Is the composite fermion a Dirac particle?

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Cold atoms meet QFT, 2015

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Plan

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- Composite fermion: quasiparticle of Fractional Quantum Hall Effect (FQHE)

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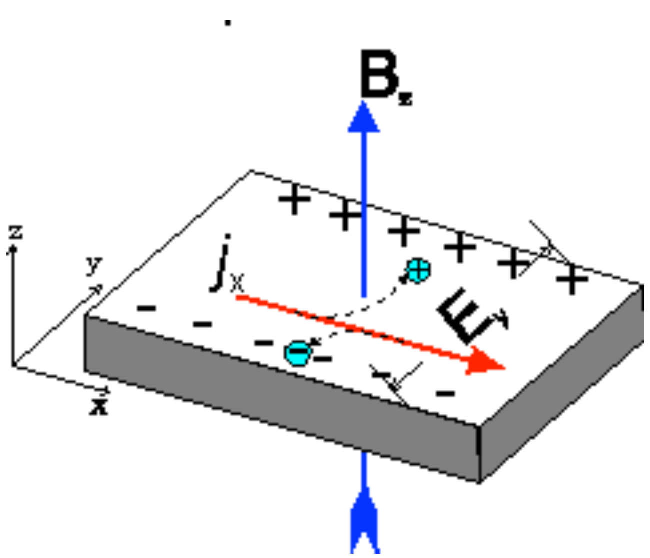
- Berry phase: new characteristic of Fermi liquid
- The old puzzle of particle-hole symmetry
- Berry phase of composite fermions

