Aspects of the phase diagram in (P)NJL-like models

Michael Buballa

TECHNISCHE
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International School of Nuclear Physics, 30th course:
“Heavy-Ion Collisions from the Coulomb Barrier to the Quark-Gluon Plasma”,
Erice, Sicily, Italy, September 16 – 24, 2008

Outline

Part I: Effects of mesonic correlations

collaborators:

- David Blaschke (Wrocław)
- Andrey E. Radzhabov (Dubna)
- Mikhail K. Volkov (Dubna)

reference: Yad. Fiz. 71, no. 10 (2008) 1 [arXiv:0705.0384]

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- PNJL model:
  - order parameters for chiral and deconfinement phase transitions
  - unconfined quarks in the hadronic phase suppressed

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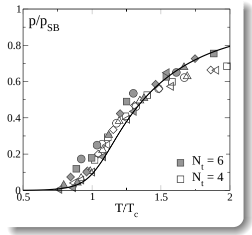

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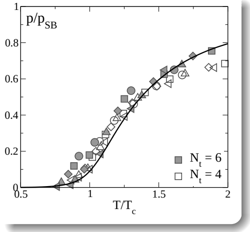

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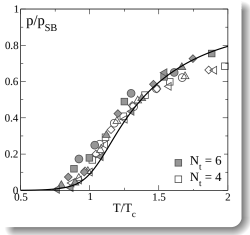

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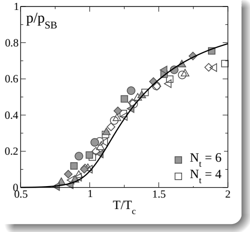

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→ include meson loops!

$1/N_c$ expansion

- thermodynamic potential:

$$\Omega(T) = \Omega^{(0)}(T) + \Omega^{(1)}(T) + \ldots$$

LO: $\Omega^{(0)} = \Omega_{mean\ field}$

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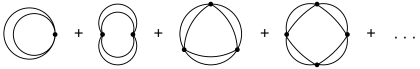

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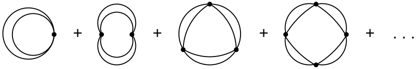
NLO: $\Omega^{(1)} \;=\; \Omega_{ring} \;=$ 

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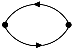
- RPA polarization loop : $\Pi_M =$ 
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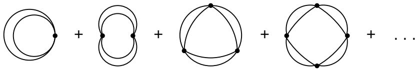

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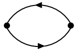
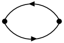
- RPA polarization loop : $\Pi_M =$ 
  - describes mesonic correlations
- chiral symmetry:
  - selfconsistent mean field + RPA (no back reaction!)

quark 4-point interaction

- local 4-point interaction:
  - ➜ (P)NJL model not renormalizable
  - ➜ $\Omega_{ring} =$  divergent,

    even after regularizing the quark loops 
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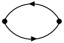
    even after regularizing the quark loops 
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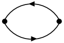
    even after regularizing the quark loops 
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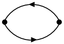
    even after regularizing the quark loops 
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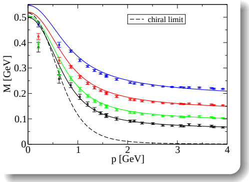
Landau-gauge QCD:

C.S. Fischer, J. Phys. G (2006)



Model

- Lagrangian:

  $$\mathcal{L} = \bar{q}(x)(i\not{D} - m_c)q(x) + \frac{G}{2}[J_\sigma^2(x) + \vec{J}_\pi^{\,2}(x)] - \mathcal{U}(\Phi, \bar{\Phi}, T)$$

- non-local quark currents:

  $$J_I(x) = \int d^4x_1 d^4x_2\, f(x_1) f(x_2)\, \bar{q}(x - x_1)\, \Gamma_I\, q(x + x_2)$$

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- parameters:
  - non-local NJL: vacuum fit (Gomez Dumm et al., PRD (2006))
  - Polyakov loop potential: log. param. (Rößner et al., PRD (2007))

Results

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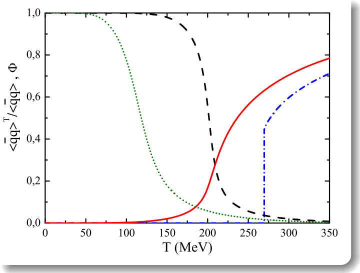
- mean-field results for $\langle \bar{q}q \rangle_{non-pert.}$ and Φ:



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  - $T_\chi \approx 115$ MeV
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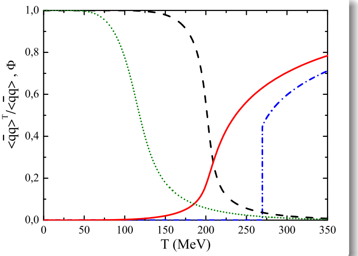

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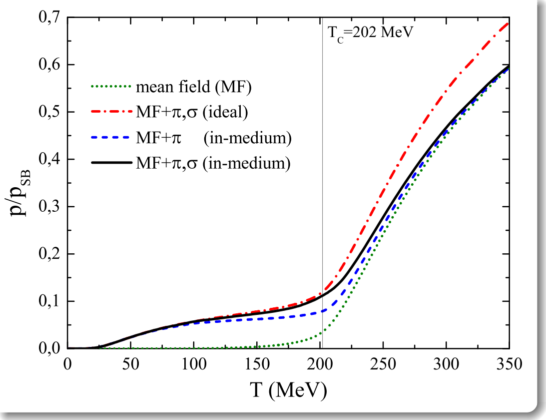
➜ strong synchronization of χ restoration and deconfinement!

