In-medium Hadrons in dense QCD

T. Hatsuda (University of Tokyo)

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- Introduction to hot/dense QCD[1]
- Ginzburg-Landau-Wilson approach to QCD phase transitions[1,2]
- Spectral continuity in dense QCD[3,4]
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(1) Yagi, T.H. & Miake, "Quark-Gluon Plasma" (Cambridge Univ. Press, 2005) Chap.2, Chap.6.

(2) T.H., Tachibana, Yamamoto & Baym, Phys. Rev. Lett. 97 (2006) 122001.

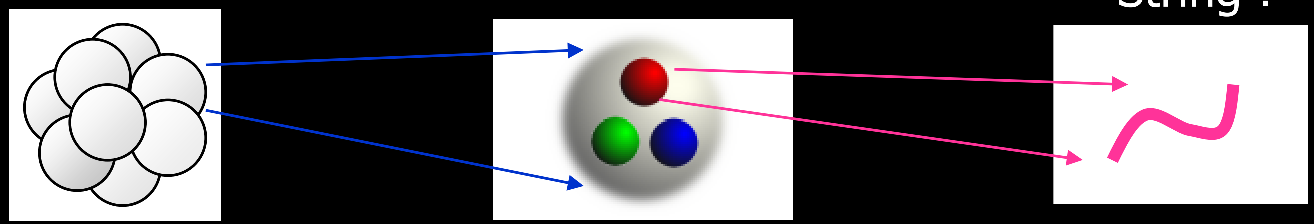
(3) Yamamoto, Tachibana, T.H. & Baym, Phys. Rev. D 76 (2007) 074001.

(4) T.H., Tachibana & Yamamoto, Phys. Rev. D 78 (2008) 011501.

Erice, Sep. 18, 2008

Building blocks of baryonic matter

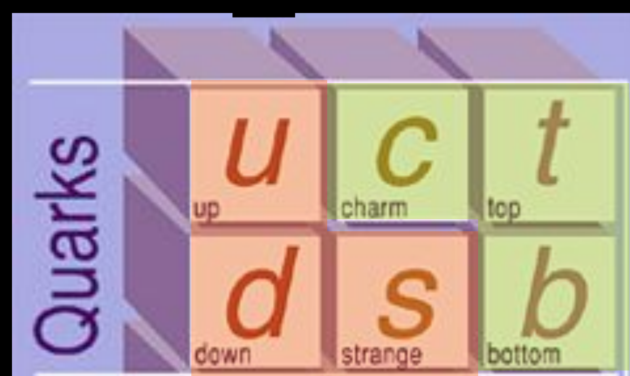
Nuclei	Hadron	String ?



$r \sim 10$ [fm]	$r \sim 1$ [fm]	$r < 10^{-4}$ [fm]

Light quarks

$m_u \sim 3$ MeV
$m_d \sim 5$ MeV
$m_s \sim 100$ MeV



Heavy quarks

$m_c \sim 1.3$ GeV
$m_b \sim 4.3$ GeV
$m_t \sim 171$ GeV

$$m_{\rm u,d,s} < (m_\rho, T_c, \mu_c) < m_{\rm c,b,t}$$

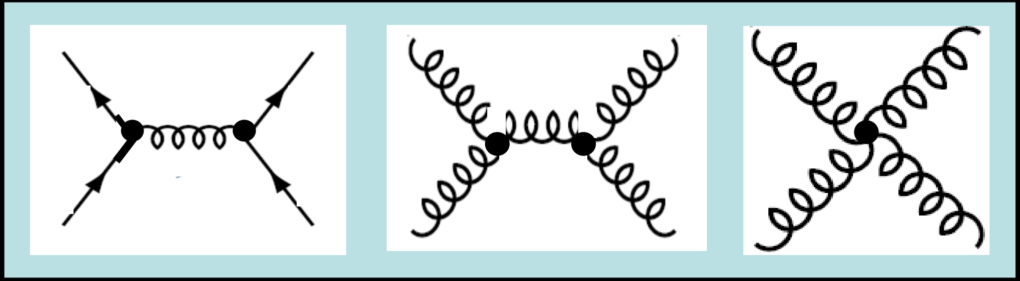
Quantum Chromo Dynamics

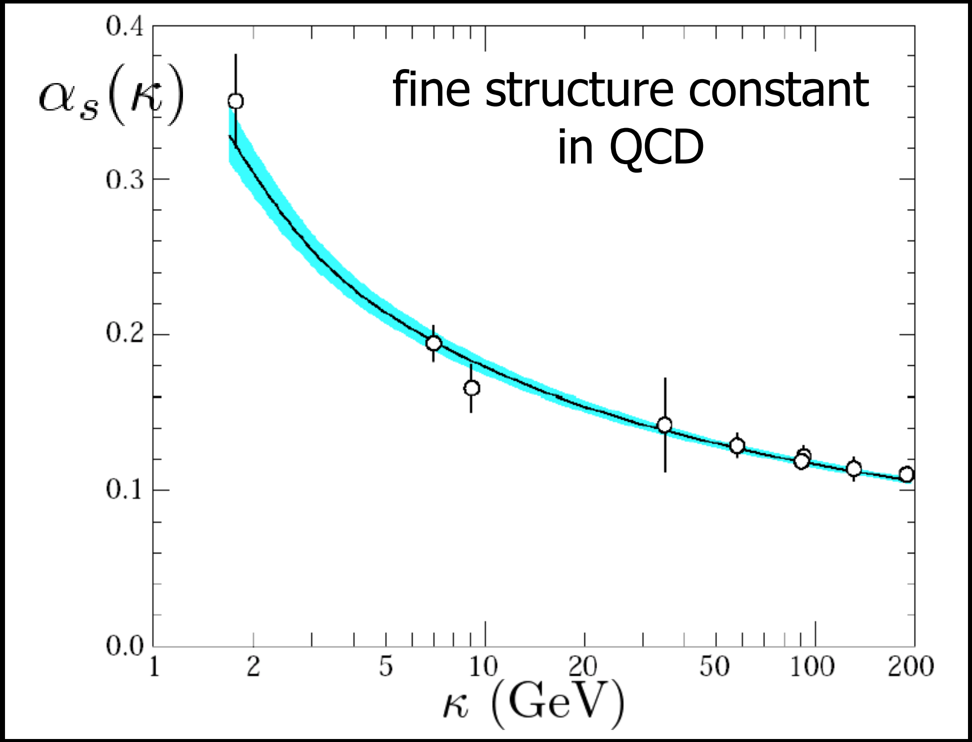
$$L = -\frac{1}{4} G^{a}_{\mu\nu} G_{a}^{\mu\nu} + \bar{q}\gamma^{\mu}(i\partial_{\mu} - g t^{a} A^{a}_{\mu})q - m\bar{q}q$$

QED: U(1) gauge theory for electric charge (**e**)

QCD: SU(3) gauge theory for color charge (**R**,**B**,**G**)

Y. Nambu





- SU(3) YM for strong interaction (Nambu '66)
- Asymptotic freedom (Gross, Wilczek & Politzer '73)
- Confinement criterion (Wilson '74)

QCD vacuum and its symmetry

Chiral basis : $q_{\rm L} = \frac{1}{2}(1-\gamma_5)q, \quad q_{\rm R} = \frac{1}{2}(1+\gamma_5)q$

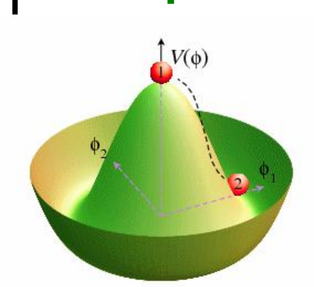
QCD Lagrangian : $\mathcal{L}_{\rm cl} = \mathcal{L}_{\rm cl}(q_{\rm L}, A) + \mathcal{L}_{\rm cl}(q_{\rm R}, A)$

classical QCD symmetry (m=0)

$$\mathcal{G} = SU(3)_C \times [SU(N_f)_L \times SU(N_f)_R] \times U(1)_B \times U(1)_A$$

Quantum QCD vacuum (m=0)

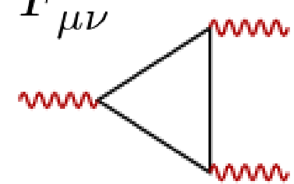
Chiral condensate : spontaneous mass generation



$$\langle \bar{q}_R q_L \rangle \neq 0$$

Axial anomaly : quantum violation of $U(1)_A$

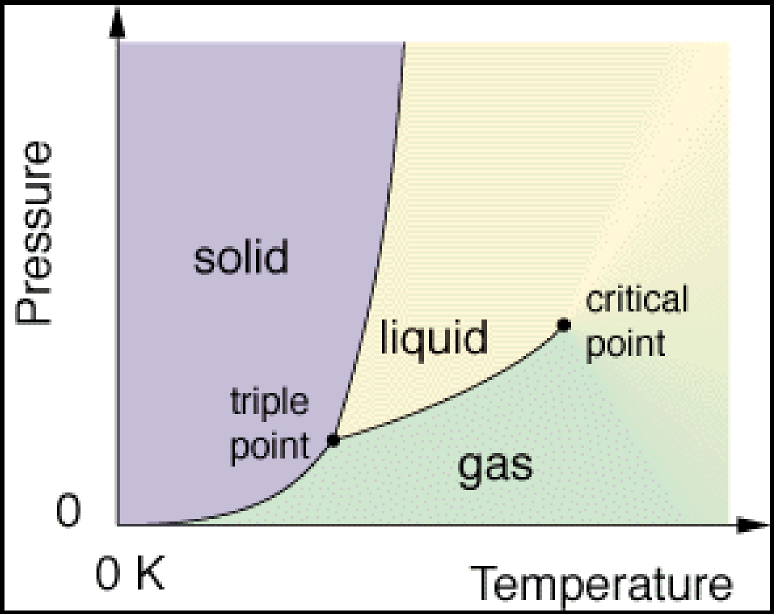
$$\partial_\mu J^\mu_A = -2N_f \frac{\alpha_s}{8\pi} F^{\mu\nu}_a \tilde{F}^a_{\mu\nu}$$



