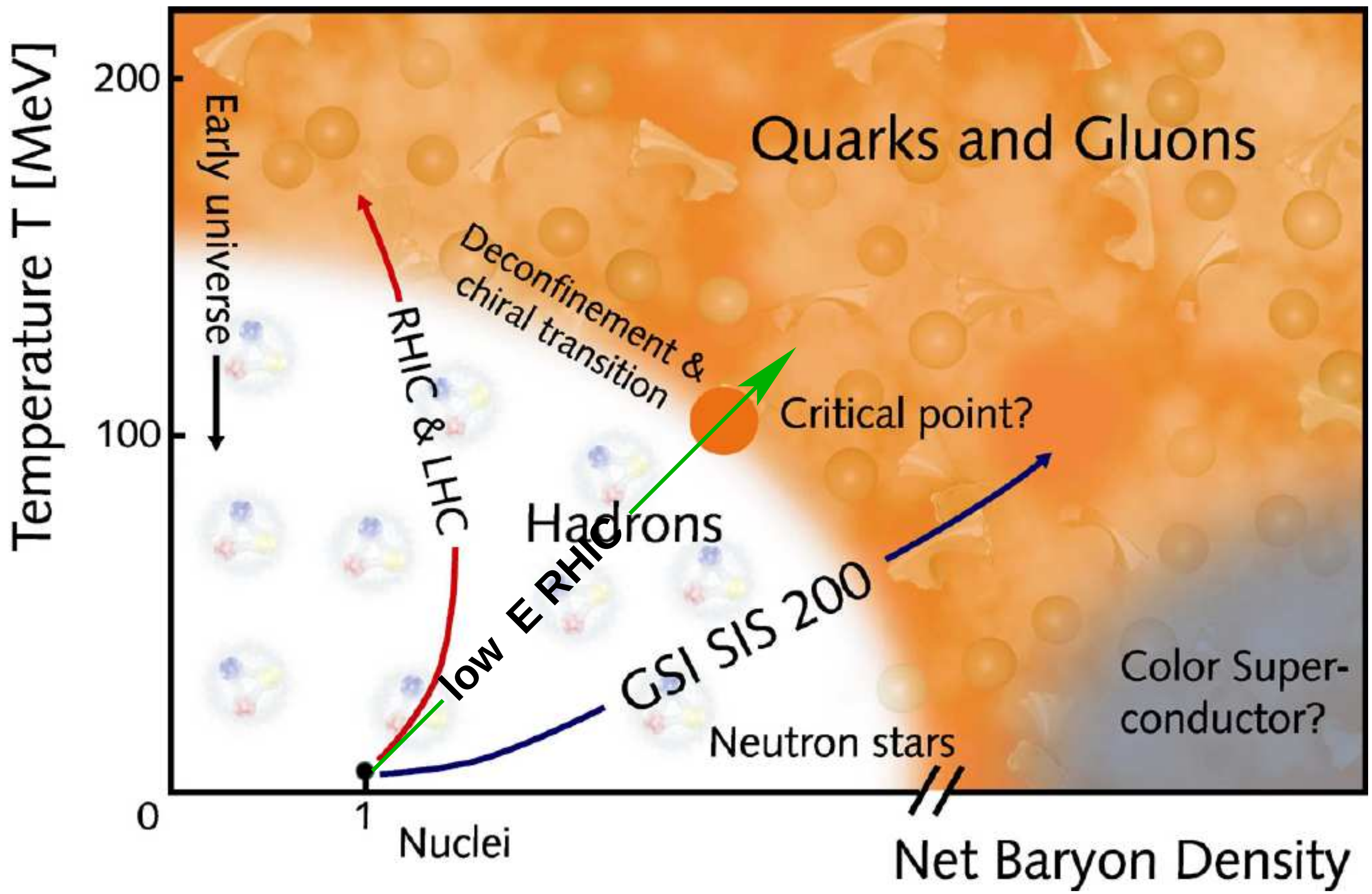
Lattice results on QCD at high-T and non-zero baryon number density

Frithjof Karsch, BNL & Bielefeld University



Outline

- Introduction:

  elementary particles at high temperature and density

- Bulk thermodynamics

  the QCD equation of state

- The QCD (phase) transition

  deconfinement and chiral symmetry restoration

- Hadronic fluctuations and finite density QCD

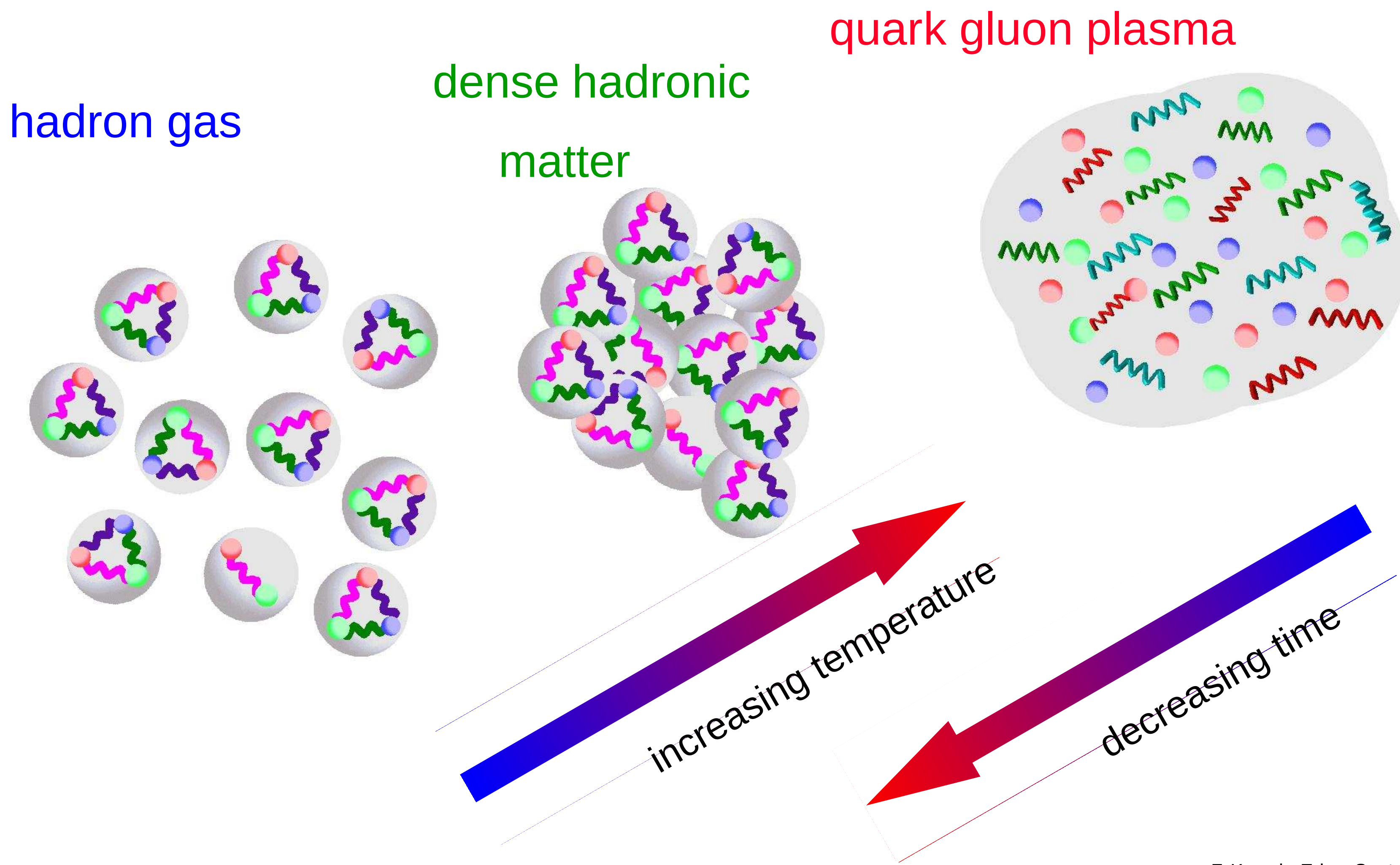
  fluctuations and correlations at vanishing chemical potential
  critical behavior at non-zero chemical potential