Hall conductivity/resistivity

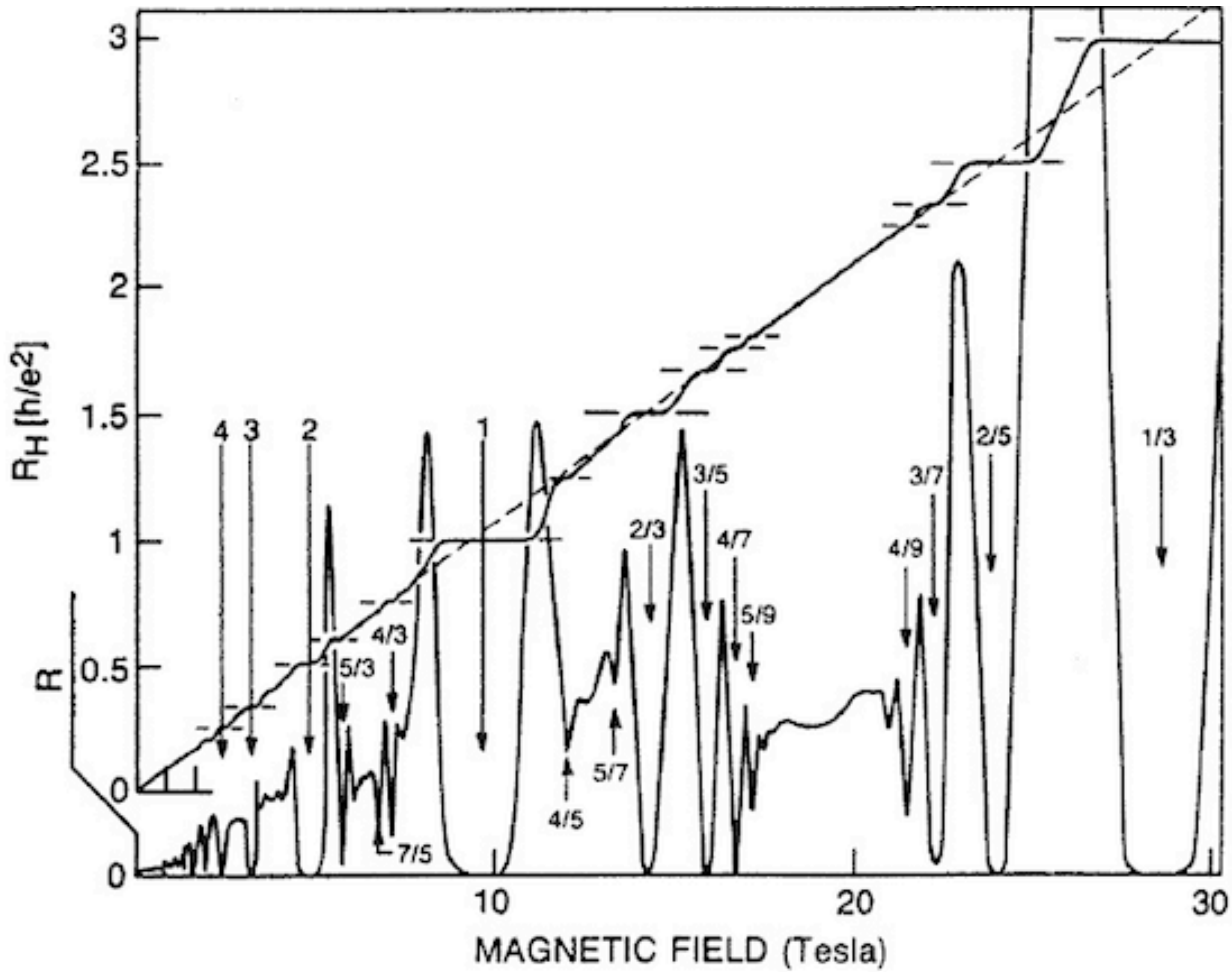


$$j_i = \sigma_{ij} E_j$$

$$E_i = \rho_{ij} j_j$$

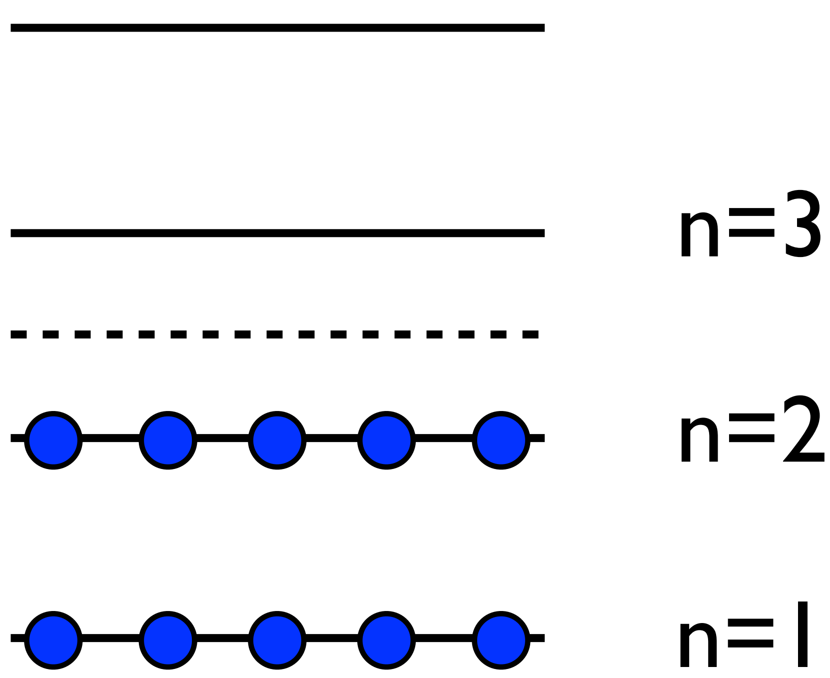
$$i, j = x, y$$

Fractional QH effect



Integer quantum Hall state

- electrons filling n Landau levels

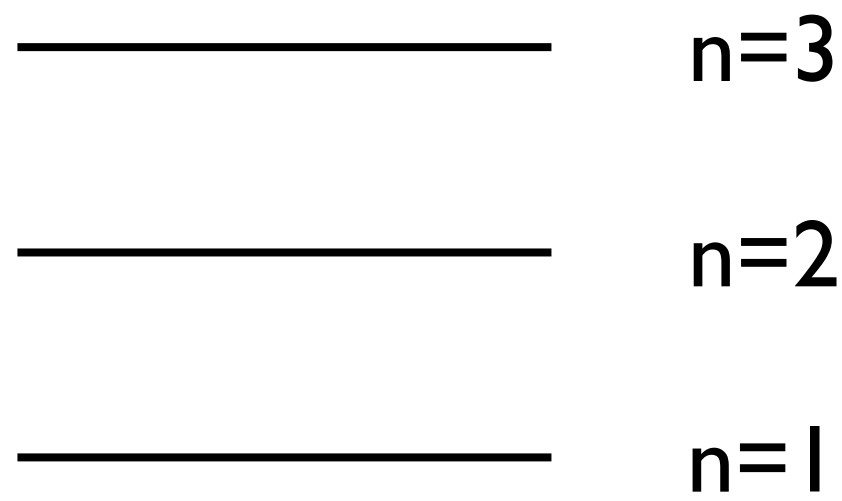


$$n_{2\mathrm{D}} = n\frac{eB}{2\pi\hbar}$$

$$\sigma_{xy} = \frac{en_{2\mathrm{D}}}{B} = n\frac{e^2}{2\pi\hbar}$$

Fractional QHE

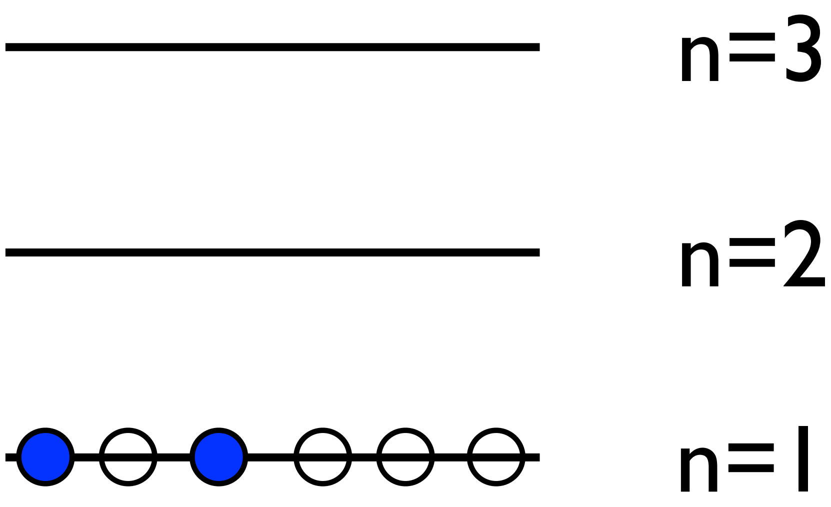
Landau levels of 2D electron in B field



Large ground-state degeneracy without interactions

Fractional QHE

Landau levels of 2D electron in B field



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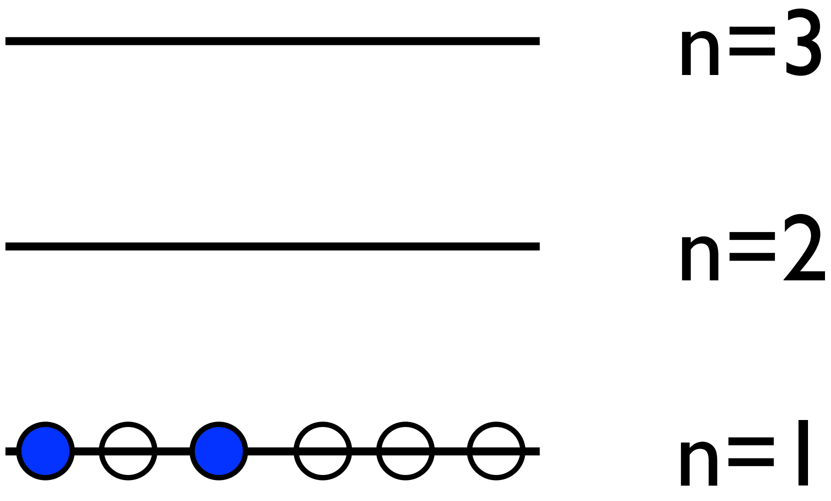
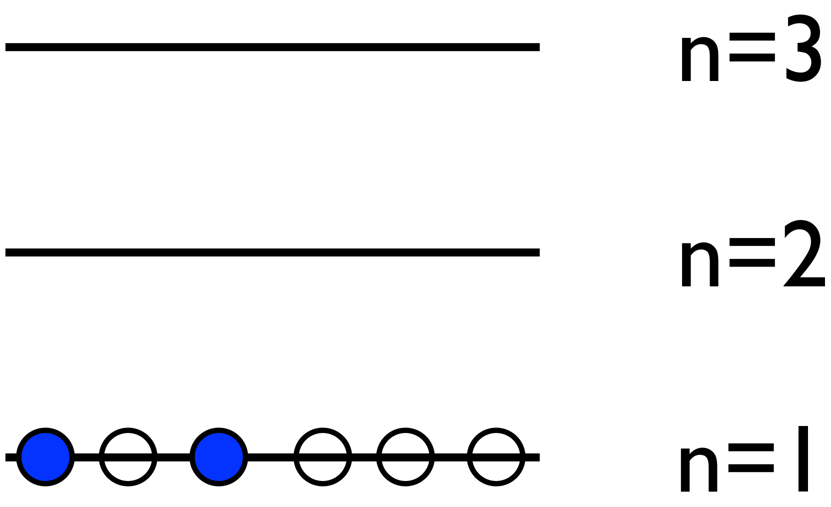


Large ground-state degeneracy without interactions

Filling fraction $$\nu = \frac{n}{B/2\pi}$$

Fractional QHE

Landau levels of 2D electron in B field

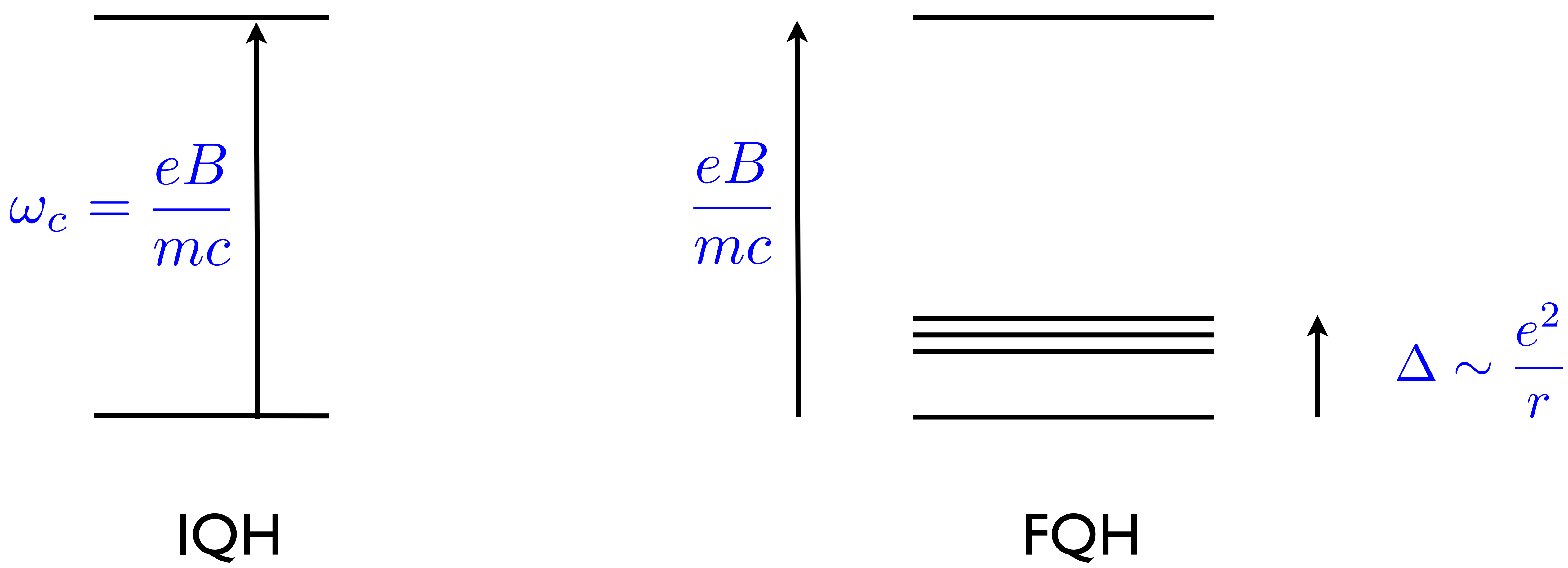


$$\nu = \frac{1}{3}$$

Large ground-state degeneracy without interactions

Filling fraction $$\nu = \frac{n}{B/2\pi}$$

Energy scales



Interesting limit: eB/mc >> Δ (m→0)
only lowest Landau level (LLL) states survives

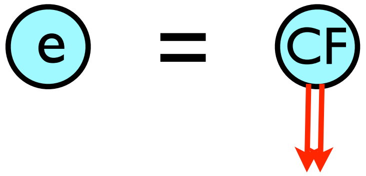
No small parameter

QHE in cold atoms

- Rapidly rotating atomic systems Wilkin Gunn 2000
- Lattice magnetic field by quadrupole potential and time modulation of tunneling Sørensen Demler Lukin 2005
- Artificial magnetic field Jaksch Zoller 2003
- Fractional Chern insulators Cooper Dalibard 2013, Yao et al 2013

Composite fermions

- Theoretical understanding of FQHE relies on the notion of the composite fermion



Mathematically

Lopez, Fradkin
Halperin, Lee, Read

$$\mathcal{L} = i\psi^\dagger(\partial_0 - iA_0 + ia_0)\psi - \frac{1}{2m}|(\partial_i - iA_i + ia_i)\psi|^2 + \frac{1}{4\pi p}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + \cdots$$

# of attached flux quanta

$$\boldsymbol{\nabla} \times \mathbf{a} = 2\pi p\, \psi^\dagger\psi$$

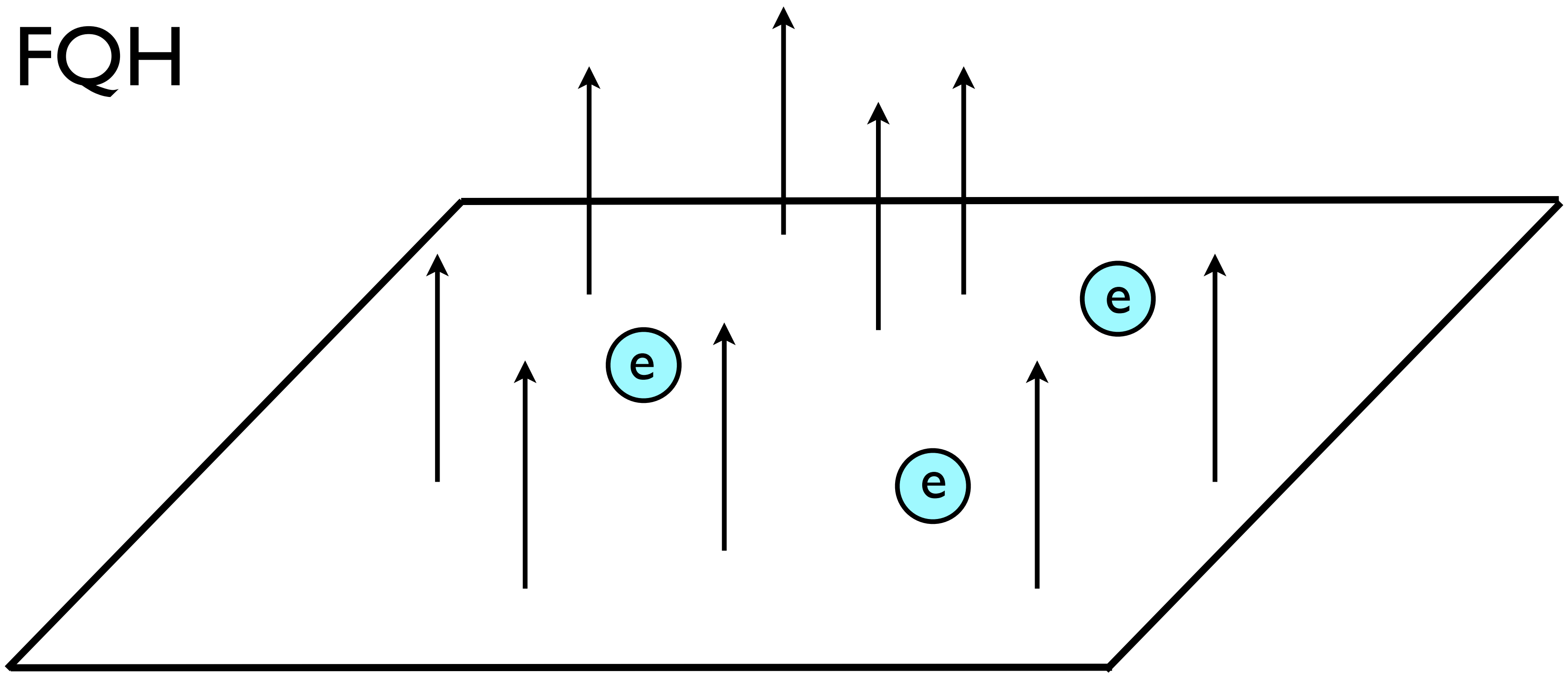
At mean field level:

