Atomic Quantum Simulation of Abelian and non-Abelian Gauge Theories

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Cold Atoms meet
Quantum Field Theory
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UNIVERSITÄT BERN
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FNSNF SWISS NATIONAL SCIENCE FOUNDATION
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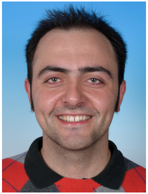
   

Innsbruck: Marcello Dalmonte, Catherine Laflamme, Peter Zoller
Bilbao: Enrique Rico Ortega

Bern: Michael Bögli, Pascal Stebler, Philippe Widmer
Berlin: Debasish Banerjee

Outline

Motivation from Nuclear and Particle Physics

Classical and Quantum Simulations of Quantum Spin Systems

Quantum Link Formulation of Lattice Gauge Theories

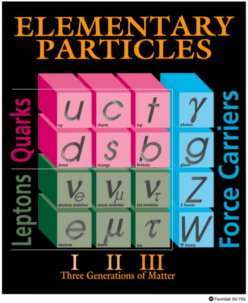
Atomic Quantum Simulators for Lattice Gauge Theories

"Nuclear Physics" from an $SO(3)$ Gauge Model via "Encoding"

Conclusions

Outline

The Standard Model — a relativistic quantum field theory



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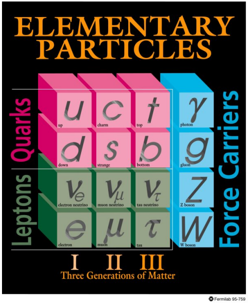
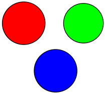


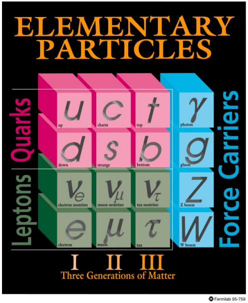
$SU(3)$ Quantum Chromodynamics (QCD)

Quarks



Gluon

The Standard Model — a relativistic quantum field theory



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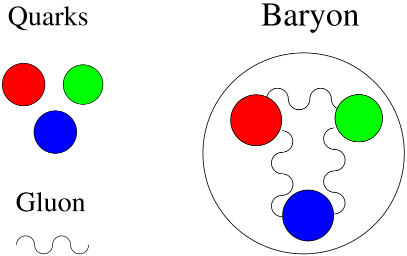

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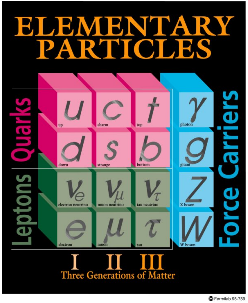

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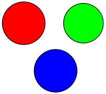
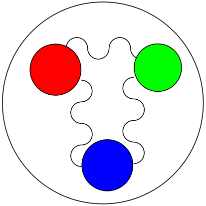
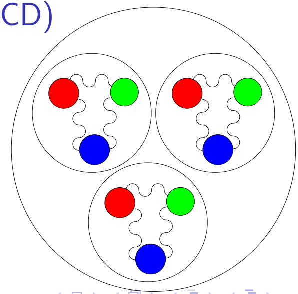


Gluon

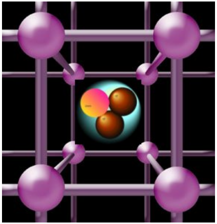
Baryon



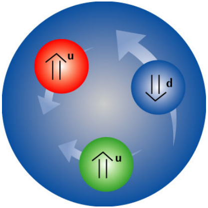
Nucleus

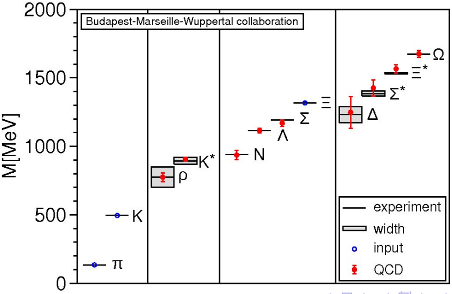


Wilson's lattice Quantum Chromodynamics (QCD) verifies confinement of quarks and gluons inside protons and neutrons