- Conclusions

From matter to elementary particles...
...to elementary particle matter

temperatures in the early universe after 10^{-6} sec: $\sim 10^{12}$ K

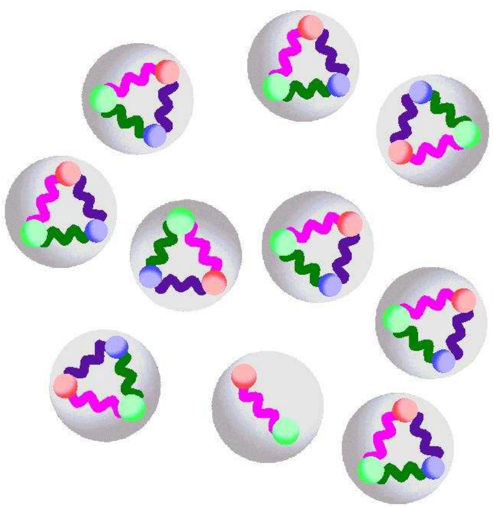
density of neutron stars: $\sim$ (3-10)-times nuclear matter density



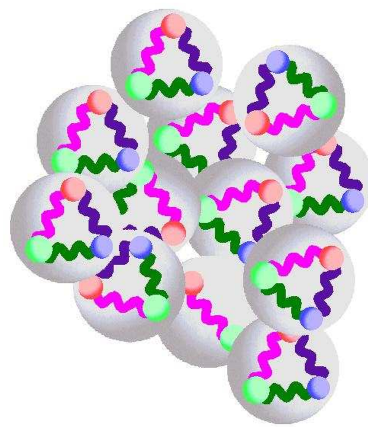
From Hadronic Matter to the Quark Gluon Plasma

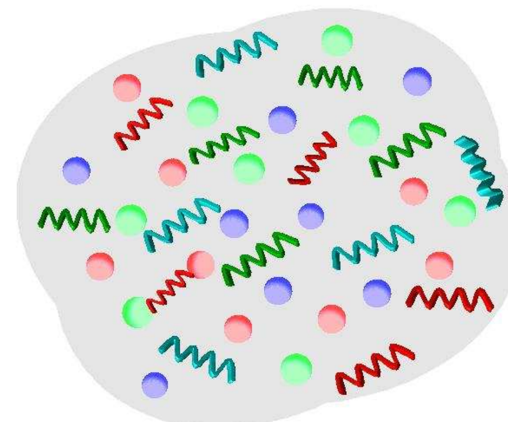
with the help of QCD?

hadron gas



dense hadronic matter





J.C. Collins, M.J. Perry, Superdense Matter: Neutrons and asymptotically free quarks? PRL 34 (1975) 1353

N. Cabibbo, G. Parisi, Exponential Hadronic Spectrum and Quark Liberation, PL B59 (1975) 67

QuantumChromoDynamics (Fritsch, Gell-Mann, 1972)

n_f quarks;
$(N_c^2 - 1)$ gluons;

confinement;
asymptotic freedom;
chiral symmetry breaking;

Heavy Ion collisions at the RHIC@BNL:

AU-AU beams: $\sqrt{s} = 130,\ 200$ GeV/A

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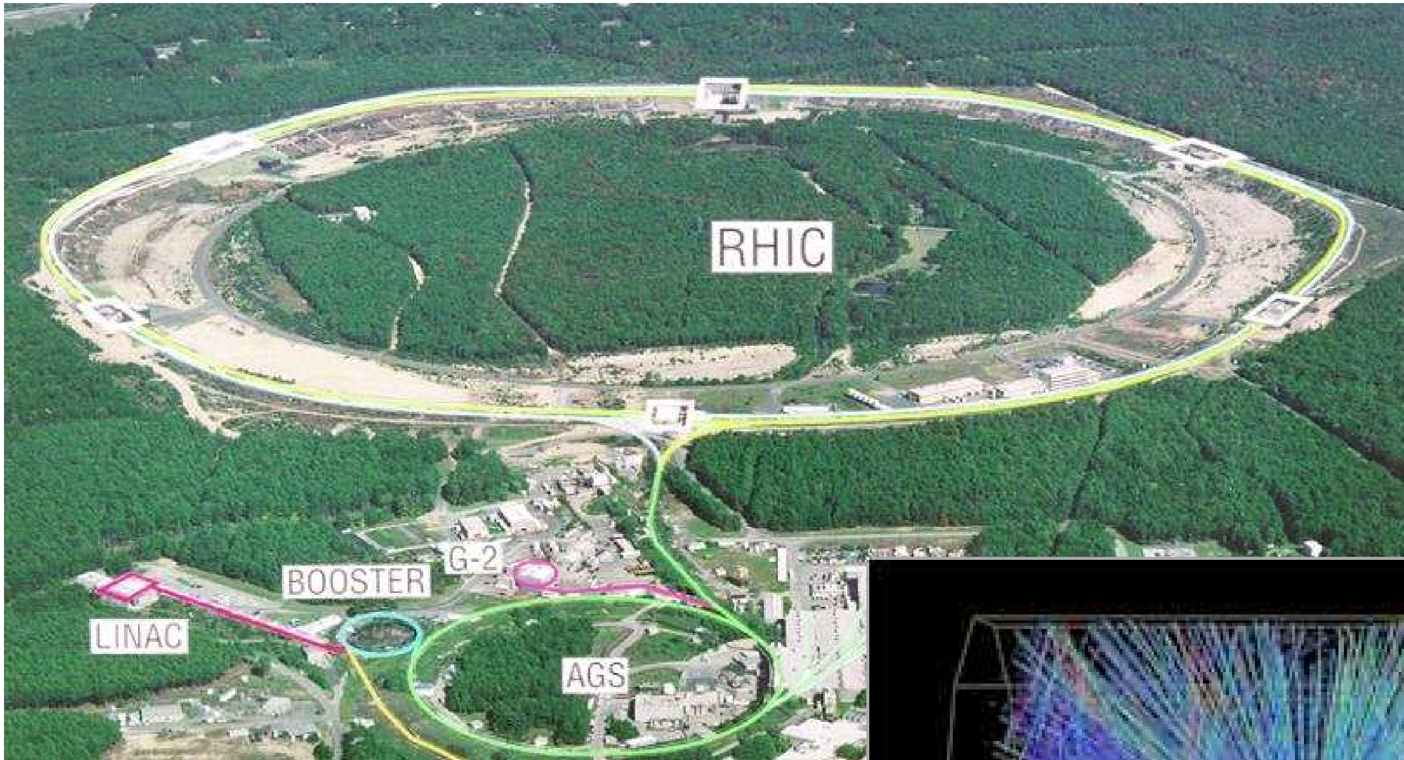
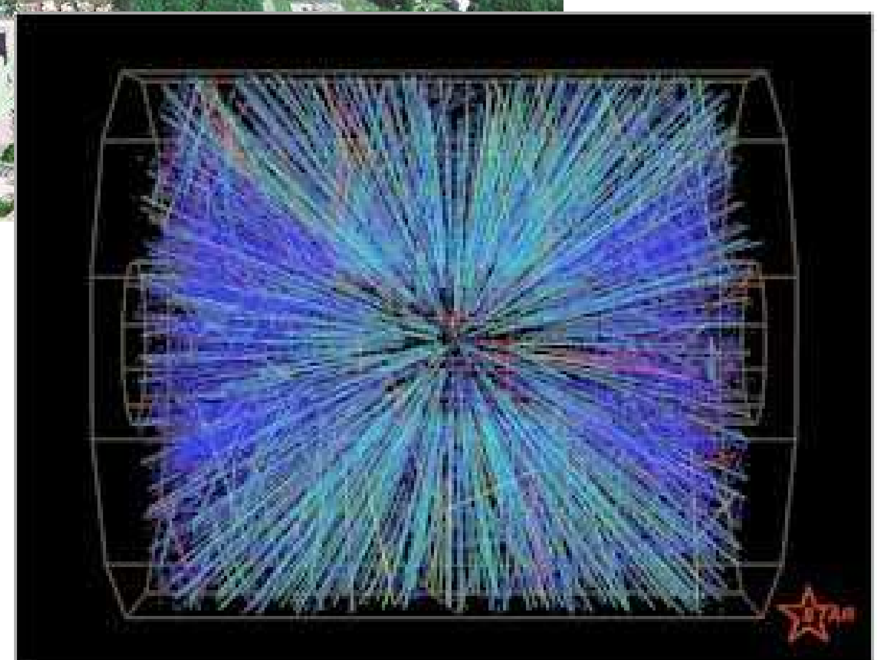


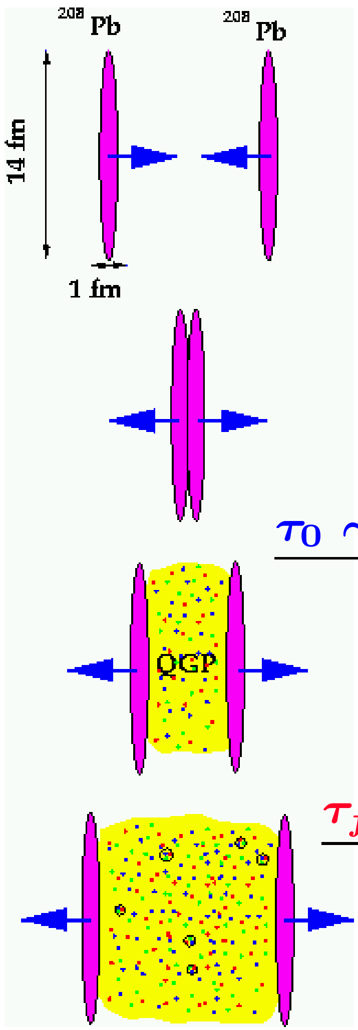


AU-AU beams: $\sqrt{s} = 130,\ 200$ GeV/A

estimated temperature: $T_0 \simeq (1.5\text{-}2)T_c$
estimated initial energy density:
$\epsilon_0 \simeq (5 - 15)$ GeV/fm^3
need EoS to estimate $T_0,\ \epsilon_0$

Creating hot and dense matter in heavy ion collisions

Creating a QGP in A-A Collisions (RHIC)



beam energy: $\mathbf{200\ GeV/A}$ (for Au)
$\sim \mathcal{O}(\mathbf{1000})$ particles/event at central rapidity

initial (thermalized) energy density
$\boldsymbol{\epsilon(\tau_0) \sim 10\ \mathrm{GeV/fm^3}}$

$\boldsymbol{\tau_0 \sim (0.5 - 1.0)}$fm

initial temperature;
$\boldsymbol{\sim 1.5\ T_c}$;
$\boldsymbol{\sim 250\ \mathrm{MeV}}$

baryon density
$\boldsymbol{\mu_B \simeq 50\ \mathrm{MeV}}$

$\boldsymbol{\tau_f} = ?$

phase transition at $\boldsymbol{T_c \simeq 170\ \mathrm{MeV}}$
back to the ordinary QCD vacuum

observable properties of QGP?