Meson contributions to the pressure

- scaled pressure: mean field and beyond



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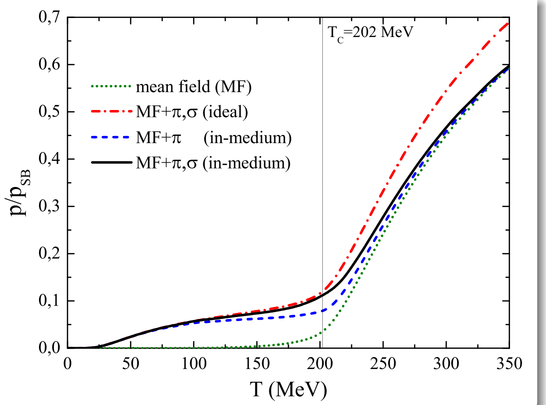

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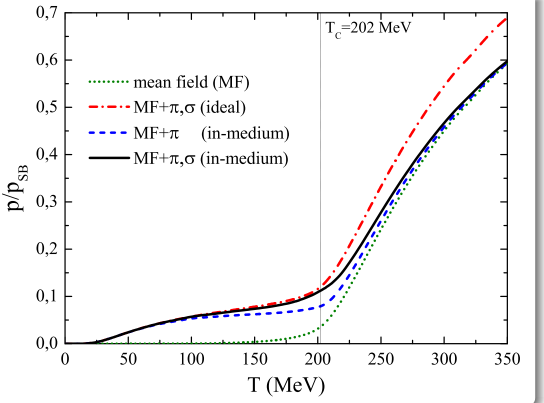

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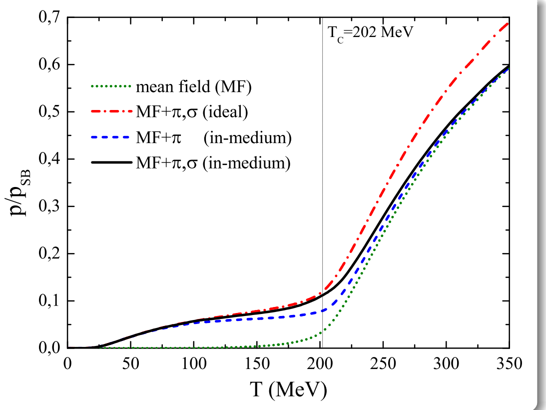

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- $T > T_c$:
  gradual convergence
  to mean field

Lattice calculations

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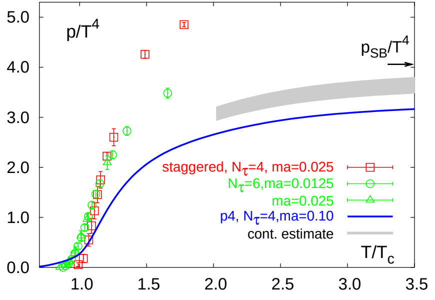

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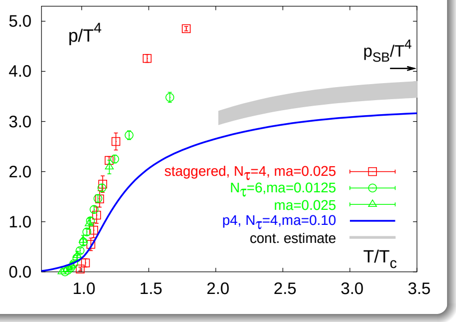

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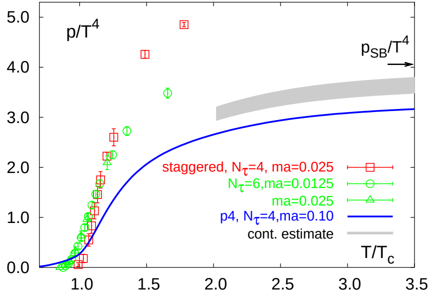

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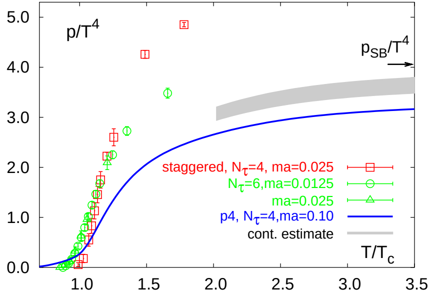

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- integration method:

  ➜ undetermined constant

  choice: $\frac{p}{T^4}\Big|_{T=0.6\ T_c} \stackrel{!}{=} 0$

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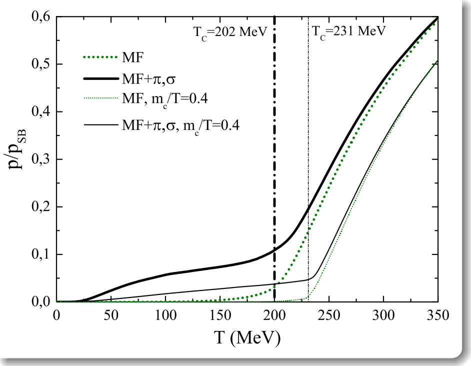

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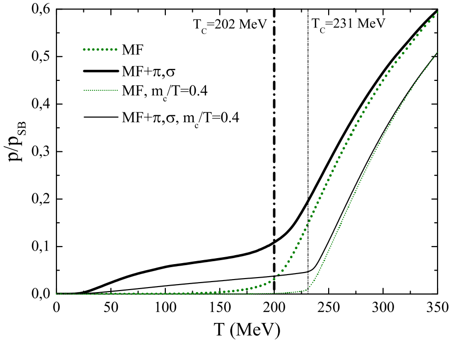

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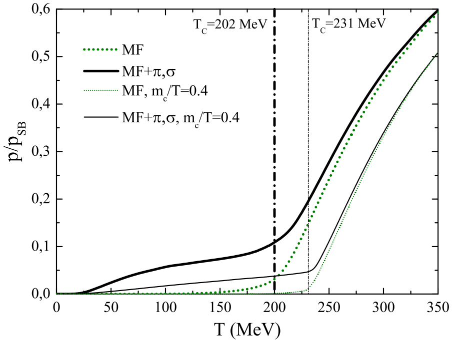

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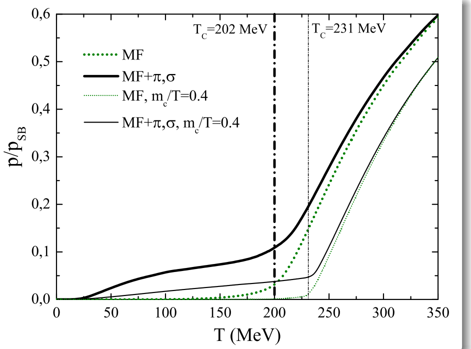

- mean field:
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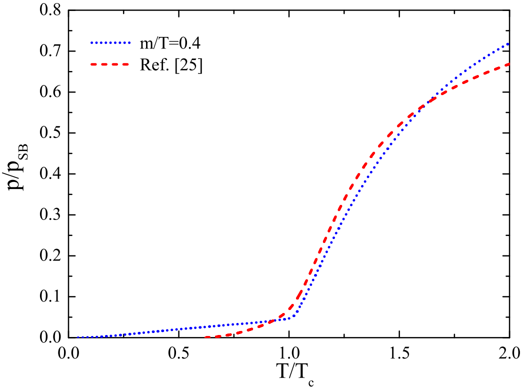
- overall picture unchanged

Comparison with lattice results

- scaled pressure vs. T/T_c:



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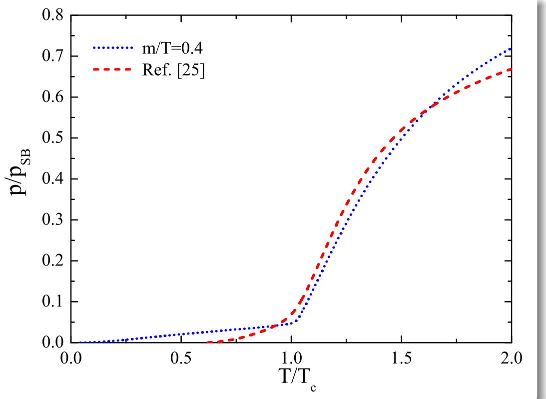

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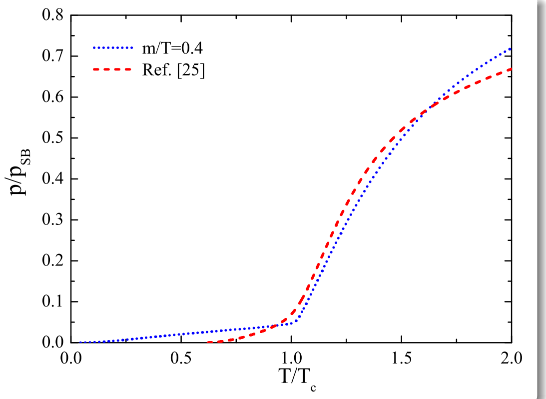

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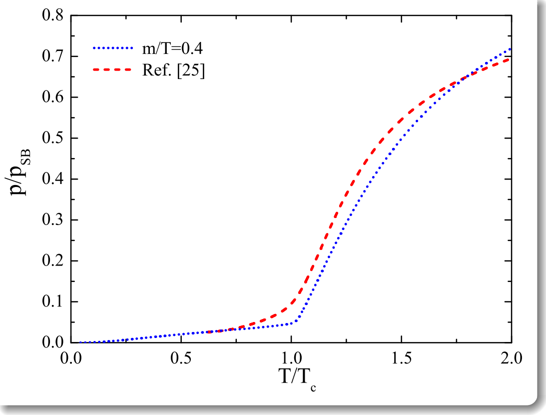