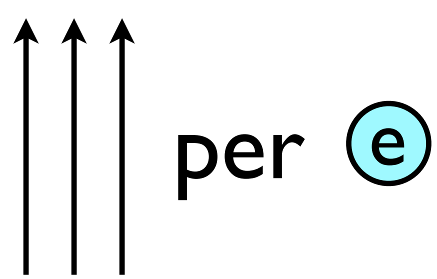
$$B_{\rm eff} = B - b = B - 2\pi p\, n$$

$$\nu_{\rm eff}^{-1} = \nu^{-1} - p$$

Composite fermion

$\nu = 1/3$ FQH





Composite fermion

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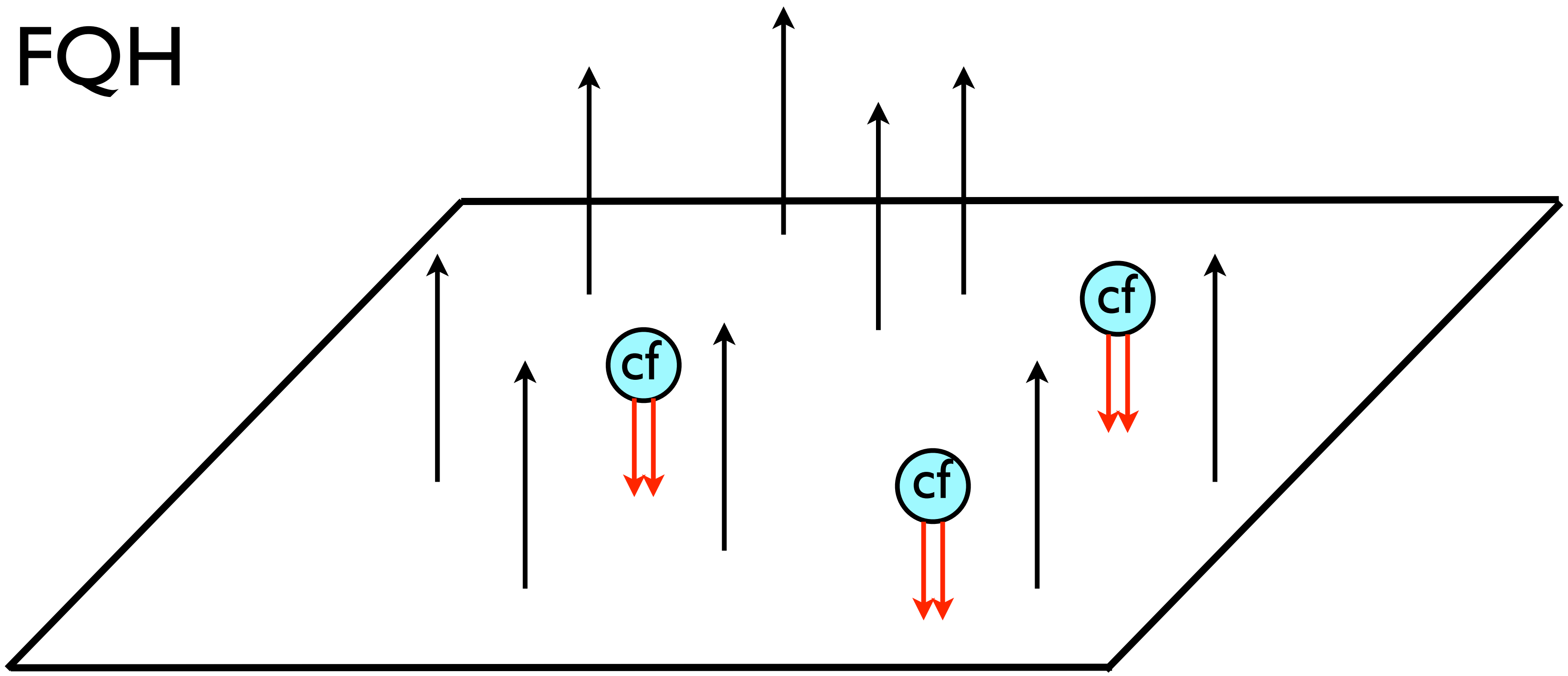
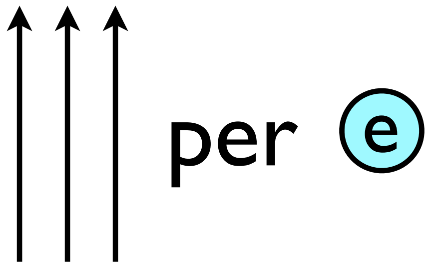
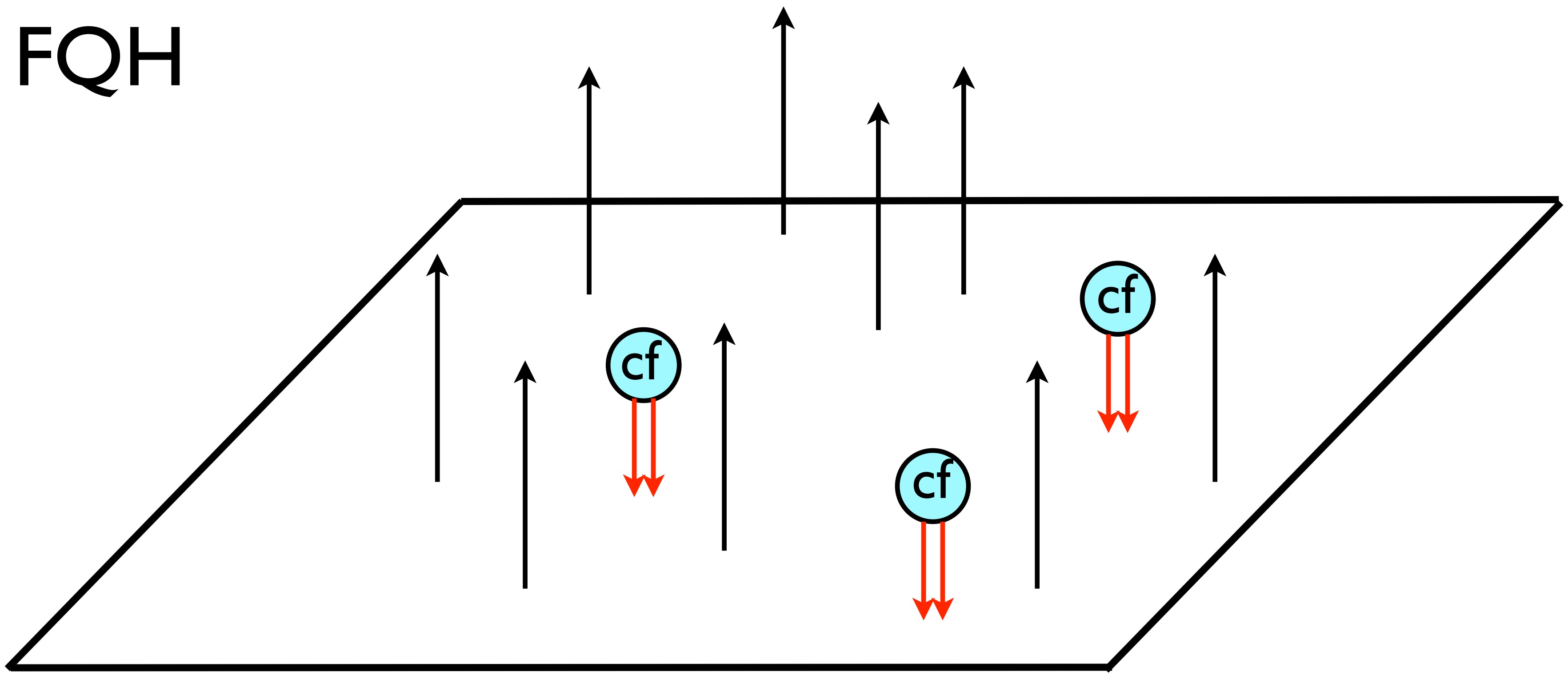




Composite fermion

$\nu = 1/3$ FQH



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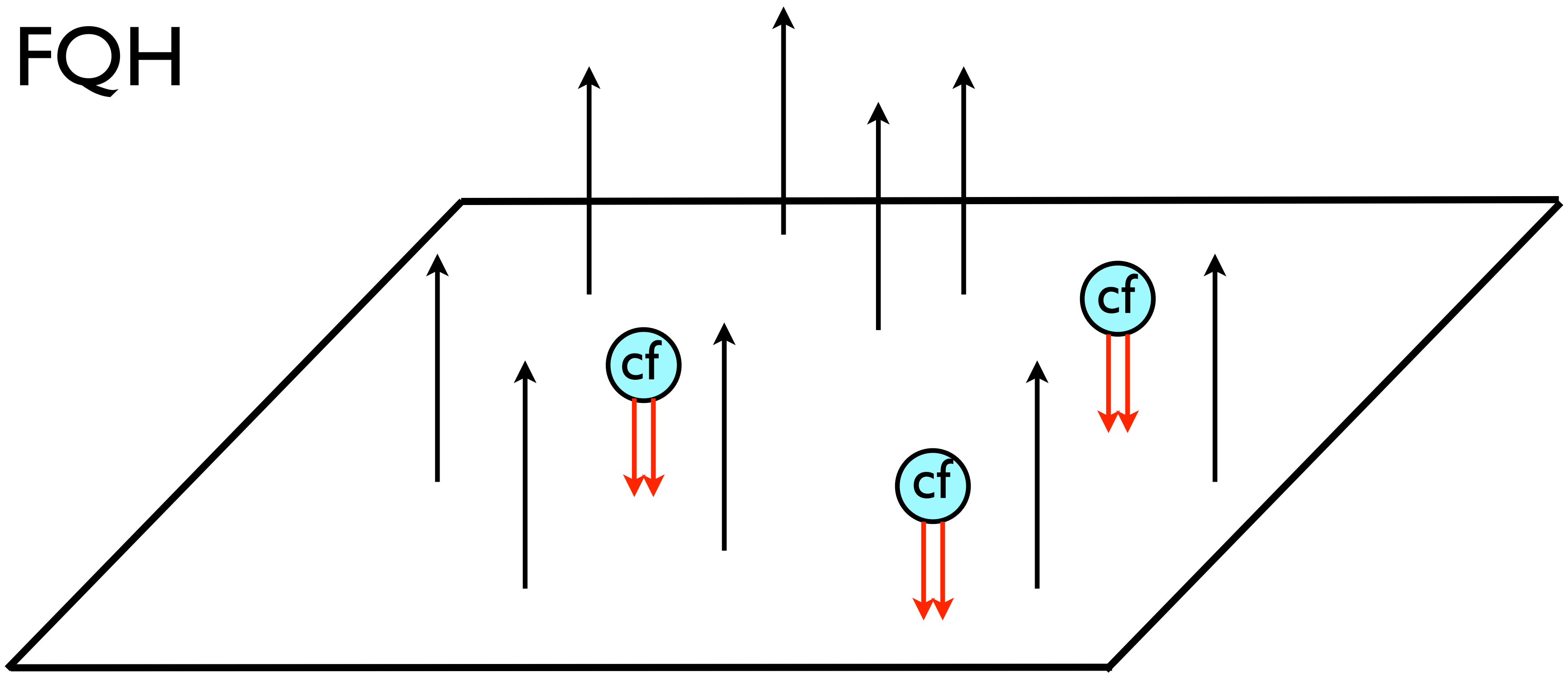
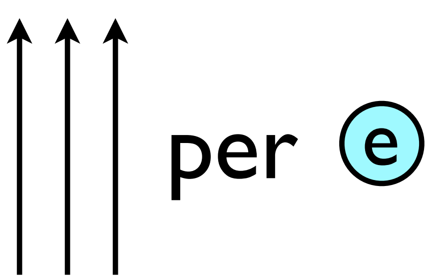
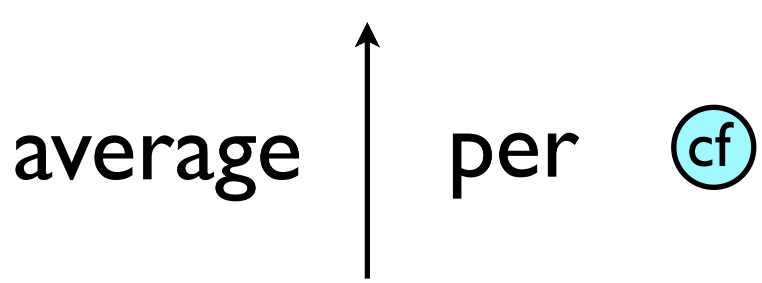
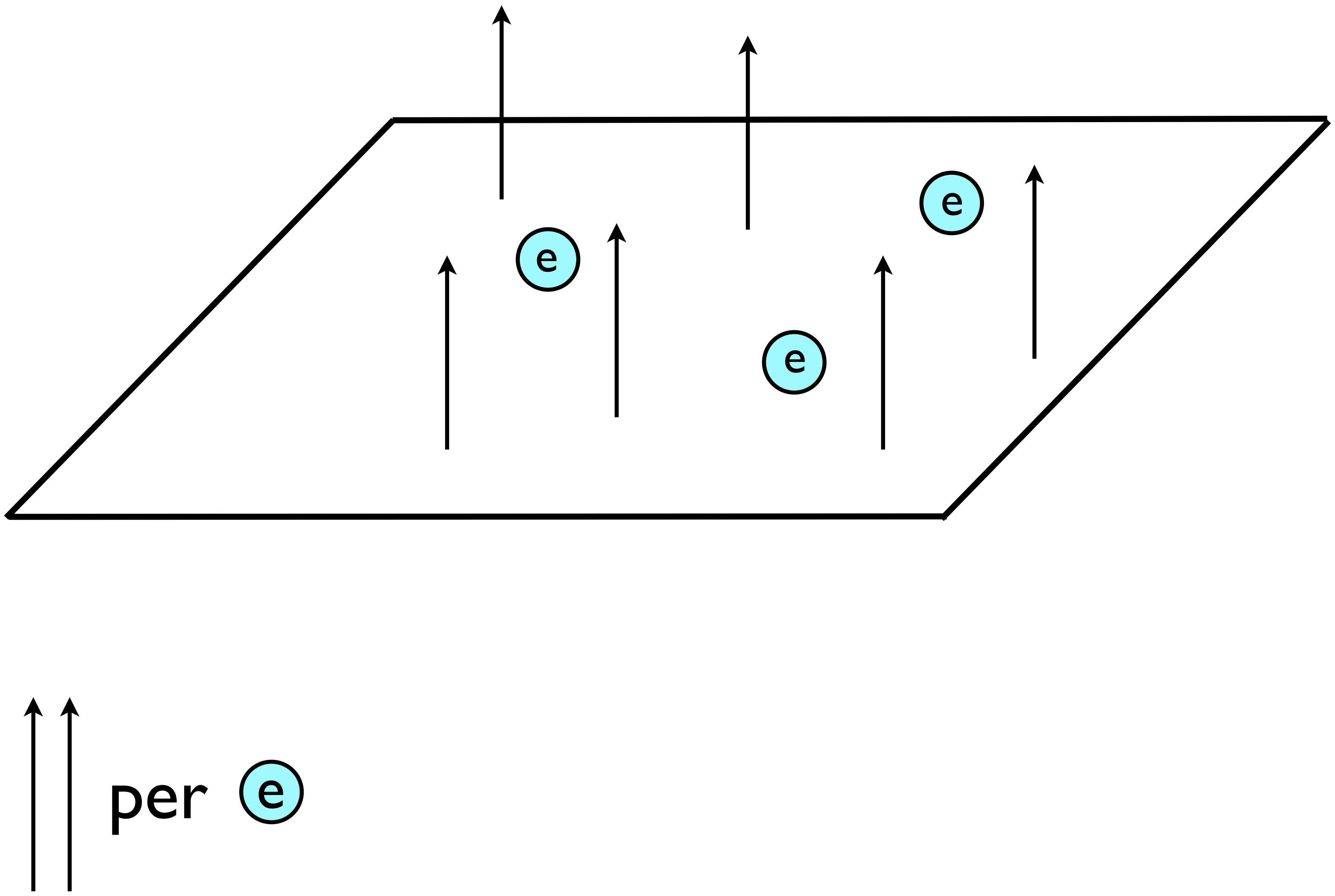




