3D Hydrodynamics and Quasi-2D Thermodynamics in Strongly Correlated Fermi Gases

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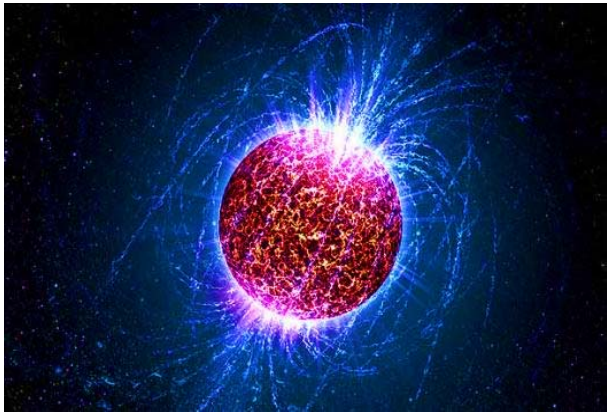
Outline

Topics

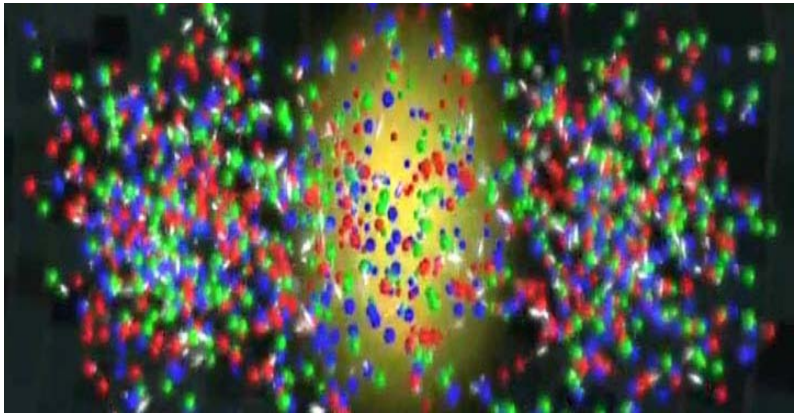
- *Scale Invariance in Expanding Fermi gases:*
  - *Ballistic* expansion of a *hydrodynamic* gas
- *Shear Viscosity measurement*
  - *Local* viscosity from trap-averaged viscosity
- *Spin-imbalanced Quasi-two-dimensional Fermi gas*
  - *Failure* of true 2D-BCS theory
  - 2D-polaron model of the thermodynamics
  - Phase-transition to a balanced core

Why Study Strongly Interacting Fermi Gases?

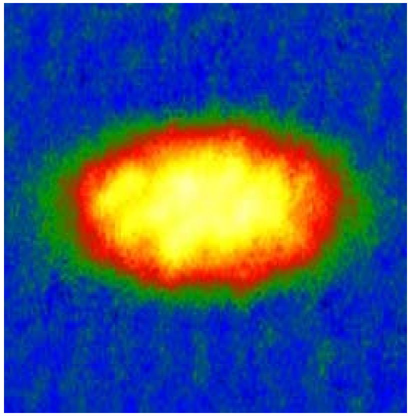
Strongly Interacting Fermionic Systems



Neutron Star



Quark Gluon Plasma



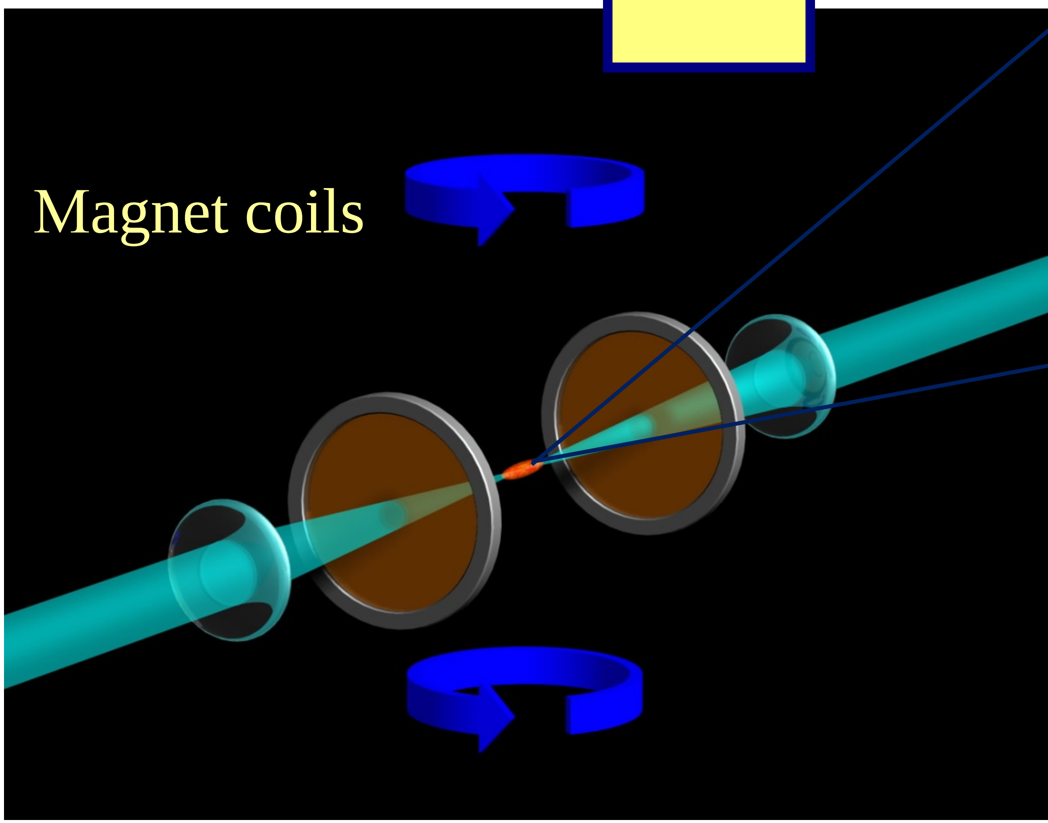
Ultra-Cold Fermi Gas

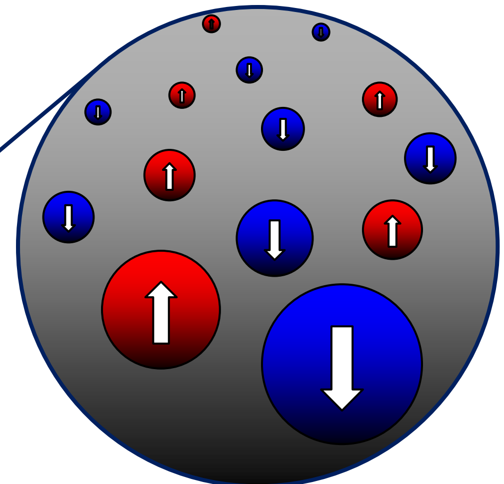
Layered High Temperature Superconductors

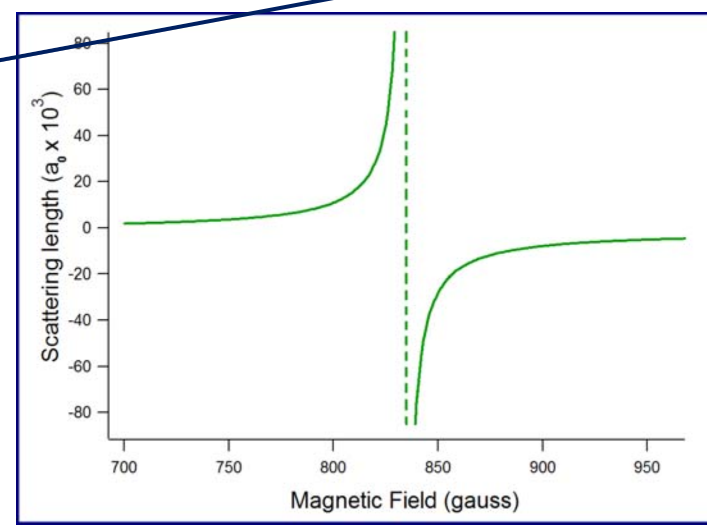
Optically Trapped Fermi gas

Our atom: Fermionic

^{6}Li

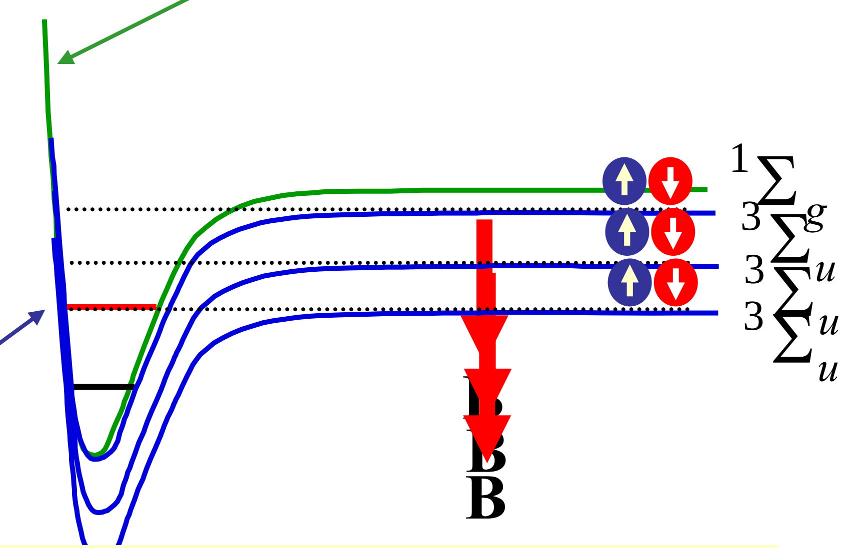






Feshbach Resonance

Singlet Diatomic Potential: Electron Spins Anti-Parallel



Triplet

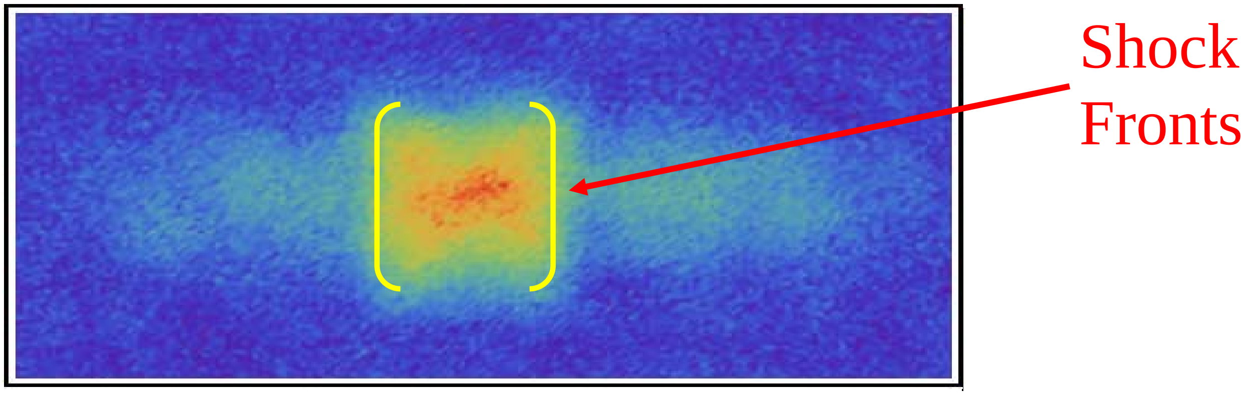
SCALE INVARIANT!

832 G-Resonant Scattering!

$$\sigma_{\text{coll}} = 4\pi \bar{\lambda}_{dB}^{2}$$

Strong Interactions: *Shock waves* in Fermi gases

- Trapped gas is divided into two clouds with a repulsive optical potential.
- The repulsive potential is extinguished, the two clouds accelerate towards each other and collide.

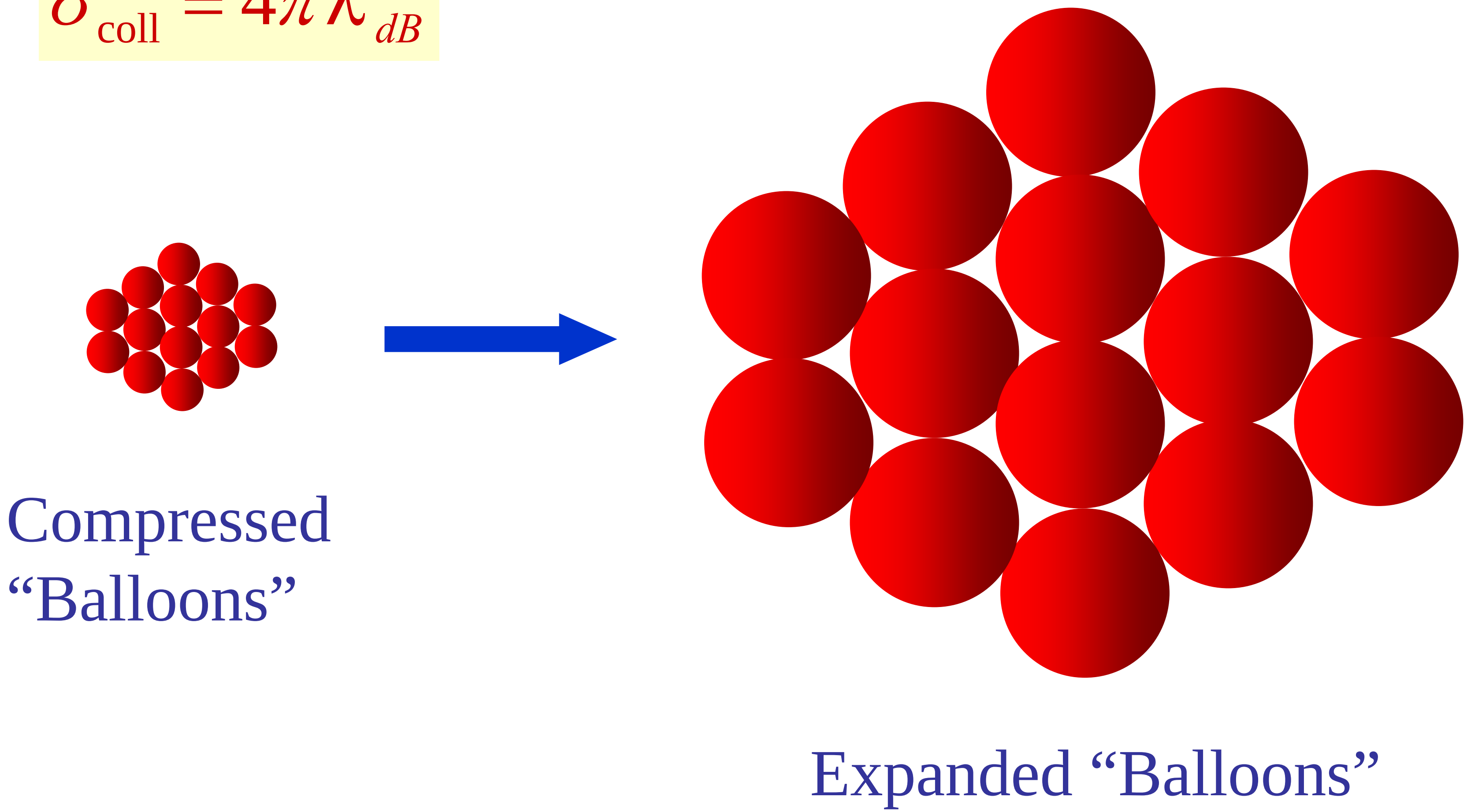


Really strong interactions!