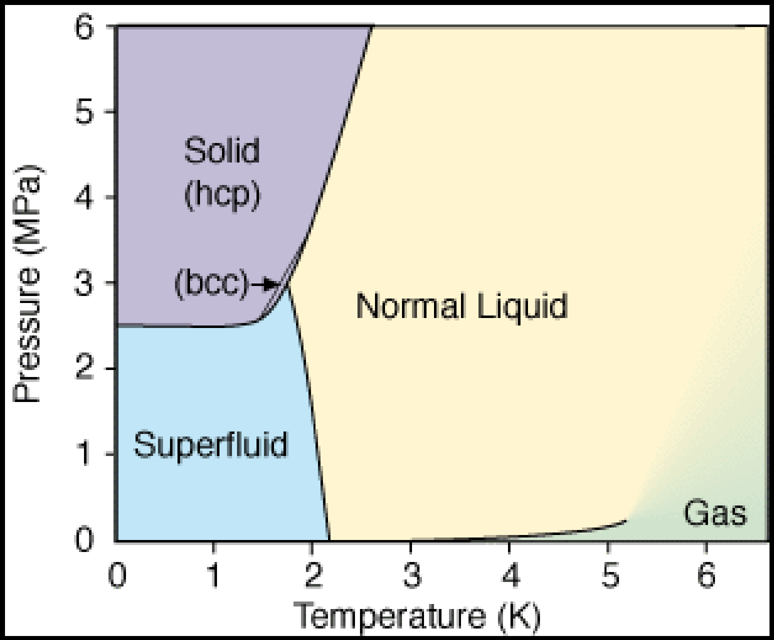
$$SU(3)_C \times SU(N_f)_{L+R} \times U(1)_B$$

Phases in QCD

H_2O



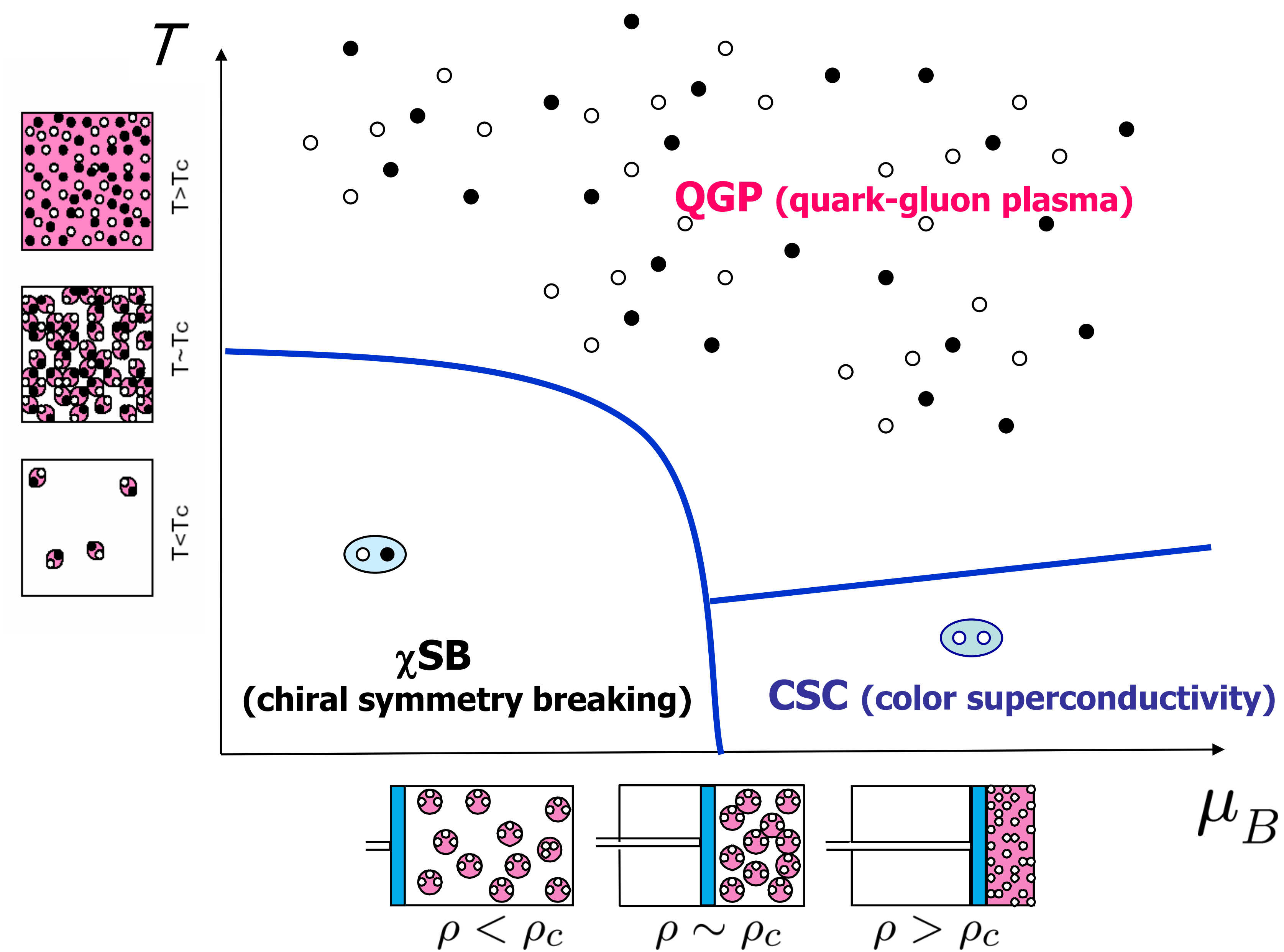
^4He



http://boojum.hut.fi/research/theory/typicalpt.html

"The whole is more than the sum of its parts." Aristotle, Metaphysica 10f-1045a

Schematic phase diagram in QCD

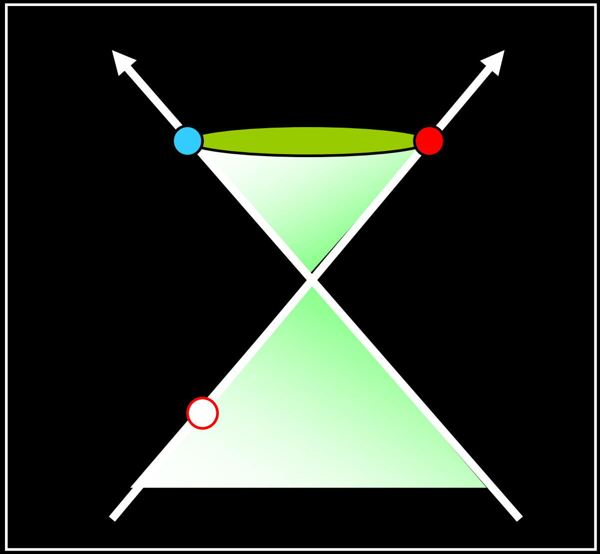


Dirac mass vs. Majorana mass

$$\Psi = (q, q^C)^t$$

$$L_{\mathrm{eff}} = \frac{1}{2}\overline{\Psi}\begin{pmatrix} i\gamma\cdot\partial - \Phi & \overline{\Delta} \\ \Delta & i\gamma\cdot\partial + \Phi \end{pmatrix}\Psi$$

Nambu-Gor’kov (cond-mat)
= Hartree-Fock-Bogoliubov (nucl-th)
= Dirac-Majorana (hep-ph)



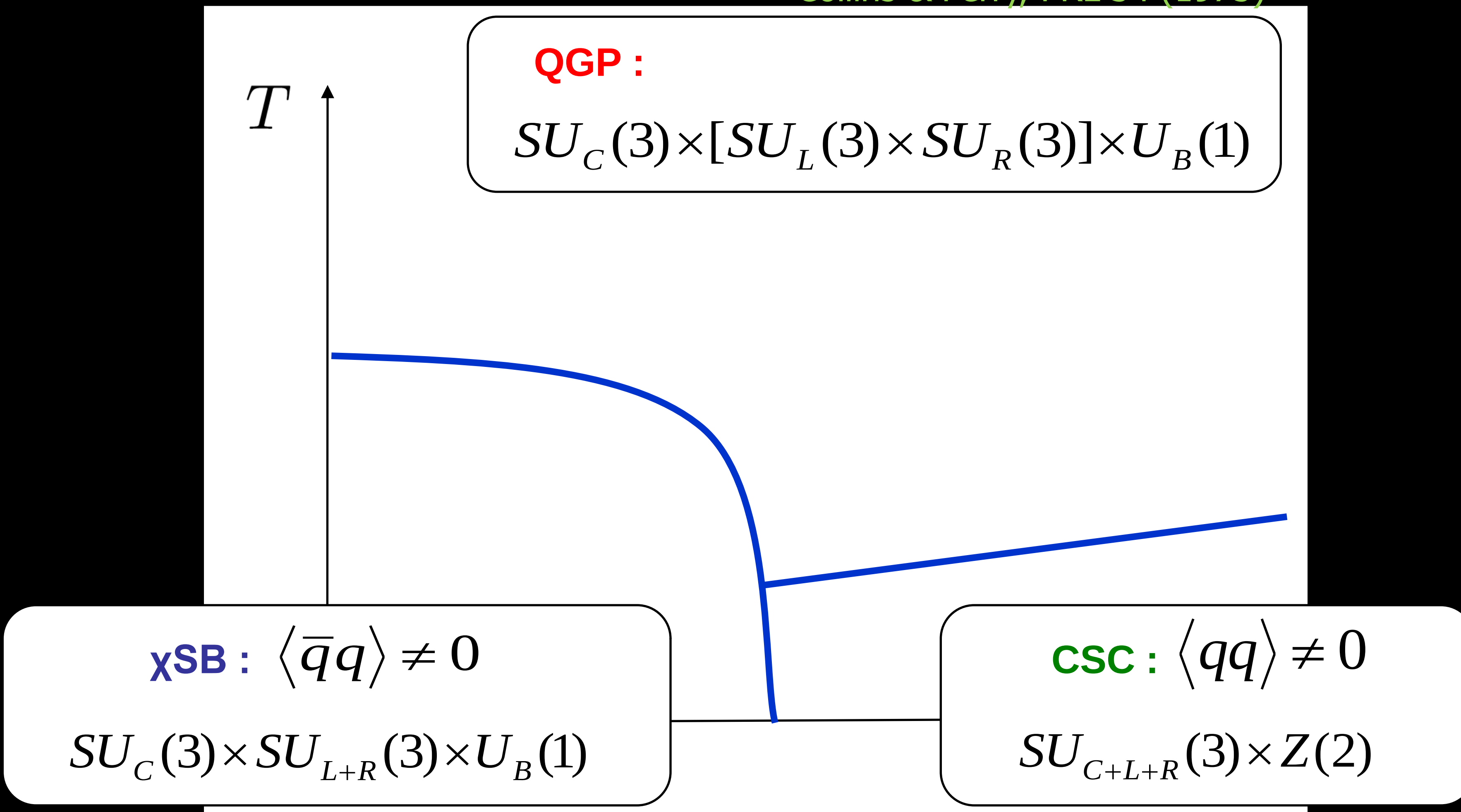
$$\Phi_{ij} \sim \left\langle \bar{q}_j q_i \right\rangle, \quad \Delta_{ij}^{ab} \sim \left\langle q_i^a C q_j^b \right\rangle$$

