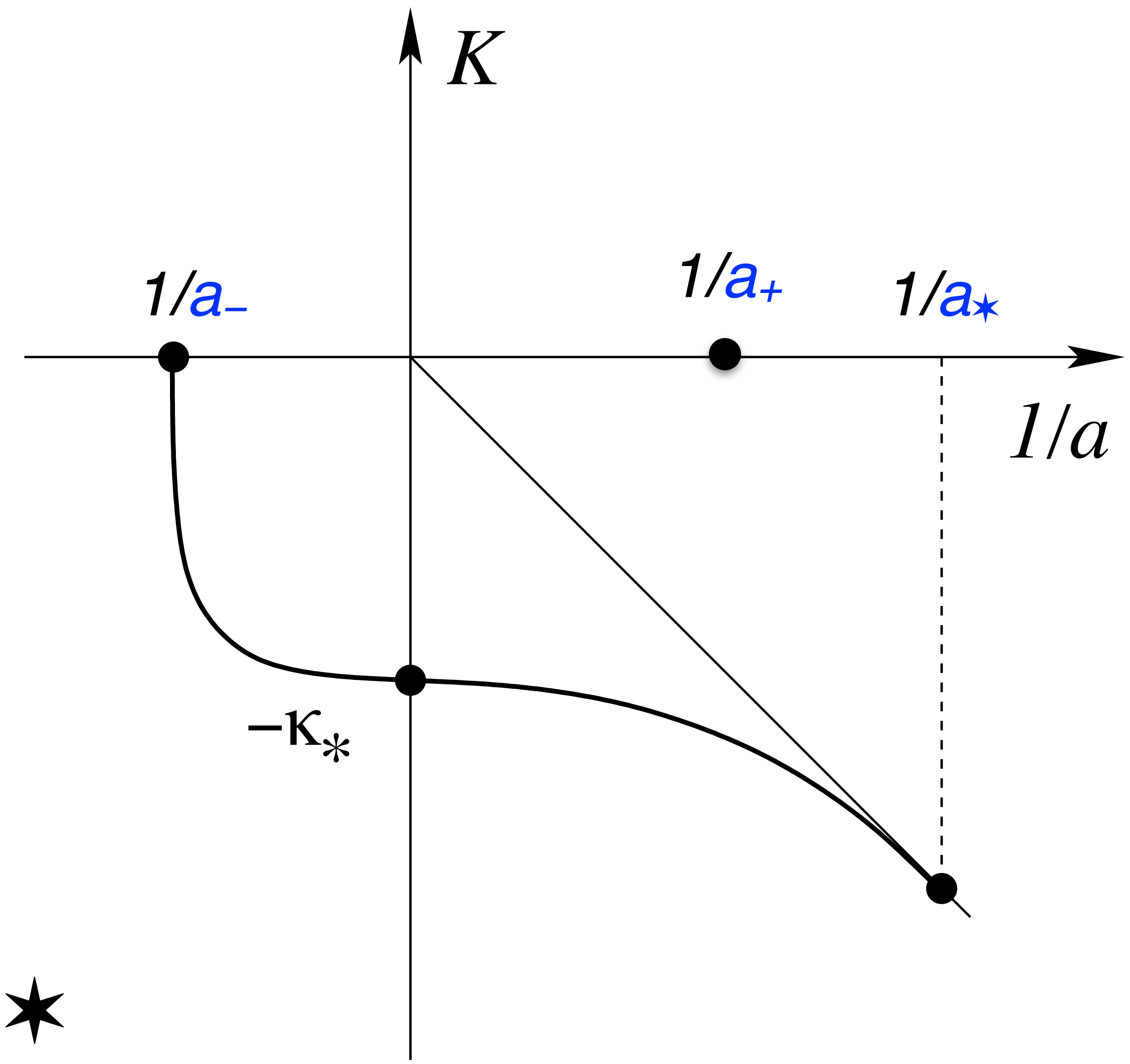
$a = a_*$: Efimov trimer at atom+dimer threshold

$a = a_+$: interference minimum in 3-atom recombination at threshold

Features associated with other branches

differ by powers of discrete scaling factor $\lambda = 22.69$

Zero-Range Limit



Efimov features: $a_{i,n}$ $i = -,+,\star$

$n = 0,1,2,\ldots$

ratios of Efimov features are universal numbers

$$a_{i,n} = \lambda^n \, \theta_i \, \kappa_\star^{-1}$$

universal ratios: $\theta_- = -1.508$

$\theta_+ = 0.3165$

$\theta_\star = 0.07076$

discrete scaling factor: $\lambda = 22.69$

Range Corrections

empirical prescription by Gattobigio and Kievsky (and Garrido)
arXiv:1212.3457 (arXiv:1306.1711)