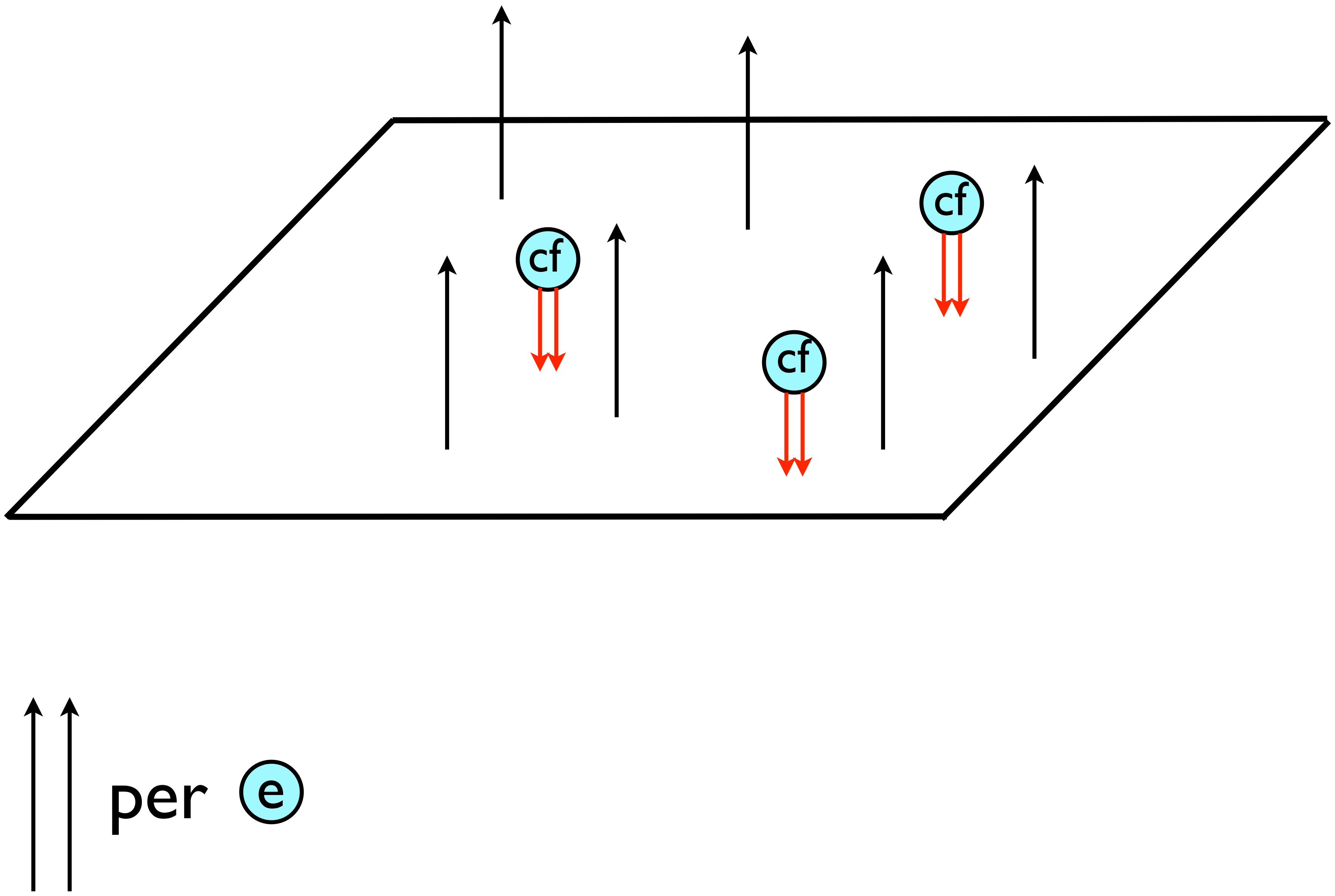


IQHE of CFs

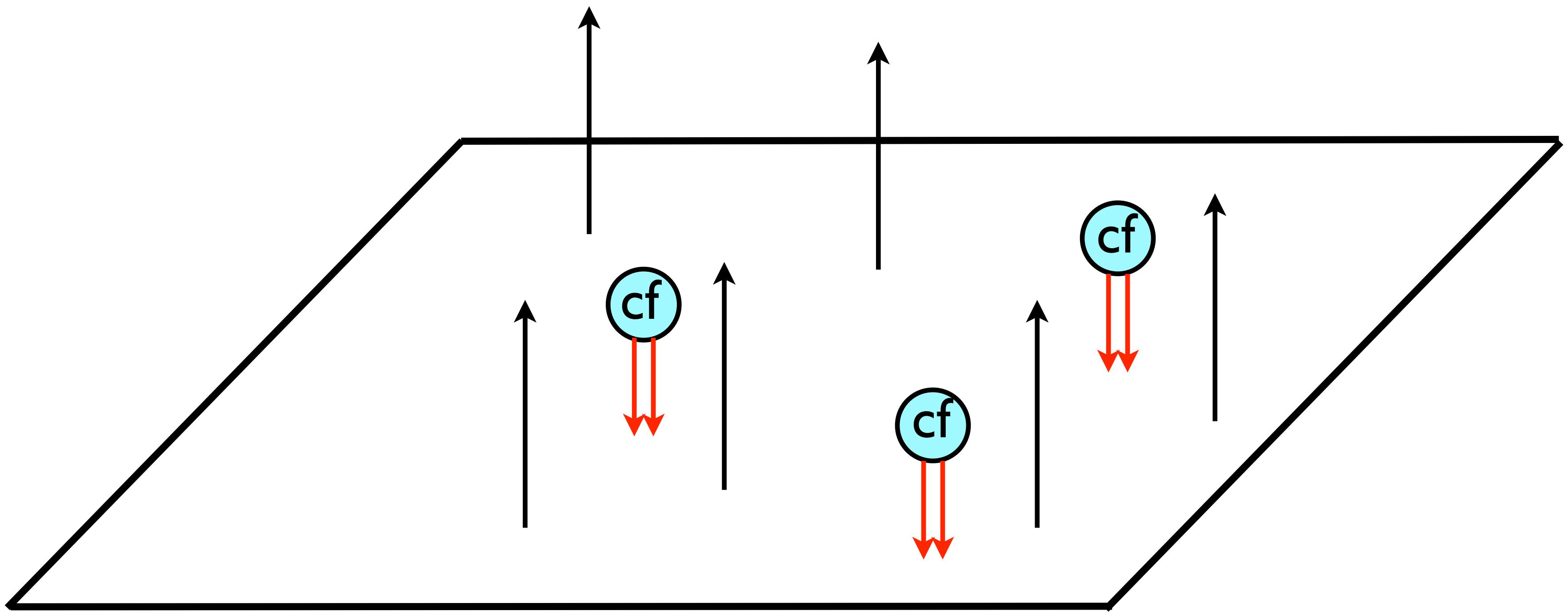
nu=1/2 state

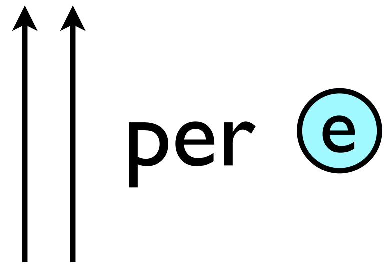


nu=1/2 state



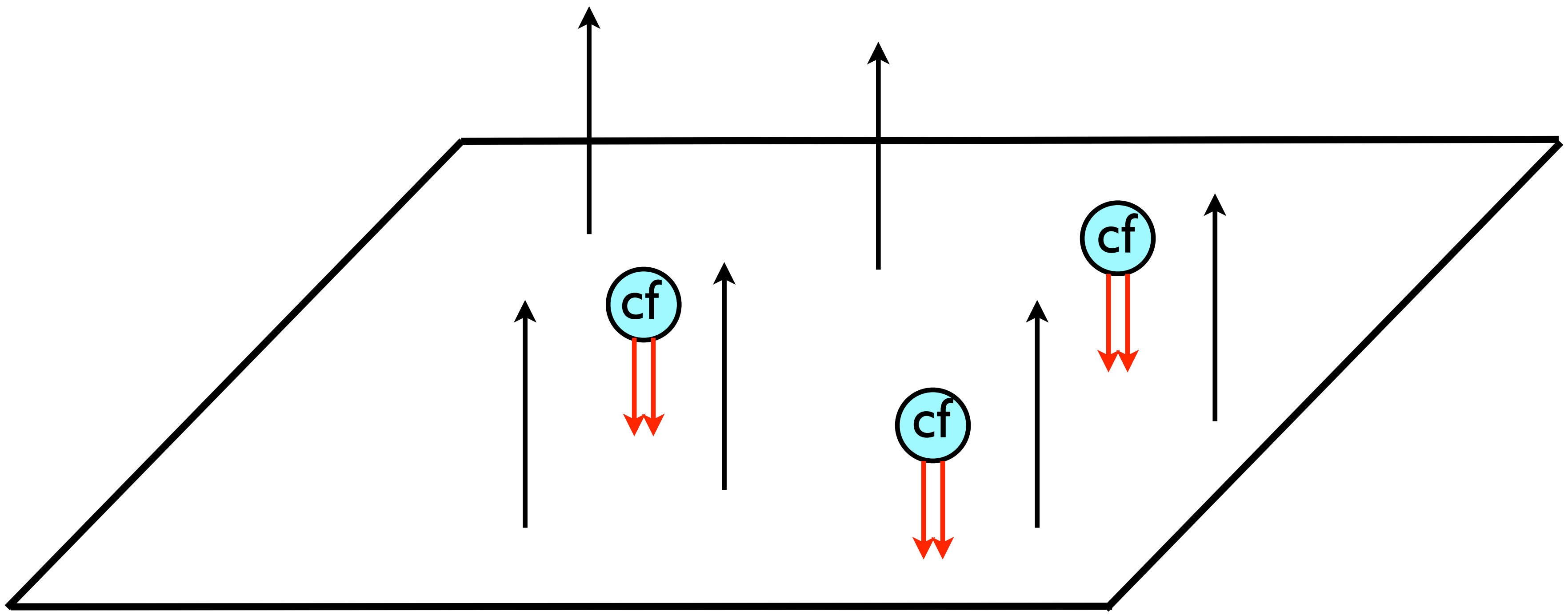
nu=1/2 state

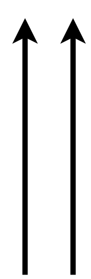




Zero B field for cf

nu=1/2 state





per e

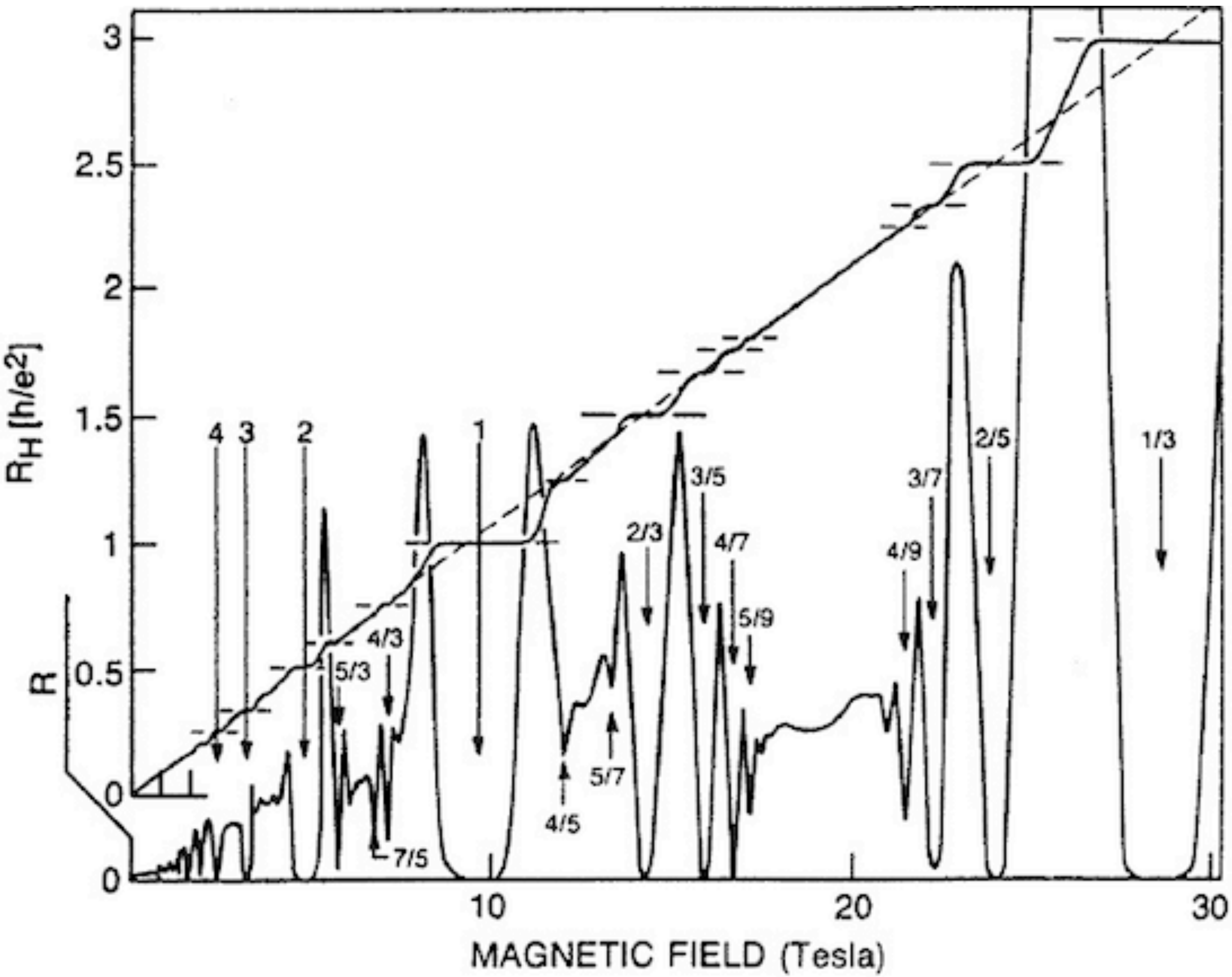
Zero B field for cf

CFs form a Fermi liquid

Fermi liquid of CFs

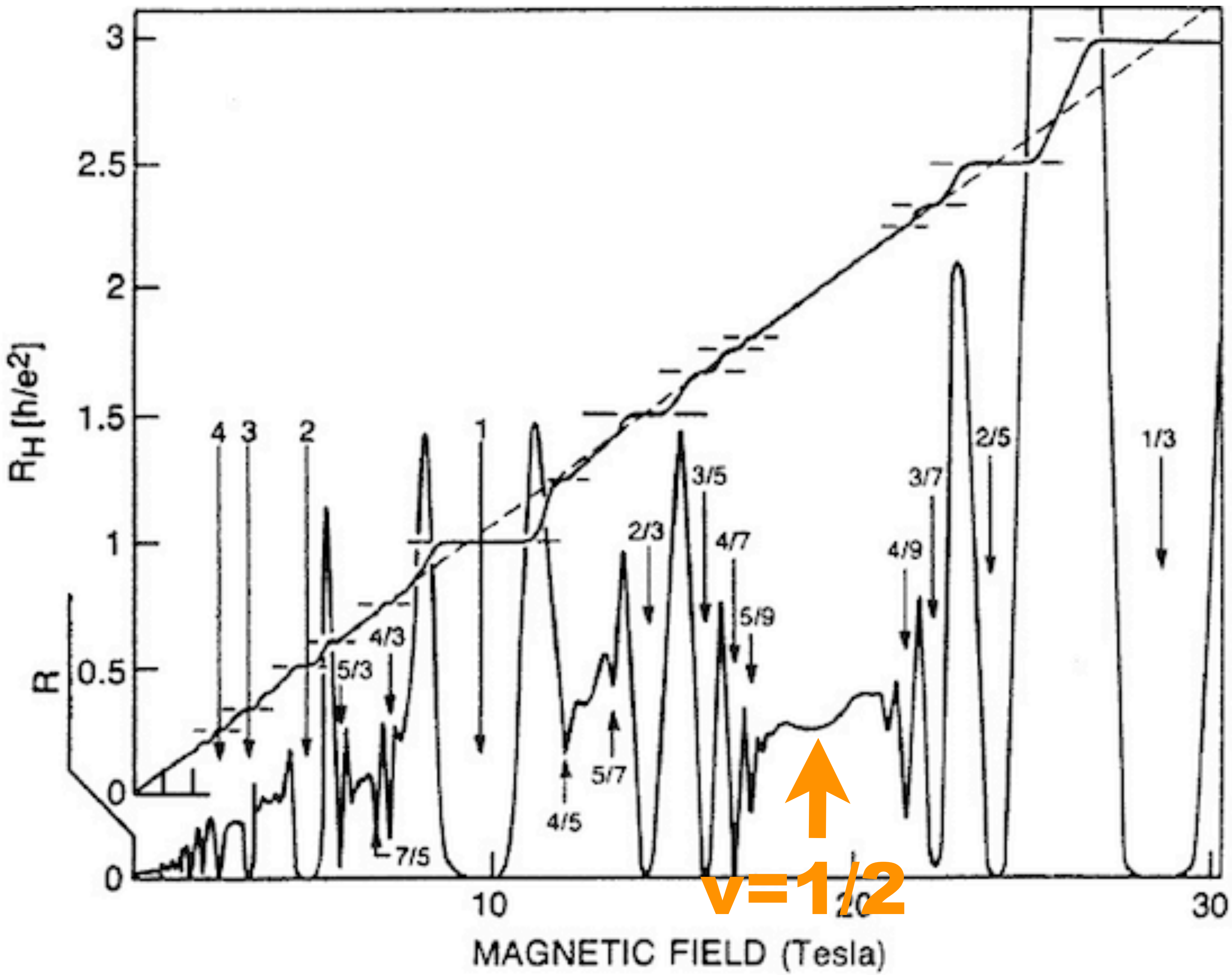
- The theory of the nu=1/2 state as a Fermi liquid of CFs was developed by Halperin, Lee, Read (HLR)