Dirac mass   Majorana mass

$$SU(3)_C \times \left[SU(3)_L \times SU(3)_R \right] \times U(1)_B$$

Symmetry realization in hot/dense QCD (for $m_{u,d,s}$=0 case)

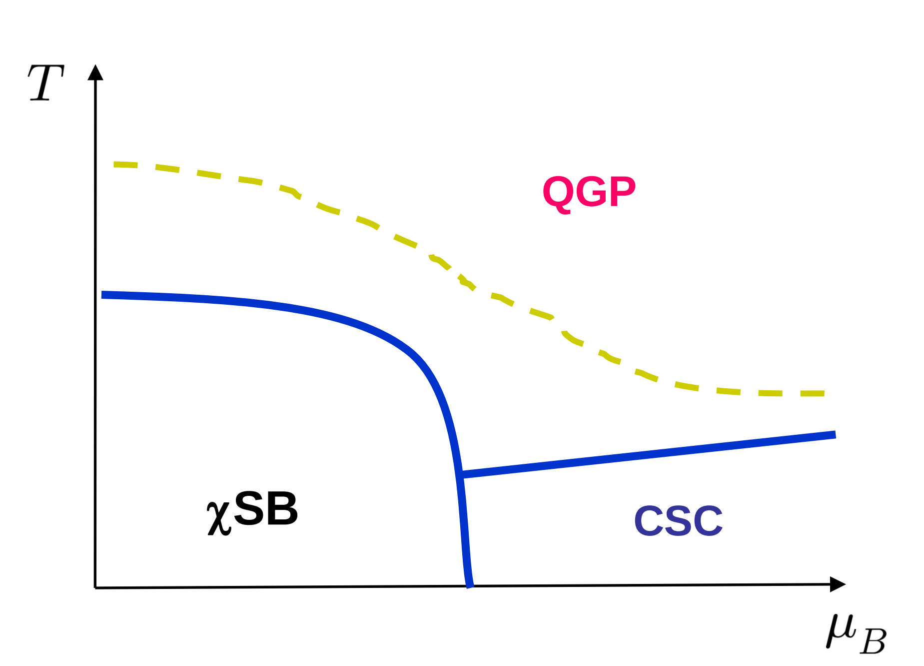
Collins & Perry, PRL 34 (1975)

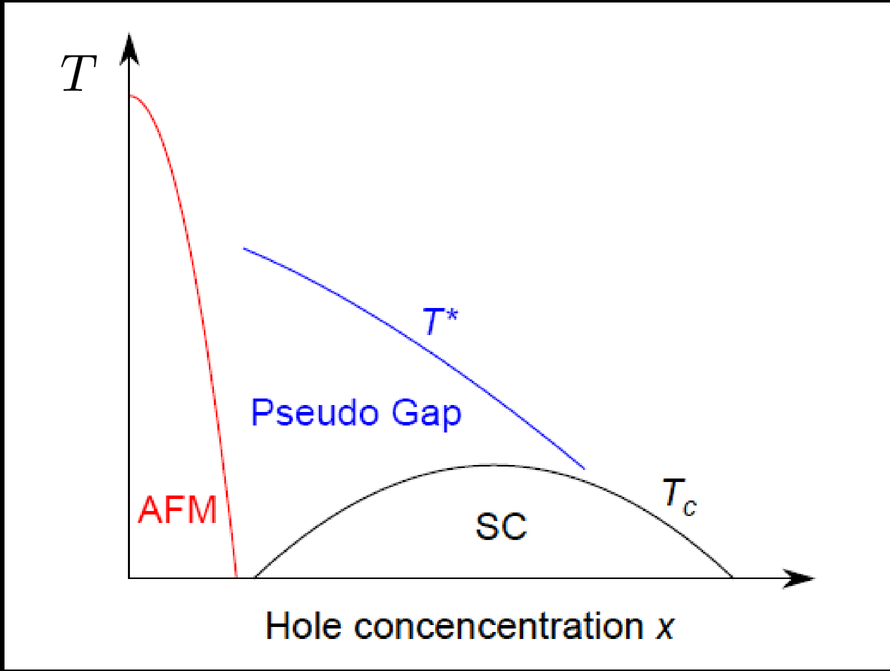


Nambu, PRL 4 (1960)

Alford, Rajagopal & Wilczek, NP B537 (1999)

QCD and high temperature superconductivity (HTS)





<u>Common features in QCD, HTS, and cold atoms</u>

1. Competition between different orders
2. Strong coupling

- Babaev, Int. J. Mod. Phys. A16 ('01)
- Kitazawa, Nemoto, Kunihiro, PTP ('02)
- Abuki, Itakura & Hatsuda, PRD ('02)
- Chen, Stajic, Tan & Levin, Phys. Rep. ('05)
- Baym, Hatsuda, Tachibana & Yamamoto (2008)

PRL 97, 122001 (2006) PHYSICAL REVIEW LETTERS week ending 22 SEPTEMBER 2006

New Critical Point Induced By the Axial Anomaly in Dense QCD

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We study the interplay between chiral and diquark condensates within the framework of the Ginzburg-Landau free energy, and classify possible phase structures of two and three-flavor massless QCD. The QCD axial anomaly acts as an external field applied to the chiral condensate in a color superconductor and leads to a crossover between the broken chiral symmetry and the color superconducting phase, and, in particular, to a new critical point in the QCD phase diagram.

DOI: 10.1103/PhysRevLett.97.122001 PACS numbers: 12.38.−t, 26.60.+c

PRL 99, 130406 (2007) PHYSICAL REVIEW LETTERS week ending 28 SEPTEMBER 2007

Superfluidity and Magnetism in Multicomponent Ultracold Fermions

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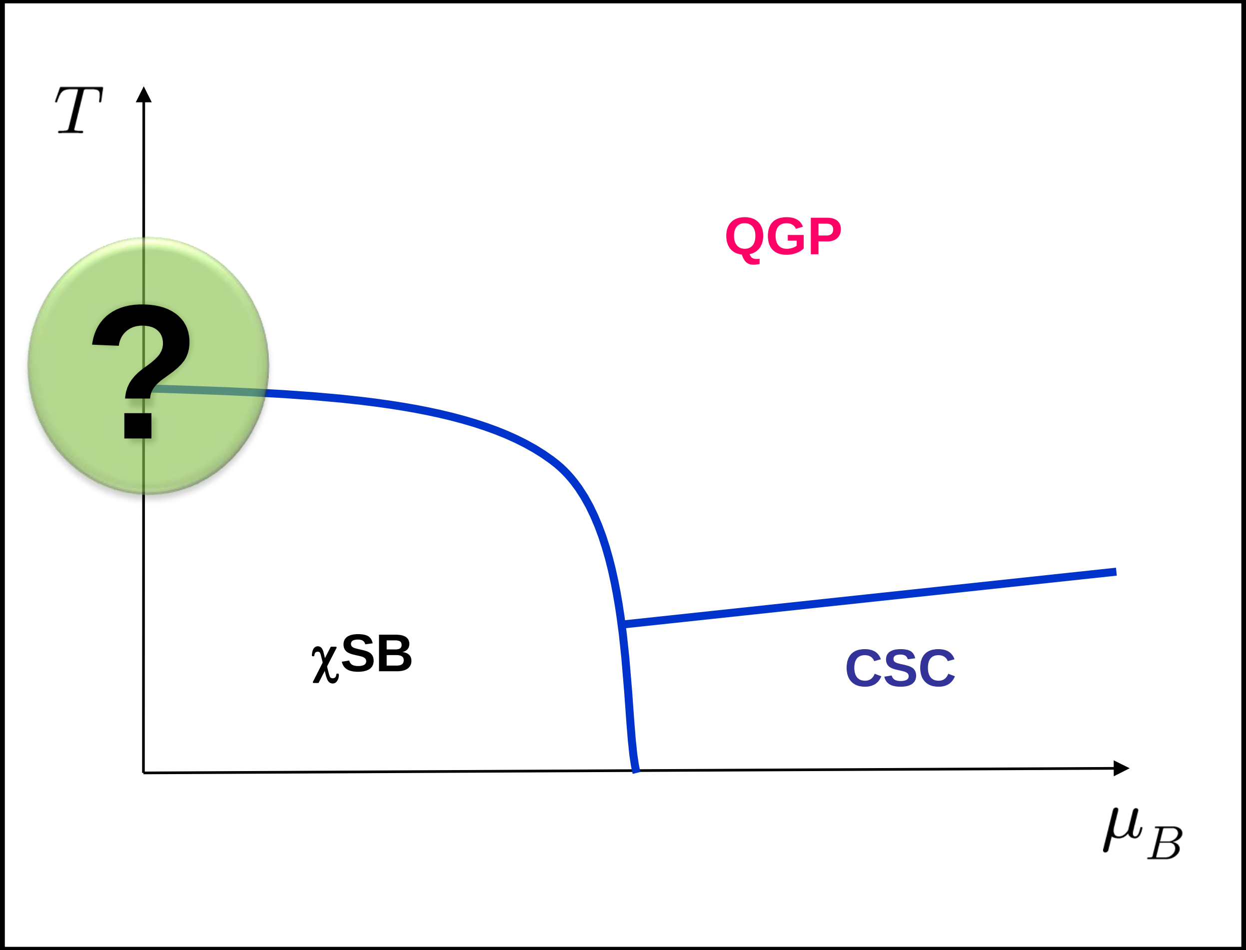
(Received 2 May 2007; published 28 September 2007)

We study the interplay between superfluidity and magnetism in a multicomponent gas of ultracold fermions. Ward-Takahashi identities constrain possible mean-field states describing order parameters for both pairing and magnetization. The structure of global phase diagrams arises from competition among these states as functions of anisotropies in chemical potential, density, or interactions. They exhibit first and second order phase transition as well as multicritical points, metastability regions, and phase separation. We comment on experimental signatures in ultracold atoms.