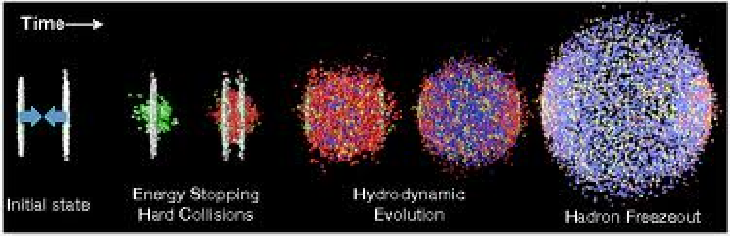


and confirms the experimentally observed hadron spectrum





Can heavy-ion collision physics or nuclear astrophysics benefit from quantum simulations in the long run?



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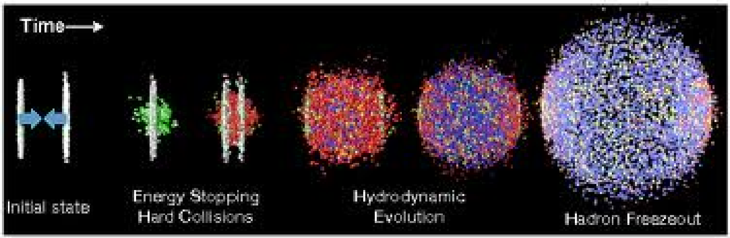

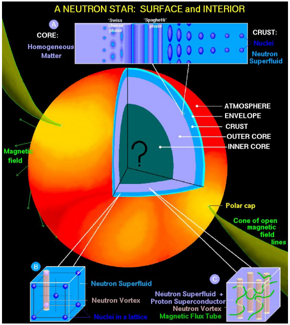

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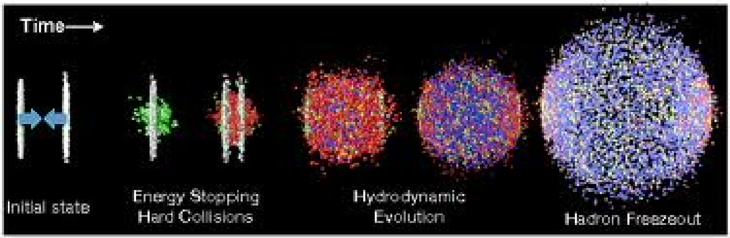
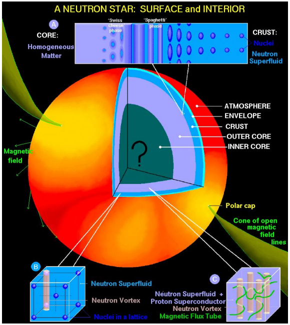
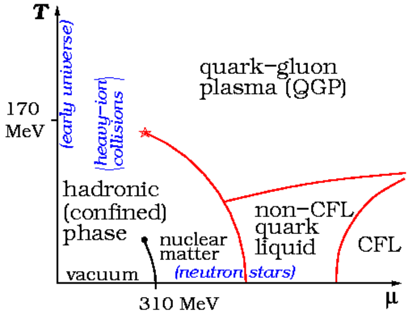
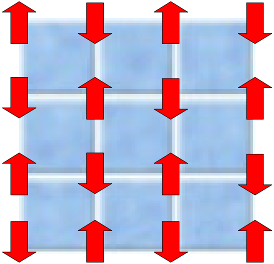






Outline

The spin $\frac{1}{2}$ quantum Heisenberg model



Quantum spins $[S_x^a, S_y^b] = i\delta_{xy}\varepsilon_{abc}S_x^c$ and their Hamiltonian