- No small expansion parameter: p~1
- Difficulty with energy scales in the limit m→0
- Nevertheless, abundant experimental evidence for a Fermi liquid behavior of the nu=1/2 state

nu=1/2 state



R_H [h/e²]
R
3
2.5
2
1.5
1
0.5
0
0
4
3
2
1
5/3
4/3
7/5
4/5
5/7
2/3
3/5
4/7
5/9
4/9
3/7
2/5
1/3
10
20
30
MAGNETIC FIELD (Tesla)

nu=1/2 state

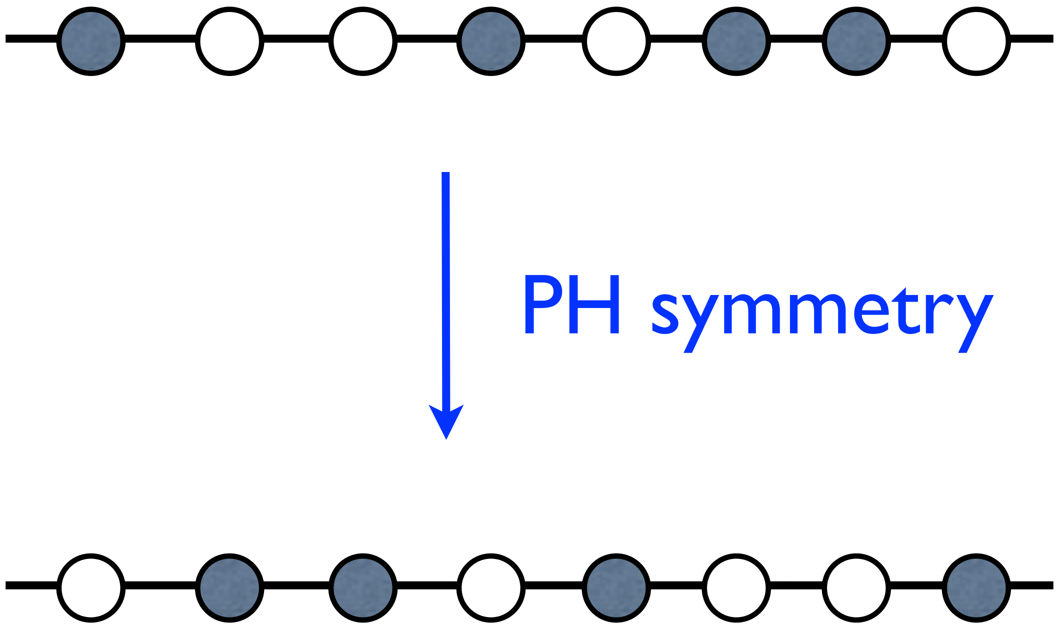


3
2.5
2
1.5
1
0.5
0
0
R
R_H [h/e²]
4
3
2
1
5/3
4/3
7/5
4/5
5/7
2/3
3/5
4/7
5/9
4/9
3/7
2/5
1/3
v=1/2
10
20
30
MAGNETIC FIELD (Tesla)