DOI: 10.1103/PhysRevLett.99.130406 PACS numbers: 05.30.Jp, 03.75.Mn, 03.75.Ss

Chiral Transition at Finite T



How to study QCD phase transition ?

Ginzburg-Landau-Wilson (GLW) approach : model independent, analytic

1. Topological structure of the phase diagram
2. Order of the phase transition
3. Critical properties

Recipe

$$Z = \int [d\sigma] \exp\left(- \int d\mathbf{x}\, \mathcal{L}_{\rm eff}(\sigma(\mathbf{x}); K) \right)$$

$\sigma(\mathbf{x})$: Order parameter field

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\nabla\sigma)^2 + \sum_n a_n(K)\sigma^n$$

Same symmetry with underlying theory

$K = \{ T, m, \mu, \ldots \}$: External parameters

Ginzburg-Landau = Saddle point approximation

Wilson = Fluctuations in renormalization group method

Caution

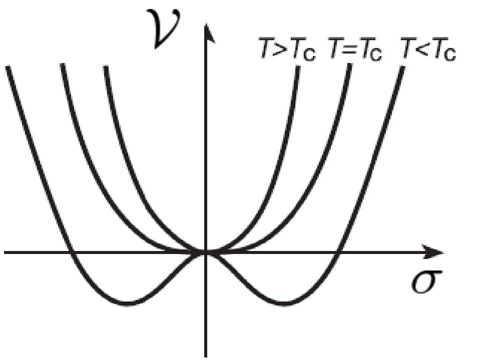
- Valid for continuous or weak 1st order transitions
- Choice of $\sigma(\mathbf{x})$ is an “art”
- Results should be eventually checked by lattice QCD

Some examples of GL potential

- 2nd order phase transition

$$\mathcal{V} = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^4$$

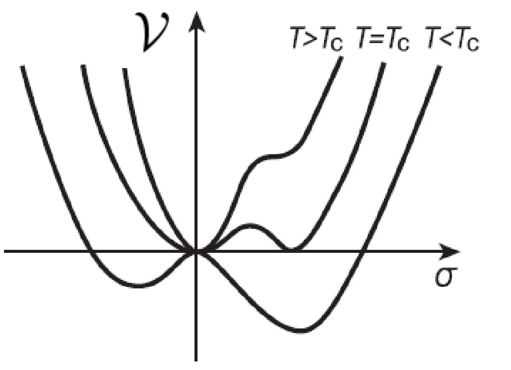
Z(2) Ising model
N_f=2 QCD



- 1st order phase transition

$$\mathcal{V} = \frac{1}{2}a\sigma^2 - \frac{1}{3}c\sigma^3 + \frac{1}{4}b\sigma^4$$

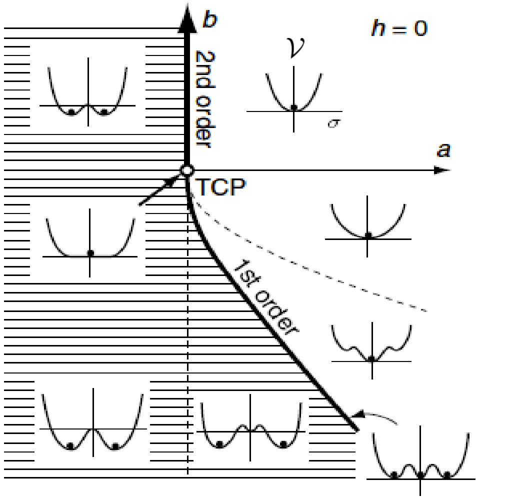
Z(3) Potts model
N_f=3 QCD



- Tri-critical behavior

$$\mathcal{V} = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^4 + \frac{1}{6}c\sigma^6$$

Meta-magnet
N_f=2+1 QCD



GLW analysis of hot QCD

Pisarski & Wilczek ('84)

Symmetry: $SU(3)_C \times [SU(N_f)_L \times SU(N_f)_R] \times U(1)_B \times \cancel{U(1)_A}$

Chiral field: $\Phi_{ij} \sim \frac{1}{2}\bar{q}^j(1-\gamma_5)q^i = \bar{q}_{\mathrm{R}}{}^j q^i_{\mathrm{L}}$

Chiral transformation: $\Phi \to e^{-2i\alpha_A} V_L \, \Phi \, V_R^\dagger$

$$\mathcal{L}_{\mathrm{eff}} = \left.\begin{array}{l} \frac{1}{2}\,\mathrm{tr}\;\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\,\mathrm{tr}\;\Phi^\dagger\Phi \\ + \frac{b_1}{4!}\left(\mathrm{tr}\;\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\,\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2 \end{array}\right\} \mathrm{SU}(N_f)_L \mathrm{x} \mathrm{SU}(N_f)_R \mathrm{x} \mathrm{U}(1)_A$$

$$\left.-\frac{c}{2}\left(\det\Phi + \det\Phi^\dagger\right)\right\} \mathrm{SU}(N_f)_L \mathrm{x} \mathrm{SU}(N_f)_R$$

$$\left.-\frac{1}{2}\,\mathrm{tr}\;h(\Phi+\Phi^\dagger).\right\} \text{quark mass term}$$

Axial anomaly

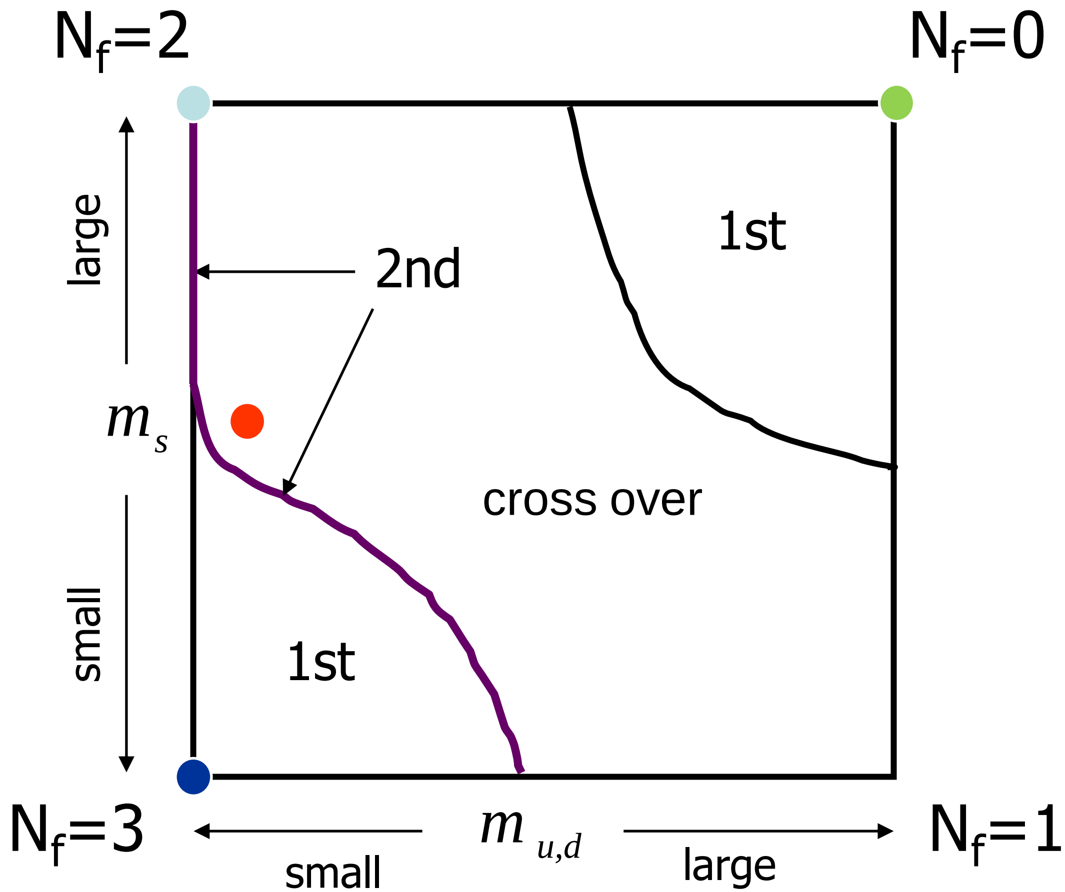
Order of the thermal QCD transition (μ=0)

Svetitsky & Yaffe, NPB210 ('82)

Pisarski and Wilczek, PRD29 ('84)

$$\mathcal{V}=\frac{a}{2}\phi^2+\frac{b}{4}\phi^4-h\phi$$

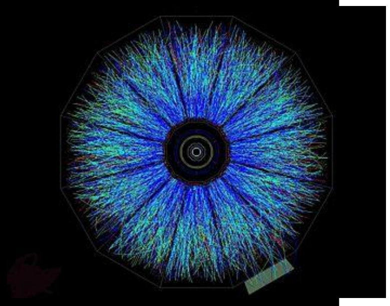
$$\mathcal{V}=\frac{a}{2}L^2-\frac{c}{3}L^3+\frac{b}{4}L^4-hL$$