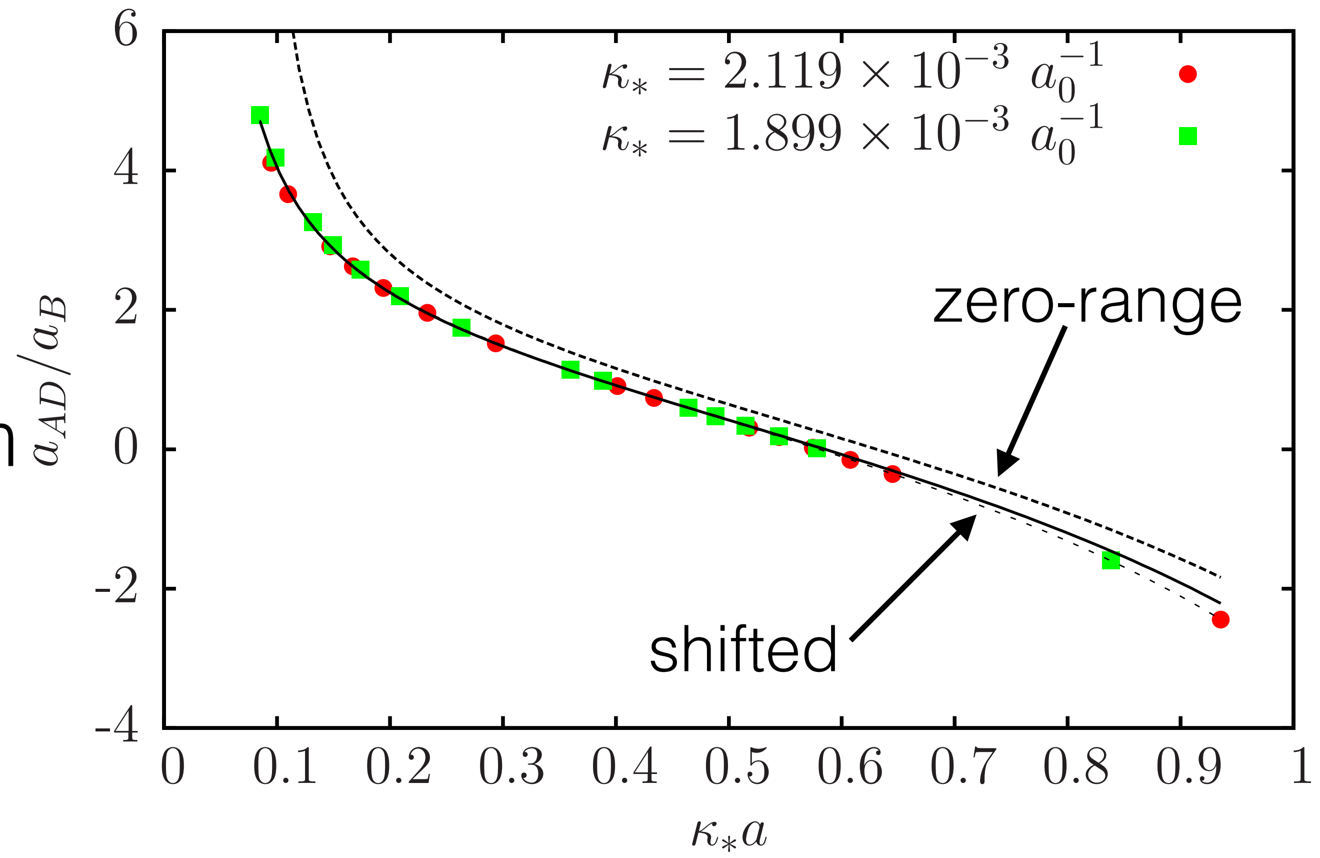
Range corrections can be largely taken into account by simple changes in the zero-range formulas

1. eliminate scattering length *a* in favor of
   inverse binding momentum a_B
   of universal dimer (or virtual state)

2. shift Efimov parameter

$$\kappa_* \longrightarrow \kappa_* + \Gamma/a$$

where Γ is an empirical parameter that depends on the system and on the observable



NLO Range Corrections

Ji, Platter, Phillips arXiv:1106.3837

expand Efimov features to 1st order in range

$$1/a_{i,n} = \lambda^{-n}\theta_i^{-1}\kappa_* + (\xi_{i,n} + \eta_{i,n}J)\,\kappa_*^2 r_s$$

r_s = S-wave effective range
$\xi_{i,n}$, $\eta_{i,n}$ are universal numbers
J is non-universal (can be determined by a 2nd 3-body input)

Ji, Braaten, Platter, Phillips arXiv:1506.02334

$a_{i,n}$ has simple dependence on *n*

$$a_{i,n} = \lambda^n\theta_i\kappa_*^{-1} + (J_i - n\sigma)r_s$$

$\sigma = 1.095$ is a universal number
differences $J_i - J_j$ are universal numbers
one J_i must be determined by a 2nd 3-body input

NLO Range Corrections

$$a_{i,n} = \lambda^n \theta_i \, \kappa_*^{-1} + (J_i - n\sigma) r_s$$

$$= \lambda^n \theta_i \, \kappa_*^{-1} \left[1 - n\sigma \frac{r_s}{\lambda^n \theta_i \kappa_*^{-1}} \right] + J_i r_s$$

$n\ \sigma\ r_s$ term can be absorbed
into Efimov parameter κ_*

$$\kappa_* \left[1 + n\sigma \frac{r_s}{\lambda^n \theta_i \kappa_*^{-1}} \right] \approx \kappa_* \left[1 + \log(|a_{i,n}| \kappa_*) \frac{\sigma}{\log \lambda} \frac{r_s}{a_{i,n}} \right]$$

correction term is proportional to $r_s/a_{i,n}$
depends logarithmically on momentum scale $1/|a_{i,n}|$