"measured" in experiment;
using Bjorken formula

hydrodynamic expansion
at constant $\boldsymbol{S}$, $\boldsymbol{N_B}$

need EoS: $\boldsymbol{p(\epsilon) \ \Rightarrow \ v_s}$
(transport coefficients)

hydro: $\boldsymbol{\epsilon(\tau)}$
lattice QCD: $\boldsymbol{\epsilon(T)}$
$\boldsymbol{\Rightarrow \ \epsilon(\tau_0),\ T_f \equiv T_c,\ \tau_f}$

LGT-EoS and hydro-expansion

- simple 1-d hydro: $\partial_\mu T^{\mu\nu} = 0 \;\Rightarrow\; \frac{\mathrm{d}\epsilon}{\mathrm{d}\tau} = -\frac{\epsilon + p}{\tau}$
  - ideal gas EoS: $p/\epsilon = 1/3$

$$\frac{\epsilon(\tau)}{\epsilon(\tau_0)} = \left(\frac{\tau_0}{\tau}\right)^{4/3} \quad \Rightarrow \quad \tau_f = \tau_0 \left(\frac{\epsilon(\tau_0)}{\epsilon(\tau_c)}\right)^{3/4}$$

  - lattice EoS: $p/\epsilon < 1/3$ $\quad\Rightarrow$ slows down expansion;
    $\Rightarrow$ increases plasma lifetime

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}\tau} = -\frac{4}{3}\frac{\epsilon}{\tau}\left(1 - \frac{0.3}{1 + 0.2\ \epsilon\ \mathrm{fm}^3/\mathrm{GeV}}\right)$$

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$\epsilon(\tau_0 = 1\ \mathrm{fm}) \simeq 10\mathrm{GeV}/\mathrm{fm}^3 \;\Rightarrow\; \tau_f \simeq 5.5\ \mathrm{fm}\ \ (\epsilon = 3p)$
$\simeq 7\ \mathrm{fm}$ (LGT EoS)

Thermodynamics on Supercomputers QCDOC and BlueGene/L at BNL

NYBlue:

100 Teraflops peak,

(10-20)% sustained;

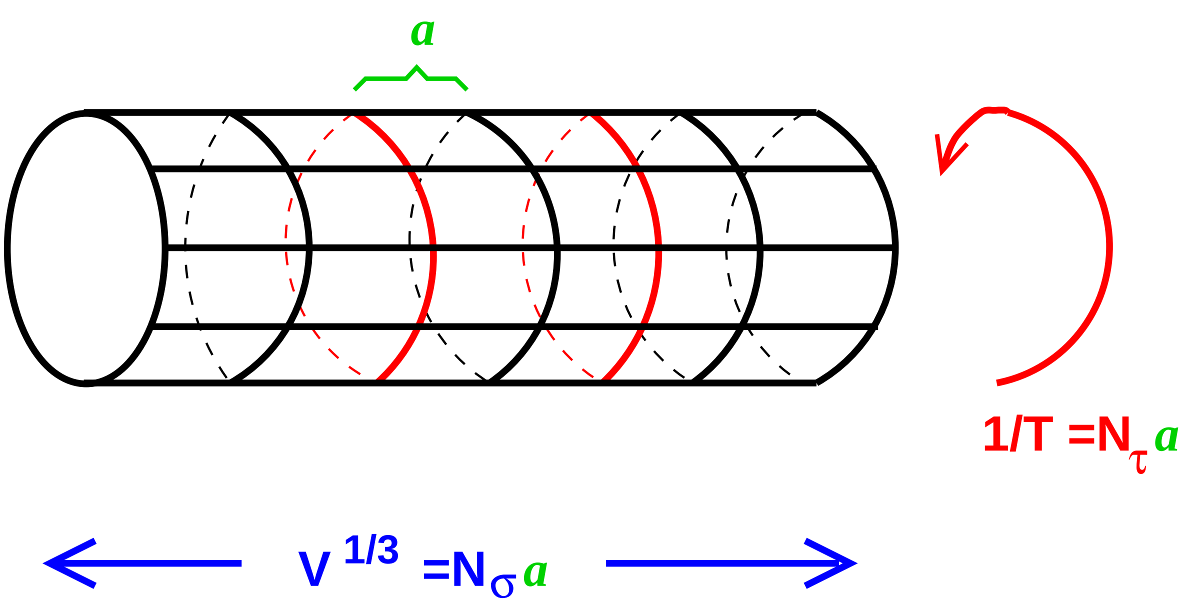
used since $\sim$ June 2007

US/RBRC QCDOC

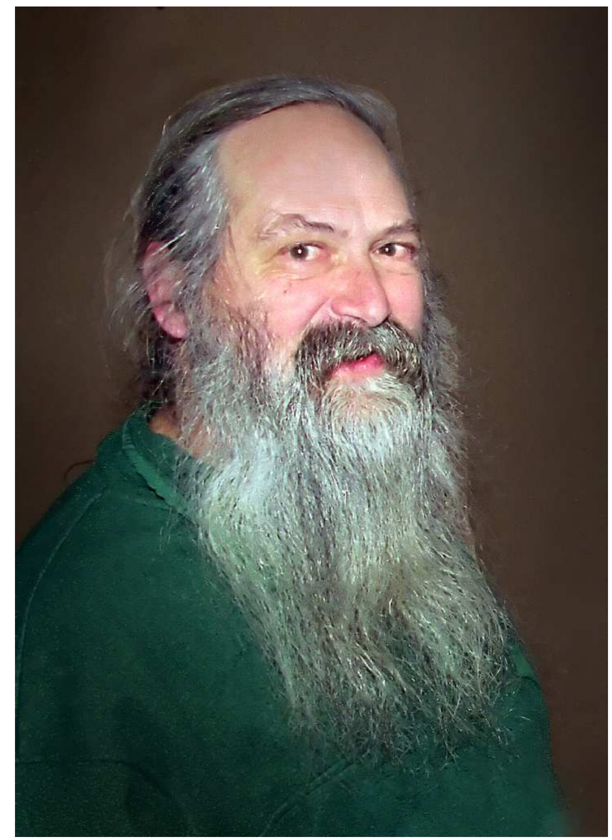
20.000.000.000.000 ops/sec

$\sim$ 40 TFlops for QCD-Thermodynamics
$\sim$ 10 times more CPU-time than for previous studies of the EoS

Analyzing hot and dense matter on the lattice: $\mathbf{N}_\sigma^3 \times \mathbf{N}_\tau$