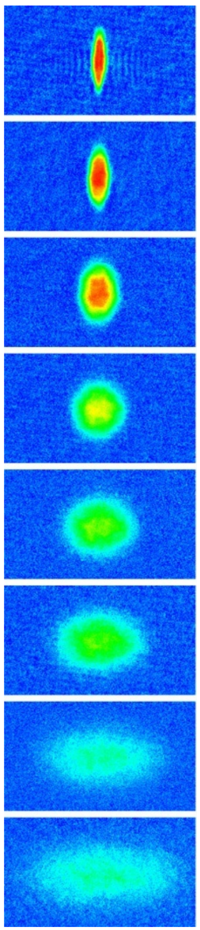
Scale Invariance

in Expanding *Resonant* Fermi Gases

$$\sigma_{\text{coll}} = 4\pi \lambdabar_{dB}^{2}$$



Scale-invariance: Connecting *Strongly* to *Weakly* Interacting



- Anti-de Sitter-Conformal Field Theory Correspondence: Connects strongly interacting fields in 4-dimensions to weakly interacting gravity in 5-dimensions.

- Can we connect elliptic flow of a resonant gas to the ballistic flow of an *ideal* gas in 3D?

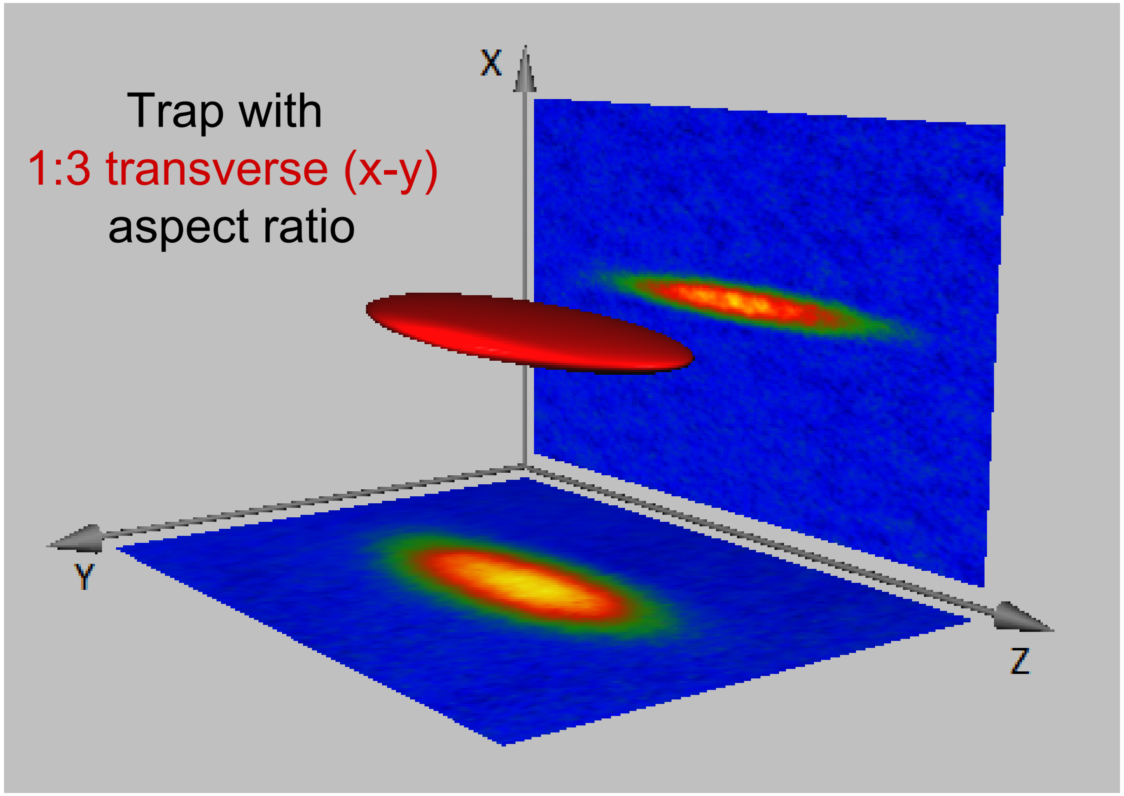
For both, the pressure is 2/3 of the energy density:

$$\boxed{\Delta p \equiv p - \frac{2}{3}\varepsilon = 0}$$

Scale Invariant?

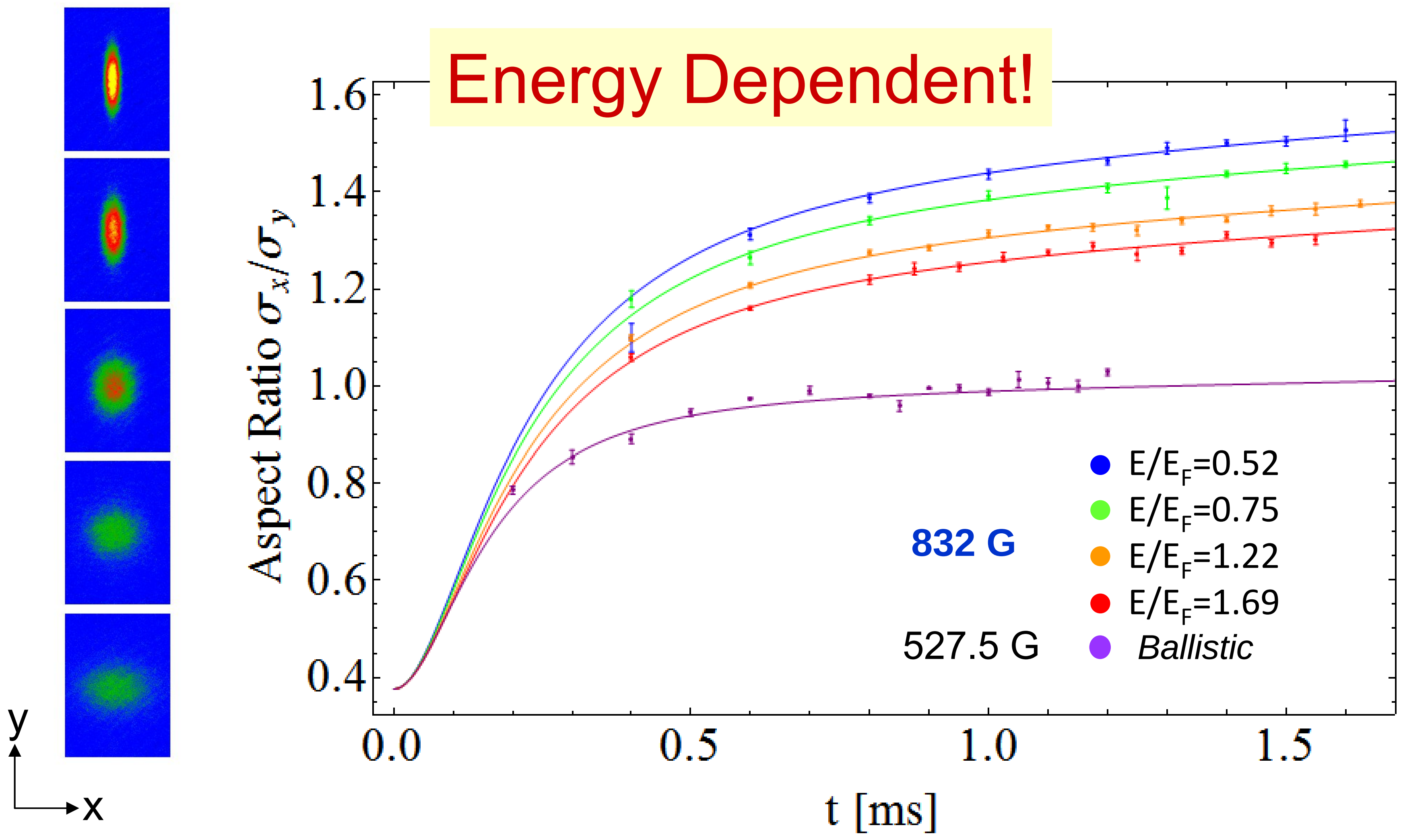
<u>Elliptic Flow</u>: Observe 2 dimensions + time

Measuring expanding clouds in 3D



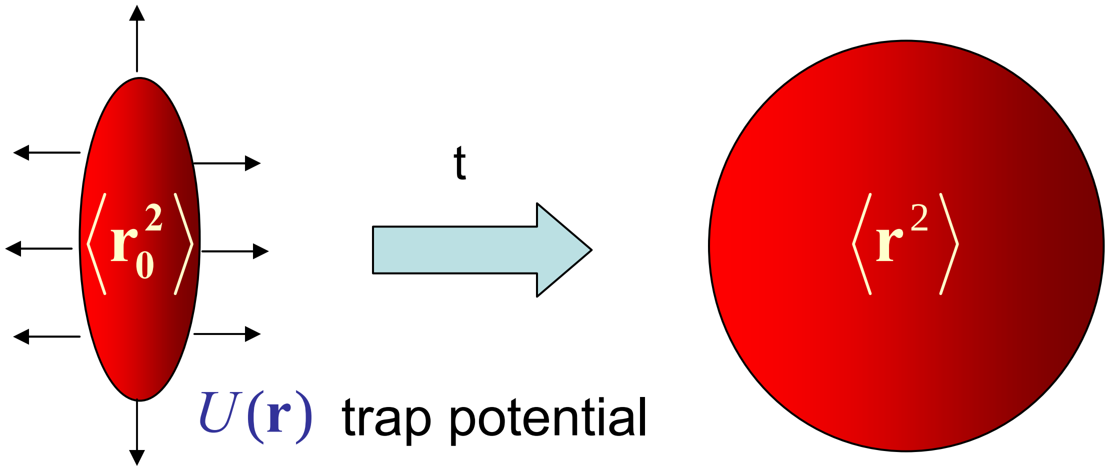
- Measure *all three* cloud radii using two cameras.

Transverse *Aspect Ratio* versus Time for a Unitary Fermi gas



Scale Invariance: Ideal Gas

<u>Ideal gas:</u> $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ Ballistic flow



$$\langle \mathbf{r}^2 \rangle = \langle \mathbf{r}^2 \rangle_0 + t^2 \langle \mathbf{v}^2 \rangle_0$$

Cloud average

How does the *mean square radius* evolve in time? $\langle \mathbf{r}^2 \rangle = \langle x^2 + y^2 + z^2 \rangle$

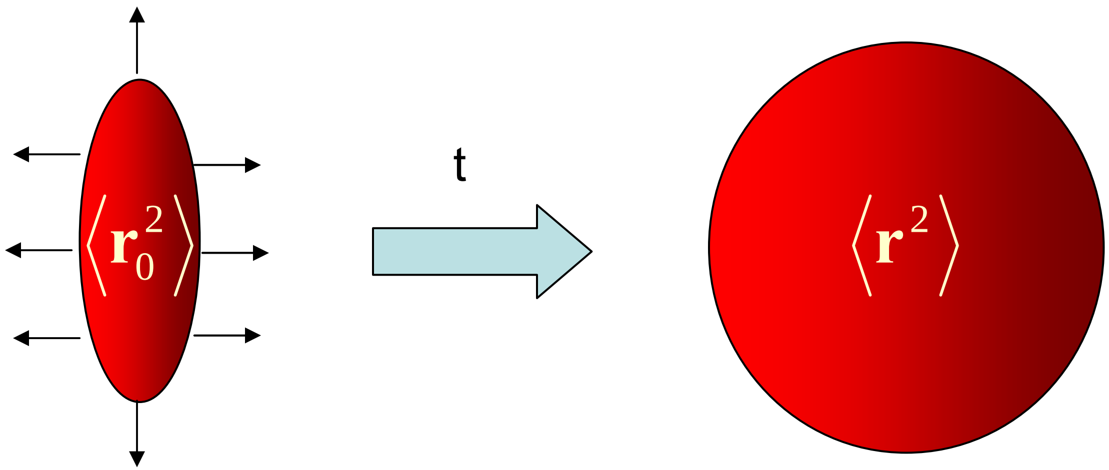
Virial Theorem:

$$m\langle \mathbf{v}^2 \rangle_0 = \langle \mathbf{r} \cdot \nabla \mathbf{U} \rangle_0$$

$$\boxed{\langle \mathbf{r}^2 \rangle = \langle \mathbf{r}^2 \rangle_0 + \frac{t^2}{m} \langle \mathbf{r} \cdot \nabla \mathbf{U} \rangle_0}$$

Ballistic Flow

Defining Scale Invariant Flow



$$\langle \mathbf{r}^2 \rangle = \langle x^2 + y^2 + z^2 \rangle$$

Cloud average

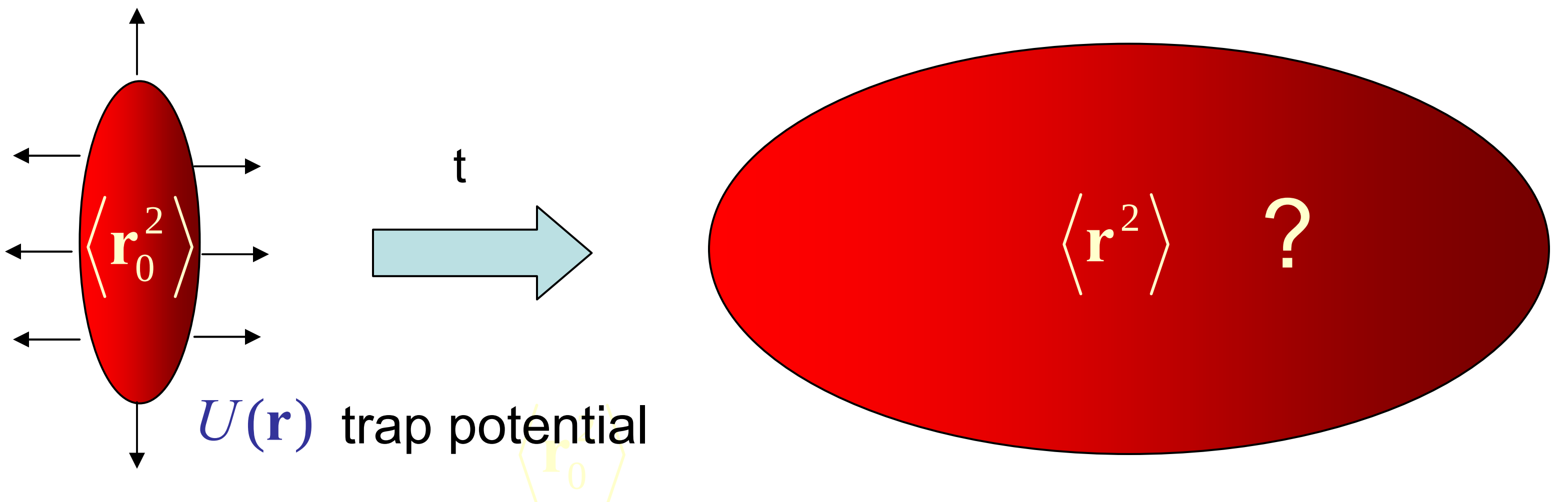
$$\boxed{\frac{m\left(\langle \mathbf{r}^2 \rangle - \langle \mathbf{r}^2 \rangle_0\right)}{\langle \mathbf{r} \cdot \nabla \mathbf{U} \rangle_0} = t^2}$$

t = expansion time

Defines Scale Invariant Flow!