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

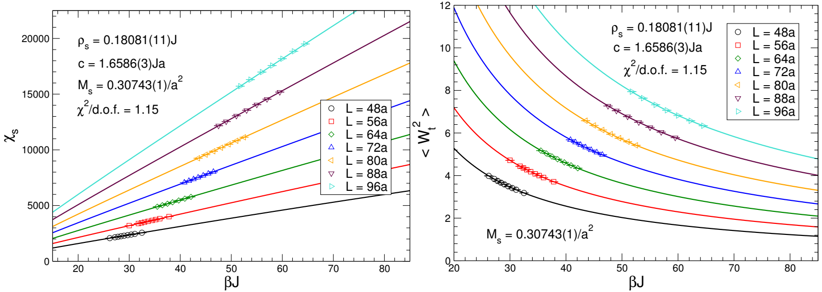
Partition function at inverse temperature $\beta = 1/T$

$$Z = \mathrm{Tr} \exp(-\beta H)$$

Low-energy effective action for antiferromagnetic magnons

$$S[\vec{e}] = \int_0^\beta dt \int d^2x \, \frac{\rho_s}{2} \left(\partial_i \vec{e} \cdot \partial_i \vec{e} + \frac{1}{c^2} \partial_t \vec{e} \cdot \partial_t \vec{e} \right)$$

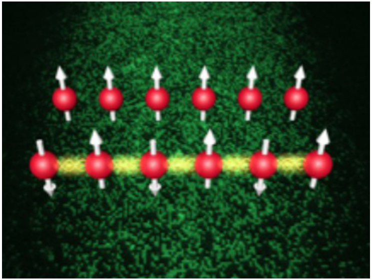
Fit to analytic predictions of effective theory

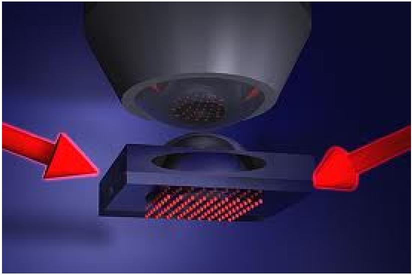


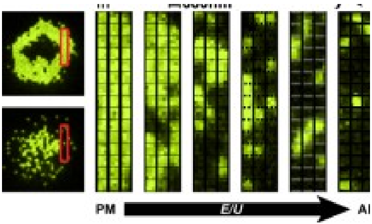
$$\mathcal{M}_s = 0.30743(1), \quad \rho_s = 0.18081(11)J, \quad c = 1.6586(3)Ja$$

UJW, H.-P. Ying (1994); F.-J. Jiang, UJW (2010)

Optical lattice quantum simulation of quantum spin systems







J. Simon, W. S. Bakir, R. Ma, M. E. Tal, P. M. Preis, M. Greiner, Nature 472 (2011) 307.

Outline

Different descriptions of dynamical Abelian gauge fields:

Maxwell's classical electromagnetic gauge fields

$$\vec{\nabla} \cdot \vec{E}(\vec{x}, t) = \rho(\vec{x}, t), \quad \vec{\nabla} \cdot \vec{B}(\vec{x}, t) = 0, \quad \vec{B}(\vec{x}, t) = \vec{\nabla} \times \vec{A}(\vec{x}, t)$$

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Quantum Electrodynamics (QED) for perturbative treatment

$$E_i(\vec{x}, t) = -i \frac{\partial}{\partial A_i(\vec{x}, t)}, \quad [E_i, A_j] = i\delta_{ij}, \quad \left[\vec{\nabla} \cdot \vec{E} - \rho\right] |\Psi[A]\rangle = 0$$

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Wilson's $U(1)$ lattice gauge theory for classical simulation

$$U_{xy} = \exp\left(ie \int_x^y d\vec{l} \cdot \vec{A}\right) = \exp(i\varphi_{xy}) \in U(1), \quad E_{xy} = -i \frac{\partial}{\partial \varphi_{xy}},$$

$$[E_{xy}, U_{xy}] = U_{xy}, \quad \left[\sum_i (E_{x,x+\hat{i}} - E_{x-\hat{i},x}) - \rho\right] |\Psi[U]\rangle = 0$$