Running Efimov Parameter

Renormalization of effective field theory
with 1st-order range corrections
implies that the Efimov parameter
runs logarithmically with the momentum scale Q
at a rate proportional to r_s/a

$$\bar{\kappa}_*(Q, a) = (Q/\kappa_*)^{-\gamma r_s/a}\kappa_*$$

constant in exponent: $\gamma = 0.351$
γ is a universal number
to numerical accuracy, $\gamma = \log(\lambda)/\sigma$
where σ is universal number in NLO range correction

Summary
Range Corrections for Efimov Features

- simple pattern in 1st-order range corrections

$$a_{i,n} = \lambda^n \theta_i \kappa_*^{-1} + (J_i - n\sigma) r_s$$

- running Efimov parameter
  runs with the momentum scale Q at a rate proportion to r_s/a

$$\bar{\kappa}_*(Q, a) = (Q/\kappa_*)^{-\gamma r_s/a} \kappa_*$$

- explains the empirical prescription for range corrections
  introduced by Gattobigio and Kievsky

$$\kappa_* \longrightarrow \kappa_* + \Gamma/a$$

Oscillating Magnetic Field near a Feshbach Resonance

Association of Atoms into Universal Dimers
using an Oscillating Magnetic Field
Christian Langmack, D. Hudson Smith, Eric Braaten
Phys. Rev. Lett. 114, 103002 (2015) [arXiv:1406.7313]

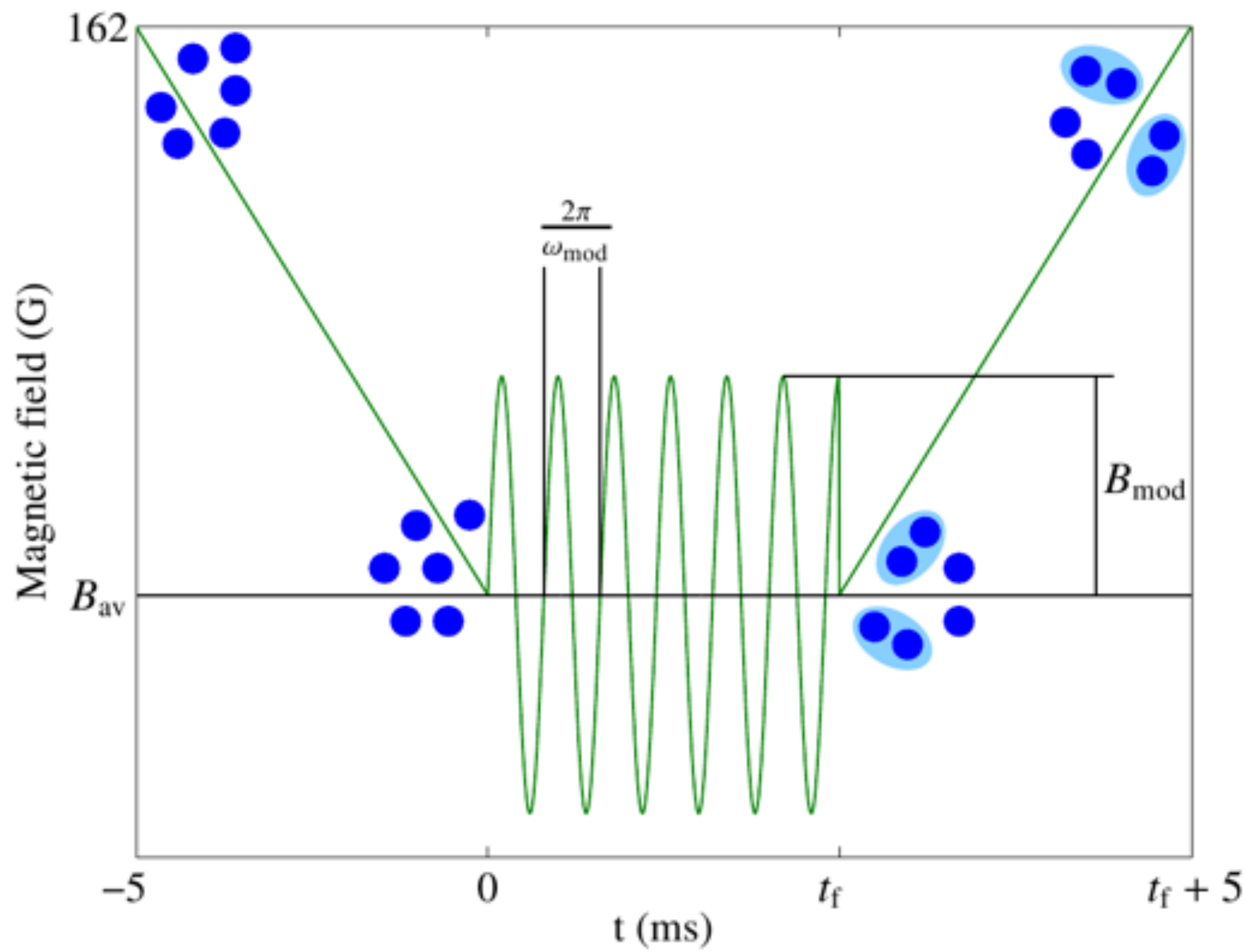
Harmonic and Subharmonic Association
of Universal Dimers in a Thermal Gas
Abhishek Mohapatra, Eric Braaten
arXiv:1504.06573