Scale Invariance: Resonant Gas

Hydrodynamic gas: Elliptic flow



How does the *mean square radius* evolve in time? $\langle \mathbf{r}^2 \rangle = \langle x^2 + y^2 + z^2 \rangle$

Hydrodynamic Expansion

From the Navier-Stokes and continuity equations, it is easy to show that a single component fluid obeys:

Trap potential Pressure

$$\frac{d^2}{dt^2}\frac{m\langle x_i^2\rangle}{2} = m\langle \mathrm{v}_i^2\rangle - \langle x_i\partial_i U\rangle + \frac{1}{N}\int d^3\mathbf{r}\left(p - \eta_S\sigma_{ii} - \zeta_B \nabla\cdot\mathbf{v}\right)$$

Stream KE Shear and Bulk Viscosity

<u>Equilibrium:</u>

$$\frac{3}{N}\int d^3\mathbf{r}\; p_0 = \langle \mathbf{r}\cdot\nabla U\rangle_0$$

Measured from the cloud profile and trap parameters

Need to find the volume integral of the pressure:

$$\boxed{p = \frac{2}{3}\varepsilon + \Delta p}$$

Global Energy Conservation

Just after the optical trap is abruptly extinguished*: t = 0^+ :

Stream KE (t)

$$\boxed{\frac{1}{N}\int d^3\mathbf{r}\,\mathcal{E} + \frac{m}{2}\left\langle \mathbf{v}^2 \right\rangle = \frac{1}{N}\int d^3\mathbf{r}\,\mathcal{E}_0}$$

Internal Energy (t)

Internal Energy (t = 0^+)

Scale Invariant Expansion

Using the hydrodynamic equations and energy conservation it is easy to show that

$$\boxed{\frac{d^2}{dt^2}\frac{m\langle \mathbf{r}^2\rangle}{2} = \langle \mathbf{r}\cdot\nabla\mathbf{U}\rangle_0 + \frac{3}{N}\int d^3\mathbf{r}\,(\Delta\mathbf{p}-\Delta\mathbf{p}_0) - \frac{3}{N}\int d^3\mathbf{r}\,\zeta_B \nabla\cdot\mathbf{v}}$$

Initial trap potential

Conformal symmetry breaking Δp

Bulk viscosity

$$\boxed{\Delta p \equiv p - \tfrac{2}{3}\varepsilon}$$

Scale Invariance!

Resonant gas $\Delta p \equiv p - \frac{2}{3}\varepsilon = 0$

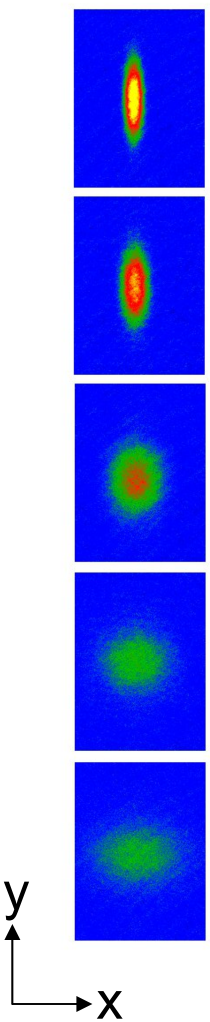
The bulk viscosity also vanishes so

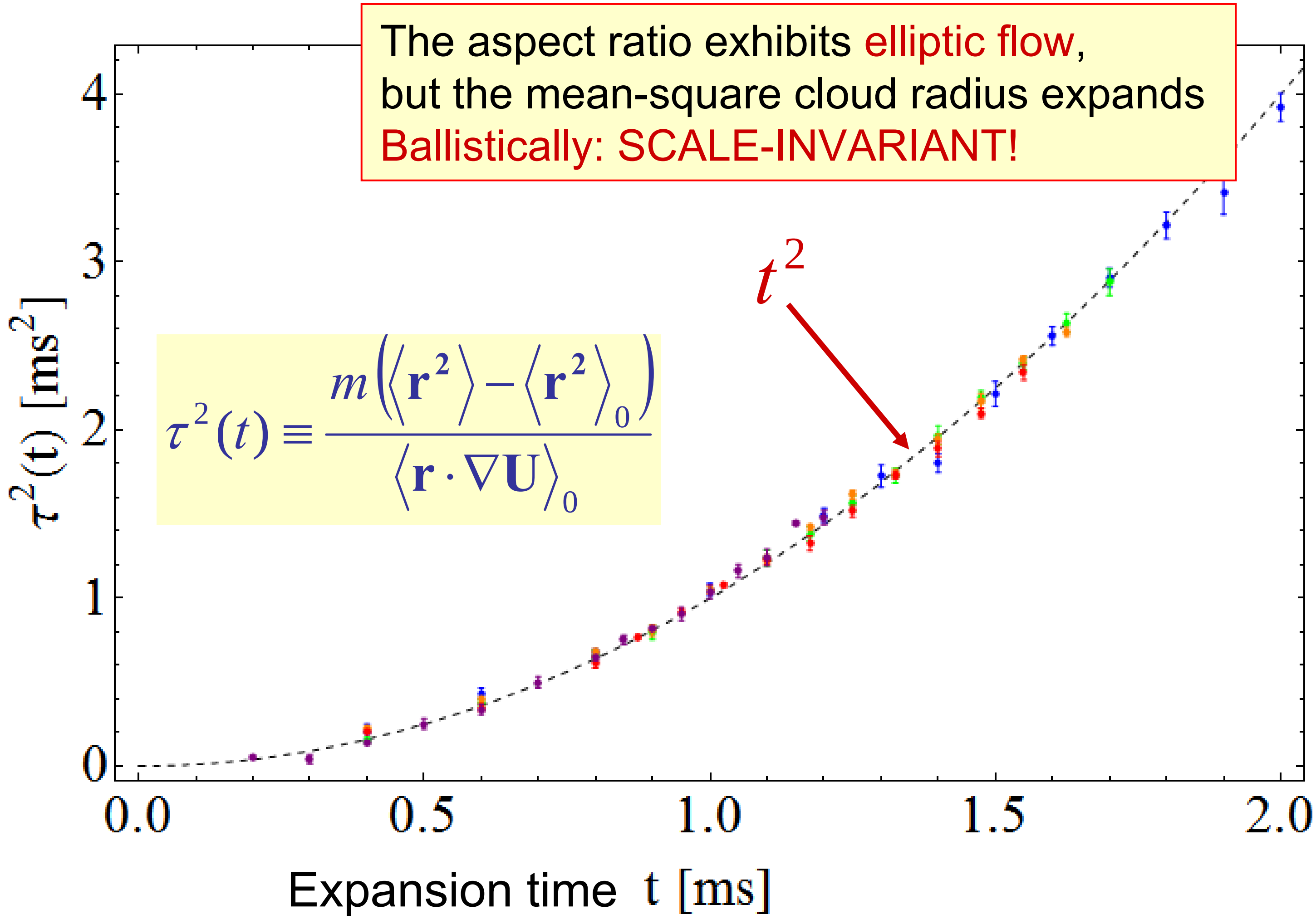
$$\boxed{\left\langle \mathbf{r}^2 \right\rangle = \left\langle \mathbf{r}^2 \right\rangle_0 + \frac{t^2}{m}\left\langle \mathbf{r}\cdot\nabla\mathbf{U} \right\rangle_0}$$

Ballistic Flow!

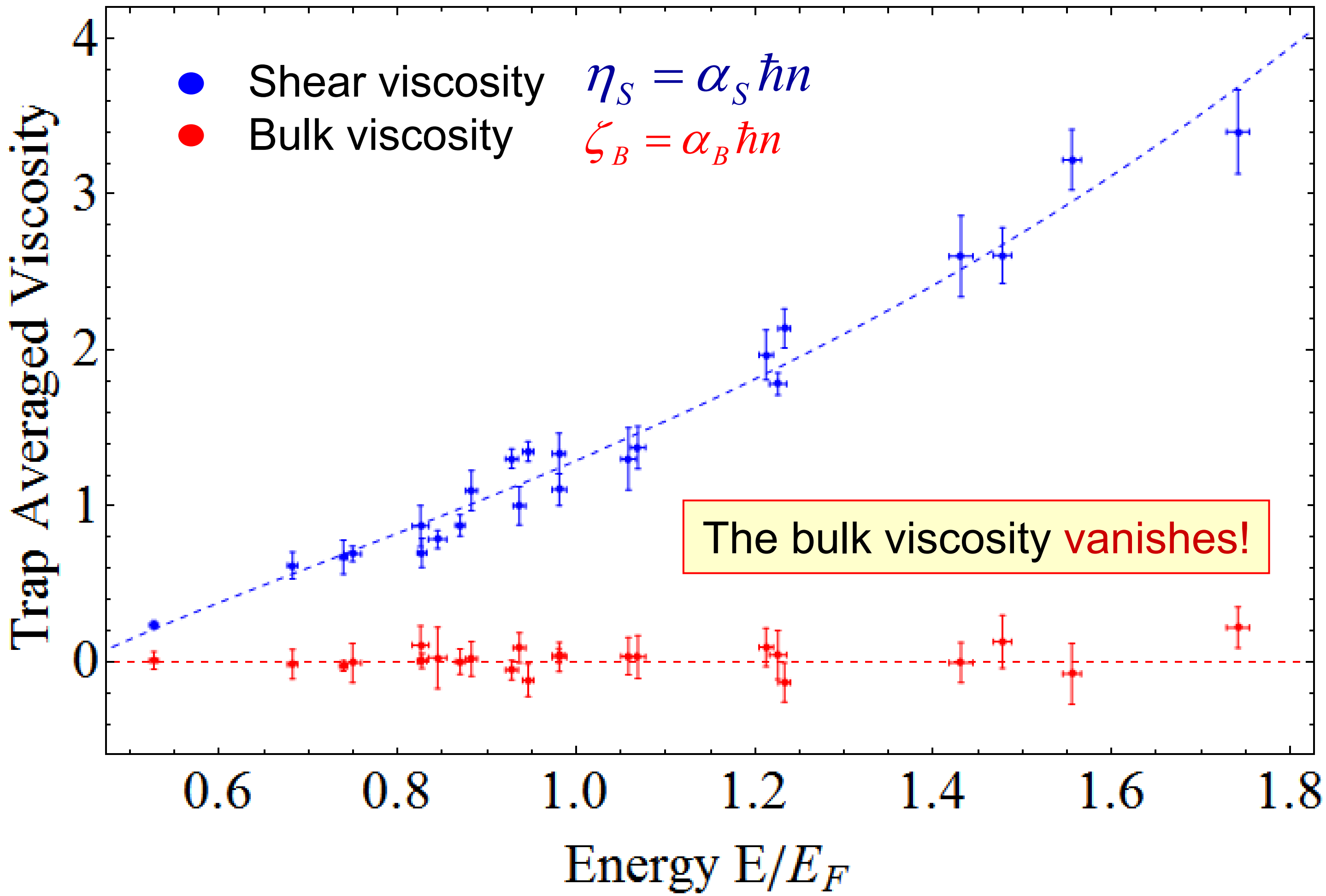
Can we observe *ballistic* flow of an *elliptically* expanding gas?

Scale-invariant "Ballistic" Expansion of a Resonant Fermi gas





Shear and Bulk Viscosity: Unitary Fermi Gas



Quantum Viscosity

Viscosity: $\eta = \alpha \hbar n$ n = density (particles/cc)

dimensionless shear viscosity coefficient

Water:	n = 3.3 x 10^{22}	$\eta = 300\ \hbar n$
Air:	n = 2.7 x 10^{19}	$\eta = 6000\ \hbar n$
Fermi gas:	n = 3.0 x 10^{13}	$\eta = 0.4\ \hbar n$
Nuclear Matter:	n = 3.0 x 10^{38}	$\eta = ?\ \hbar n$

Measuring Shear Viscosity: Scaling Approximation

$$n(x,y,x,t)=\frac{n_0(x/b_x,y/b_y,z/b_z)}{\Gamma}$$

$$\Gamma=b_xb_yb_z$$

Volume scale factor

$$\mathrm{v}_i=x_i\dot{b}_i/b_i$$

Velocity field is linear in the spatial coordinates

$$\langle x_i^2\rangle=\langle x_i^2\rangle_0 b_i^2(t)$$

$$\langle \mathrm{v}_i^2\rangle=\langle x_i^2\rangle_0 \dot{b}_i^2(t)$$

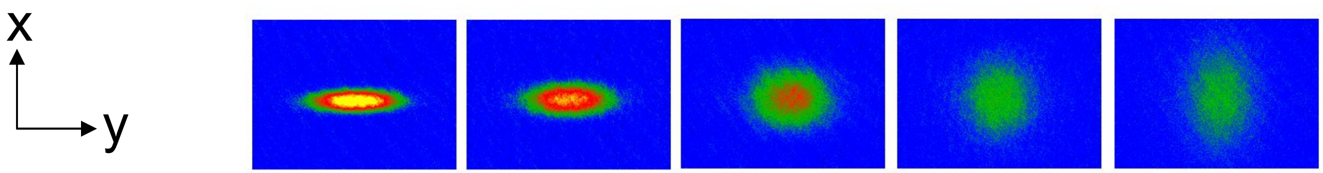
$$\overline{\omega_i^2}\equiv\frac{\langle \mathbf{r}\cdot\nabla U\rangle_0}{3m\langle x_i^2\rangle_0}$$

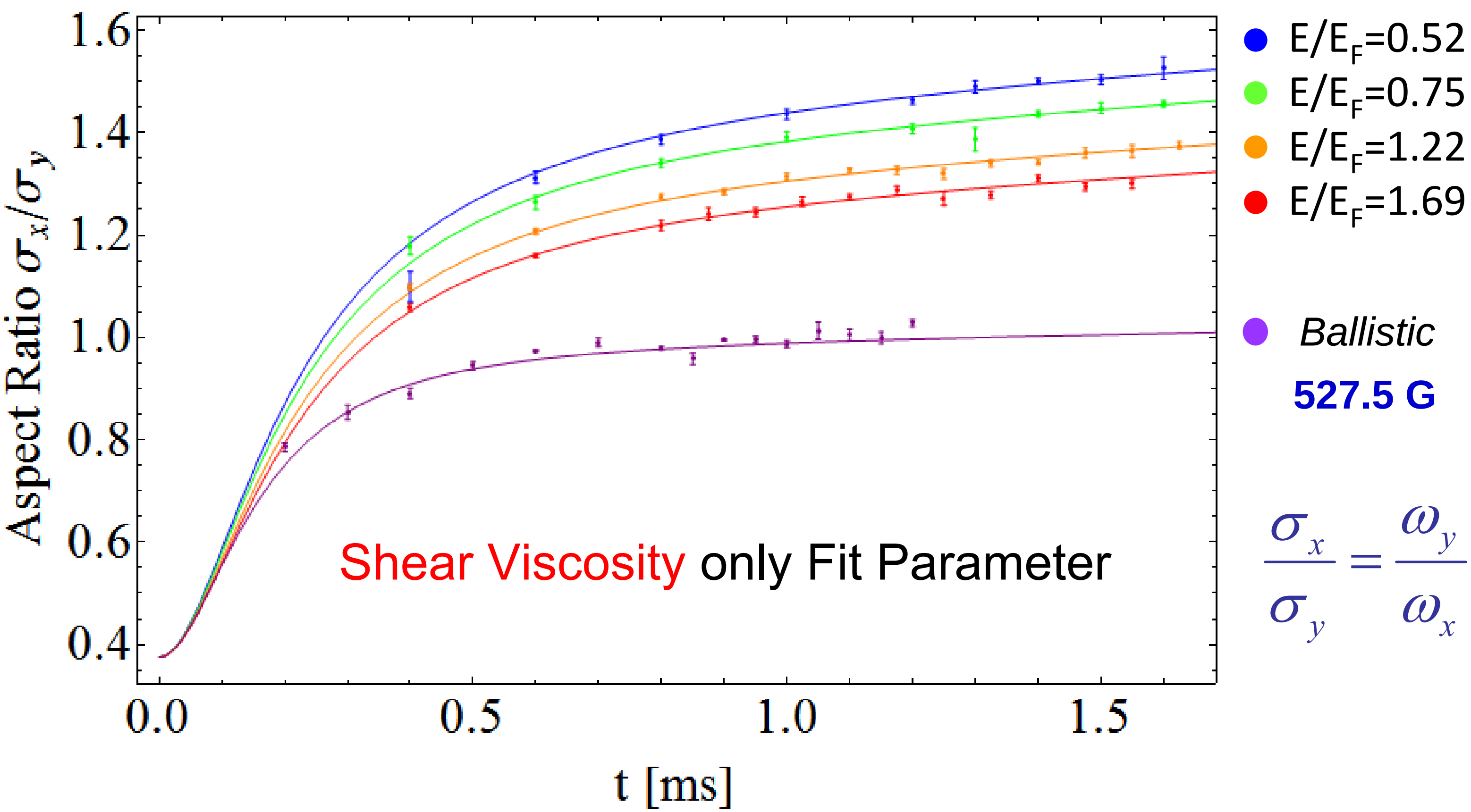
$$\sigma_{ii}=2\frac{\dot{b}_i}{b_i}-\frac{2}{3}\frac{\dot{\Gamma}}{\Gamma}$$