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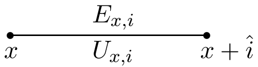
$U(1)$ quantum link models for quantum simulation

$$U_{xy} = S^+_{xy}, \quad U^\dagger_{xy} = S^-_{xy}, \quad E_{xy} = S^3_{xy},$$

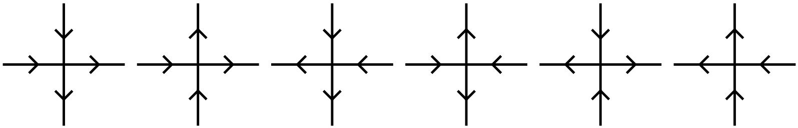
$$[E_{xy}, U_{xy}] = U_{xy}, \quad [E_{xy}, U^\dagger_{xy}] = -U^\dagger_{xy}, \quad [U_{xy}, U^\dagger_{xy}] = 2E^\dagger_{xy}$$

$U(1)$ gauge fields from spins $\frac{1}{2}$

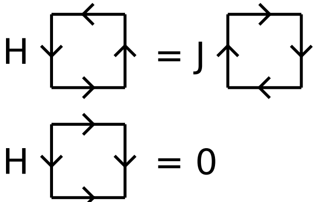


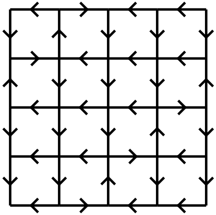
$$U = S^+, \; U^\dagger = S^-, \; E = S^3.$$

Gauss law



Ring-exchange plaquette Hamiltonian





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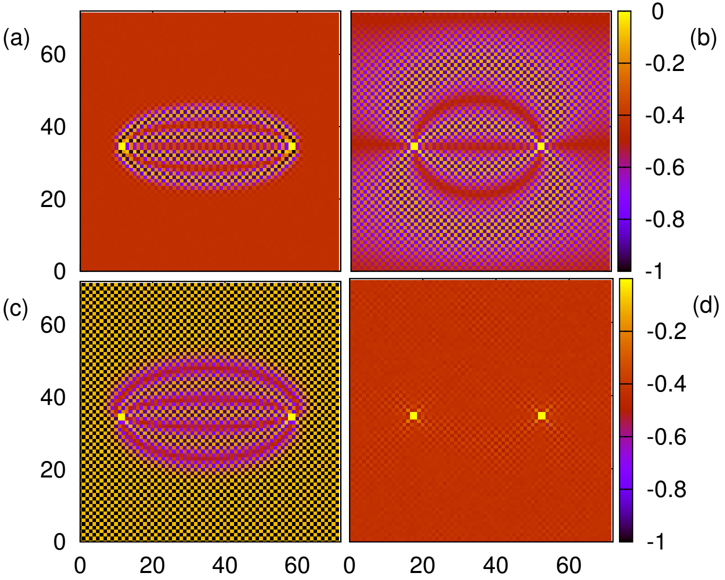
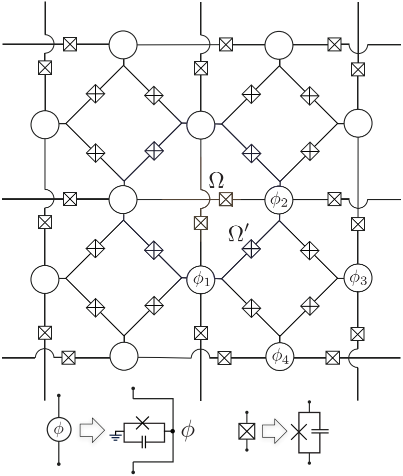
Energy density of charge-anti-charge pair $Q = \pm 2$