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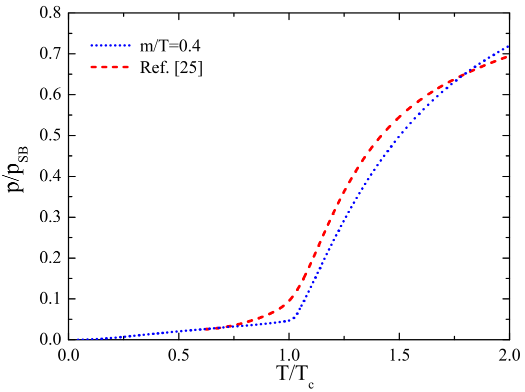

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  ➜ shift lattice points

- $T \lesssim 1.75 T_c$:

  $p_{model} < p_{lattice}$

  ➜ include more resonances!

Outlook 1

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- include further channels

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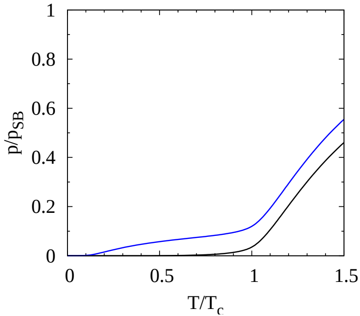
  mean field + ideal π + σ



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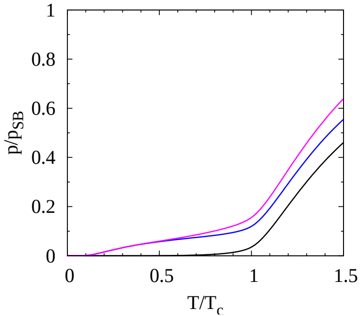

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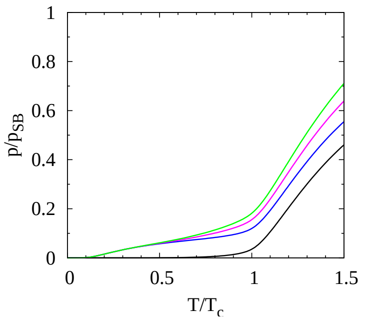

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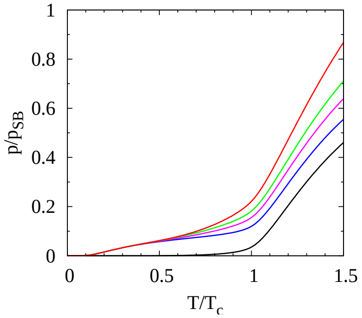

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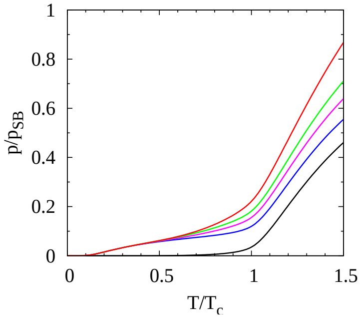

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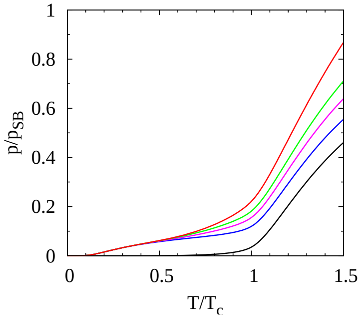

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  + $\omega + \eta + a_1$
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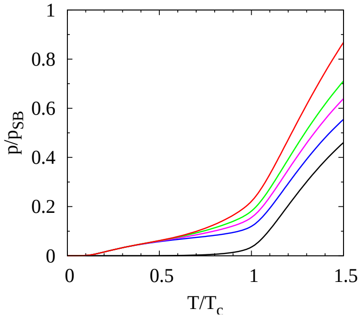
  + ...



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- finite μ

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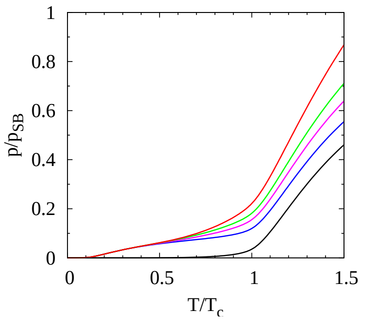

- 3 flavors
- finite μ
- back reaction ?

Part II: Assessing the phase diagram via Taylor expansion

collaborators:

- David Scheffler (Darmstadt)
- Jochen Wambach (Darmstadt)

TECHNISCHE UNIVERSITÄT DARMSTADT

reference: D. Scheffler, BSc thesis, TU Darmstadt, 2007.