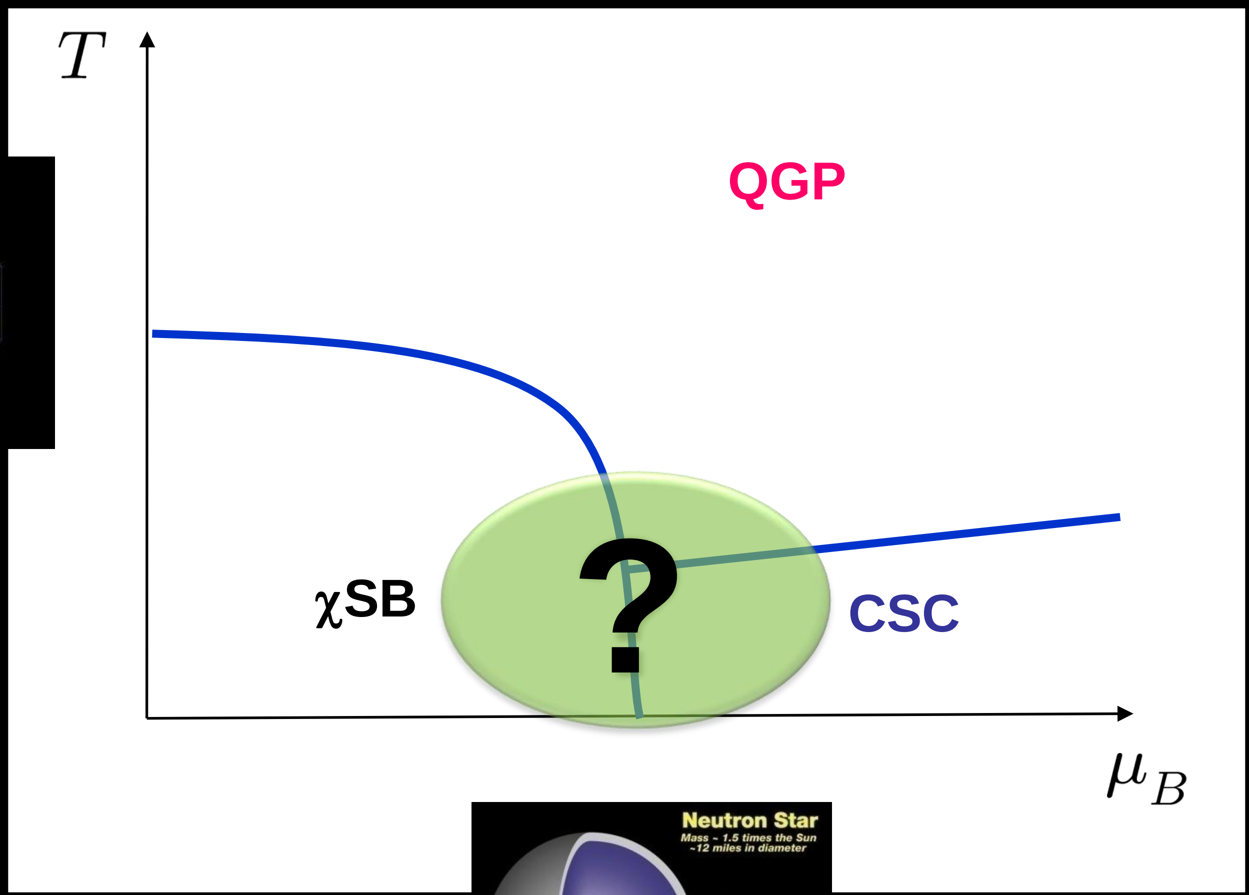


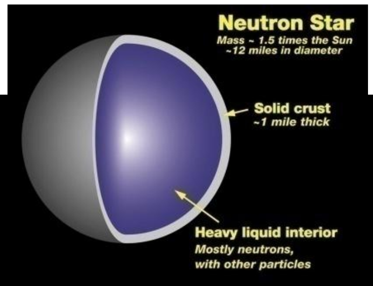
$$\mathcal{V}=\frac{a}{2}\sigma^2-\frac{c}{3}\sigma^3+\frac{b}{4}\sigma^4-h\sigma$$

$$\mathcal{V}=-A\chi+\frac{a}{2}\chi^2+\frac{b}{4}\chi^4$$

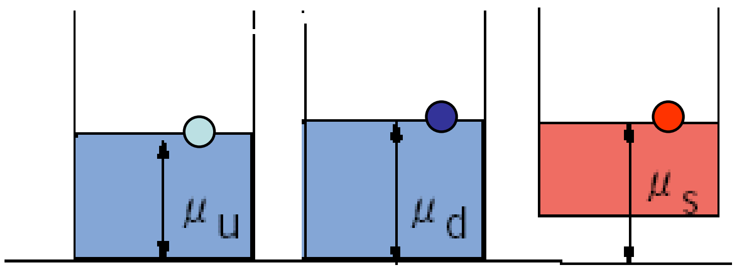
Chiral-super interplay at finite μ







CSC at high μ



$$(d_L)_{ia} \sim \epsilon_{ijk}\epsilon_{abc}(q_L)^j_b C (q_L)^k_c$$

$$(d_R)_{ia} \sim \epsilon_{ijk}\epsilon_{abc}(q_R)^j_b C (q_R)^k_c$$

flavor color

major differences from the standard BCS superconductor

1. Relativistic fermi system
   color-magnetic int. dominant

   Son, PRD59 ('99),
   Schafer & Wilczek, PRD60 ('99)
   Pisarski & Rischke, PRD61 ('00)

$$|d| \sim \varepsilon_F \; e^{-c/\sqrt{\alpha_s}}$$

High T_c : $T_c/\varepsilon_F \sim 0.1$
Compact pair : r~ 1-10 fm

2. Color-flavor entanglement

$$d_{ia}$$

Various phases (c.f. Ice, ^{3}He)
CFL, 2SC, dSC, uSC, etc

GL analysis for chiral-super interplay in QCD (N_f=3)

Symmetry: $SU(3)_C \times [SU(3)_L \times SU(3)_R] \times U(1)_B \times U(1)_A$

Chiral field:

$$\Phi_{ij} \sim (\bar{q}_R)^j_a (q_L)^i_a$$

$$\Phi \to \mathrm{e}^{-2i\alpha_A} V_L \Phi V_R^\dagger$$

diquark field:

$$(d_L)_{ia} \sim \epsilon_{ijk}\epsilon_{abc}(q_L)^j_b C (q_L)^k_c$$

$$d_L \to e^{2i\alpha_A} e^{2i\alpha_B} V_L d_L V_C^{\mathrm{T}}$$

$$\mathcal{V}(\Phi, d) = \mathcal{V}_\chi(\Phi) + \mathcal{V}_d(d_L, d_R) + \mathcal{V}_{\chi d}(\Phi, d_L, d_R)$$

Pisarski & Wilczek, PRD29 ('84)

- Iida & Baym, PRD63 ('01)
- Iida, Matsuura, Tachibana & TH, PRD71 ('05)

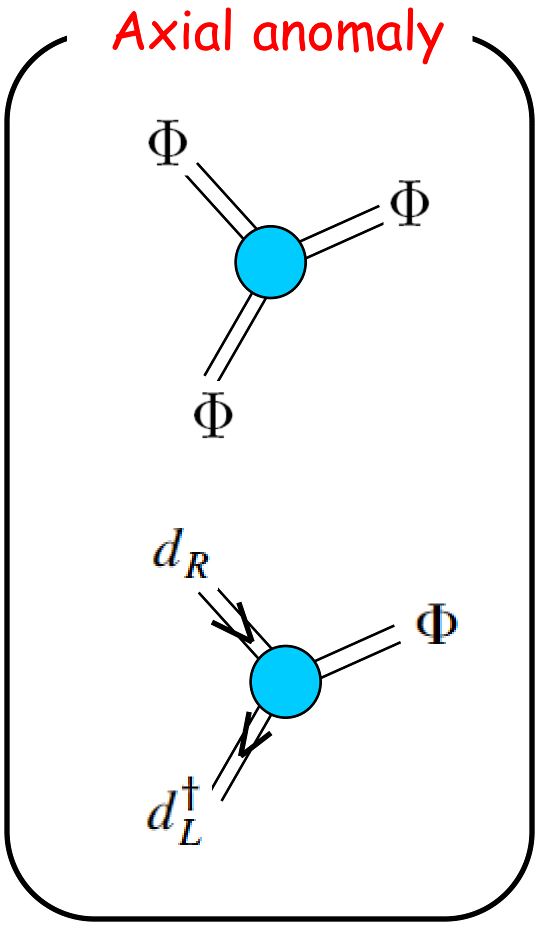
Yamamoto, TH, Tachibana & Baym, PRL97 ('06)

Complete classification of the GL potential (m=0)

$$\mathcal{V}_\chi = \frac{a_0}{2}\mathrm{tr}\ \Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\ \Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2 \boxed{-\frac{c_0}{2}\left(\mathrm{det}\Phi + \mathrm{det}\Phi^\dagger\right)},$$

$$\begin{aligned}\mathcal{V}_d = &\ \alpha_0\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger] \\ &+ \beta_1\left([\mathrm{tr}(d_L d_L^\dagger)]^2 + [\mathrm{tr}(d_R d_R^\dagger)]^2\right) \\ &+ \beta_2\left(\mathrm{tr}[(d_L d_L^\dagger)^2] + \mathrm{tr}[(d_R d_R^\dagger)^2]\right) \\ &+ \beta_3\ \mathrm{tr}[(d_R d_L^\dagger)(d_L d_R^\dagger)] + \beta_4\ \mathrm{tr}(d_L d_L^\dagger)\mathrm{tr}(d_R d_R^\dagger)\end{aligned}$$

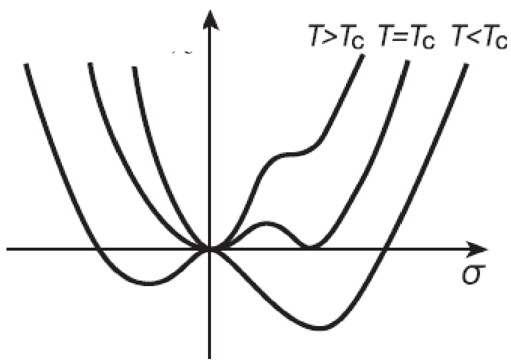
$$\begin{aligned}\mathcal{V}_{\chi d} = &\ \boxed{\gamma_1\ \mathrm{tr}[(d_R d_L^\dagger)\Phi + (d_L d_R^\dagger)\Phi^\dagger]} \\ &+\lambda_1\ \mathrm{tr}[(d_L d_L^\dagger)\Phi\Phi^\dagger + (d_R d_R^\dagger)\Phi^\dagger\Phi] \\ &+\lambda_2\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger]\cdot\mathrm{tr}[\Phi^\dagger\Phi] \\ &+\lambda_3\left(\mathrm{det}\Phi\cdot\mathrm{tr}[(d_L d_R^\dagger)\Phi^{-1}] + h.c\right)\end{aligned}$$