- Despite its success, the HLR theory suffers from a flaw: lack of particle-hole symmetry

Particle-hole symmetry

Girvin 1984

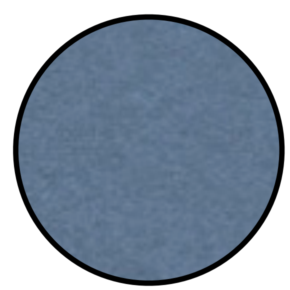


$$\nu \to 1 - \nu$$

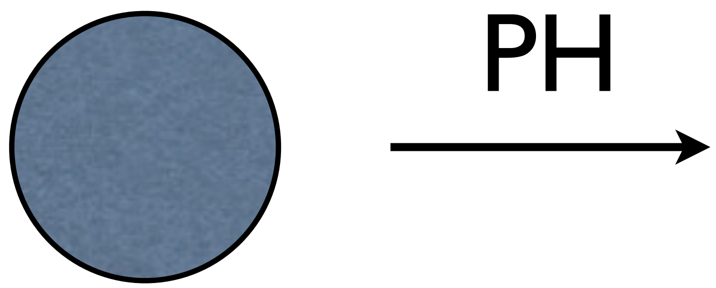
Can be formalized mathematically

exact symmetry the Hamiltonian on the LLL, when mixing of higher LLs negligible

PH symmetry of a Fermi liquid?



PH symmetry of a Fermi liquid?



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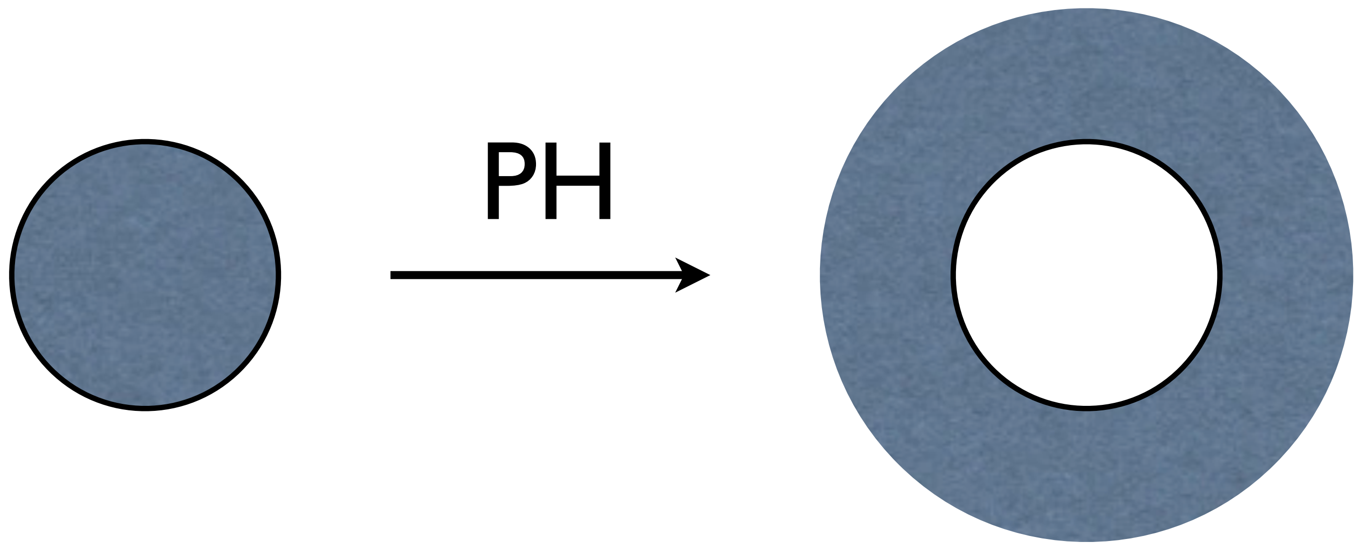
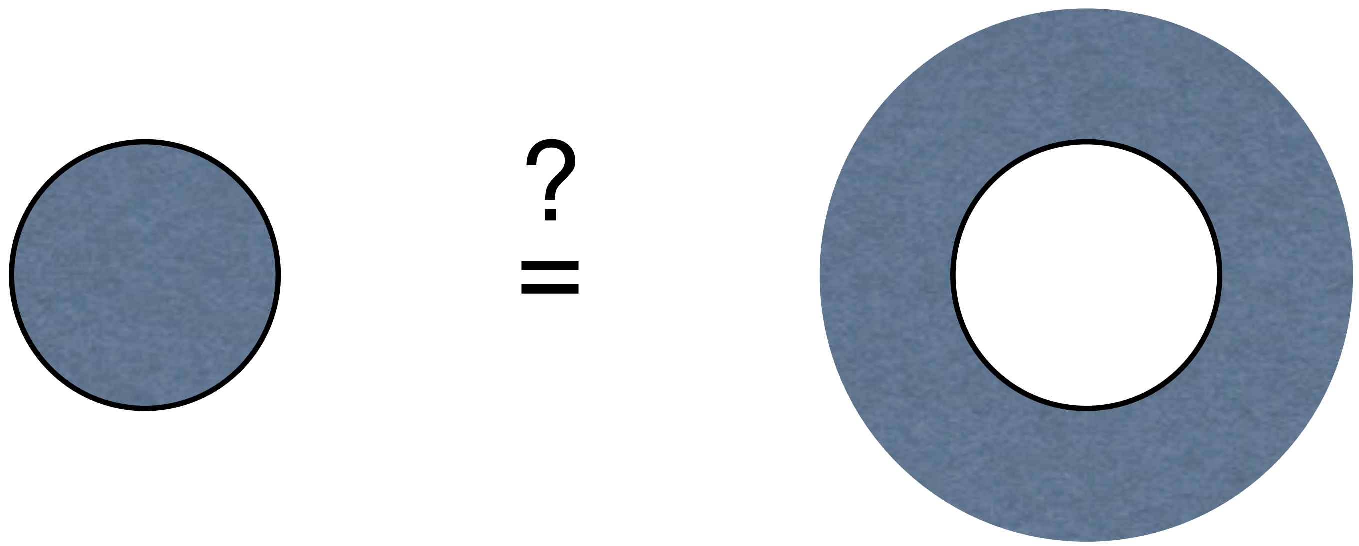
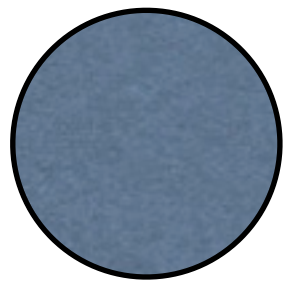


PH symmetry of a Fermi liquid?



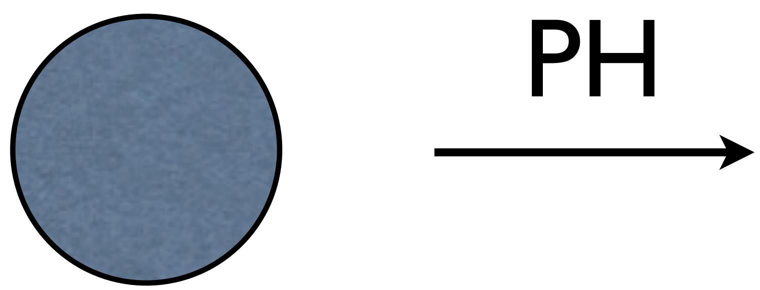
- The particle-hole asymmetry of the HLR theory has been noticed early on Kivelson et al 1997
- No conclusive resolution has emerged
- Maybe ground state at nu=1/2 breaks PH symmetry spontaneously? Barkeshli Mulligan Fisher 2015
- By now, numerical and experimental evidence: nu=1/2 state is particle-hole symmetric

The proposal



CF has Berry phase pi around the Fermi surface

The proposal



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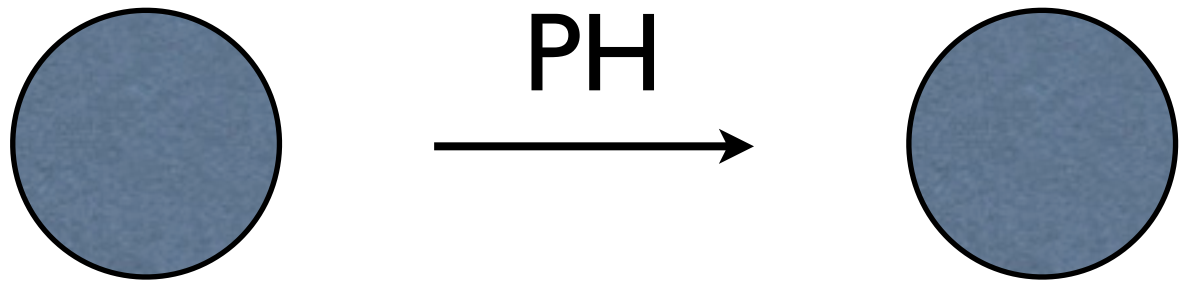
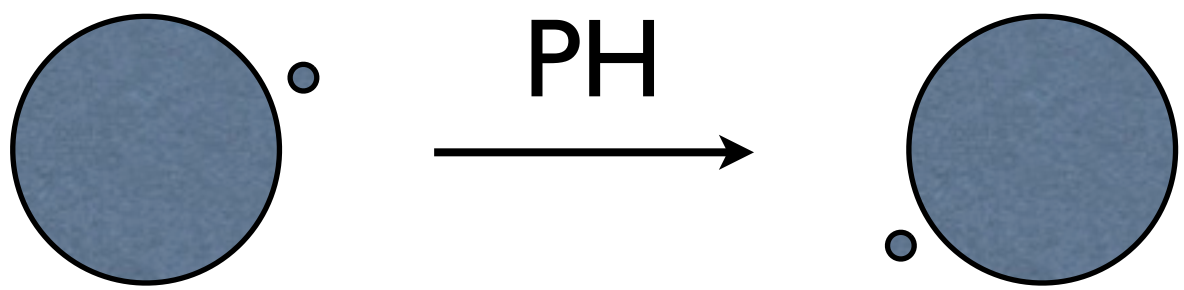
The proposal