Michael Creutz



Phys. Rev. D21 (1980) 2308

Quantum Chromo Dynamics

partition function: $Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E}$

$$\boxed{S_E = \int_0^{1/T} dx_0 \int_V d^3x\ \mathcal{L}_E(\mathcal{A},\psi,\bar{\psi},\mu)}$$

temperature volume chemical potential

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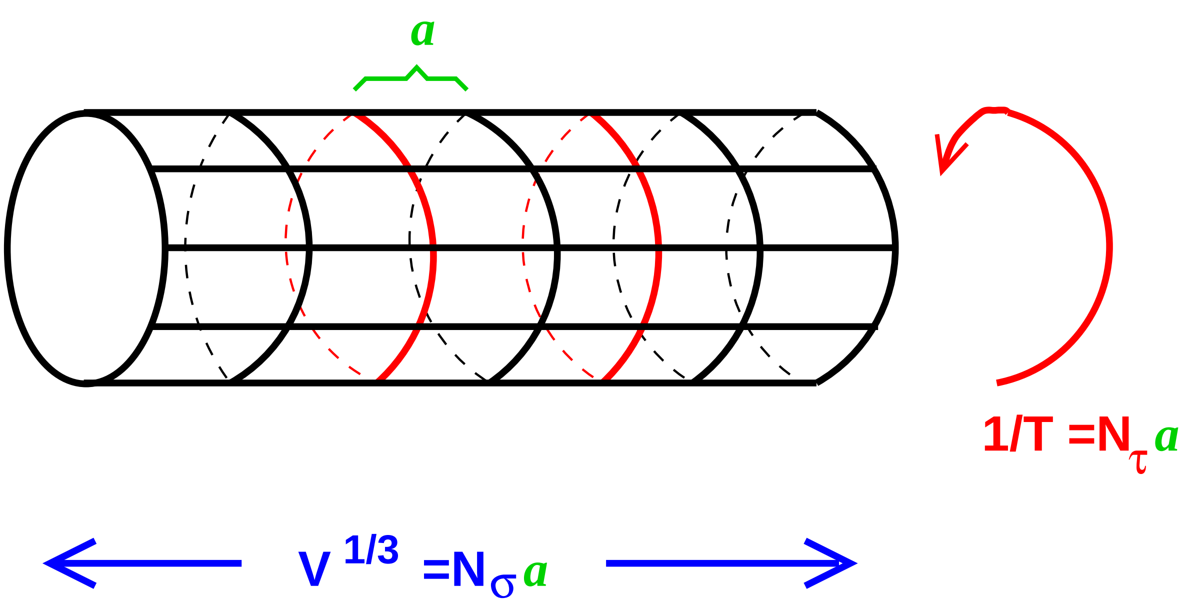

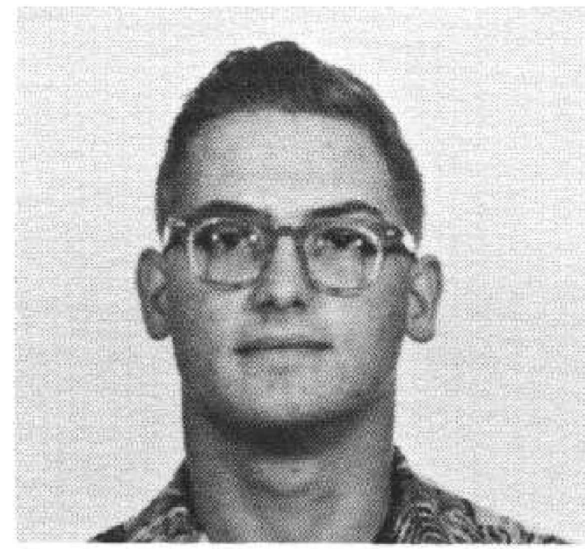

Michael J. Creutz
San Dieguito Un. H
Encinitas, Calif.

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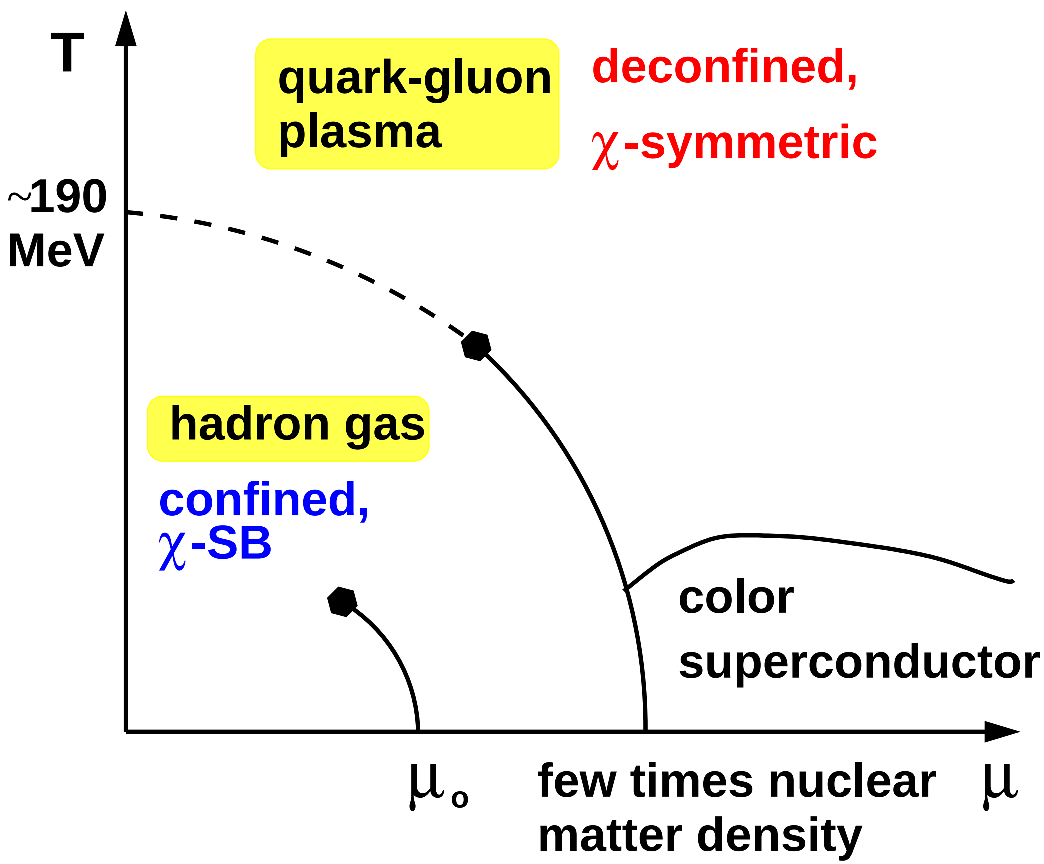
$$\boxed{S_E = \int_0^{1/T} dx_0 \int_V d^3x\ \mathcal{L}_E(\mathcal{A}, \psi, \bar{\psi}, \mu)}$$

temperature volume chemical potential

$\mathcal{O}(10^6)$ grid points;
$\mathcal{O}(10^8)$ d.o.f.;
integrate eq. of motion

Critical behavior in hot and dense matter: QCD phase diagram



continuous transition for small chemical potential and small quark masses at

$T_c \simeq (170 - 190)$ MeV

$\epsilon_c \simeq 1$ GeV/fm^3

want accurate $T_c, \epsilon_c, \ldots$ determination to make contact to HI-phenomenology