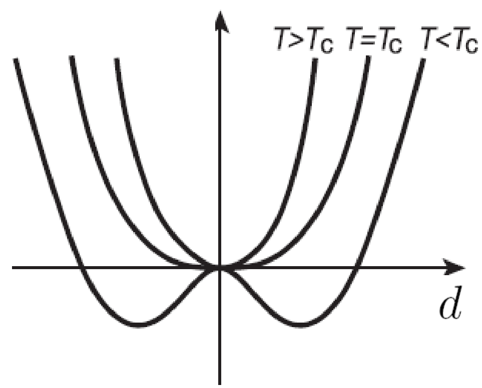


Chiral-CFL interplay in N_f=3

$$\Phi = \begin{pmatrix} \sigma & & \\ & \sigma & \\ & & \sigma \end{pmatrix} \qquad d_L = -d_R = \begin{pmatrix} d & & \\ & d & \\ & & d \end{pmatrix}$$

$$\boxed{\mathcal{V} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \gamma d^2\sigma + \lambda d^2\sigma^2}$$

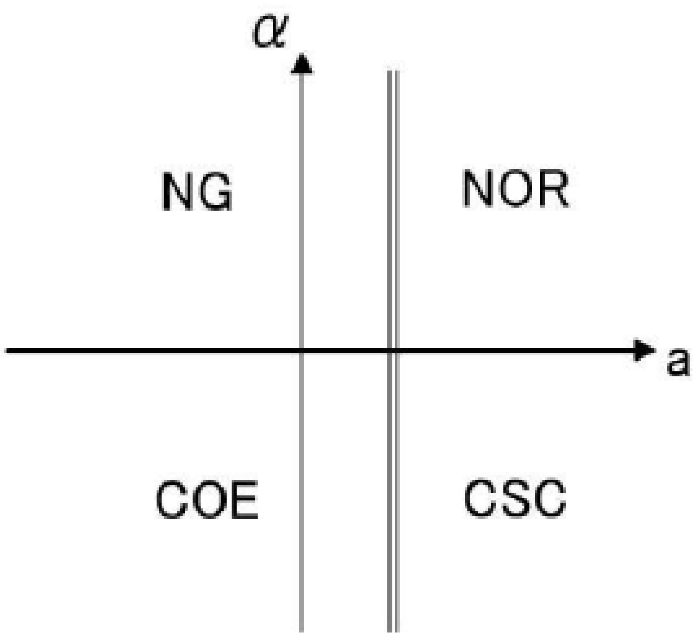


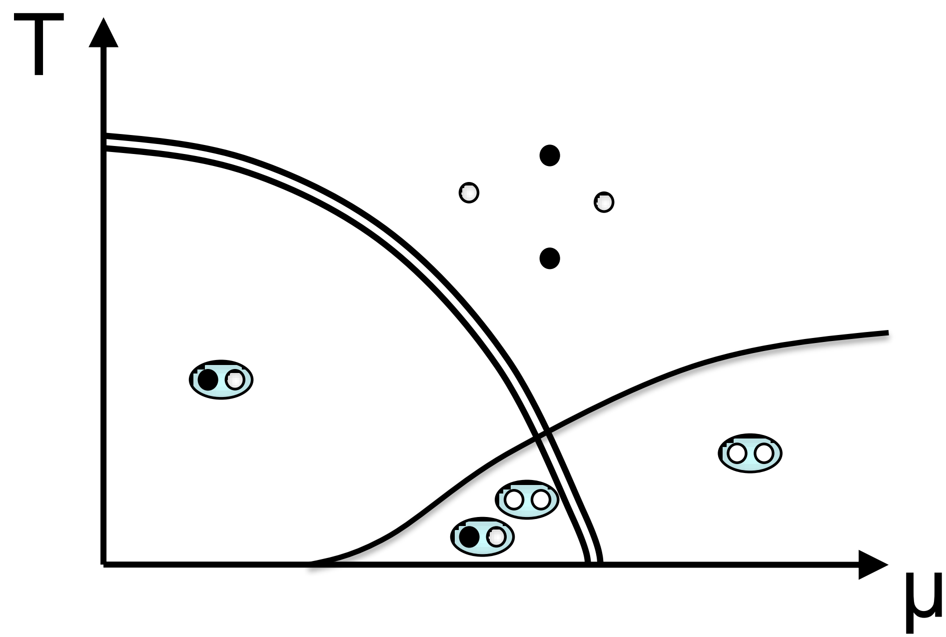


Natural parameter relations
$$\begin{cases} \beta > 0,\ b > 0 \\ \gamma \sim c > 0 \\ 1 \gg \lambda/\beta > 0 \end{cases}$$

phase diagram (without d-σ coupling)

$$\mathcal{V} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \cancel{\gamma d^2}\sigma$$

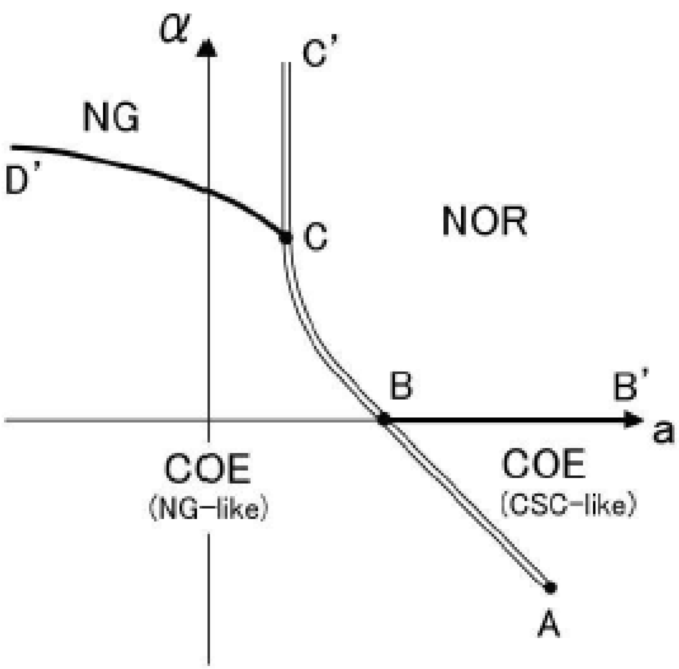


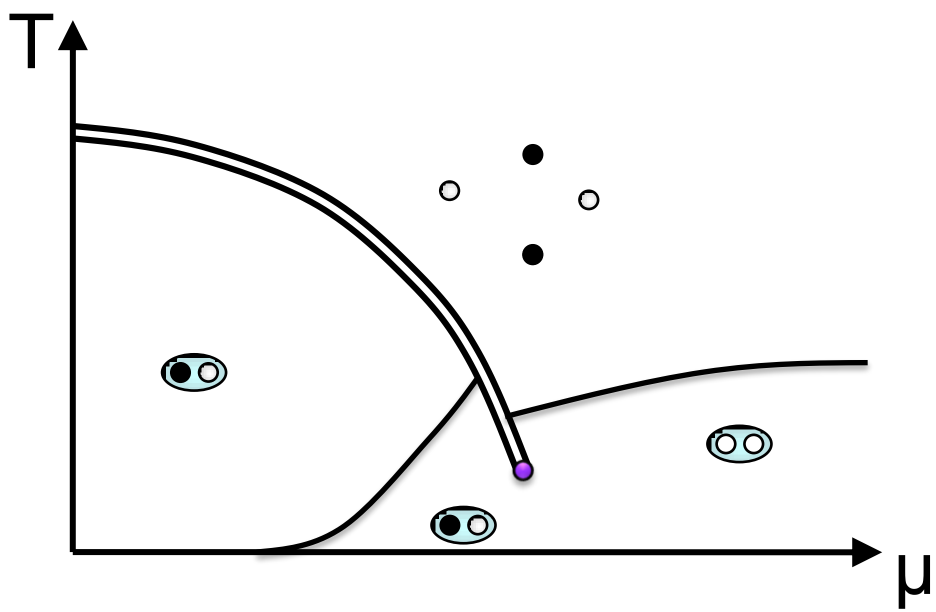


══════ : 1st order
────── : 2nd order

phase diagram (with d-σ coupling)

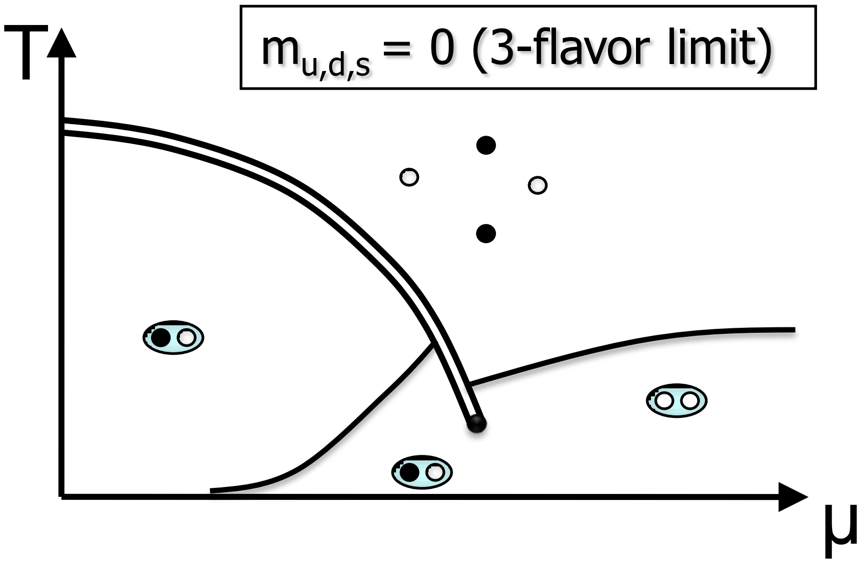
$$\mathcal{V} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \gamma d^2 \sigma$$