Motivation

- Taylor expansion of the pressure in $\frac{\mu}{T}$:

$$\frac{p}{T^4}(T,\mu) = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n,$$

$$c_n(T) = \frac{1}{n!} \frac{\partial}{\partial(\mu/T)^n} \left(\frac{p}{T^4}(T,\mu)\right)\Big|_{\mu=0}$$

  - one way to assess the finite μ regime on the lattice
  - charge conjugation: only even powers

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- lattice calculations:
  - up to order $\left(\frac{\mu}{T}\right)^6$ (e.g., Allton et al. , Phys.Rev. D71 (2005) 054508)
  - How reliable is this?
  - Can we learn anything about the critical endpoint?

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  - How reliable is this?
  - Can we learn anything about the critical endpoint?
- idea: test this in a model

Model

- two-flavor NJL model (without Polyakov loop):

  $$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G\{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\}$$

- mean field thermodynamic potential:

  $$\Omega(T,\mu;M) = \frac{(M-m)^2}{4G} - \frac{N_f N_c}{\pi^2}\Big\{ \int_0^{\Lambda} dk\, k^2\, E_k + \int_0^{\infty} dk\, k^2 \Big[T\ln(1+e^{-\frac{E_k-\mu}{T}}) + T\ln(1+e^{-\frac{E_k+\mu}{T}}) \Big] \Big\}$$

  - correct SB limit: keep thermal part unregularized
  - physical solution: $\Omega(T,\mu) \equiv \min\, \Omega(T,\mu;M)$ w.r.t. M
  - Taylor expand $\Omega(T,\mu)$ (implicit μ-dependences!)

Results

- realistic model parameters:
  - ➜ $f_\pi = 92.4$ MeV, $m_\pi = 135$ MeV, $\langle \bar{u}u \rangle^{1/3} = -241$ MeV

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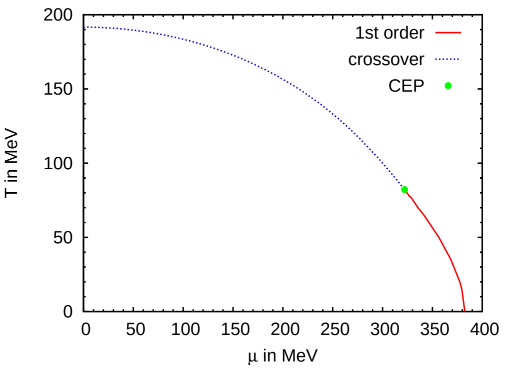

- crossover line:

  maxima of

  $\frac{\chi_{mm}}{T^2} = -\frac{1}{T^2}\frac{\partial^2 \Omega}{\partial m^2}$

  along $\frac{\mu}{T} = const.$
- endpoint:

  $T_c = 82.2$ MeV

  $\mu_c = 322.0$ MeV

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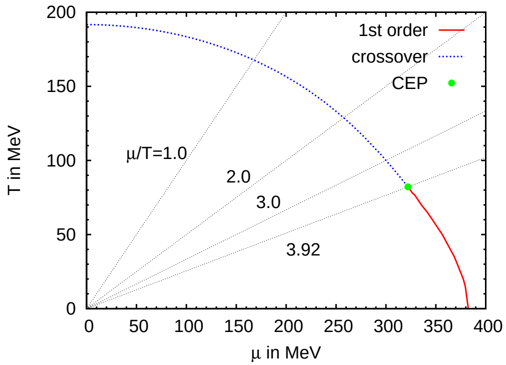

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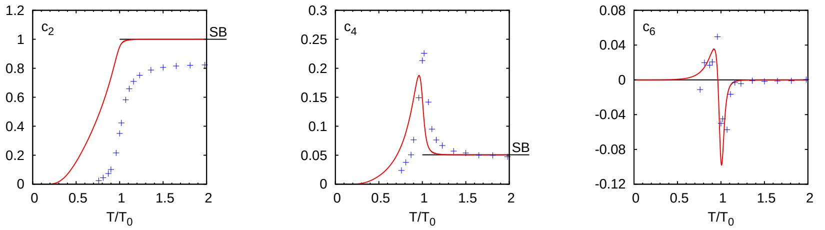
  $\mu_c = 322.0$ MeV

  $\frac{\mu_c}{T_c} = 3.92$

Results

- Taylor coefficients:



➜ qualitative agreement with lattice results (Allton et al. , PRD (2005))

Results

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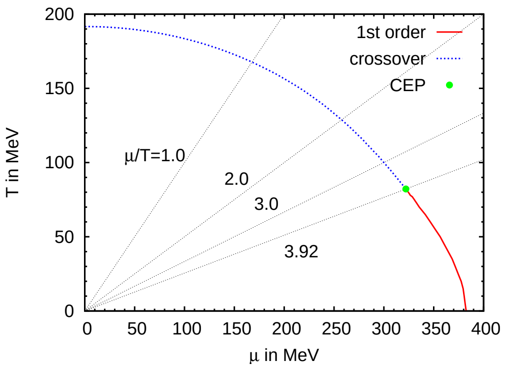