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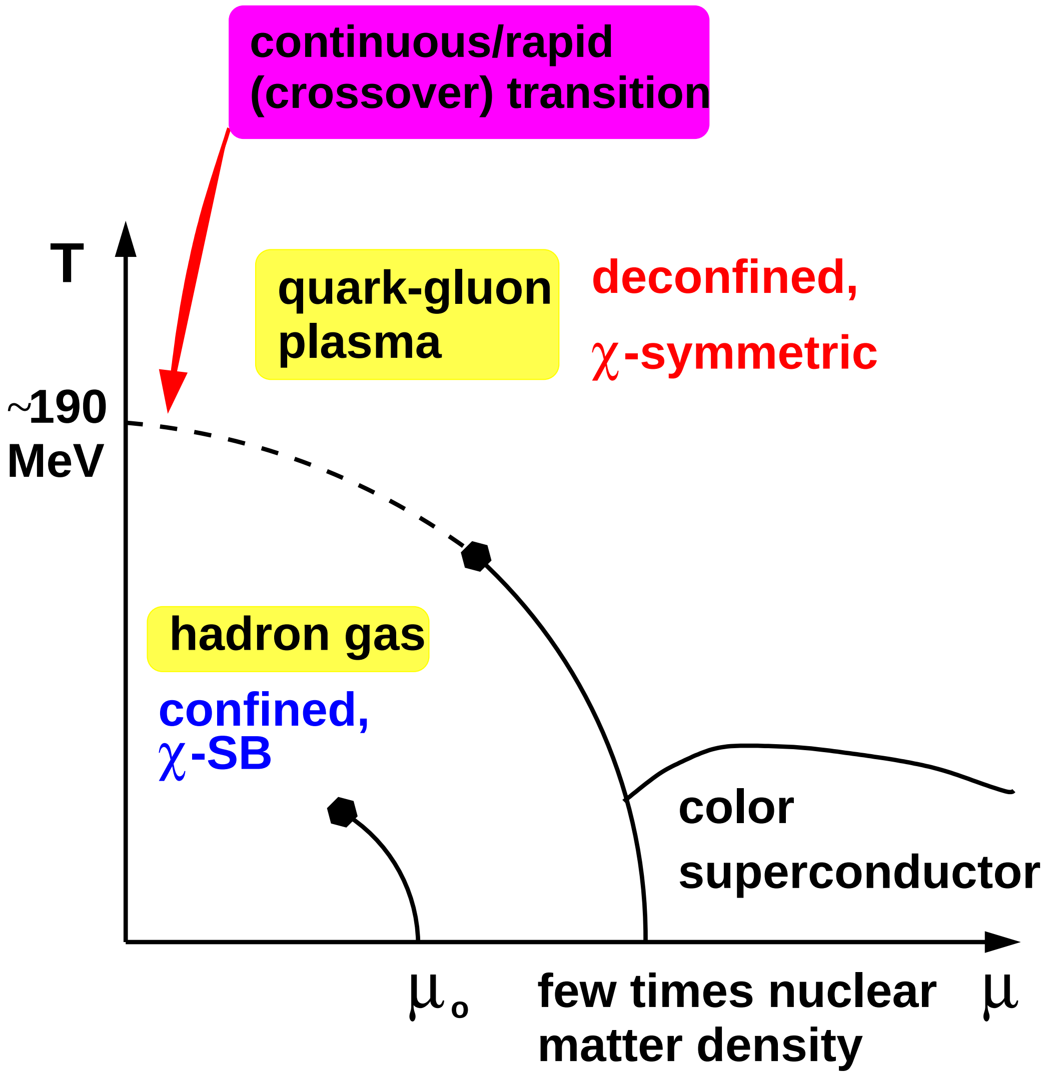

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remaining doubts on order of transition
A. Di Giacomo et al., hep-lat/0503030

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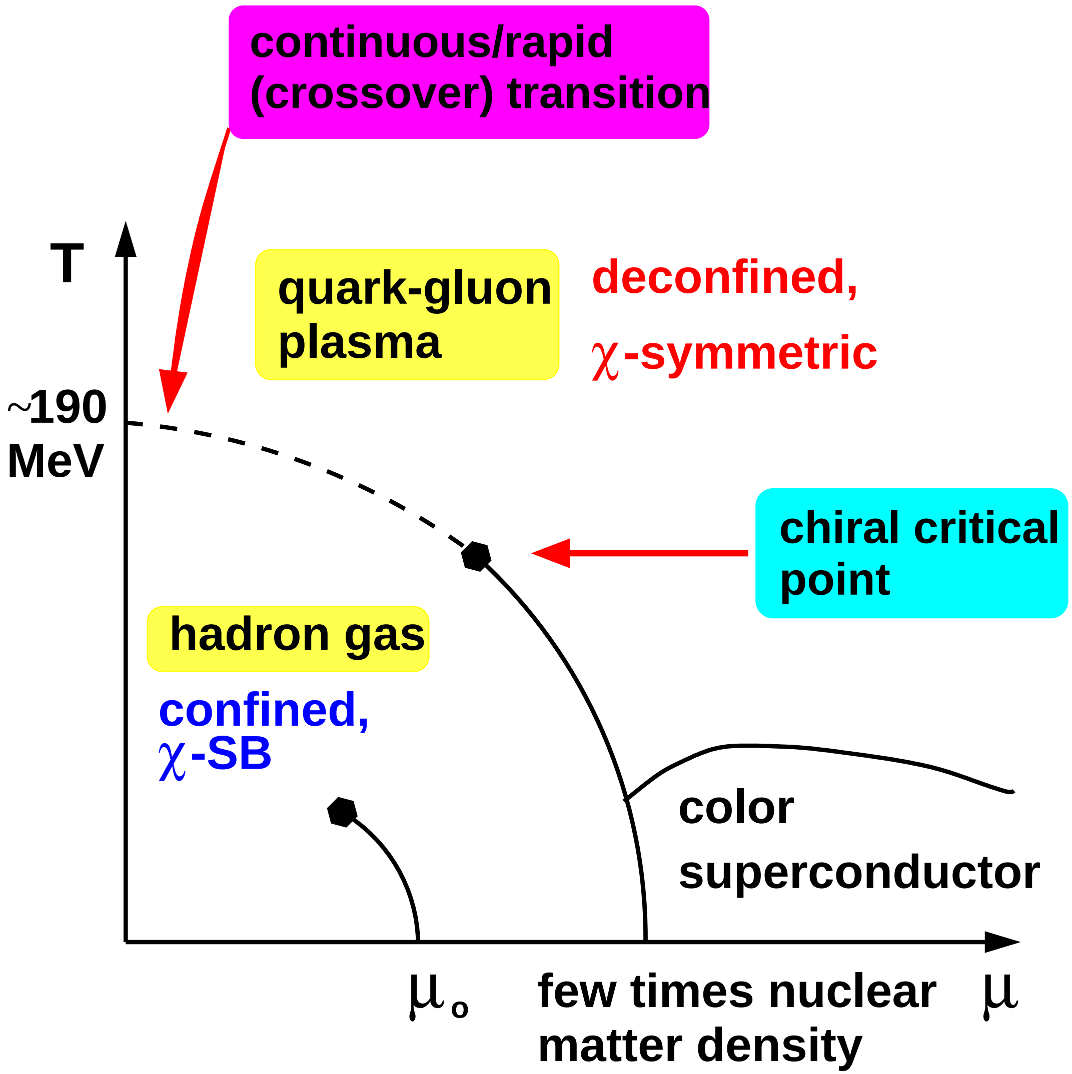