CF has Berry phase pi around the Fermi surface

The proposal



CF has Berry phase pi around the Fermi surface

Berry phase in Fermi liquids

- Original Fermi liquid theory (Landau, 1956)

$$\varepsilon = \varepsilon_0(p) + \delta\varepsilon(\mathbf{p}),$$

и $\varepsilon_0(p)$ соответствует распределению $n_0(p)$
на с δn формулой вида (см. [1])

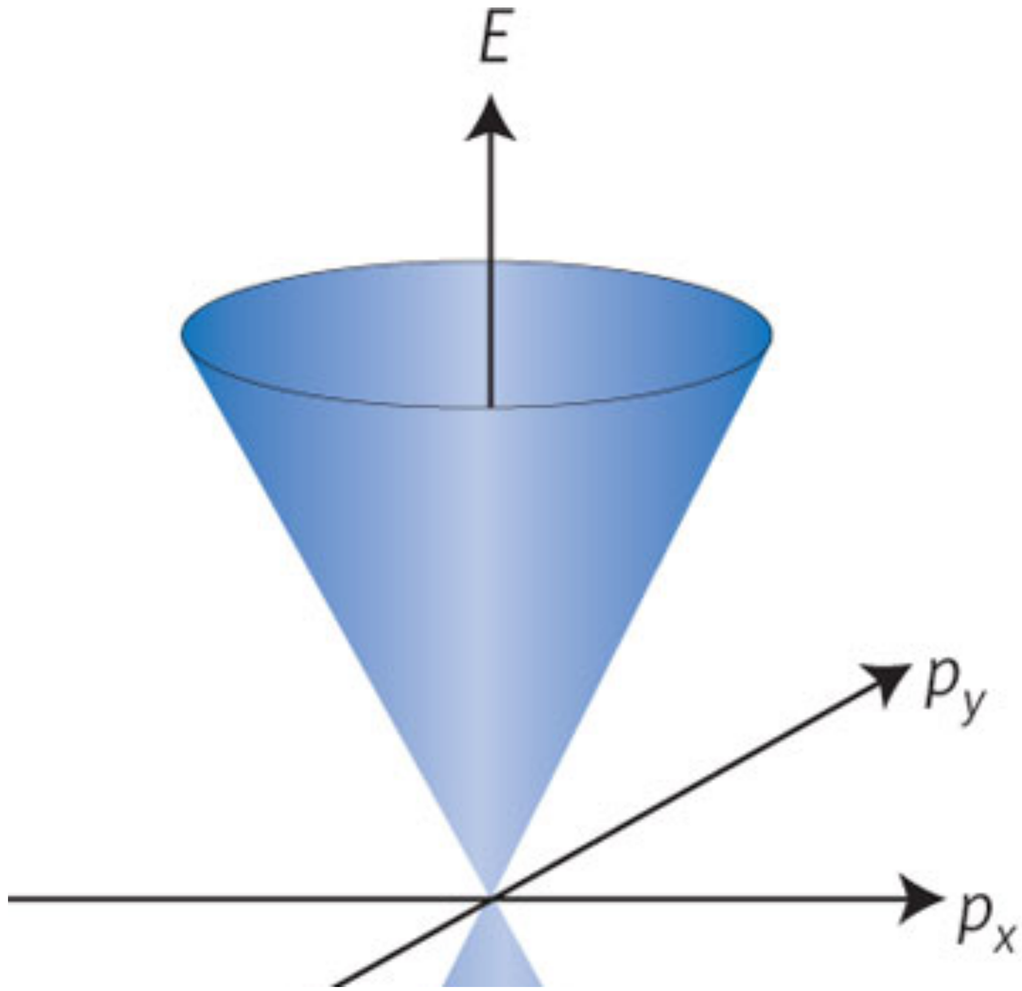
$$\delta\varepsilon(\mathbf{p}) = \mathrm{Sp}_{\sigma'} \int f(\mathbf{p}, \mathbf{p}')\, \delta n'\, d\tau', \quad d\tau = \frac{d^3\mathbf{p}}{(2\pi\hbar)^3}.$$

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}} \frac{\partial \varepsilon}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}} \frac{\partial \varepsilon}{\partial \mathbf{r}} = I(n),$$

Recent understanding: Landau's Fermi liquid theory has to be supplemented by the Berry phase of quasiparticles

Niu, Haldane, ...

Example: Dirac cone

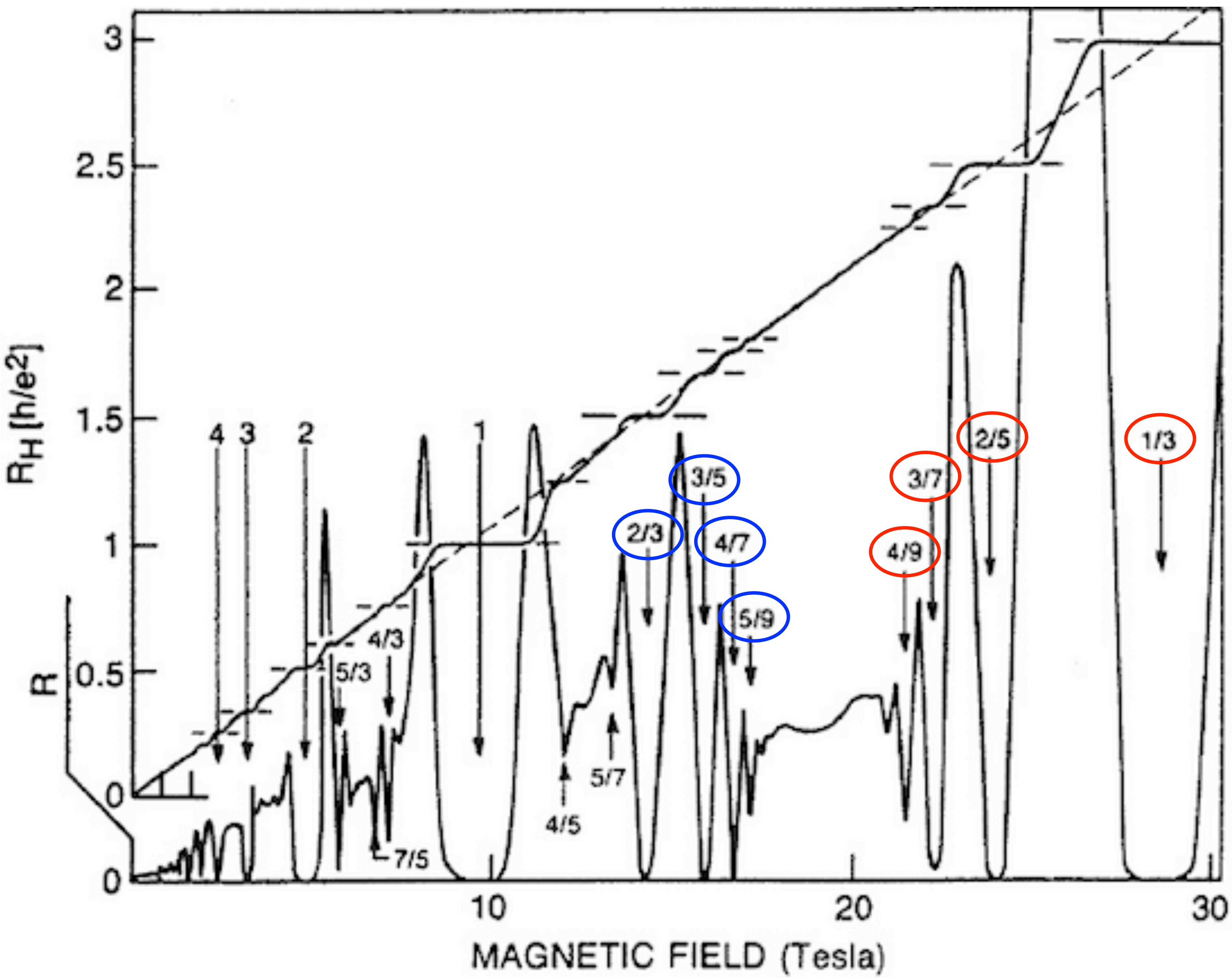


$$(\boldsymbol{\sigma}\cdot\mathbf{p})u_{\mathbf{p}} = |\mathbf{p}|u_{\mathbf{p}}$$

$$u_{\mathbf{p}}^{\dagger}\nabla_{\mathbf{p}}u_{\mathbf{p}} = i\mathbf{A}(\mathbf{p})$$