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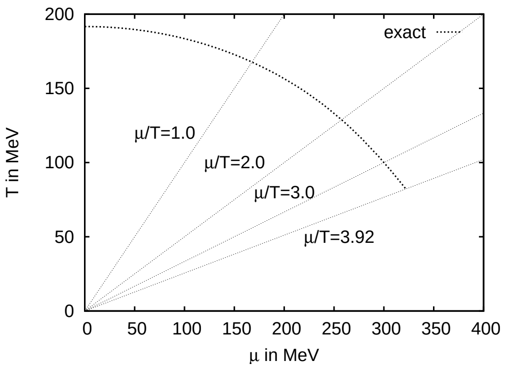
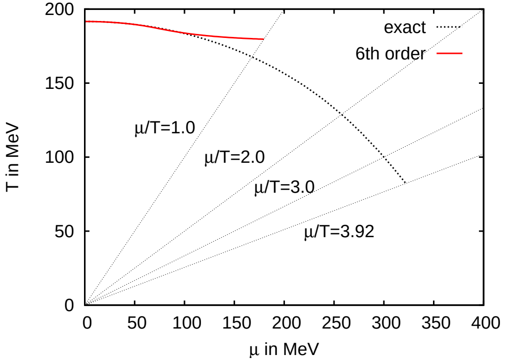


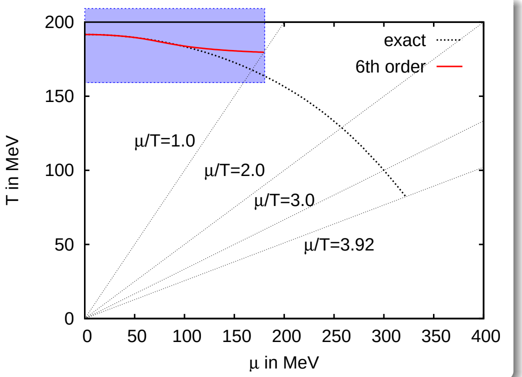
Results

- “exact” vs. 6th order:



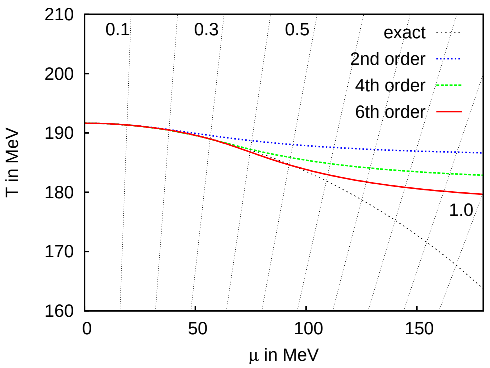
Results

- zoom in:



Results

- “exact” vs. Taylor expansion:



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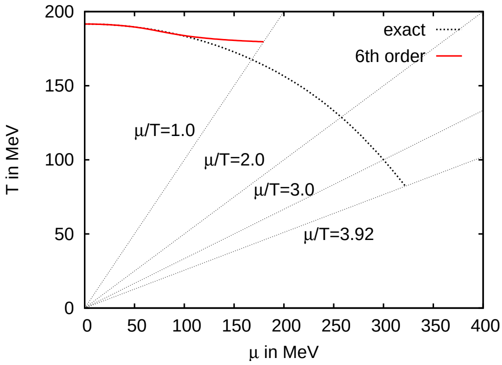
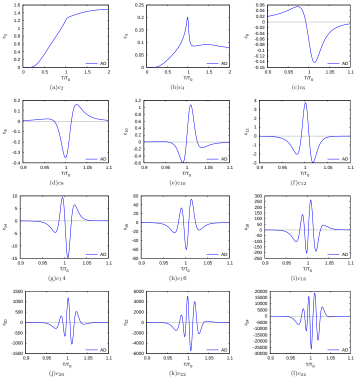


Recent developments

- arbitrary high Taylor coefficients via “algorithmic differentiation”: (B.-J. Schaefer and M. Wagner for the QM model)



(a) c_2 (b) c_4 (c) c_6 (d) c_8 (e) c_{10} (f) c_{12} (g) c_{14} (h) c_{16} (i) c_{18} (j) c_{20} (k) c_{22} (l) c_{24}

FIG. 1: Taylor coefficients c_2 to c_{24} [Quark-Meson model at physical masses ($m_\sigma = 800\,\text{MeV}$), see arXiv:0808.1491]

Outlook 2

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- include Polyakov loop
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- more chemical potentials (μ_I, μ_s)
- try to fit lattice data as good as possible
  - unphysical quark masses
  - add further channels
- play around with the endpoint
  - shift it to smaller $\frac{\mu}{T}$
  - remove it completly

Part III: The critical surface

collaborator:

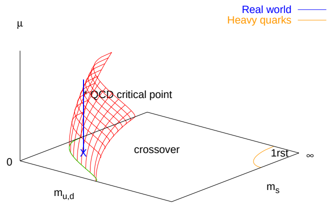
- A. Gabriela Grunfeld (Buenos Aires)

Motivation

- critical surface:

(de Forcrand & Philipsen, JHEP 0701 (2007) 077))

"standard scenario"



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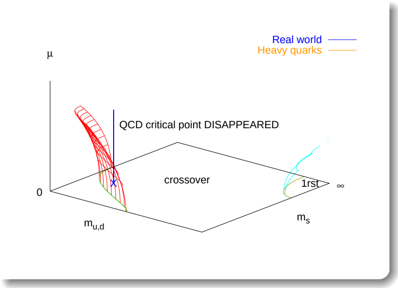