Cloud-averaged shear viscosity coefficient

$$\boxed{\ddot{b}_i=\frac{\overline{\omega_i^2}}{\Gamma^{2/3}b_i}\left[1+C_Q(t)+C_{\Delta p}(t)\right]-\frac{\hbar\langle\alpha_S\rangle\sigma_{ii}}{m\langle x_i^2\rangle_0 b_i}-\omega_{imag}^2 b_i}$$

Aspect Ratio versus Expansion Time

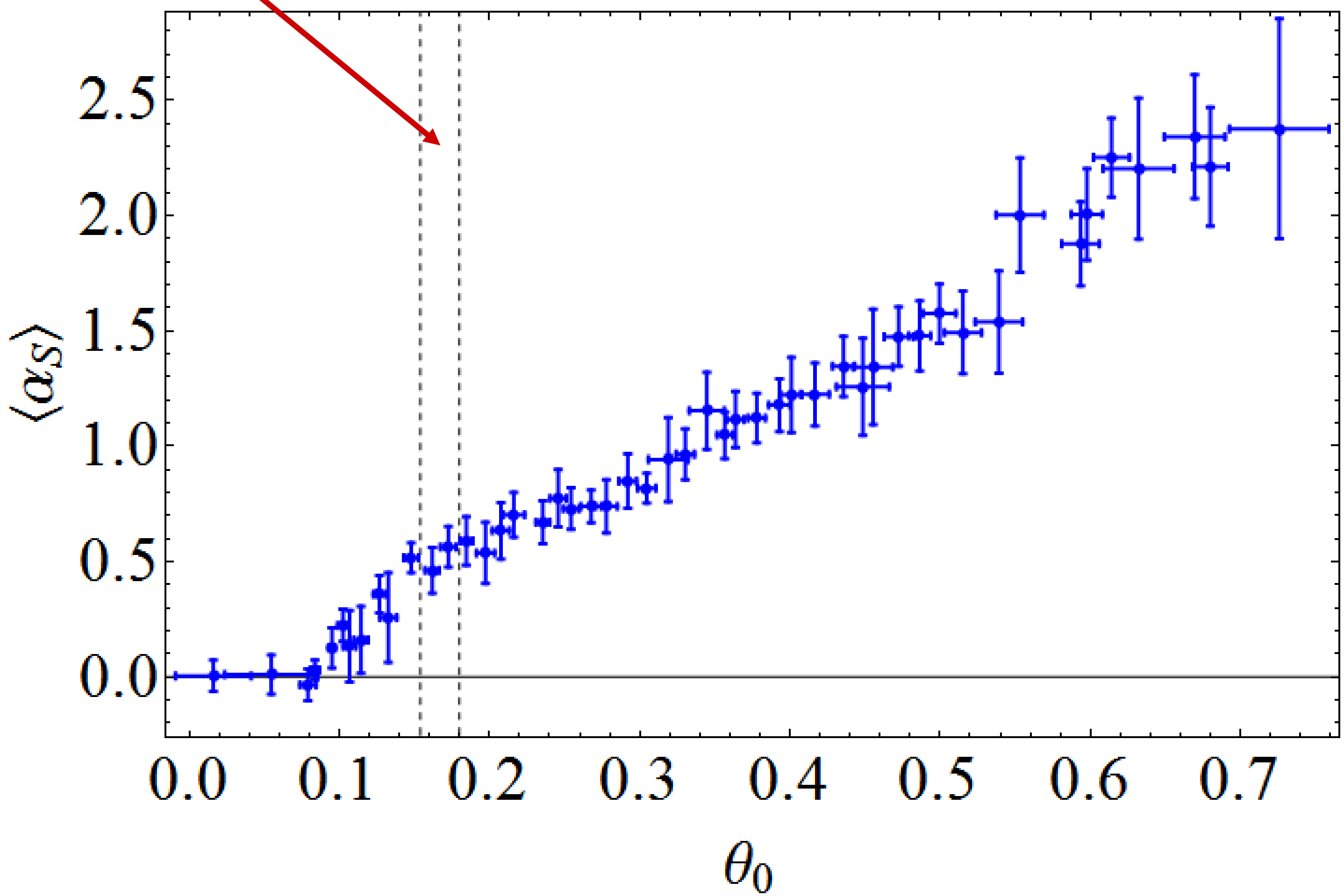




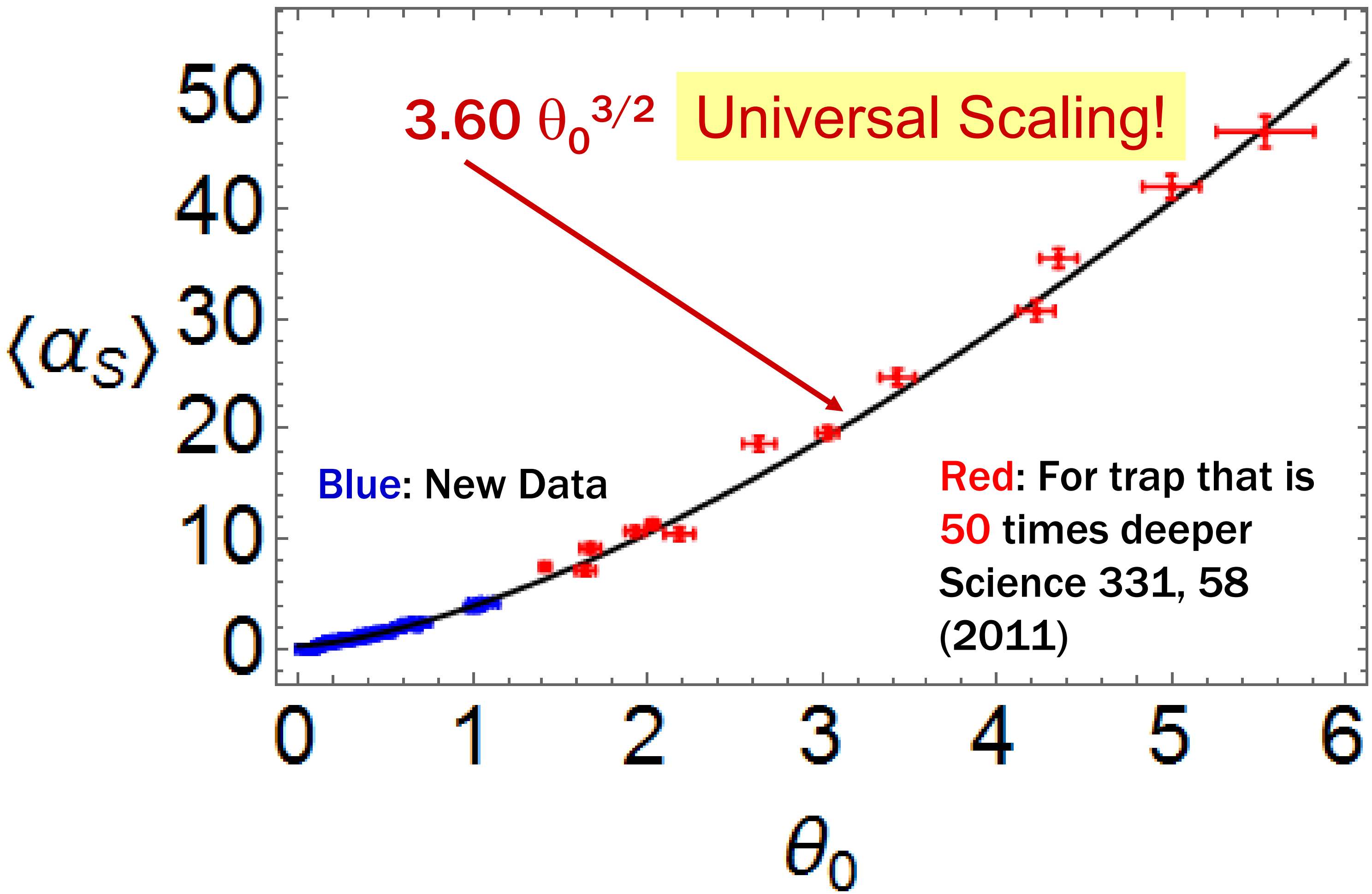
$$\frac{\sigma_x}{\sigma_y} = \frac{\omega_y}{\omega_x}\frac{b_x(t)}{b_y(t)}$$

Shear Viscosity at Resonance versus Reduced Temperature

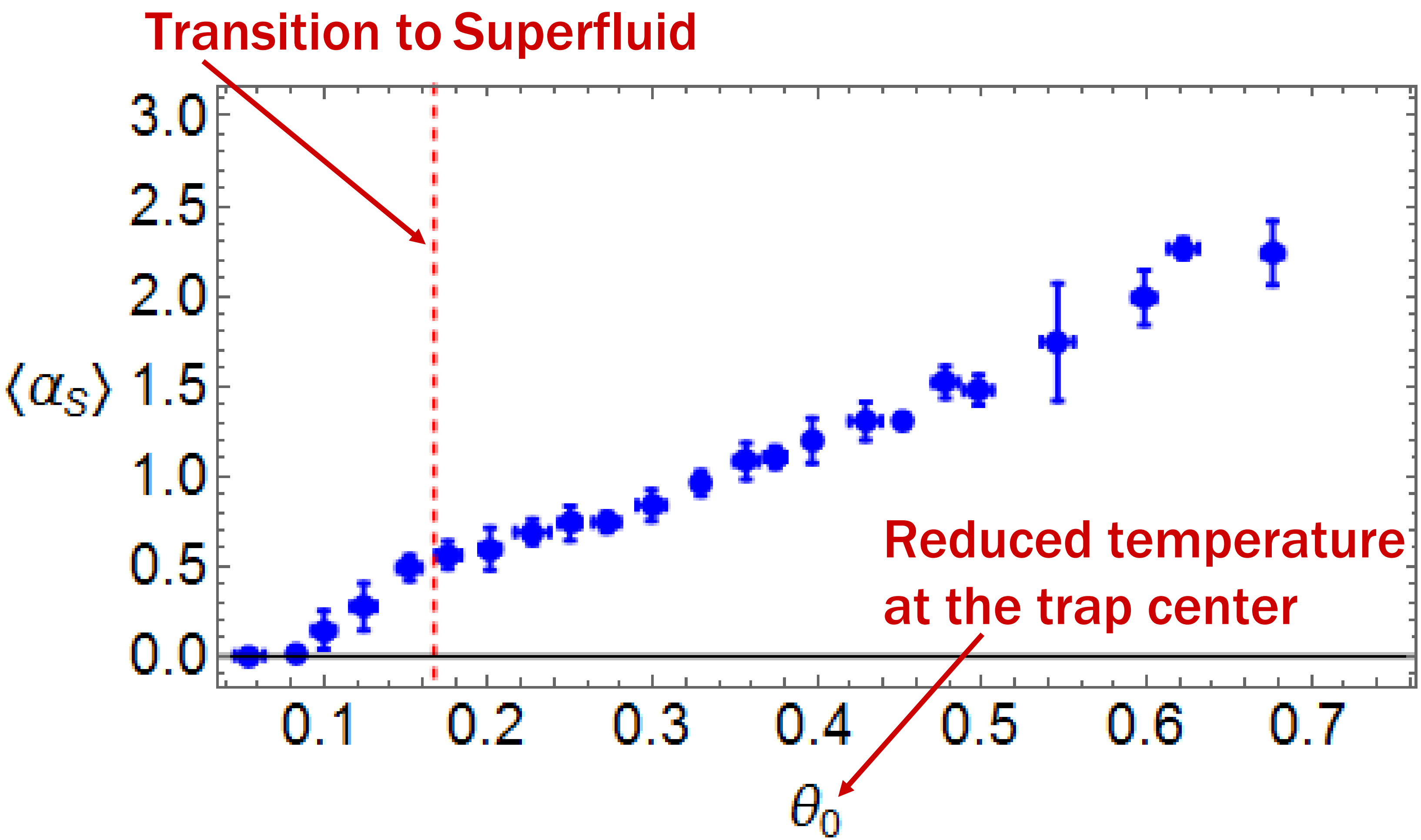
Transition to Superfluid



High Temperature Scaling of the Cloud-Averaged Viscosity



Cloud Averaged Shear Viscosity versus Reduced Temperature



*EoS from Ku et al., *Science*, 2012

Local Shear Viscosity: Linear Inverse Problem

For each trap averaged shear viscosity measured at reduced temperature θ_{0j}, we create a linear equation

$$\langle \alpha \rangle_j = \sum_i C_{ji} \alpha_i$$

Discrete Local α

$\theta_i \rightarrow \theta_{i+1}$

Equivalent matrix equation: $$\langle \boldsymbol{\alpha} \rangle = \mathbf{C} \boldsymbol{\alpha}$$

Can be solved iteratively

$$\boldsymbol{\alpha}_{m+1} = (1-\beta)\boldsymbol{\alpha}_m + \beta\, \Psi\left[\boldsymbol{\alpha}_m + \mathbf{C}^{\mathrm{T}}\left(\langle \boldsymbol{\alpha} \rangle - \mathbf{C} \bullet \boldsymbol{\alpha}_m\right)\right]$$

Determining the *Local* Shear Viscosity: Finite Volume Average

Divergent!

$$\langle \alpha_S \rangle = \frac{1}{N}\int_0^{R_C} d^3r\, \alpha(r)\, n(r) \quad \Rightarrow \quad c_1\theta_0^{3/2} = \frac{1}{N}\alpha_{3/2}\theta_0^{3/2} n_0 \frac{4\pi}{3} R_C^3$$