D. Banerjee, F.-J. Jiang, P. Widmer, UJW, JSTAT (2013) P12010.

“String theory on a chip” with superconducting circuits



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Phys. Rev. Lett. 111 (2013) 110504 (2013).
D. Marcos, P. Widmer, E. Rico, M. Hafezi, P. Rabl, UJW, P. Zoller,
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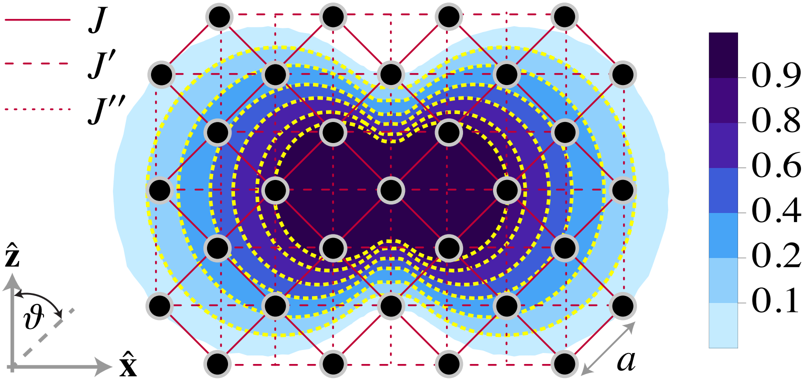
P-state excited Rydberg atoms in an optical lattice



A. G. Glaetzle, M. Dalmonte, R. Nath, I. Rousochatzakis, R. Moessner, P. Zoller, arXiv:1404.5326

Outline

Analog quantum simulator proposals

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E. Zohar, B. Reznik, Phys. Rev. Lett. 107 (2011) 275301.

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D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

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M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502; Nat. Phys. 6 (2010) 382.

L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.

L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Ann. Phys. 330 (2013) 160.

Review on quantum simulators for lattice gauge theories

UJW, Annalen der Physik 525 (2013) 777, arXiv:1305.1602.

Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \mathrm{h.c.} \right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

$$U_{x,x+1} = b_x b_{x+1}^\dagger, \;\; E_{x,x+1} = \frac{1}{2} \left(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x \right)$$

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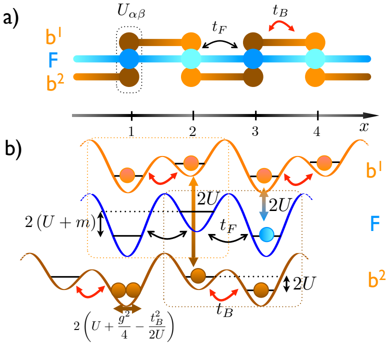
Optical lattice with Bose-Fermi mixture of ultra-cold atoms



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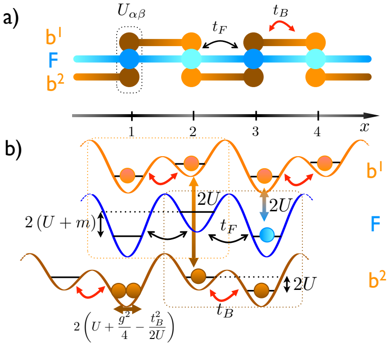
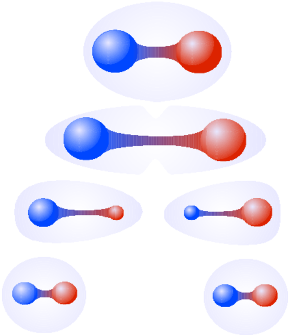
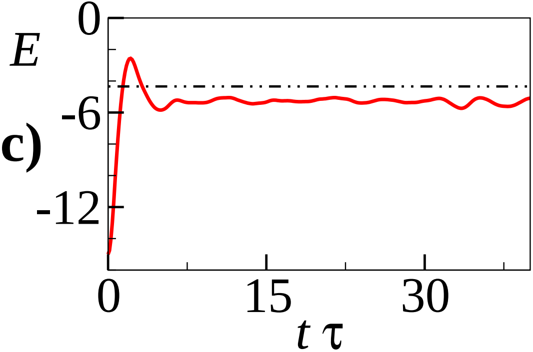




Quantum simulation of the real-time evolution of string breaking



D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, PRL 109 (2012) 175302.

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S. Kühn, J. I. Cirac, M. Banuls, PRA 90 (2014) 042305.