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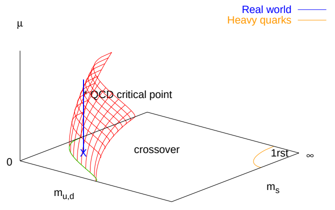
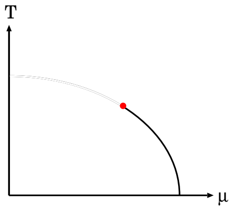


- physical masses



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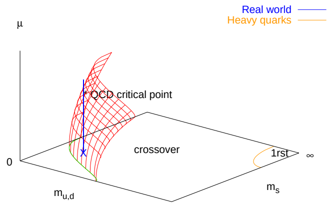
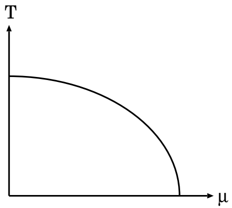


- small masses



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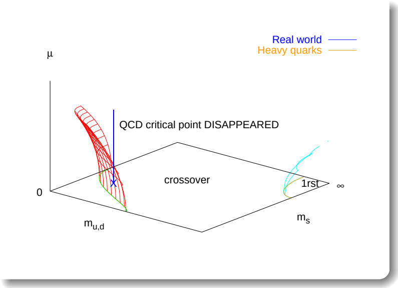

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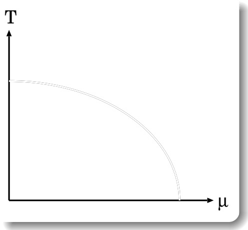

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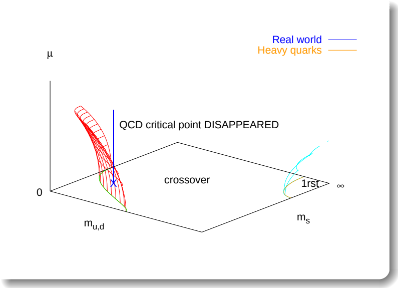

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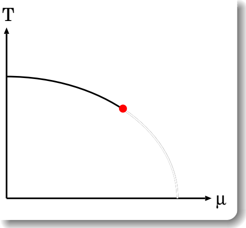

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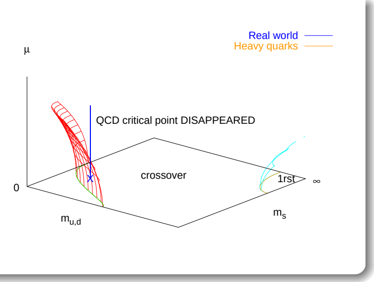

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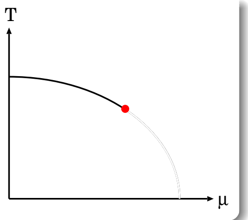


- Is there a way to realize the non-standard scenario in a model?

Model

- three-flavor PNJL model:

$$\mathcal{L} = \bar{\psi}(i\not{D} - \hat{m})\psi + \mathcal{L}_4 + \mathcal{L}_6 - \mathcal{U}(\Phi, \bar{\Phi})$$

  - 4-point interaction:
    $\mathcal{L}_4 = \frac{g_s}{2}\left\{(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2\right\}$
  - 't Hooft interaction:
    $\mathcal{L}_6 = g_D\left\{ \det_f[\bar{\psi}(1+\gamma_5)\psi] + h.c.\right\}$
  - Polyakov loop potential:

$$\mathcal{U} = b\,T\left\{54e^{-a/T}\Phi\bar{\Phi} + \ln[1 - 6\Phi\bar{\Phi} - 3\Phi(\bar{\Phi})^2 + 4(\Phi^3 + \bar{\Phi}^3)]\right\}$$

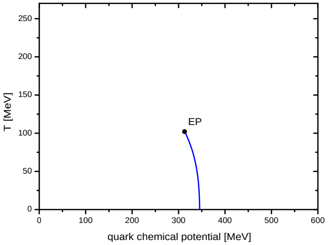
- parameters:
  - NJL part: g_s, g_D, Λ (+ quark masses) ➜ from Hatsuda & Kunihiro
  - Polyakov loop potential: a, b ➜ from Fukushima

Results

- phase diagrams

  “physical” masses:

  ($m_{ud} = 5.5$ MeV, $m_s = 135.7$ MeV)



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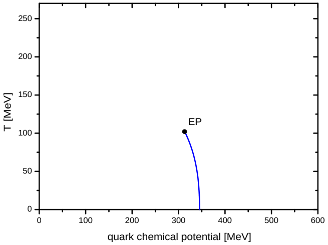

small masses:

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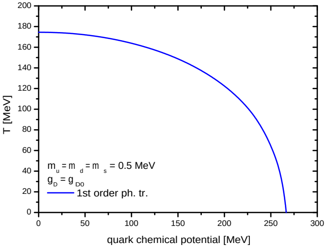