Volume

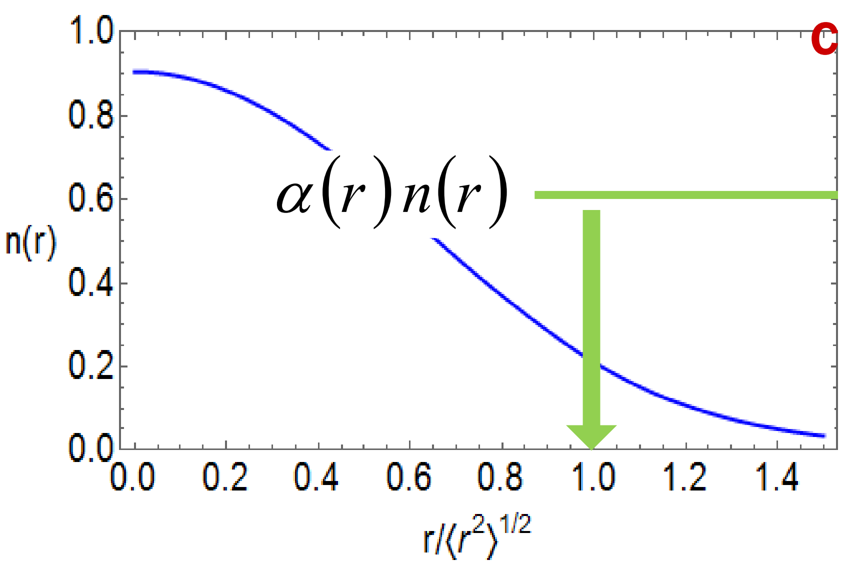
Fit from our data

$c_1 = 3.60$

from theory

$\alpha_{3/2} = 2.77$

G. M. Bruun and H. Smith, Phys. Rev. A **75**, 043612 (2007).



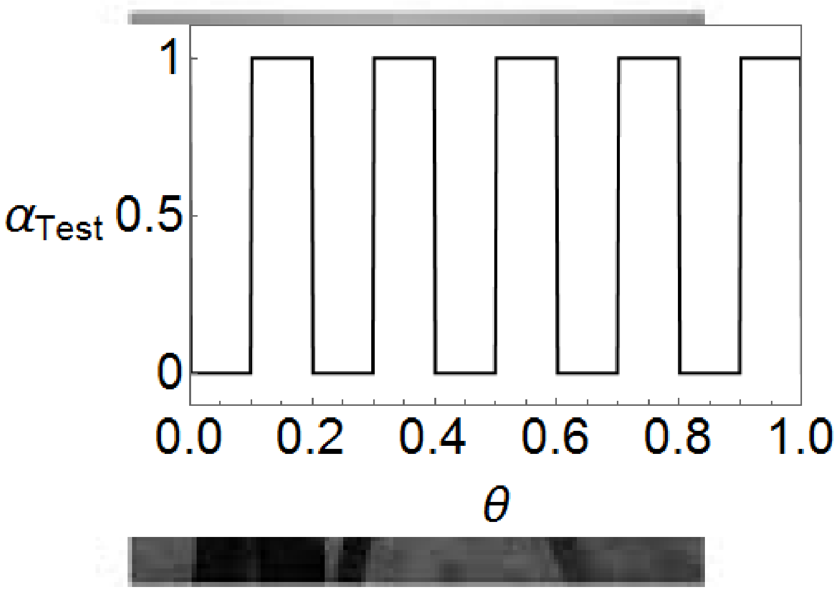
$$n_0 = N\left(\pi\frac{2}{3}\langle r^2\rangle\right)^{-3/2}$$

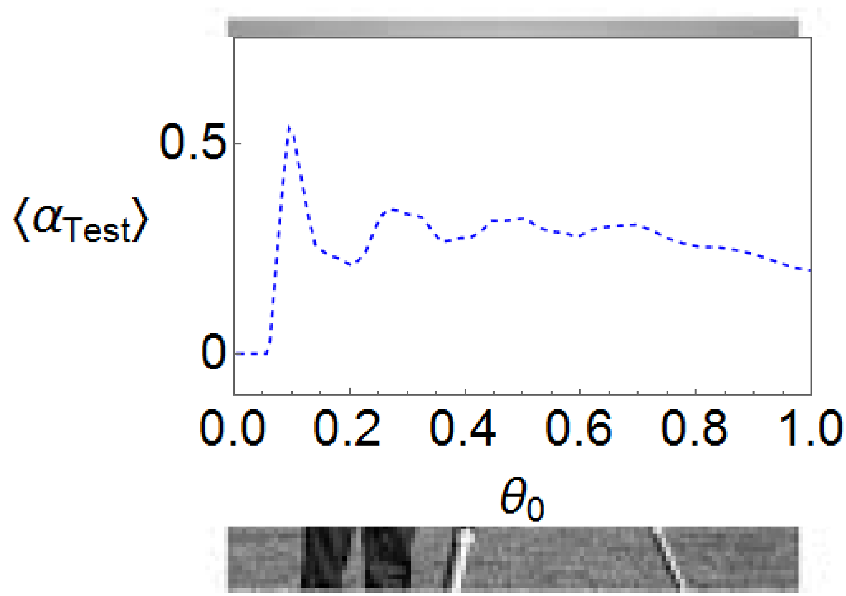
For a Gaussian density

$$R_C = 0.98\langle r^2\rangle^{1/2}$$

Image Processing Technique
Iterative Matrix Inversion

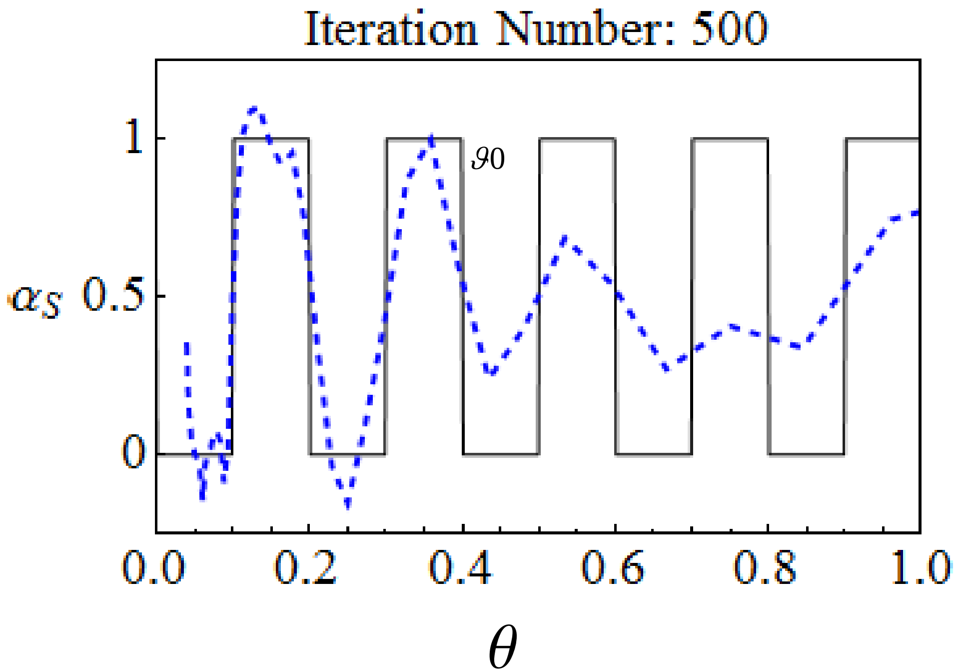
$$\boldsymbol{\alpha}_{m+1} = (1-\beta)\boldsymbol{\alpha}_m + \beta\,\Psi\left[\boldsymbol{\alpha}_m + \mathbf{C}^{\mathrm{T}}\left(\left\langle \boldsymbol{\alpha}_S \right\rangle - \mathbf{C}\bullet\boldsymbol{\alpha}_m\right)\right]$$



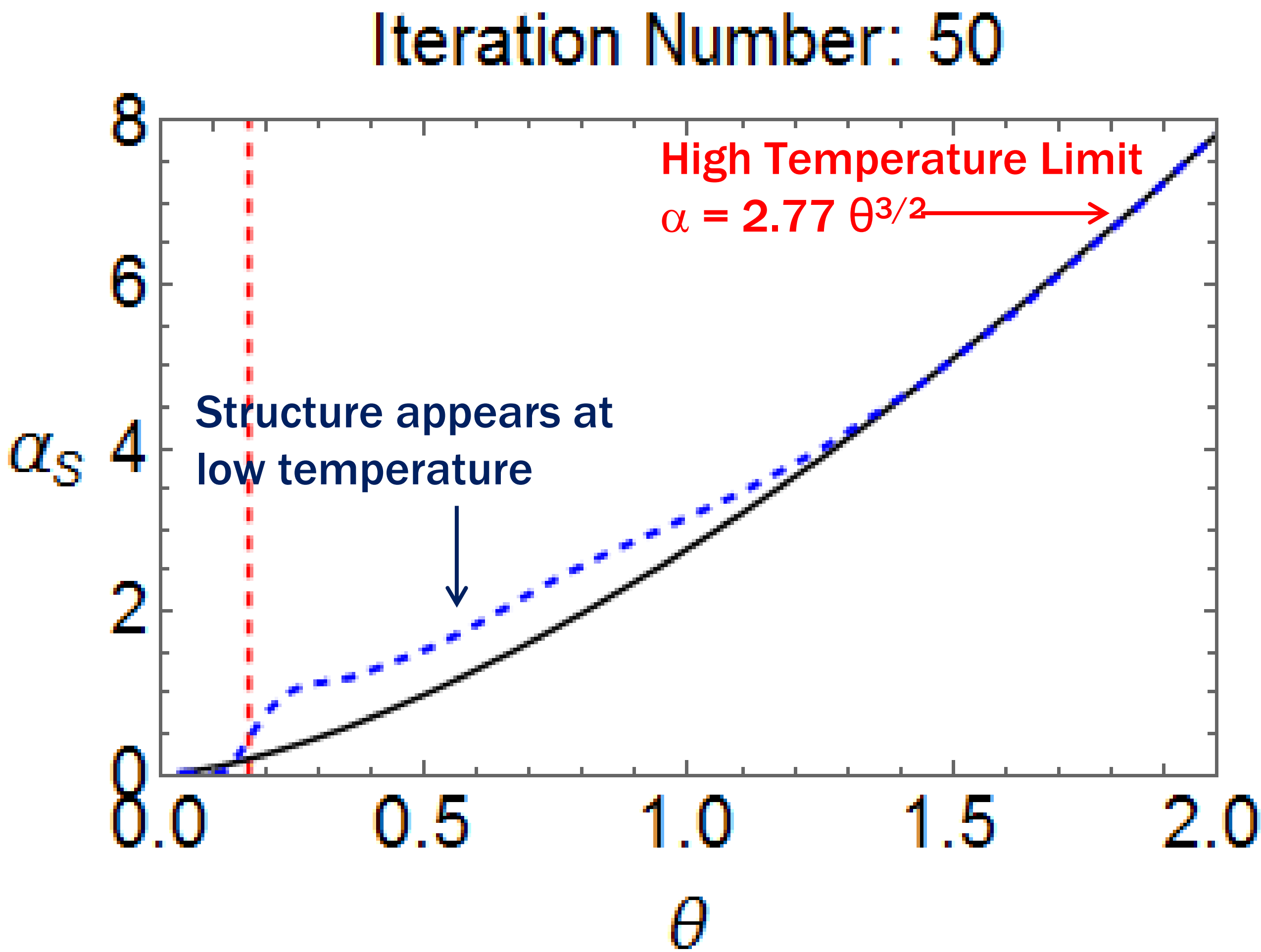


Iterative Solution for Test Function

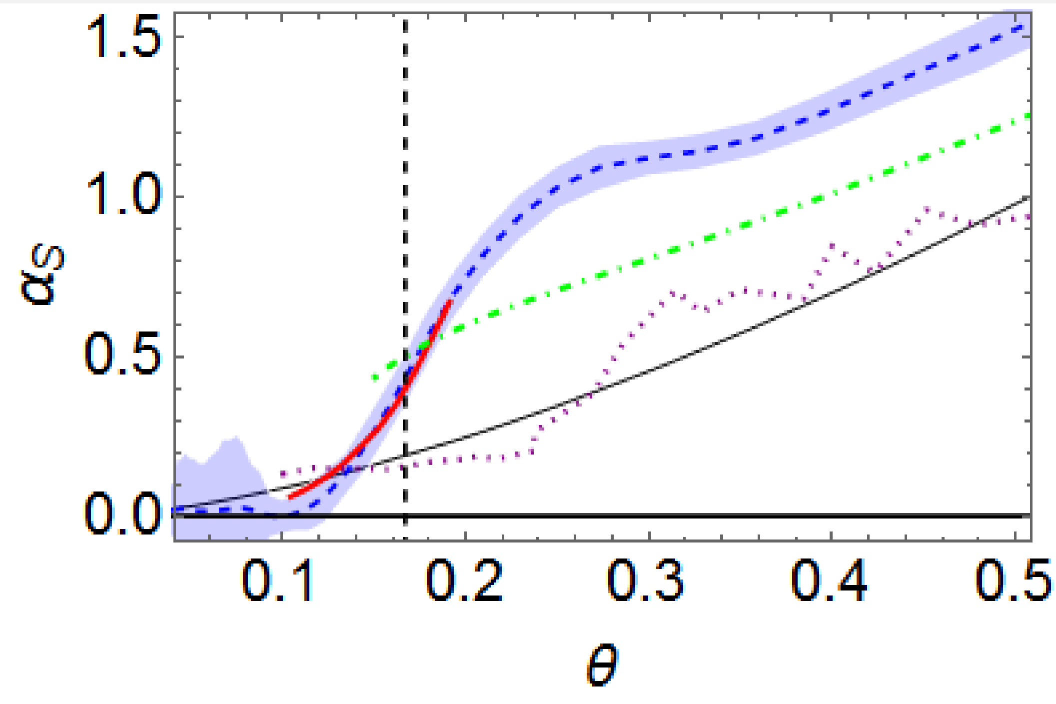
$$\boldsymbol{\alpha}_{m+1} = (1-\beta)\boldsymbol{\alpha}_m + \beta\,\Psi\left[\boldsymbol{\alpha}_m + \mathbf{C}^{\mathrm{T}}\left(\langle\boldsymbol{\alpha}_S\rangle - \mathbf{C}\bullet\boldsymbol{\alpha}_m\right)\right]$$



Local Shear Viscosity versus Reduced Temperature



Local Shear Viscosity (Comparison to Theory)



"Measured"

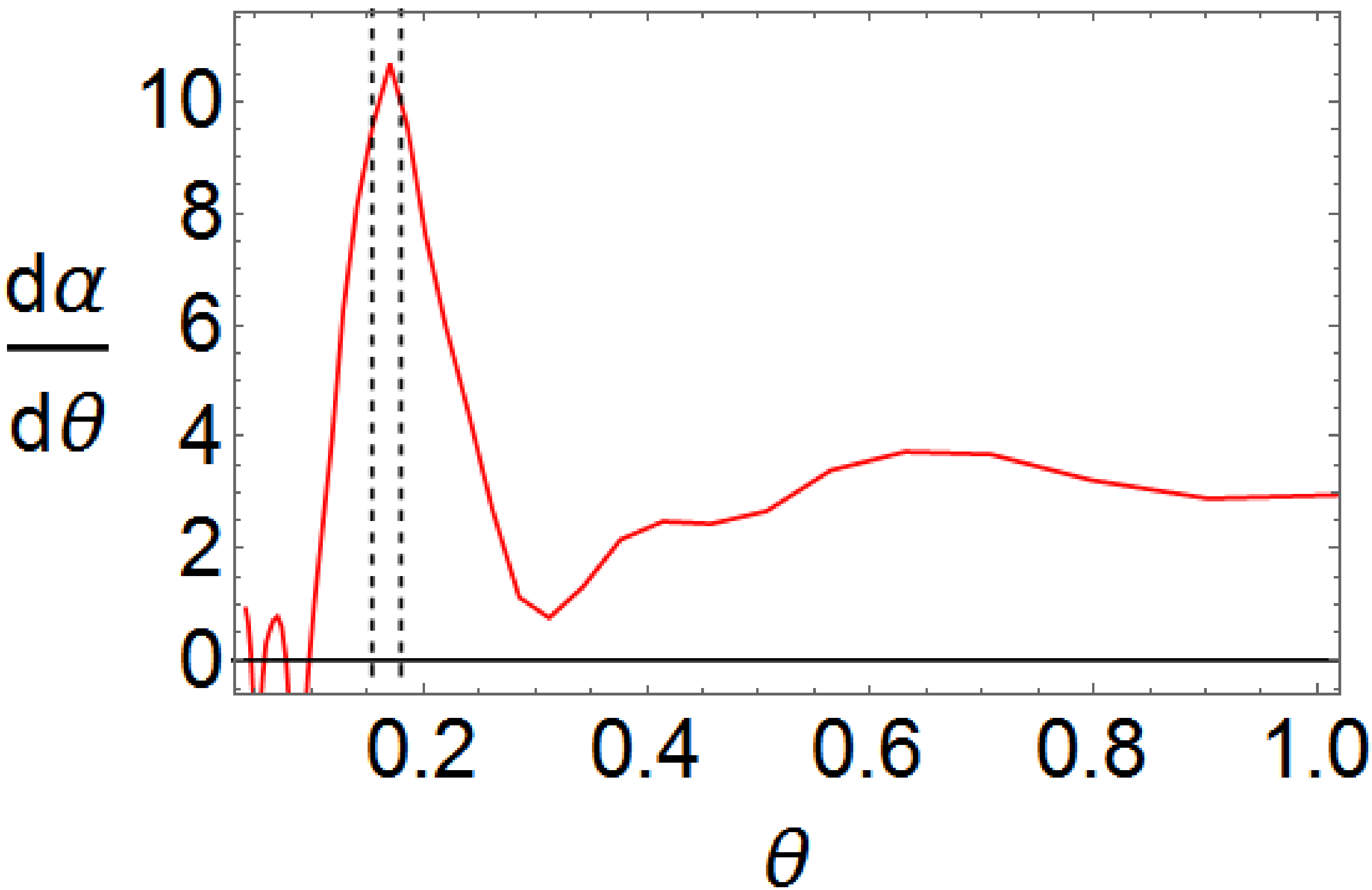
Kinetic theory
$\alpha = 2.77\ \theta^{3/2}$