Related poster

Inducing Resonant Interactions in Ultracold Atoms
with a Modulated Magnetic Field
D. Hudson Smith
arXiv:1503.0268

Oscillating Magnetic Field near a Feshbach Resonance

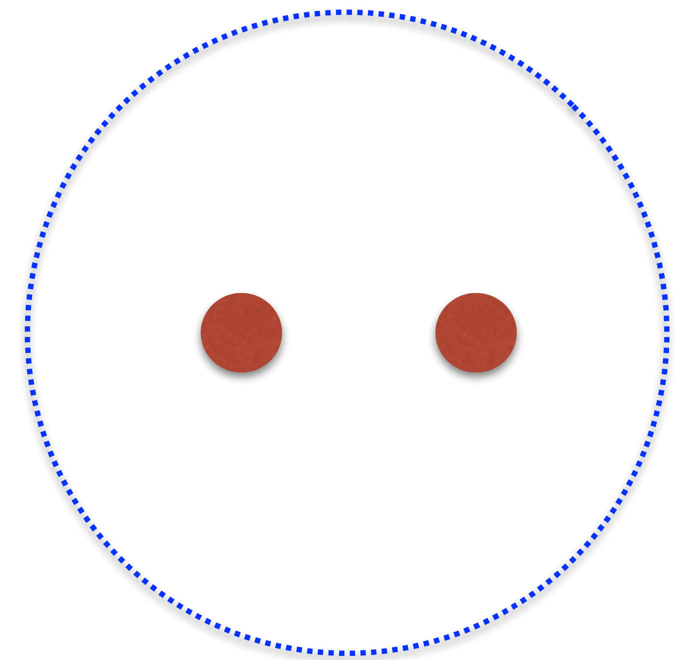
- Scattering length controlled by external magnetic field
- Wiggle magnetic field with angular frequency ω
- Induces harmonic transitions to states with $\Delta E \approx \pm\hbar\omega$
- Associate atoms into molecules by tuning ω to near binding frequency



Hanna, Koehler, and Burnett (2006)

Oscillating Magnetic Field

Associate atoms into universal dimer



JILA experiments

Thompson, Hodby, and Wieman (2005): ^{85}Rb
Papp and Wieman (2006): ^{85}Rb + ^{87}Rb