T. Pichler, M. Dalmonte, E. Rico, P. Zoller, S. Montagnero, arXiv:1505.04440.

V. Kasper, F. Hebenstreit, M. Oberthaler, J. Berges, arXiv:1506.01238.

$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij}, \ U^{ij\dagger} = S_1^{ij} - iS_2^{ij}, \ i,j \in \{1,2,\ldots,N\}, \ [U^{ij},(U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = i f_{abc} L^c, \ [R^a, R^b] = i f_{abc} R^c, \ a,b,c \in \{1,2,\ldots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U, \ [R^a, U] = U\lambda^a, \ [E, U] = U$$

Algebraic structures of different quantum link models

$U(N)$: $U^{ij}, \ L^a, \ R^a, E, \ 2N^2 + 2(N^2-1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

$SO(N)$: $O^{ij}, \ L^a, \ R^a, \ N^2 + 2\frac{N(N-1)}{2} = N(2N-1)$ $SO(2N)$ generators

$Sp(N)$: $U^{ij}, \ L^a, \ R^a, \ 4N^2 + 2N(2N+1) = 2N(4N+1)$ $Sp(2N)$ generators

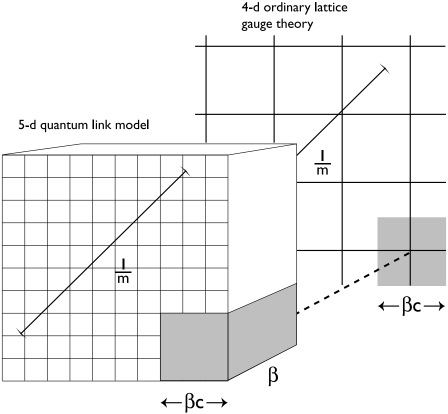
R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Low-energy effective action of a quantum link model

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \, \frac{1}{2e^2} \left(\mathrm{Tr}\ G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \mathrm{Tr}\ \partial_5 G_\mu \partial_5 G_\mu \right), \ G_5 = 0$$

undergoes dimensional reduction from $4+1$ to 4 dimensions

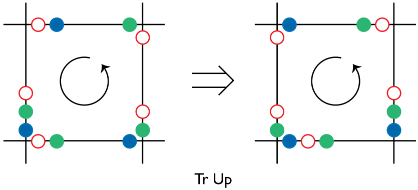
$$S[G_\mu] \to \int d^4x \, \frac{1}{2g^2} \mathrm{Tr}\ G_{\mu\nu} G_{\mu\nu}, \ \frac{1}{g^2} = \frac{\beta}{e^2}, \ \frac{1}{m} \sim \exp\left(\frac{24\pi^2\beta}{11 N e^2}\right)$$

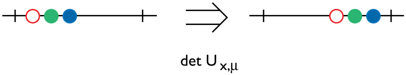


$SU(3)$ quantum link operator in terms of fermionic "rishons"

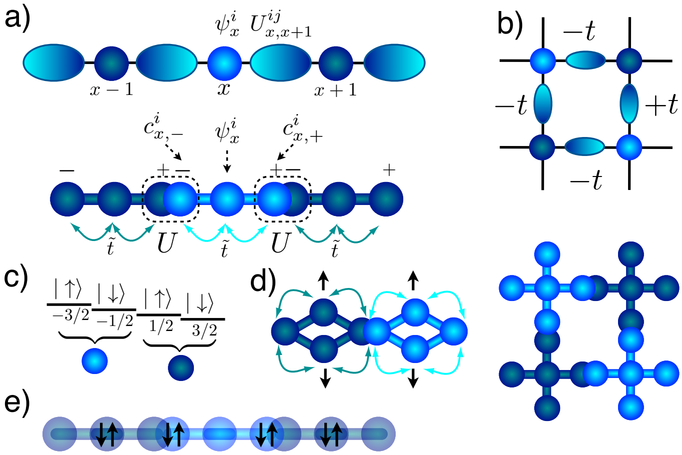
$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \; i,j \in \{1,2,3\}$$

Ring-exchange Hamiltonian as a "rishon-abacus"