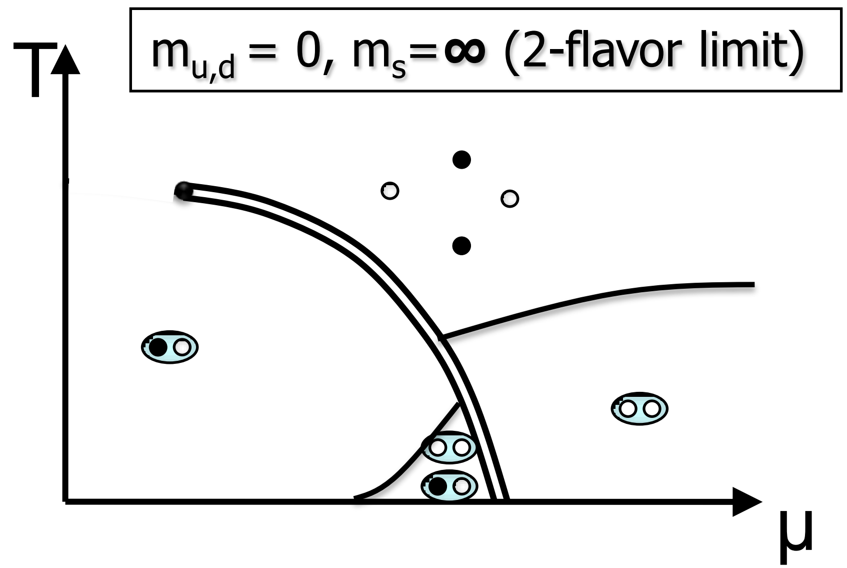


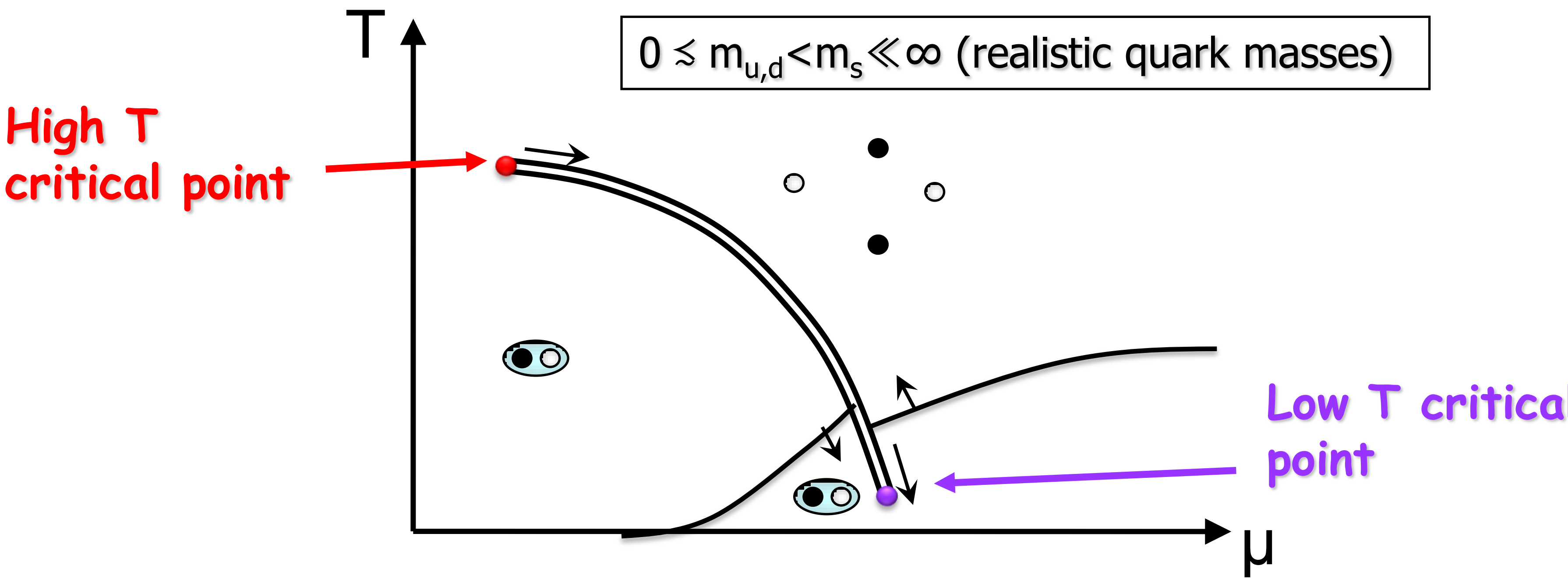


A new critical point driven by the axial anomaly

Realistic phase diagram in N_f=2+1 ?







Frequently asked questions

1. Location of the new critical point in physical unit ?

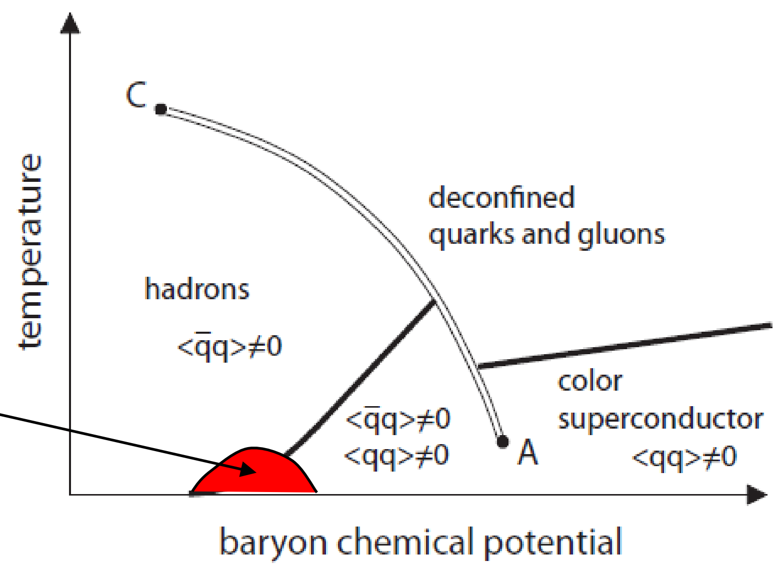
   - No definite answer at present
   - NJL & PNJL model calculation on-going (Yamamoto, Rossner, Weise, Baym &TH)

2. Connection to baryon superfluidity at low μ ?

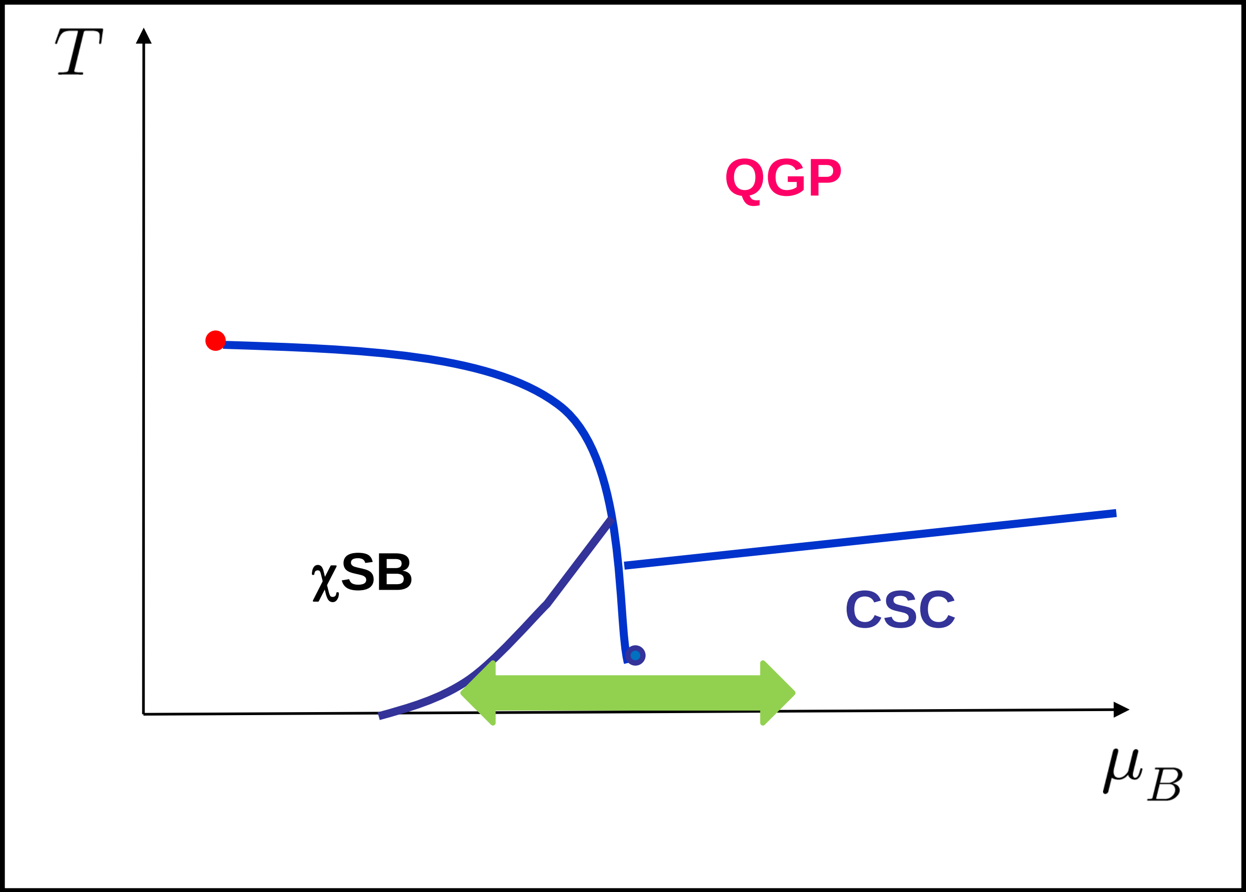
   - Mechanism for the crossover from $<q^2>$ to $<q^3q^3>$
     similar to bose-fermi mixture in cold atoms (Maeda, Baym & TH)

$< q^3 q^3 > \neq 0$

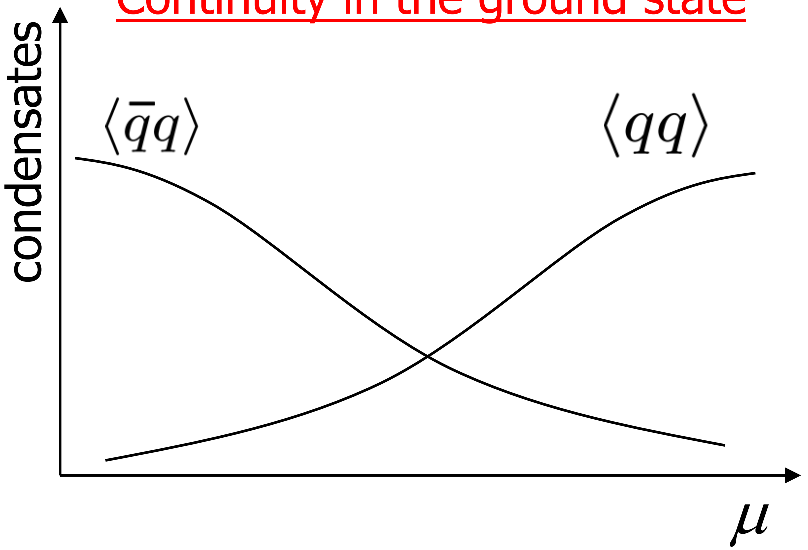
baryon superfluid



Spectral continuity at finite μ



Continuity in the ground state



Continuity in the excited state??