Enss et al., Annals 2011 (Diagrammatic Kubo formula)

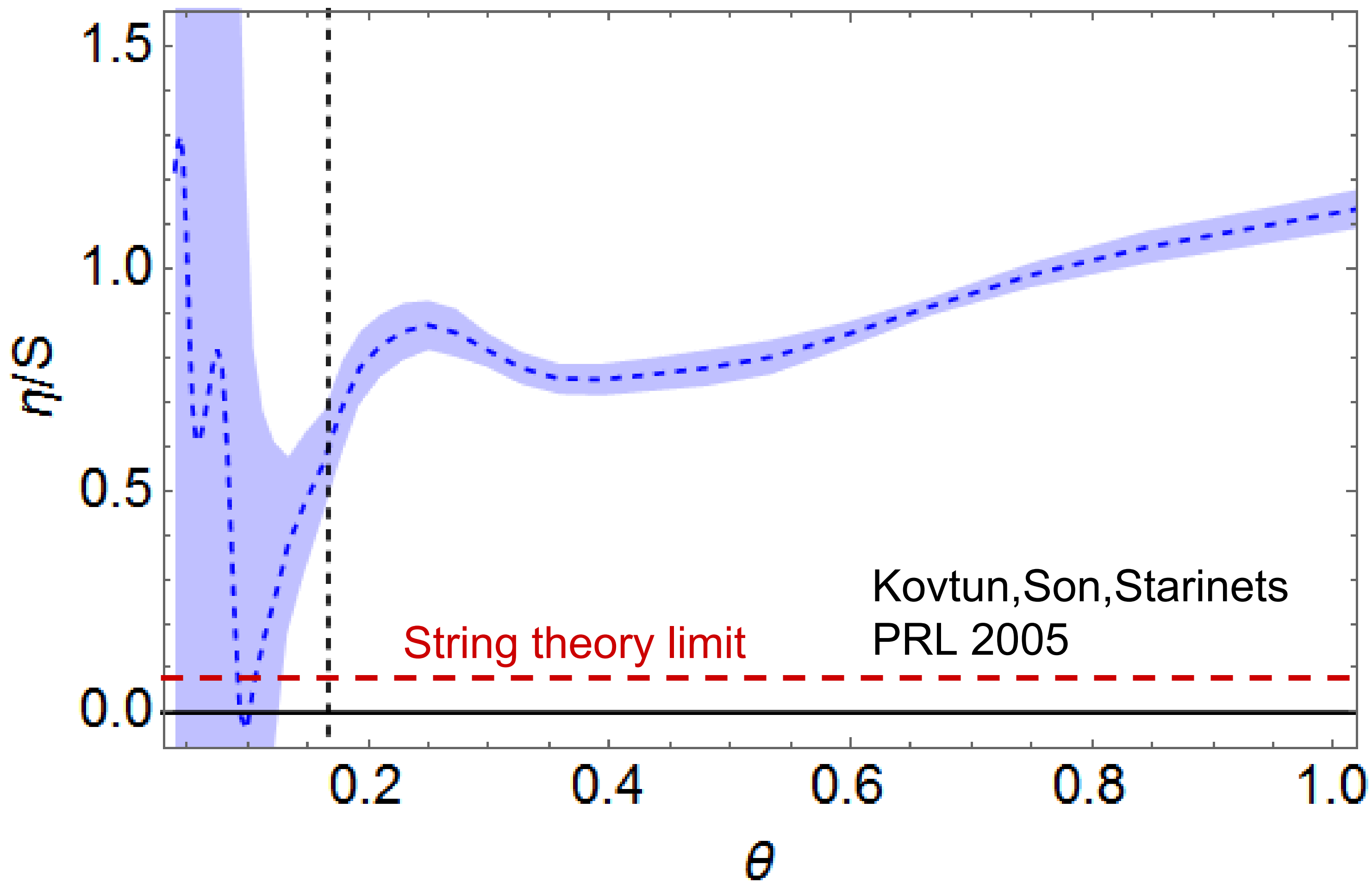
Wlazłowski et al., PRL 2012 (Monte Carlo)

Guo et al., PRL 2011

Derivative of the Local Shear Viscosity with respect to Reduced Temperature



Ratio of the Local Shear Viscosity to the Entropy Density*



*EoS from Ku et al., *Science*, 2012

Summary: 3D Hydrodynamics

- Scale-Invariant Strongly Interacting Fermi Gas
  - Tests theories of scale-invariant hydrodynamics
- Measured Cloud-Averaged Shear Viscosity
  - Single-shot method to find initial cloud size (temperature) and cloud-averaged shear viscosity self consistently
- Determined Local Shear Viscosity
  - Image processing techniques
  - Tests of non-perturbative many-body theory

Quasi-2D Fermi Gases

Search for high temperature superconductivity in layered materials:

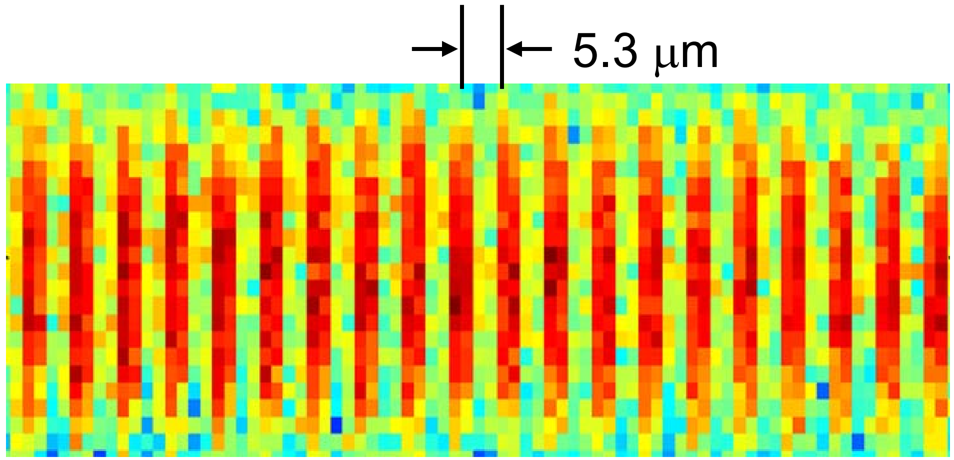
- In copper oxide and organic films, electrons are confined in a quasi-two-dimensional geometry
- Complex, strongly interacting many-body systems
- Phase diagrams are not well understood
- Exotic superfluids

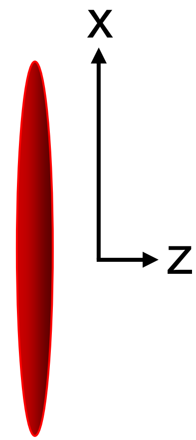
Enhancement of the superfluid transition temperature compared to true 2D materials:

- Heterostructures and inverse layers
- Quasi-2D organic superconductors
- Intercalated structures and films of transition metals

Creating a Quasi-2D Fermi gas

Two-lowest hyperfine states of fermionic ^{6}Li in CO_2 laser trap at $T < 0.2\ T_F$:





Measure Column Density:

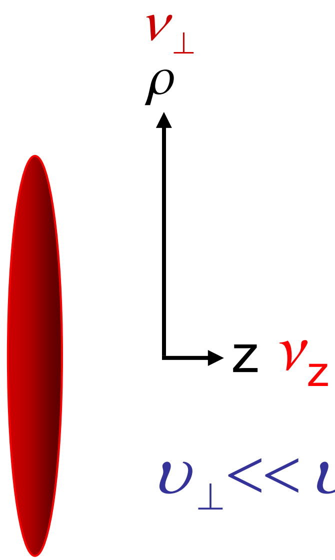
$$\boxed{n_c(x) = \int_{-\infty}^{\infty} dy\, n_{2D}(\sqrt{x^2 + y^2})}$$

N_1 = Majority #
(800 per site)

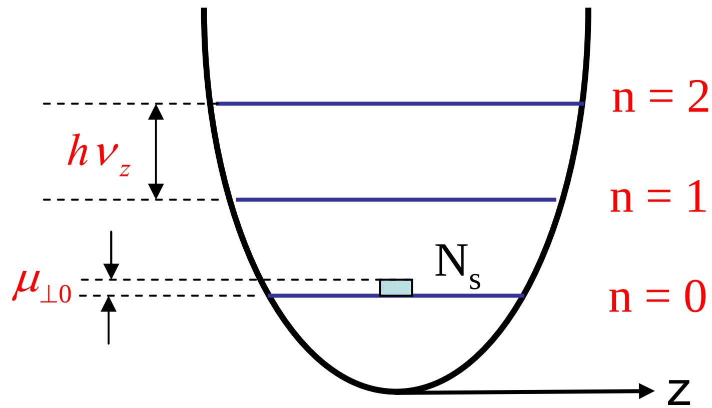
N_2 = Minority #

Transverse Density Profiles: $n_{2D}(\rho)$

Two-Dimensional Gas



$$\upsilon_\perp << \upsilon_z$$



Two-dimensional

$$N_s = \int_0^{\mu_{\perp 0}} \frac{d\varepsilon_\perp \varepsilon_\perp}{(h\upsilon_\perp)^2}$$

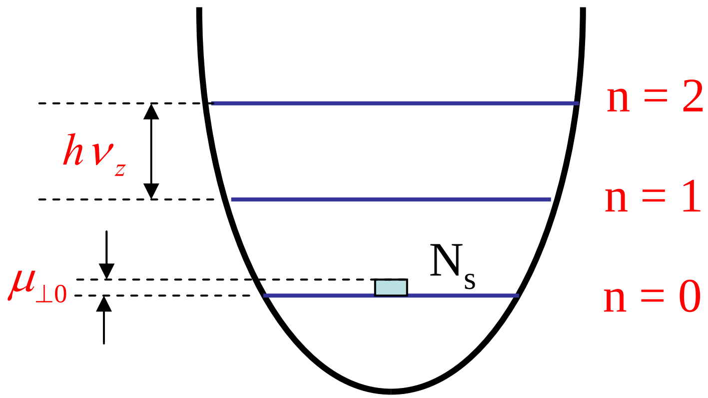
2D Density of States

$$\mu_{\perp 0} = E_F \equiv h\upsilon_\perp \sqrt{2N_s}$$

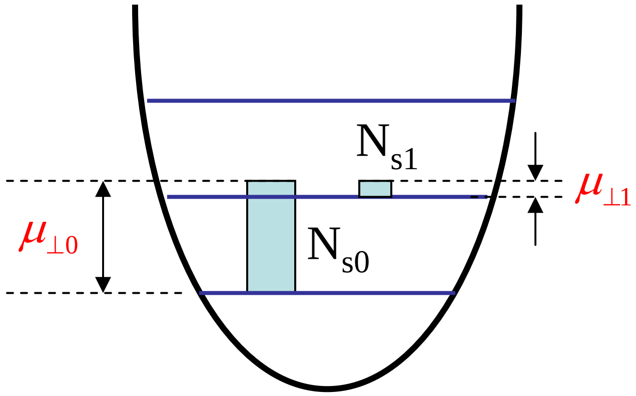
2D Transverse Fermi Energy

True 2D if: $\boxed{E_F << h\upsilon_z}$

Quasi-Two-Dimensional Gas



Two-dimensional



Quasi-two-dimensional

True 2D if: $\boxed{\mu_{\perp 0} = E_F << h\upsilon_z}$

3D if: $\boxed{\mu_{\perp 0} = E_F >> h\upsilon_z}$

Quasi-2D if: $\boxed{E_F \approx h\upsilon_z}$

What is the ground state of a quasi-2D strongly-interacting Fermi gas?