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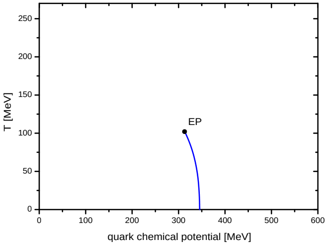

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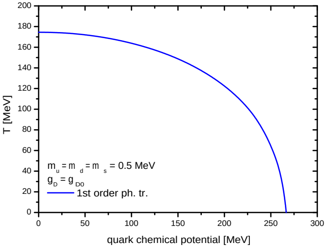
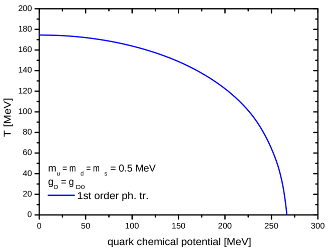


→ standard scenario

Small masses: $m_{ud} = m_s = 0.5$ MeV

- effect of the t' Hooft interaction:

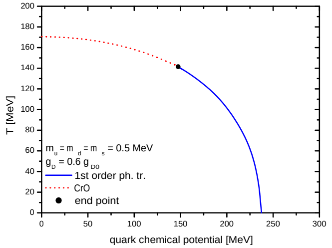
$$g_D = g_D^{(0)}$$



Small masses: $m_{ud} = m_s = 0.5$ MeV

- effect of the t' Hooft interaction:

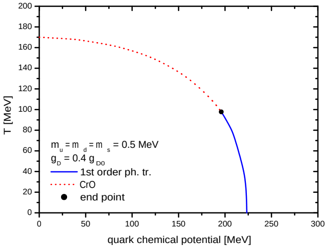
$$g_D = 0.6\, g_D^{(0)}$$



Small masses: $m_{ud} = m_s = 0.5$ MeV

- effect of the t' Hooft interaction:

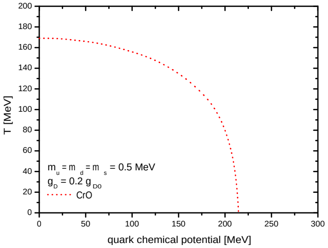
$$g_D = 0.4\, g_D^{(0)}$$



Small masses: $m_{ud} = m_s = 0.5$ MeV

- effect of the t' Hooft interaction:

$$g_D = 0.2\, g_D^{(0)}$$



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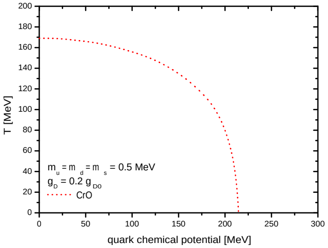
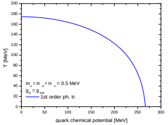
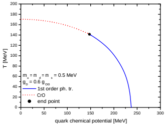
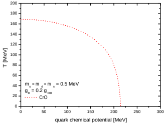






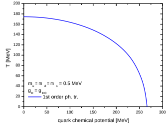


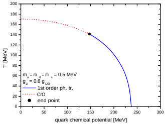
Small masses: $m_{ud} = m_s = 0.5$ MeV

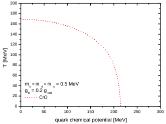
➜ idea: μ-dependent coupling constant!

$$g_D(\mu) = e^{-\mu^2/\mu_0^2}\, g_D^{(0)}$$

(K. Fukushima, PRD 77 (2008))







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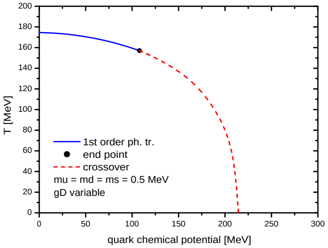

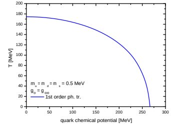

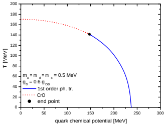

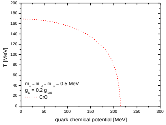

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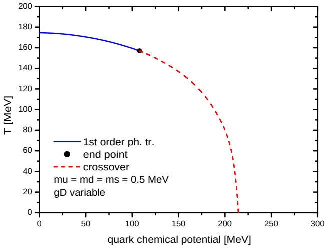
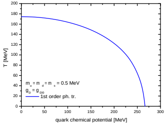
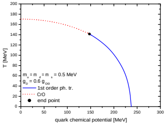
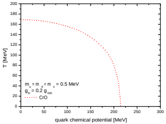




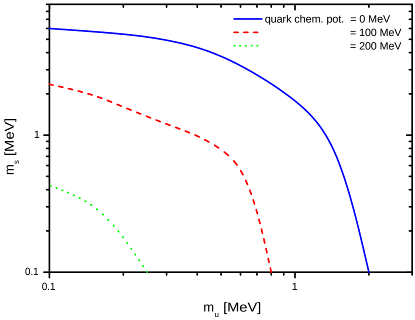




➜ nonstandard scenario!

Critical surface

- very preliminary result for the critical lines at different μ:



Conclusions & outlook 3

- The non-standard scenario can be realized in a (relatively) simple model!

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