LENS experiment

Weber et al. (2008): ^{87}Rb + ^{41}K

Innsbruck experiment

Lange et al. (2009): ^{133}Cs

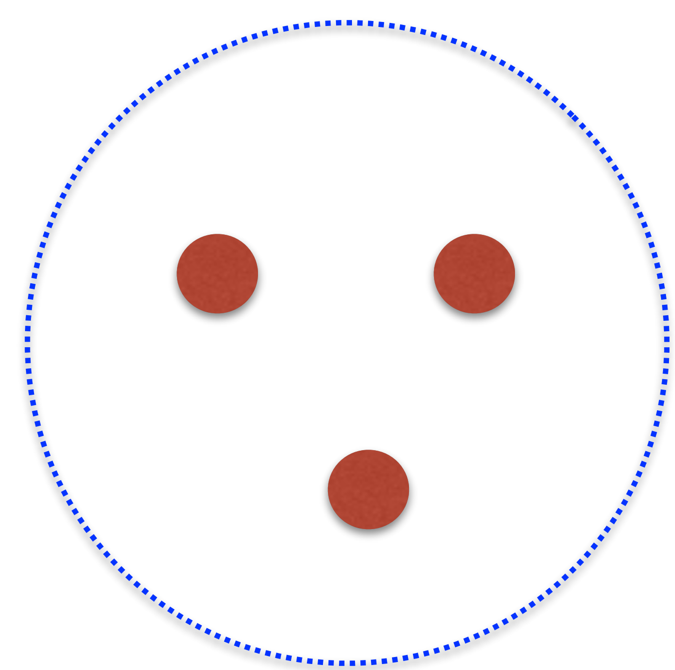
Bar-Ilan experiment

Gross et al. (2011): ^{7}Li

Rice experiment

Dyke, Pollack, and Hulet (2013): ^{7}Li

Associate atoms into Efimov trimer



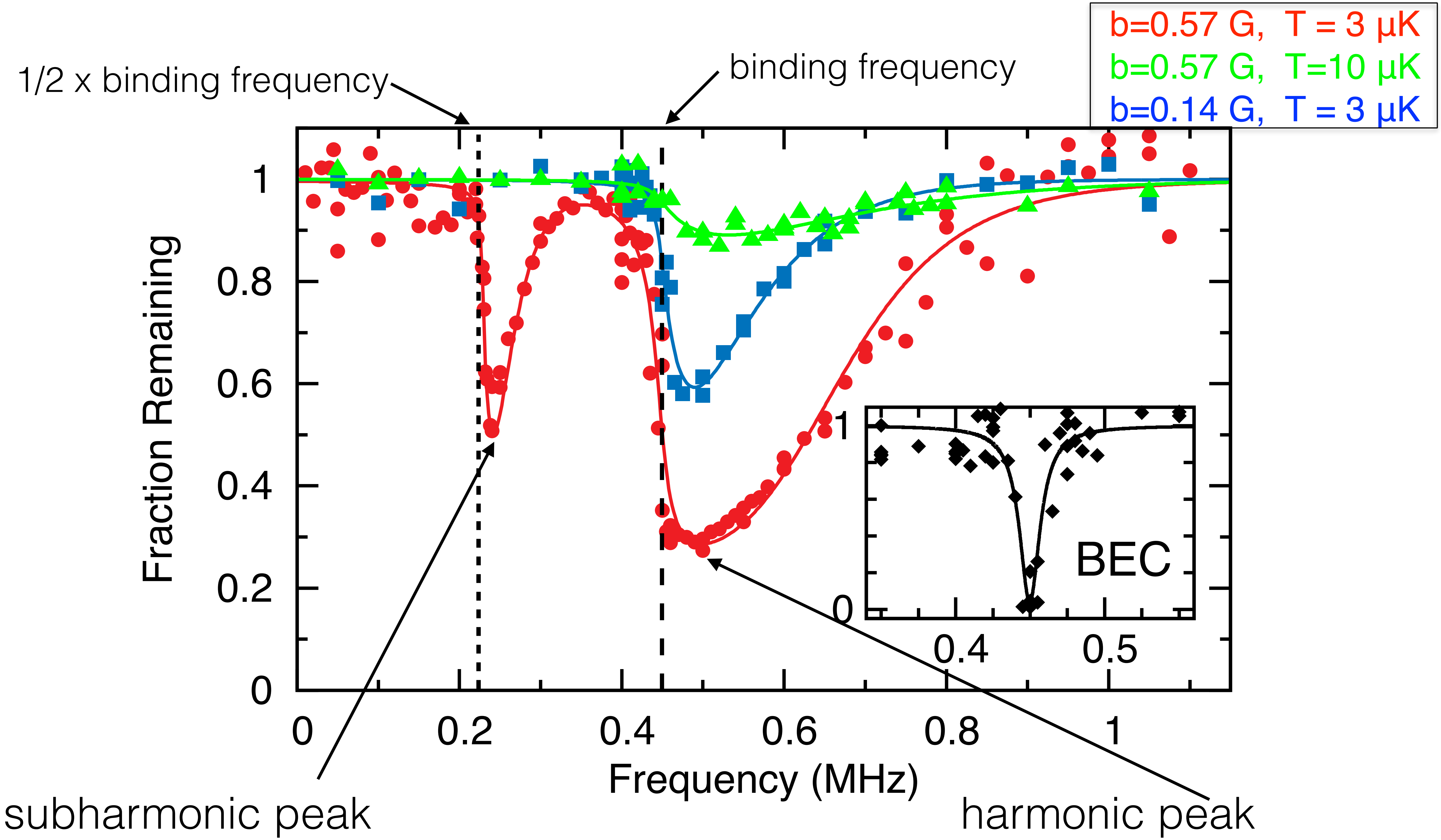
Bar-Ilan experiment

Machtev et al. (2012): ^{7}Li

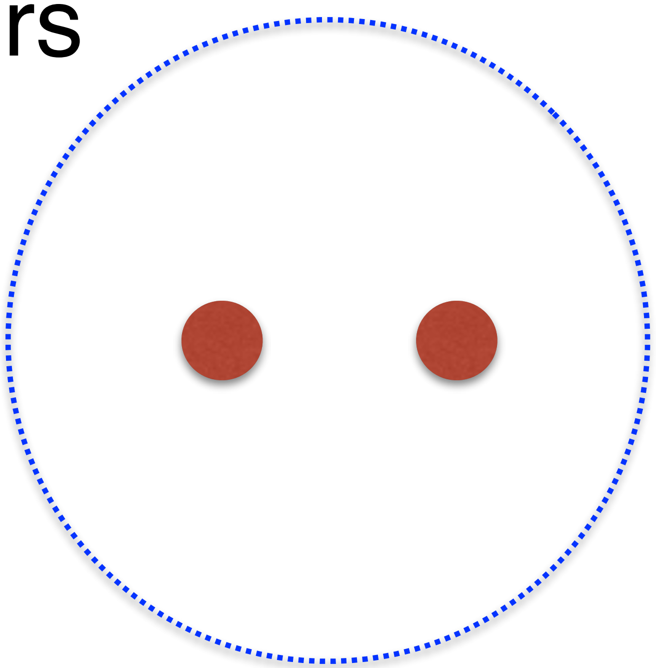
Associate ^{7}Li atoms into universal dimers

Rice experiment Dyke, Pollack, and Hulet (2013)

Observe association into dimers through loss of atoms
(dimers and additional atoms lost through inelastic collisions)



Previous theoretical work on association of atoms into universal dimers



Hanna, Koehler, and Burnett (2007)

2-channel model for 2-atom system
numerical results for harmonic association rates

Brouard and Plata (2015)

2-state model for 2-atom system
analytic approximation to deduce qualitative features
of harmonic and subharmonic association rates
numerical results for harmonic association rates

Bazak, Liverts, and Barnea (2012)

assumes transition proceeds through absorption of real photon
analytic result for harmonic association rate
for arbitrary direction of oscillating electromagnetic field
ill-defined normalization factor
frequency dependence on disagrees with our result