Column Densities versus N_2/N_1

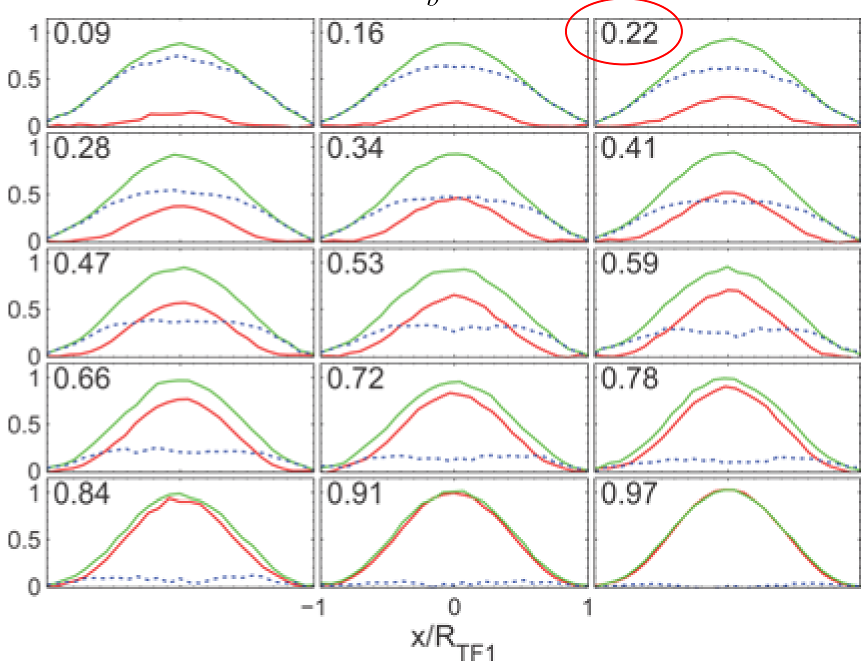
Majority state: $N_1 \cong 800$ per site

Minority state: $N_2 \leq N_1$

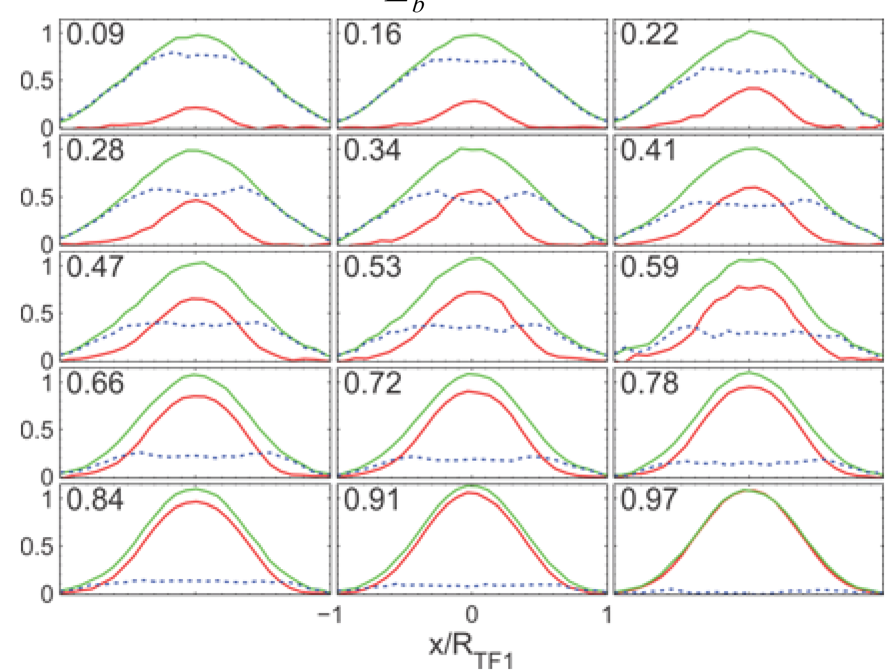
E_b = 2D dimer binding energy

E_F = 2D ideal gas Fermi energy

- 832 G $\frac{E_F}{E_b} = 6.6$



- 775 G $\frac{E_F}{E_b} = 0.75$

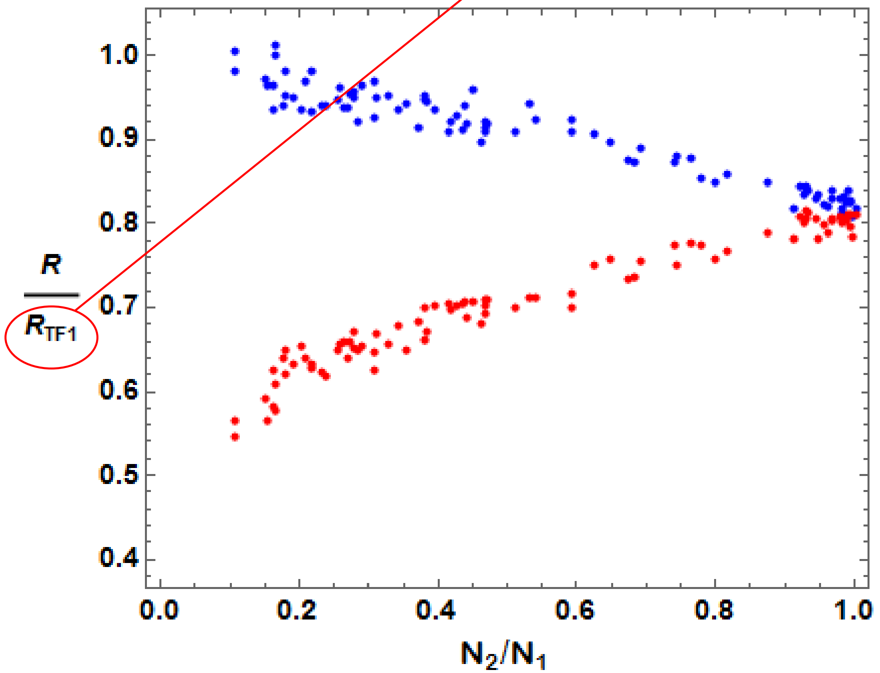


Majority and Minority Radii

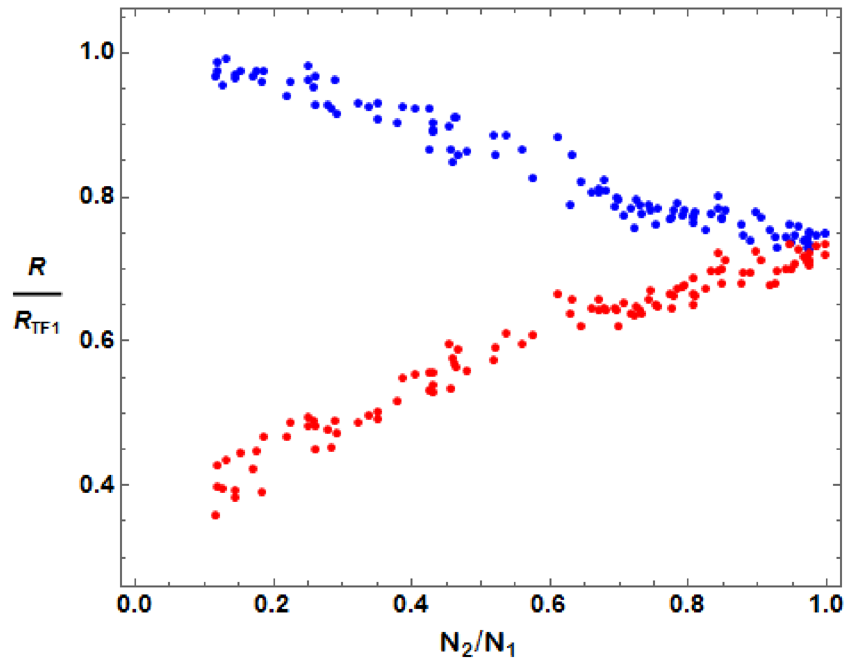
E_b = 2D dimer binding energy E_F = 2D ideal gas Fermi energy

Ideal gas Thomas-Fermi radius - Majority

$$\frac{E_F}{E_b} = 6.6$$

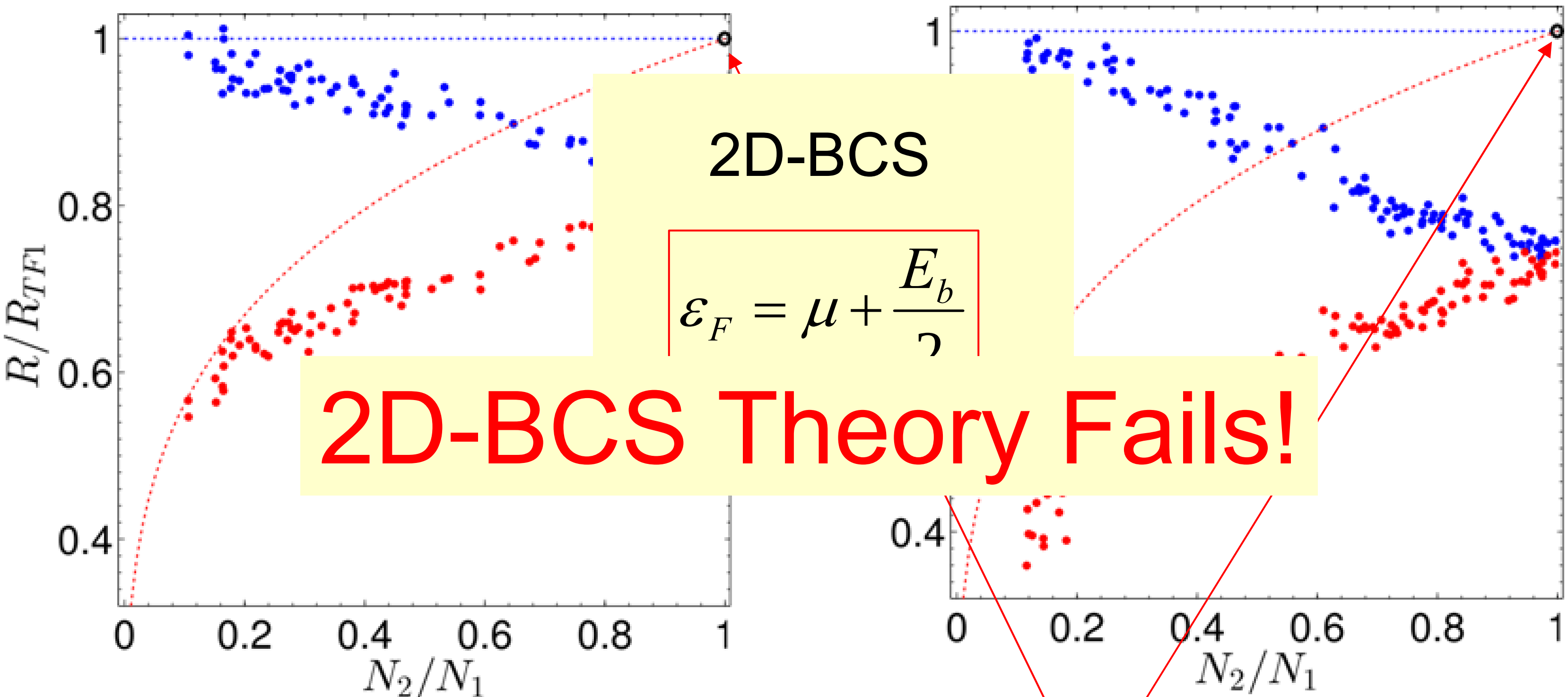


$$\frac{E_F}{E_b} = 0.75$$



Majority and Minority Radii

$\frac{E_F}{E_b} = 6.6$ — Ideal gas — $\frac{E_F}{E_b} = 0.75$



2D-BCS

$$\boxed{\varepsilon_F = \mu + \frac{E_b}{2}}$$

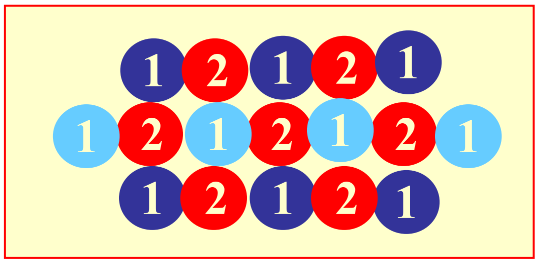
2D-BCS Theory Fails!

2D-BCS—Balanced

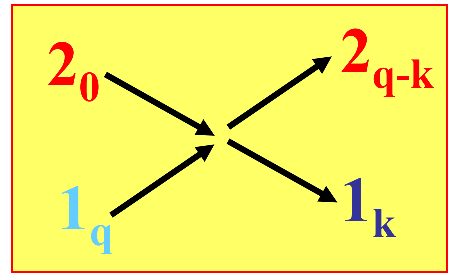
Polaron Gas Model

$1 = \uparrow$

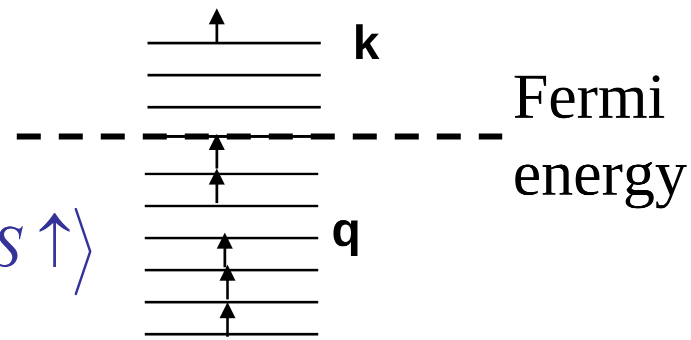
$2 = \downarrow$



2-Collision with the **1**-Fermi sea



$|FS\uparrow\rangle$



- Chevy's Fermi-polaron

$$|\psi\rangle_{\text{polaron}} = \phi_0 |\downarrow, \mathbf{p}=0\rangle |FS\uparrow\rangle + \sum_{\mathbf{kq}} \phi_{\mathbf{kq}} |\downarrow, \mathbf{q}-\mathbf{k}\rangle |FS\uparrow; 1_{\mathbf{k}\uparrow} 0_{\mathbf{q}\uparrow}\rangle$$

single spin down cloud of particle-hole pairs

2D-Polaron Thermodynamics

- Free energy density-imbalanced gas: $\boxed{f = \tfrac{1}{2} n_1 \varepsilon_{F1} + \tfrac{1}{2} n_2 \varepsilon_{F2} + n_2 E_p(2)}$

Ideal Fermi gas

Minority Polaron Energy

- Polaron energy: $\boxed{E_p(2) = y_m(q_1)\,\varepsilon_{F1}}$

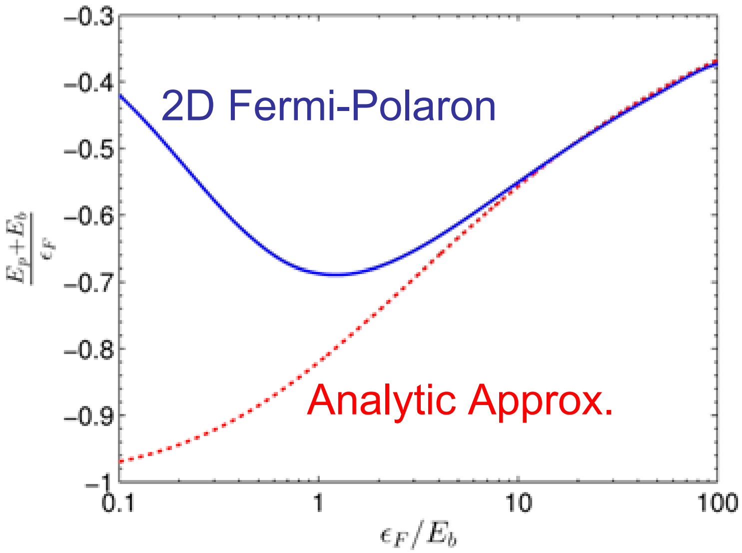
$$\boxed{\varepsilon_{F1} = \frac{2\pi\hbar^2}{m} n_1} \qquad \boxed{q_1 \equiv \frac{\varepsilon_{F1}}{E_b}} \qquad \boxed{y_m(q_1) = \frac{-2}{\log(1+2q_1)}}$$

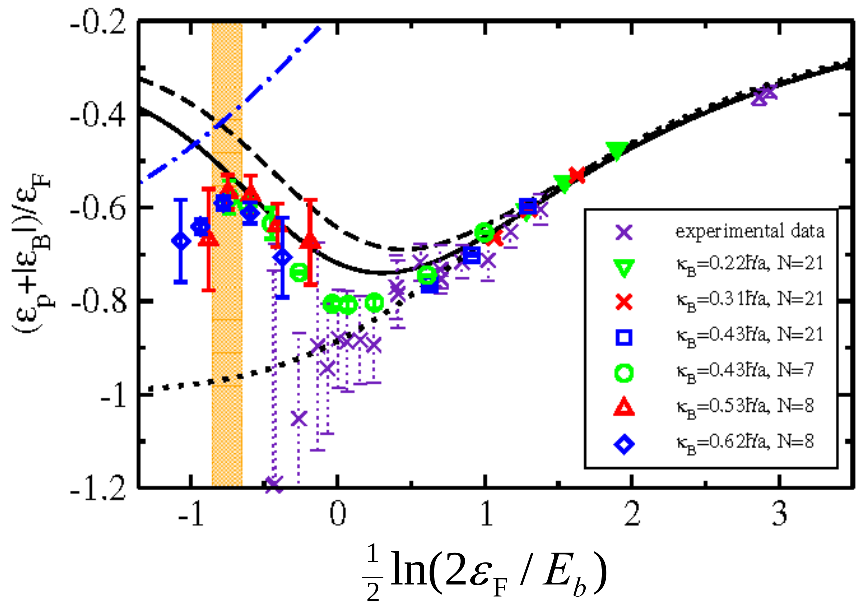
Klawunn and Recati 2011

- Chemical potentials: $\mu_1 = \dfrac{\partial f}{\partial n_1} \qquad \mu_2 = \dfrac{\partial f}{\partial n_2}$