excitation	Low μ	High μ
NGs	π(8) & H	π′ (8) & H
Vectors	V (9)	gluons (8)
Fermions	baryons (8)	Quarks (9)

Schafer and Wilczek, PRL 82 (1999)

Explicit realization of spectral continuity

- Generalized Gell-Mann-Oakes-Renner relation :

$$m_{\tilde{\pi}}^2 \simeq \frac{m_q}{f_\pi^2 + f_{\pi'}^2}\left[\alpha\langle \bar{q}q\rangle + \beta\langle qq\rangle^2\right]$$

Yamamoto, Tachibana, Baym + T.H., PR D76 ('07)

- Gauge invariant method to show the continuity of vector mesons

In-medium QCD sum rules

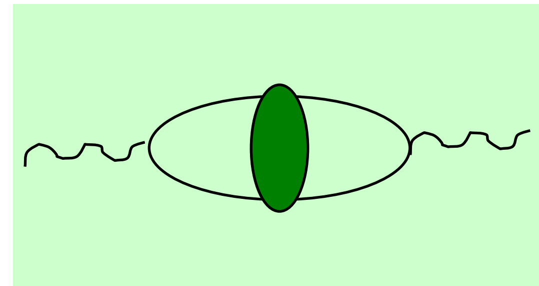
T.H., Tachibana and Yamamoto, PRD78 (2008)

QCD sum rules in the superconducting medium

- Vector current: $J_\mu^{(8)} = \bar{q}\tau^a\gamma_\mu q$, $J_\mu^{(1)} = \bar{q}\tau^0\gamma_\mu q$

- Current correlation function:

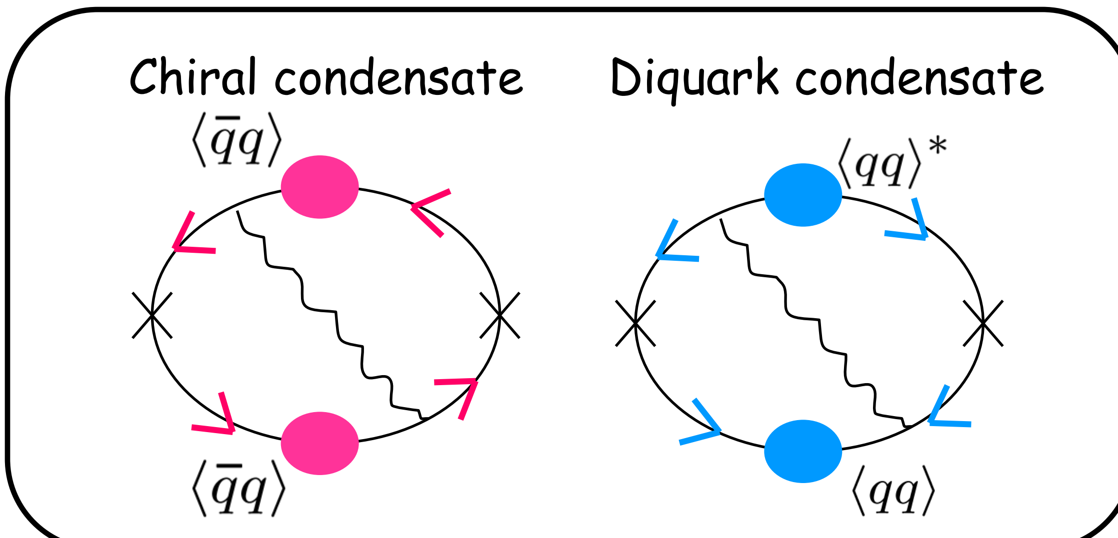
$$\Pi_{\mu\nu}(q) \;=\; i\int d^4x e^{iqx}\langle RJ_\mu(x)J_\nu(0)\rangle$$



- Operator Product Expansion (OPE) up to $\mathcal{O}(1/Q^6)$:

4-quark condensate

$$\langle(\bar{q}\Gamma q)(\bar{q}\Gamma q)\rangle$$



Mass formula from Finite Energy Sum Rules

At low density:

$$\boxed{\left(m_{\mathrm{V}}\right)^2 \to \left(\frac{448\pi^3\alpha_s}{27}\langle\bar{q}q\rangle^2\right)^{1/3}}$$

At intermediate density:

$$\boxed{\left(m_{\mathrm{V}}^{(8)}\right)^2 \simeq \frac{56\pi^3\alpha_s}{81\mu^4}\left(\langle\bar{q}q\rangle^2 + \frac{15}{7}\langle qq\rangle^2\right)}$$

$$\boxed{\left(m_{\mathrm{V}}^{(1)}\right)^2 \simeq \frac{1}{f}\frac{56\pi^3\alpha_s}{81\mu^4}\left(\langle\bar{q}q\rangle^2 - \frac{66}{7}\langle qq\rangle^2\right)}$$

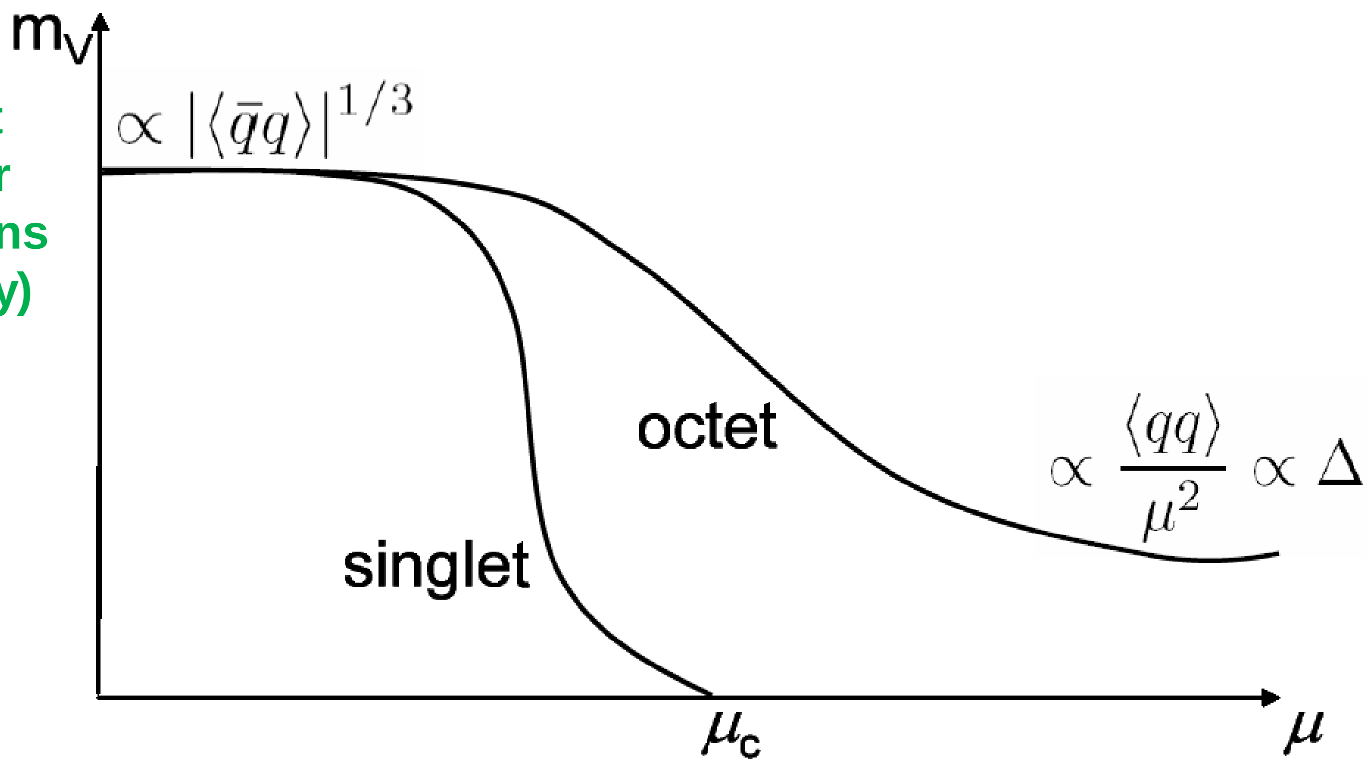
At high density:

$$\boxed{m_{\mathrm{V}}^{(8)} \to \sqrt{\frac{20}{3}}\,\Delta \simeq 2.6\Delta}$$

Spectral continuity of vector mesons

T.H., Tachibana and Yamamoto,
PRD78 (2008)

Nonet vector mesons (heavy)



Octet vector mesons (light)

Octet gluons in CFL: $m_g = 1.362\Delta$

Gusynin & Shovkovy, NPA700 (2002)
Malekzadeh & Rischke, PRD73 (2006)

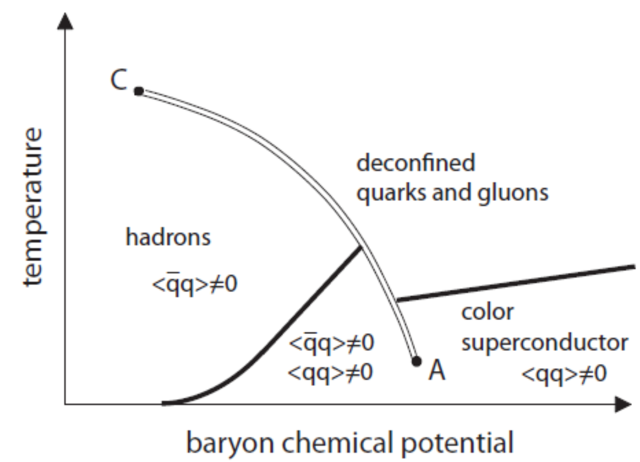
Summary and Future

1. QCD phase structure

- Three major phases in QCD: χSB, QGP and CSC
- Axial anomaly plays crucial roles everywhere
- Close similarity with high Tc supercond. & multi-comp. cold atoms

2. Chiral-super interplay driven by axial anomaly

- A new critical point at low T and high μ
- Continuity of χSB phase and CSC phase



3. Spectral continuity in high density QCD

- Pions are pions.
- Vector mesons are gluons.

4. Future

- Real location of the new critical point ?
- How to detect it in lab. experiment : it's a FAIR question !
- Tabletop simulations of high density QCD using cold atoms ?