Scattering length *a*
controlled by magnetic field *B* near Feshbach resonance

$$a(B) = a_{\mathrm{bg}} \left[1 - \frac{\Delta}{B - B_0} \right]$$

Oscillation of longitudinal magnetic field $B(t) = \bar{B} + b \sin(\omega t)$

implies time-dependent "scattering length"

$$a(t) = a(\bar{B}) + a'(\bar{B})\, b \sin(\omega t) + \ldots$$

Treat as a time-dependent perturbation.
Deduce perturbing Hamiltonian from … Tan's adiabatic relation
from … quantum field theory

Deduce perturbing Hamiltonian from Tan's adiabatic relation

Adiabatic relation $$\frac{dE}{d(1/a)} = -\frac{\hbar^2}{4\pi m} C$$ where C is the contact

change in energy from small change in inverse scattering length

$$\Delta E = -\frac{\hbar^2}{4\pi m} C \Delta(1/a)$$

Perturbing Hamiltonian is proportional to the contact operator!

$$H_{\rm pert}(t) = -\frac{\hbar^2}{4\pi m} \left(\frac{1}{a(t)} - \frac{1}{\bar{a}} \right) C$$

Langmack, Smith, Braaten arXiv:1406.7313

Quantum Field Theory for constant scattering length

interaction Hamiltonian density operator $$\mathcal{H}_{\rm int} = \frac{g_0}{m}\psi_1^\dagger\psi_2^\dagger\psi_2\psi_1$$

bare coupling constant
scales as $1/\Lambda$

$$g_0 = \frac{4\pi}{1/a - (2/\pi)\Lambda}$$

matrix elements of operator $\psi_1^\dagger\psi_2^\dagger\psi_2\psi_1$
have ultraviolet divergences that scale as Λ^2

matrix elements of H_{int} have ultraviolet divergence that scales as Λ
cancelled by kinetic energy density

contact density operator $$\mathcal{C} = g_0^2\psi_1^\dagger\psi_2^\dagger\psi_2\psi_1$$

has finite matrix elements as $\Lambda \to \infty$

Quantum Field Theory for time-dependent scattering length

interaction Hamiltonian density operator $$\mathcal{H}_{\rm int} = \frac{g_0}{m}\psi_1^\dagger\psi_2^\dagger\psi_2\psi_1$$

time-dependent bare coupling constant
scales as $1/\Lambda$

$$g_0(t) = \frac{4\pi}{1/a(t) - (2/\pi)\Lambda}$$

expand g_0 in powers of b $$g_0(t) = \bar{g}_0 - \frac{\bar{g}_0^2}{4\pi}\left(\frac{1}{a(t)} - \frac{1}{\bar{a}}\right) + \dots$$

Perturbing Hamiltonian density proportional to contact density!

$$\mathcal{H}_{\rm pert}(t) = -\frac{\hbar^2}{4\pi m}\left(\frac{1}{a(t)} - \frac{1}{\bar{a}}\right)\mathcal{C}$$

Mohapatra, Braaten arXiv:1504.06573

Fermi's Golden Rule

Perturbing Hamiltonian

$$H_{\text{pert}}(t) = -\frac{\hbar^2}{4\pi m}\left(\frac{1}{a(t)} - \frac{1}{\bar{a}}\right) C$$

Harmonic transition rate

$$\Gamma_1(\omega) = \frac{\hbar^2}{64\pi^2 m^2 a_{\text{bg}}^2}\left(\frac{b\Delta}{(\Delta + B_0 - \bar{B})^2}\right)^2 \sum_f \left|\langle f|C|i\rangle\right|^2 \sum_{\pm} 2\pi\delta(\omega_{fi} \pm \omega).$$

normalization determined by Feshbach resonance parmaters

transition matrix element of contact operator

frequency delta function

Langmack, Smith, Braaten arXiv:1406.7313

Subharmonic transition rate

from 2nd order perturbation theory in the contact operator

Mohapatra, Braaten arXiv:1504.06573

Analytic results for transition rates in thermal gas

- Thermal equilibrium with Boltzmann statistics
- Local density approximation
  reduces matrix element of operator operator
  to matrix element of contact density operator in homogeneous system
- Diluteness
  reduces many-body matrix element of contact density operator
  to two-body matrix element
- Large scattering length
  calculate two-body matrix elements of contact density operator
  analytically in terms of a

Analytic results for Association rates

into universal dimers in thermal gas of fermionic atoms

Harmonic association rate

$$\Gamma_1(\omega) = \frac{2\sqrt{2}\hbar^2}{m^2 a_{\mathrm{bg}}^2 \bar{a}} \left(\frac{b\Delta}{(\Delta + B_0 - \bar{B})^2} \right)^2 \left(\int d^3r\, n_1(\mathbf{r}) n_2(\mathbf{r}) \right) \frac{\lambda_{\mathrm{T}}^3 \kappa(\omega)}{\omega} \exp(-\beta\hbar^2\kappa^2(\omega)/m).$$