- Pressure: $p = n_1\mu_1 + n_2\mu_2 - f$

Polaron Energy: 2D-Fermi Polaron vs Analytic Approximation





Meera M. Parish and Jesper Levinson
Phys. Rev. A 87, 033616 (2013)

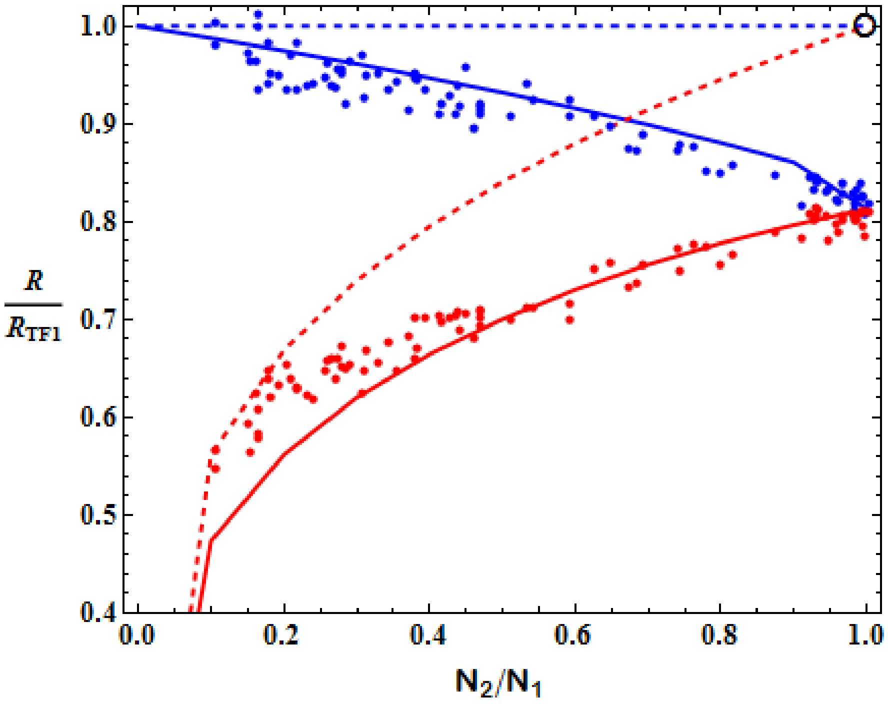
Shahin Bour, Dean Lee, H.-W. Hammer, and Ulf-G. Meiner
arXiv:1412.8175v2 (2014)

Lianyi He, Haifeng Lu, Gaoqing Cao, Hui Hu, Xia-Ji Liu
arXiv:1506.07156v1 (2015)

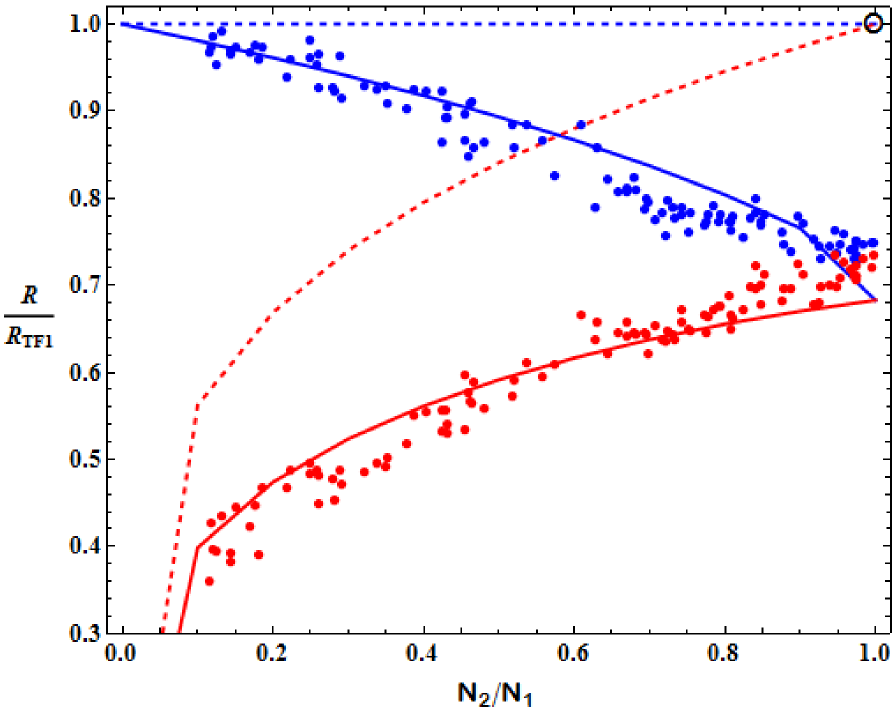
Majority and Minority Radii

Ideal gas — Polaron model — ○ 2D-BCS—balanced

$$\frac{E_F}{E_b} = 6.6$$



$$\frac{E_F}{E_b} = 0.75$$



Measuring the 2D Pressure

Force balance: $$-\frac{\partial P}{\partial \rho} - n_{2D}(\rho)\frac{\partial U}{\partial \rho} = 0$$

Harmonic Trap: $$U(\rho) = \tfrac{1}{2} m\omega_\perp^2 \rho^2$$

Pressure at trap center

$$\boxed{P(0) = \frac{m\omega_\perp^2}{2\pi} N}$$

Ideal 2D Pressure
(for density n_{2D})

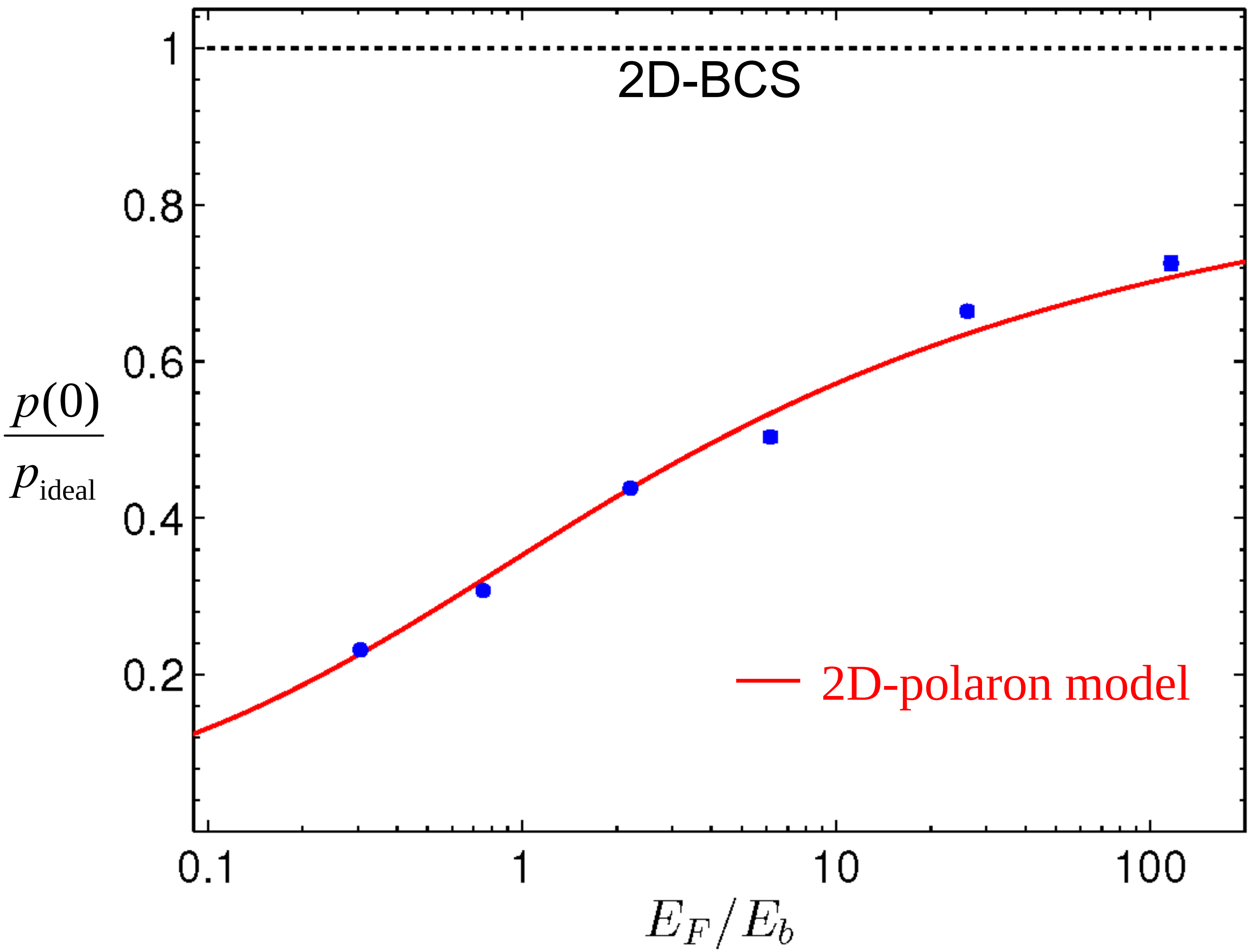
$$P_{\text{ideal}} = \frac{1}{2} n_{2D}\varepsilon_F$$

$$\boxed{\frac{P(0)}{P_{\text{ideal}}(0)} = \frac{n_{\text{ideal}}^2(0)}{n_{2D}^2(0)}}$$

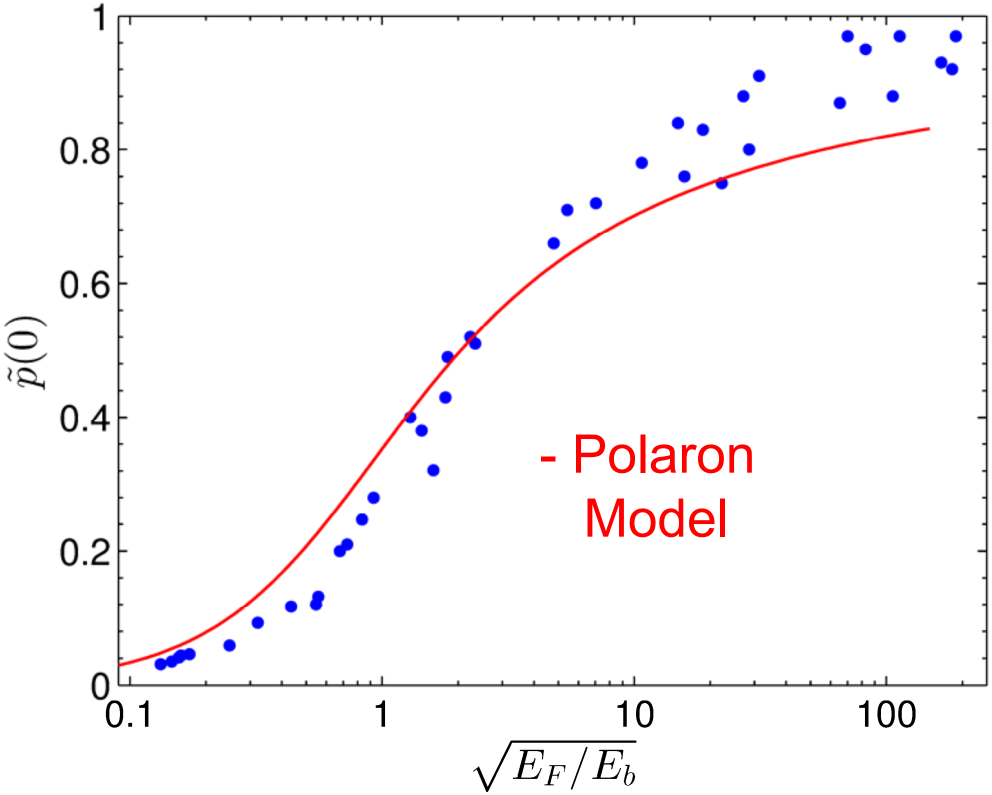
= 1 for an Ideal Fermi gas:

Determine $n_{2D}(0)$ from the column densities.

2D-Pressure Balanced Gas



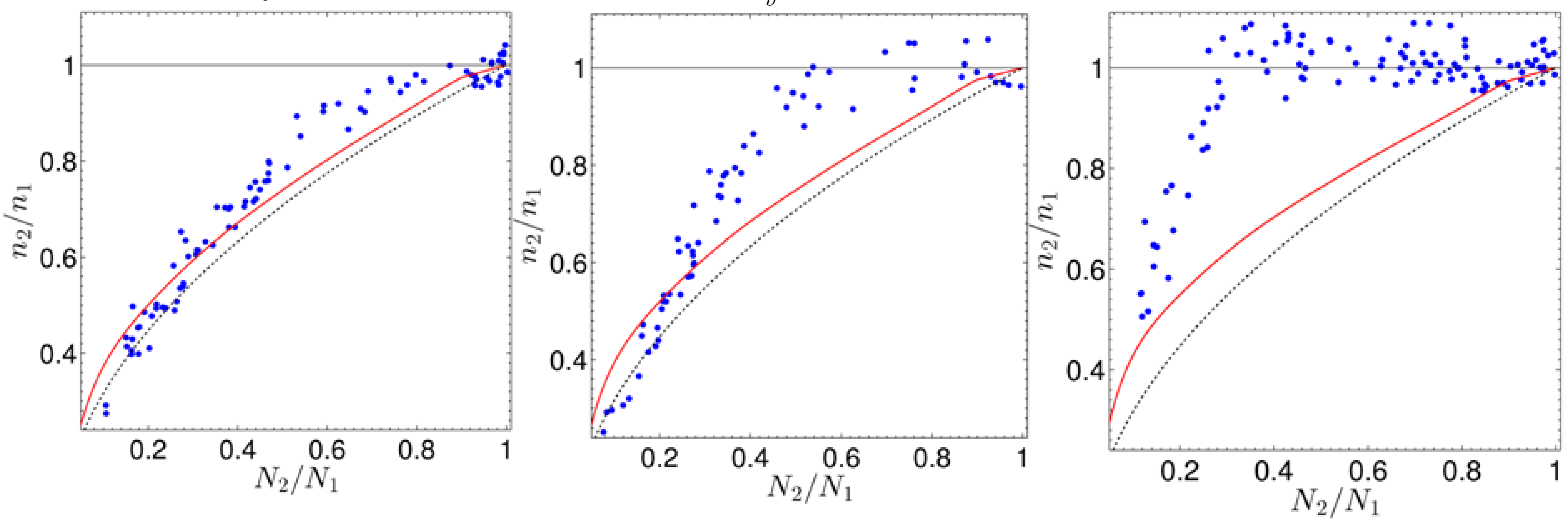
2D-Pressure—Wide Range



A. Turlapov, Phys. Rev. Lett. **112**, 045301 (2014).

2D-Central Density Ratio

$\frac{E_F}{E_b} = 6.6$ $\frac{E_F}{E_b} = 2.1$ $\frac{E_F}{E_b} = 0.75$



Polaron model

Ideal gas

- Transition to balanced core:
- Not predicted!

Summary: Quasi-2D Fermi Gas

- 2D BCS theory for a <u>True</u> 2D system *fails* in the <u>Quasi-2D</u> regime.
- 2D Polaron model explains several features of the density profiles in the quasi-2D regime.
- 2D Polaron model with the analytic approximation is too crude to predict the transition to a balanced core.
- Measurements with imbalanced mixtures provide the first benchmarks for predictions of the phase diagram for quasi-2D Fermi gases.

Birthday Party Dec-2014