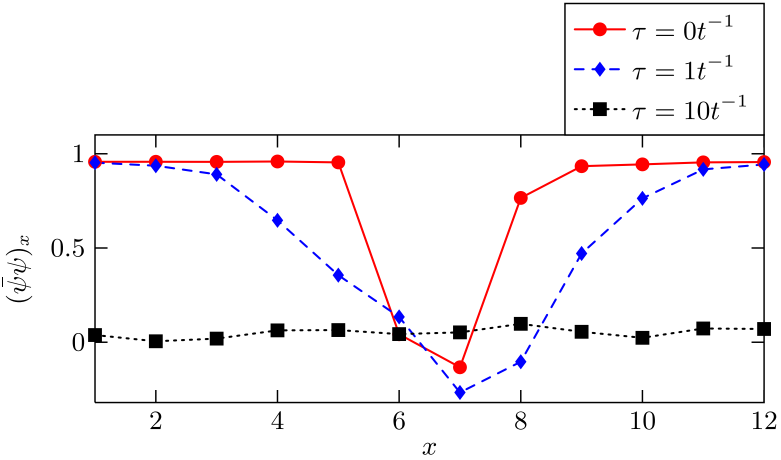


Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin



D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

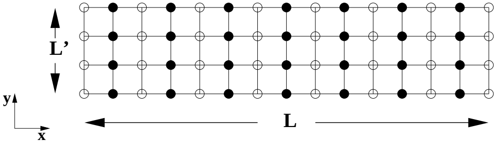
Expansion of a “fireball” mimicking a hot quark-gluon plasma



How to reach the continuum limit?

Ladder of $SU(N)$ quantum spins $[T_x^a, T_y^b] = i\delta_{xy} f_{abc} T_x^c$ embodied with alkaline-earth atoms.

$$H = -J \sum_{x \in A} [T_x^a T_{x+\hat{1}}^{a*} + T_x^a T_{x+\hat{2}}^{a}] - J \sum_{x \in B} [T_x^{a*} T_{x+\hat{1}}^{a} + T_x^{a*} T_{x+\hat{2}}^{a*}]$$



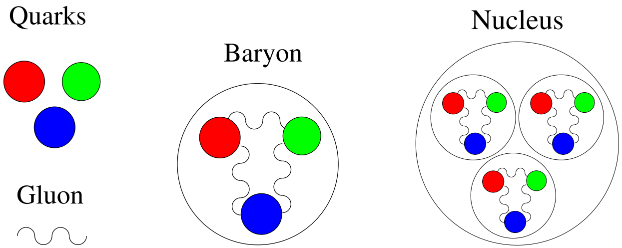
Very large correlation length $\xi \propto \exp(4\pi L' \rho_s / cN) \gg L'$.
Reduction to the $(1+1)$-d $\mathbb{C}P(N-1)$ model at $\theta = n\pi$.

$$S[P] = \int_0^\beta dt \int_0^L dx \, \mathrm{Tr} \left\{ \frac{1}{g^2} \left[\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P \right] - nP \partial_x P \partial_t P \right\}$$

M. Dalmonte, C. Kraus, C. Laflamme, E. Rico, UJW, P. Zoller, in preparation

Outline

Nuclear Physics from $SU(3)$ QCD



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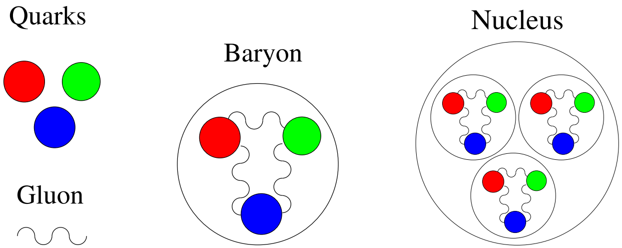
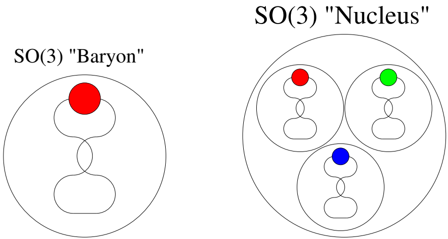


"Nuclear Physics" in an $SO(3)$ lattice gauge theory?