Subharmonic association rate

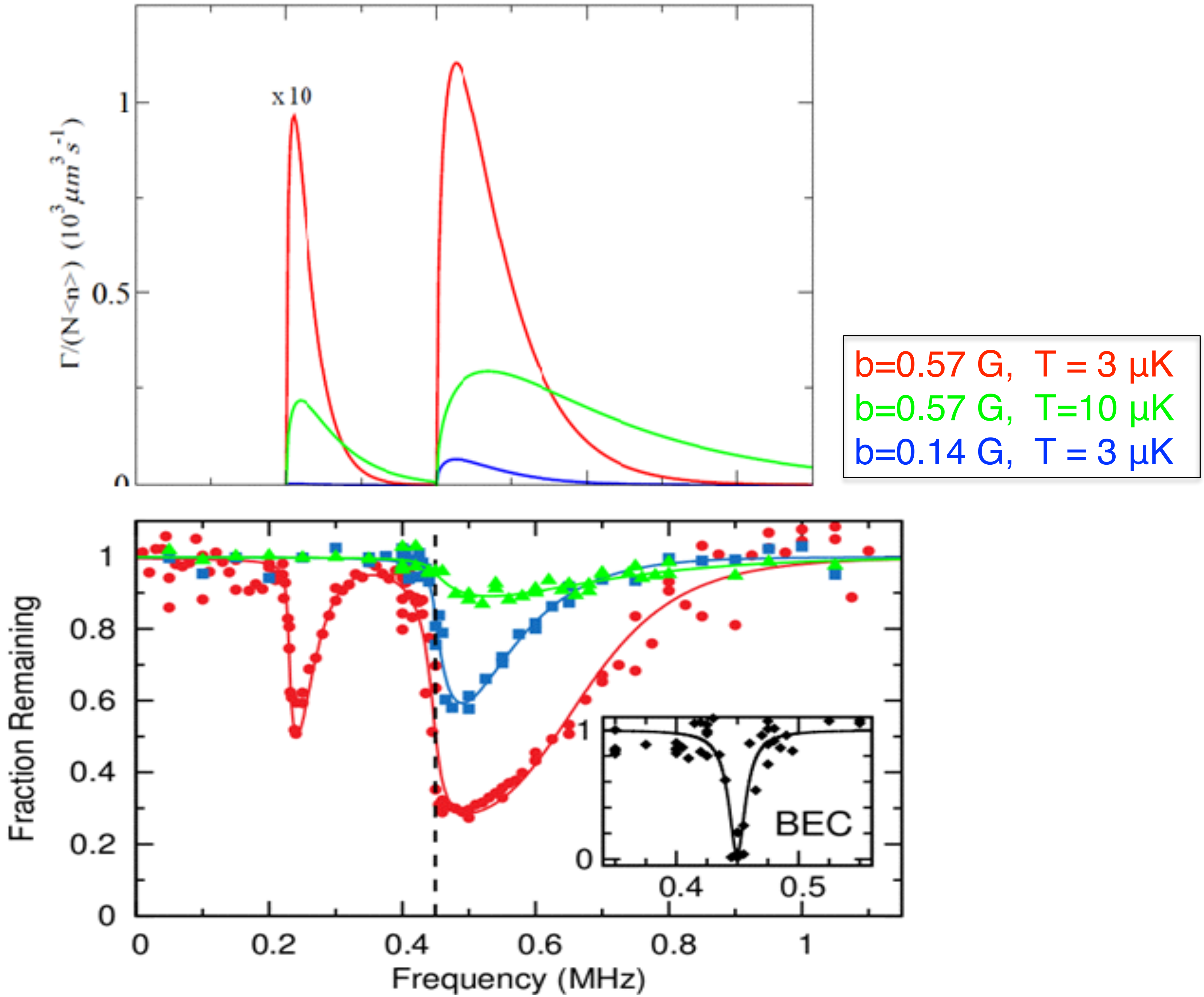
$$\Gamma_2(\omega) = \frac{\sqrt{2}\hbar^2 \bar{a}}{4m^2 a_{\mathrm{bg}}^4} \left(\frac{b\Delta}{(\Delta + B_0 - \bar{B})^2} \right)^4 \left(\int d^3r\ \ n_1(\mathbf{r}) n_2(\mathbf{r}) \right) \frac{\lambda_T^3 \kappa(2\omega)}{\omega} \exp\left(-\beta\hbar^2\kappa^2(2\omega)/m\right) \left(\frac{1}{1+\sqrt{1-m\omega\bar{a}^2/\hbar}} - \frac{2}{m\omega\bar{a}^2/\hbar} \right)^2.$$

- analytic
- absolutely normalized
- depends on Feshbach resonance parameters
  local atom number densities
  modulation frequency ω
  temperature

Association rate and Rice data

Association rate for producing dimers

Fraction of atoms remaining (after variable holding time)



Analytic results for Dissociation rates
into fermionic atoms in thermal gas of universal dimers

Harmonic dissociation rate

$$\Gamma_1(\omega) = \boxed{\frac{\hbar^2}{m^2 a_{\rm bg}^2 \bar{a}} \left(\frac{b\Delta}{(\Delta + B_0 - \bar{B})^2} \right)^2} \left(\int d^3r \, n_{\rm D}(\mathbf{r}) \right) \boxed{\frac{(m\omega/\hbar - 1/\bar{a}^2)^{1/2}}{\omega}}.$$