continuous transition for small chemical potential and small quark masses at

$T_c \simeq (170 - 190)$ MeV

$\epsilon_c \simeq 1$ GeV/fm^3

2nd order phase transition; Ising universality class

$T_c(\mu)$ under investigation

location of CCP uncertain: volume and quark mass dependence

remaining doubts on order of transition
A. Di Giacomo et al., hep-lat/0503030

want accurate $T_c, \epsilon_c, ...$ determination to make contact to HI-phenomenology

Calculating the EoS on lines of constant physics (LCP)

- The interaction measure for $N_f = 2+1$ $\Leftrightarrow$ Trace Anomaly

$$\frac{\epsilon - 3p}{T^4} = T\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{p}{T^4}\right) = \left(a\frac{\mathrm{d}\beta}{\mathrm{d}a}\right)_{LCP} \frac{\partial p/T^4}{\partial \beta}$$

$$= \left(\frac{\epsilon - 3p}{T^4}\right)_{gluon} + \left(\frac{\epsilon - 3p}{T^4}\right)_{fermion} + \left(\frac{\epsilon - 3p}{T^4}\right)_{\hat{m}_s/\hat{m}_l}$$

- The pressure

$$\left.\frac{p}{T^4}\right|_{\beta_0}^{\beta} = \int_0^T \mathrm{d}T\, \frac{1}{T}\left(\frac{\epsilon - 3p}{T^4}\right)$$

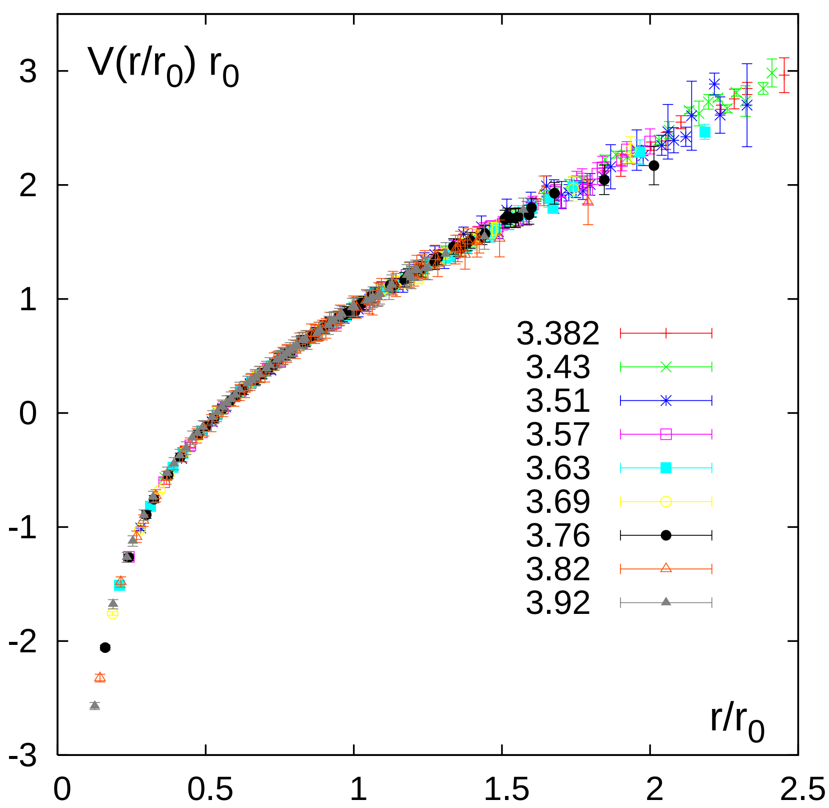
- need T-scale, $aT = 1/N_\tau$ and its relation to the gauge coupling $a \equiv a(\beta)$

N.B.: $a(\beta)$ is only defined through physical observables
$\Rightarrow$ choose a simple one

$T = 0$ scale setting using the heavy quark potential

use r_0 or string tension to set the scale for $T = 1/N_\tau a(\beta)$

$$V(r) = -\frac{\alpha}{r} + \sigma r \quad , \quad r^2 \frac{\mathrm{d}V(r)}{\mathrm{d}r}\Big|_{r=r_0} = 1.65$$



no significant cut-off dependence when cut-off varies by a factor 5

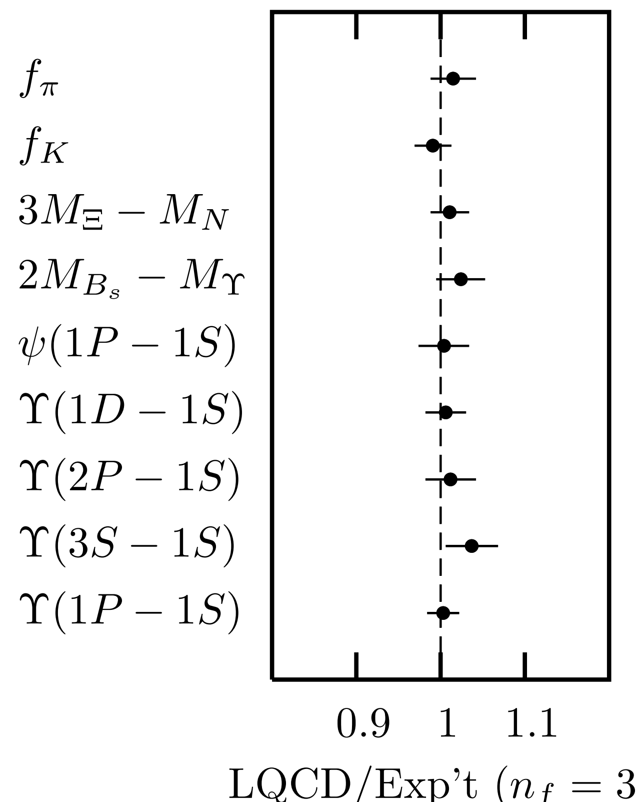
i.e. from the transition region on $N_\tau = 4$ lattices ($a \simeq 0.25$ fm) to that on $N_\tau = 20$ lattices ($a \simeq 0.05$ fm) !!