$$\oint \mathbf{A}\cdot d\mathbf{p} = \pi$$

Jain's sequences



$$\nu = \frac{n+1}{2n+1} \qquad \nu = \frac{n}{2n+1}$$

Standard flux attachment: $\nu_{\text{eff}}^{-1} = \nu^{-1} - p$

$$\nu = \frac{n}{2n+1} \qquad \nu_{\text{eff}} = n$$

$$\nu = \frac{n+1}{2n+1} \qquad \nu_{\text{eff}} = -(n+1)$$

In the new picture, these two fractions correspond to

$$\nu_{\text{CF}} = \pm\left(n + \frac{1}{2}\right)$$

IQHE in graphene

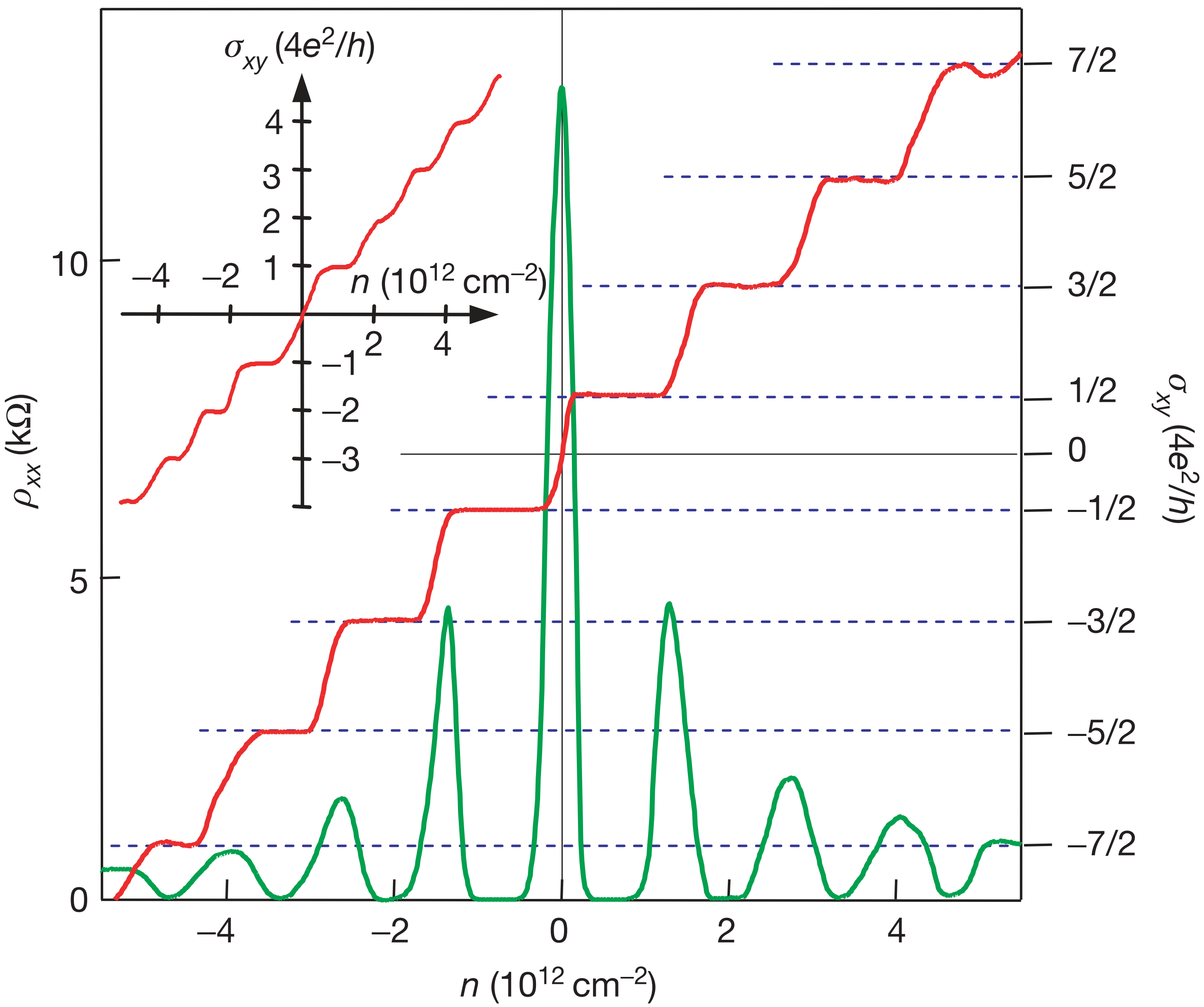


Figure 4 | QHE for massless Dirac fermions. Hall conductivity σ_{xy} and longitudinal resistivity ρ_{xx} of graphene as a function of their concentration at $B = 14\,\mathrm{T}$ and $T = 4\,\mathrm{K}$. $\sigma_{xy} \equiv (4e^2/h)\nu$ is calculated from the measured

$$\sigma_{xy} = \left(n + \frac{1}{2}\right)\frac{e^2}{2\pi\hbar}$$

Alternative to flux attachment

- Flux attachment breaks PH symmetry
- Alternative: fermionic particle-vortex duality

$$\mathcal{L}_A = i\bar{\Psi}\gamma^\mu(\partial_\mu - iA_\mu)\Psi$$

$$\mathcal{L}_B = i\bar{\psi}\gamma^\mu(\partial_\mu + 2ia_\mu)\psi + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda$$

Particle-vortex duality

DTS; Metlitski, Vishwanath; Senthil, Wang

original fermion	composite fermion
magnetic field	density
density	magnetic field

$$S = \int d^3x \left[i\bar{\psi}\gamma^\mu(\partial_\mu + 2ia_\mu)\psi + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda + \cdots \right]$$

$$j^\mu = \frac{\delta S}{\delta A_\mu} = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$$

$$\frac{\delta S}{\delta a_0} = 0 \longrightarrow \langle \psi\bar{\gamma}^0\psi \rangle = \frac{B}{4\pi}$$

Jain's sequences again

$$2\nu_B = \frac{1}{2\nu_A}$$

$$\nu_A = \nu - \frac{1}{2}$$

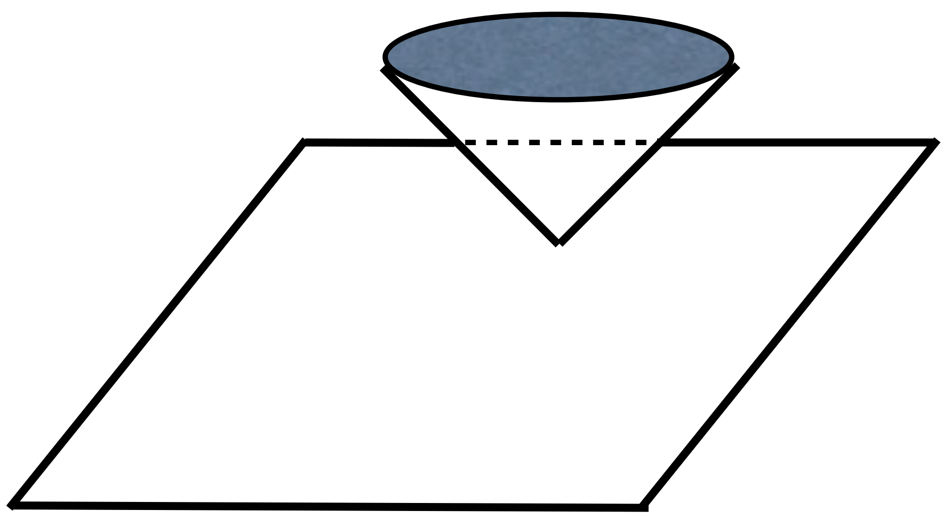
$$\nu = \frac{n}{2n+1} \rightarrow \nu_B = n + \frac{1}{2}$$

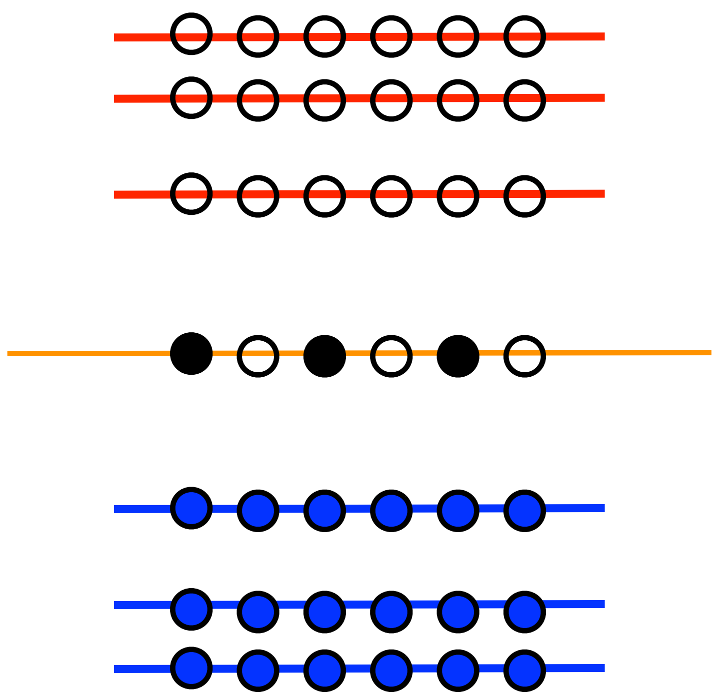
$$\nu = \frac{n+1}{2n+1} \rightarrow \nu_B = -\left(n + \frac{1}{2}\right)$$

Comments on particle-vortex duality

- Bosonic counterpart: duality between XY model and abelian Higgs model
  - strong numerical evidence
  - specific for d=3, N=1
- Fermionic particle-vortex duality: no numerical evidence (yet?) at zero B field
  - small N: chiral symmetry breaking in dual theory
  - strong interactions needed for original fermions?
  - magnetic field quenches kinetic energy,

Relativistic model with FQHE





$\mu = 0$

$$S = \int d^3x\, i\bar{\psi}\gamma^\mu(\partial_\mu - iA_\mu)\psi - \frac{1}{4e^2}\int d^4x\, F_{\mu\nu}^2$$

Low-energy description of ground state at zero chemical potential, finite B field

Consequences

- Exact particle hole symmetry in linear response
  - at $\nu = \frac{1}{2}$, $\sigma_{xy} = \frac{1}{2}$ exactly (HLR: ρ_{xy}=2)
- New particle-hole symmetric gapped nonabelian state at ν=1/2:

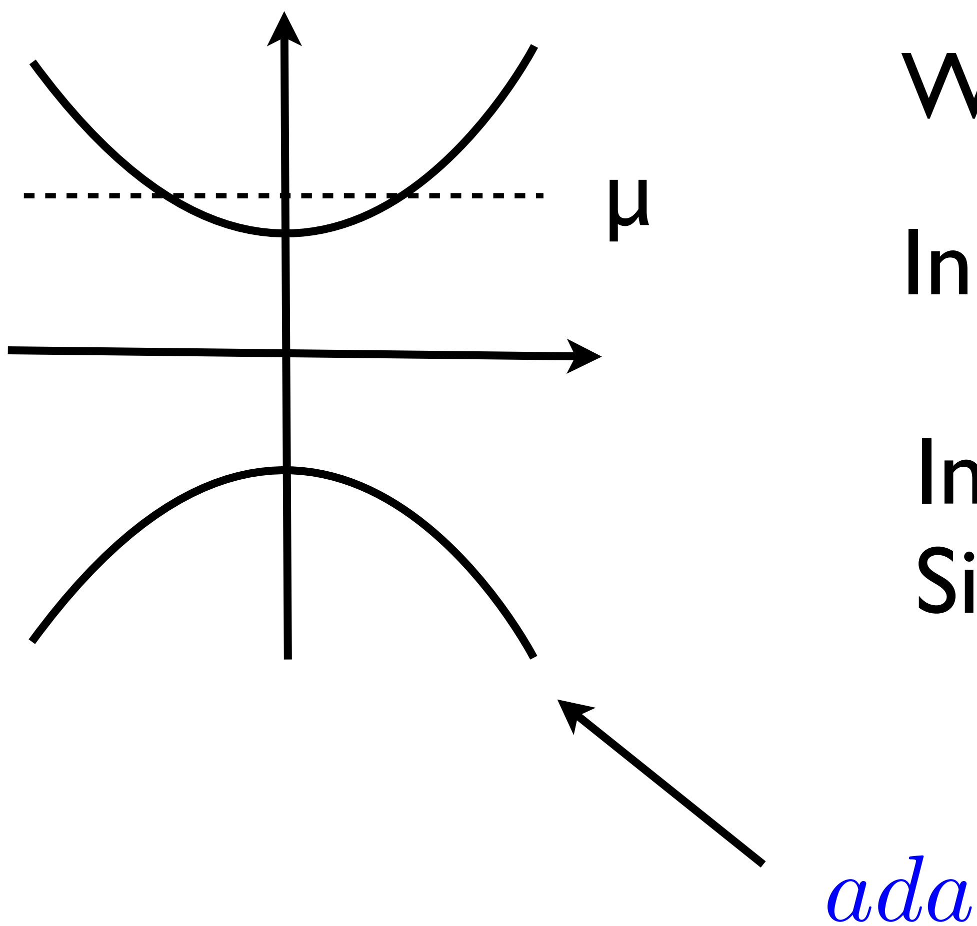
$$\langle \epsilon^{\alpha\beta} \psi_\alpha \psi_\beta \rangle \neq 0$$

Pfaffian and anti-Pfaffian states: pairing of Dirac CFs with angular momentum 2 and -2

Dirac composite fermions

- Emergent gauge field
- No Chern-Simons interaction *ada*
    - *ada* would break CP and CT
- Composite fermion without flux attachment
- composite fermions have Berry phase π around Fermi surface

HLR theory as the NR limit



When CP is broken, CF has mass

In the NR limit: NR action for CF

Integrating out Dirac sea: Chern-Simons interaction between CF

Standard HLR theory is reproduced
Particle-hole symmetry broken by the CF Dirac mass

Conclusion and open questions

- PH symmetry: a challenge for CF picture
- Proposal: Dirac CF with gauge, non-CS interaction
- particle-vortex duality instead of flux attachment
- experimentally verifiable consequences
- Open questions:
  - derivation of the effective theory
  - experimental measurement of the Berry phase: cold atoms?