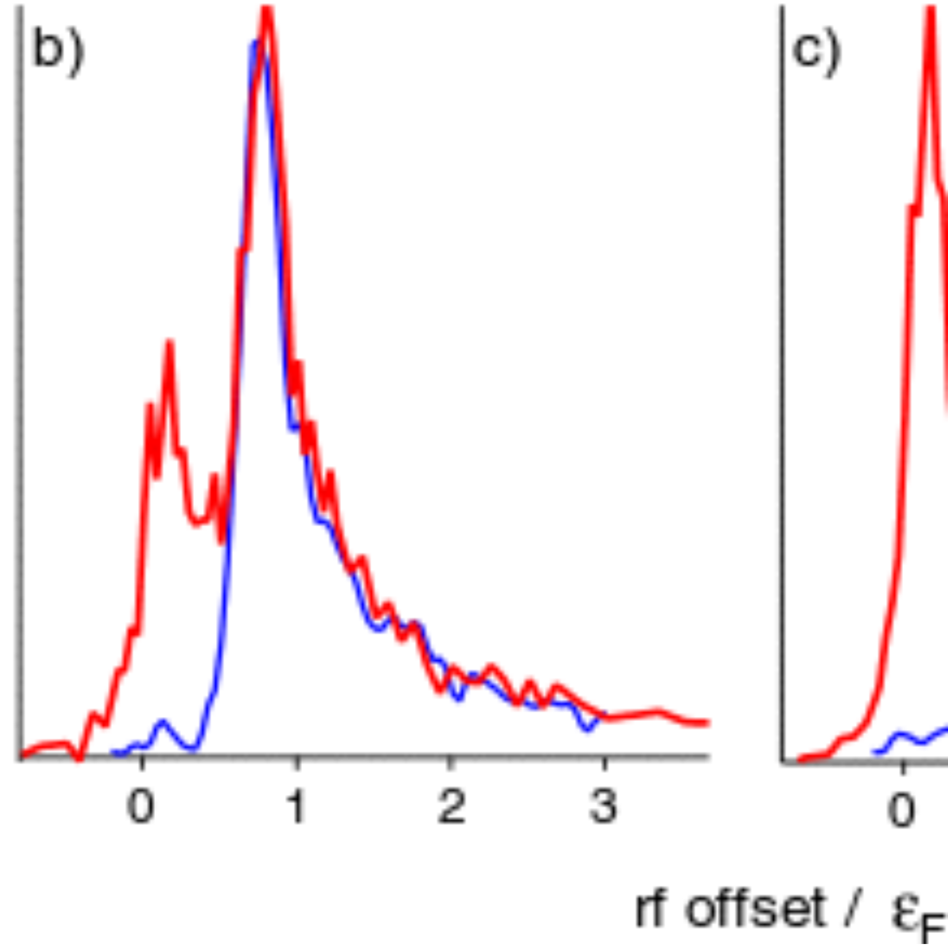
Subharmonic dissociation rate

$$\Gamma_2(\omega) = \boxed{\frac{\hbar^2 \bar{a}}{8 m^2 a_{\rm bg}^4} \left(\frac{b\Delta}{(\Delta + B_0 - \bar{B})^2} \right)^4} \left(\int d^3r \; n_{\rm D}(\mathbf{r}) \right) \boxed{\frac{\kappa(2\omega)}{\omega} \left(\frac{1}{1 + \sqrt{1 - m\omega\bar{a}^2/\hbar}} - \frac{2}{m\omega\bar{a}^2/\hbar} \right)^2}$$

- analytic
- absolutely normalized
- depends on Feshbach resonance parameters
  local dimer number density
  modulation frequency ω
- independent of temperature

Dissociation of Cooper pairs
into fermionic atoms in a Fermi gas

best measurement of the
pairing gap at unitarity:
from spin-imbalanced
rf spectroscopy
MIT group: Schiroztek et al. (2008)



no direct measurement of pairing gap
at unitarity for the balanced Fermi gas

oscillating magnetic field can be used dissociate Cooper pairs

calculation of the dissociation rate of Cooper pairs
would allow a direct measurement of the pairing gap

Summary
Oscillating Magnetic Field near a Feshbach Resonance

- oscillating magnetic field near a Feshbach resonance
  can be treated as a time-dependent perturbing Hamiltonian
  proportional to the contact operator

- transition rate from Fermi's Golden Rule
  involves transition matrix elements of the contact operator

- association rate for universal dimer in a thermal gas
  can be calculated analytically

- dissociation rate of Cooper pairs in a Fermi gas
  can be calculated analytically in the BCS limit
  (and perhaps in the unitary limit?)

- strong motivation for using oscillating magnetic field
  for direct measurements of pairing gap

Summary
Range Corrections for Efimov Features

- simple pattern in 1st-order range corrections

$$a_{i,n} = \lambda^n \theta_i \kappa_*^{-1} + (J_i - n\sigma) r_s$$

- running Efimov parameter
  runs with the momentum scale Q at a rate proportion to r_s/a

$$\bar{\kappa}_*(Q,a) = (Q/\kappa_*)^{-\gamma r_s/a} \kappa_*$$

- explains the empirical prescription for range corrections
  introduced by Gattobigio and Kievsky

$$\kappa_* \longrightarrow \kappa_* + \Gamma/a$$