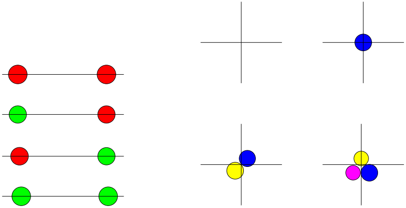


1-d $SO(3)$ quantum link model with adjoint triplet-fermions

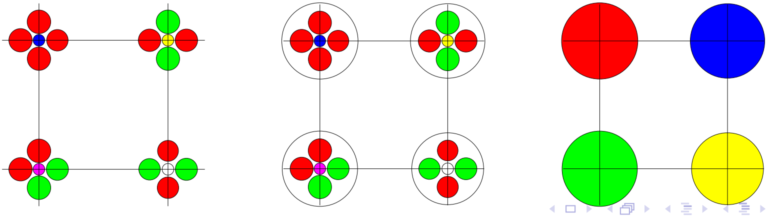
$$H = -t \sum_x \left[\psi_x^{i\dagger} O_{x,x+1}^{ij} \psi_{x+1}^j + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^{i\dagger} \psi_x^i$$

$SO(3)$ quantum links

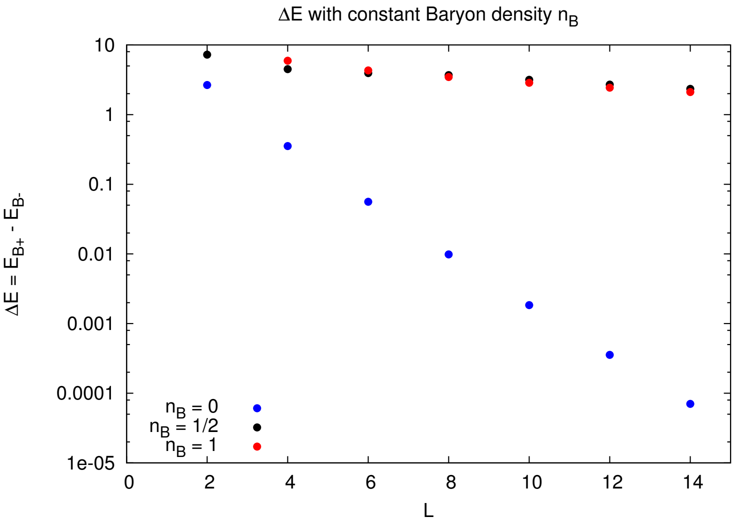
$$O_{x,x+1}^{ij} = \sigma_{x,L}^i \sigma_{x+1,R}^j$$



Encoding manifestly gauge invariant states obeying Gauss' law

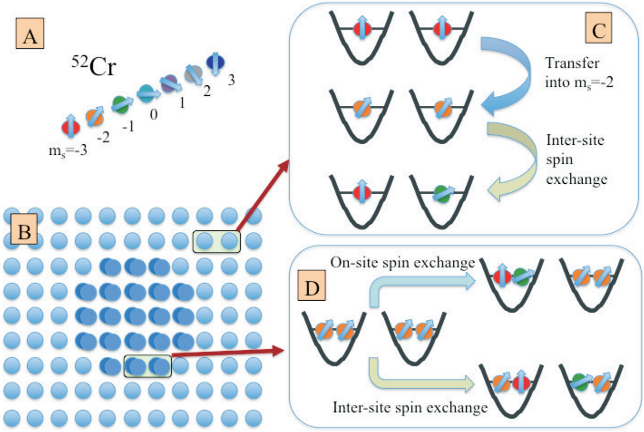


Restoration of chiral symmetry at baryon density $n_B \geq \frac{1}{2}$



M. Dalmonte, E. Rico, D. Banerjee, M. Bögli, P. Stebler, UJW, P. Zoller, in preparation

Implementation with magnetic atoms (e.g. Cr), whose dipolar interactions allow spin-spin interactions without superexchange



A. de Paz, A. Sharma, A. Chotia, E. Marechal, J. H. Huckans, P. Pedri, L. Santos, O. Gorceix, L. Vernac, and B. Laburthe-Tolra, Phys. Rev. Lett. 111 (2013) 185305.

Outline

Conclusions

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• The path towards quantum simulation of QCD will be a long one.

However, with a lot of interesting physics along the way.