scales extracted from 'gold plated observables'

- high precision studies of several experimentally well known observables in lattice calculations with staggered (asqtad) fermions led to convincing agreement $\Rightarrow$ gold plated observables
- simultaneous determination of $\boldsymbol{r_0/a}$ in these calculations determines the scale $\boldsymbol{r_0}$ in MeV

knowing any of these experimentally accessible quantities accurately from a lattice calculation is equivalent to knowing $\boldsymbol{r_0}$, which is a fundamental parameter of QCD

C.T.H. Davies et al., PRL 92 (2004) 022001
A. Gray et al., PRD72 (2005) 094507



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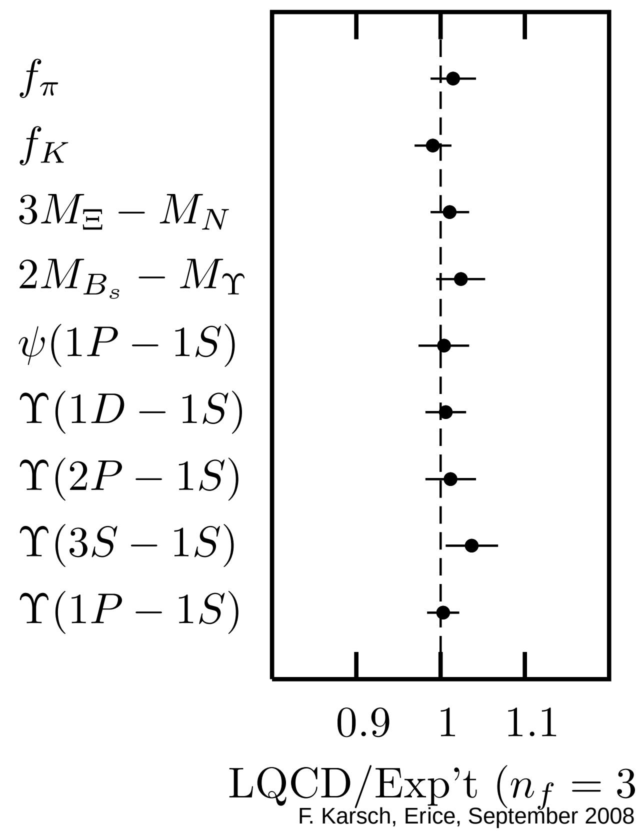
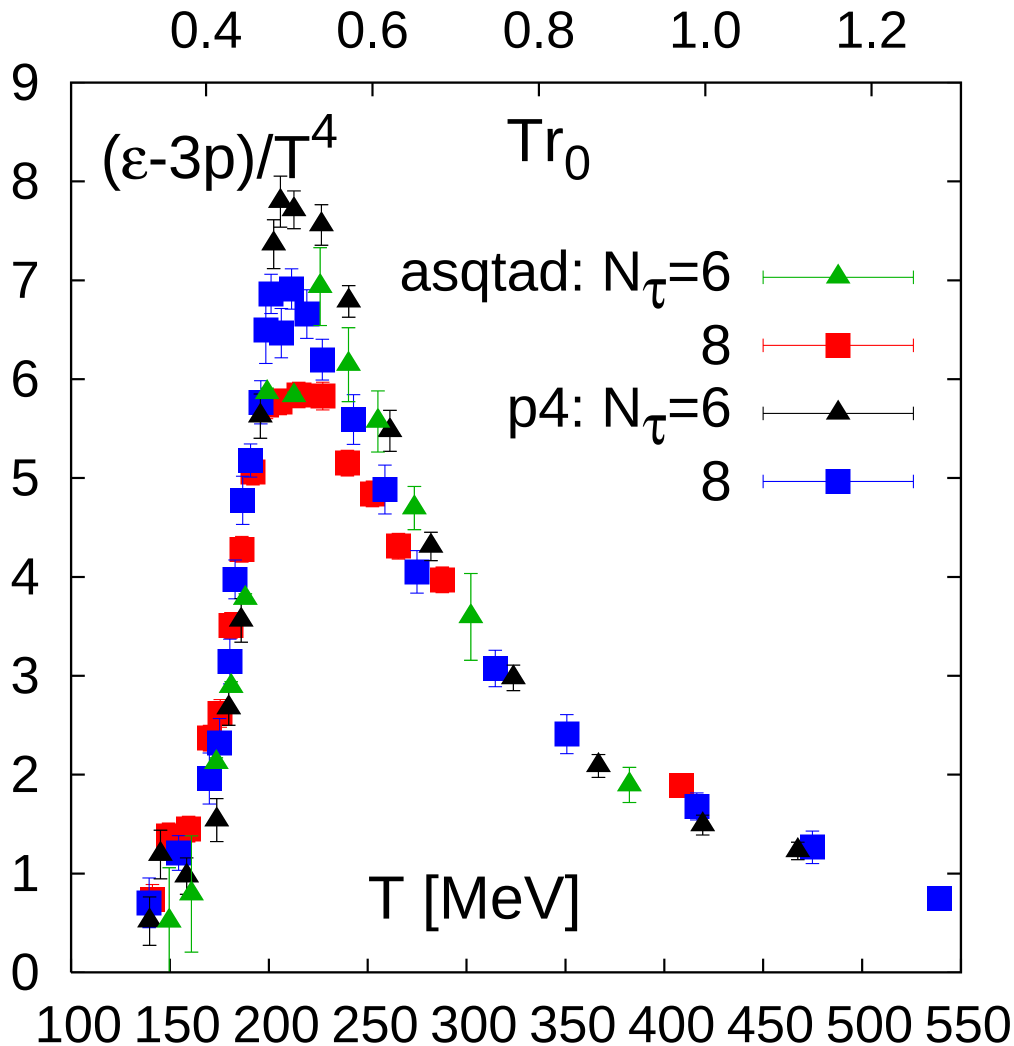
we use $r_0 = 0.469(7)$ fm determined from quarkonium spectroscopy
A. Gray et al, Phys. Rev. D72 (2005) 094507



C.T.H. Davies et al., PRL 92 (2004) 022001
A. Gray et al., PRD72 (2005) 094507

$(\epsilon - 3p)/T^4$ on LCP



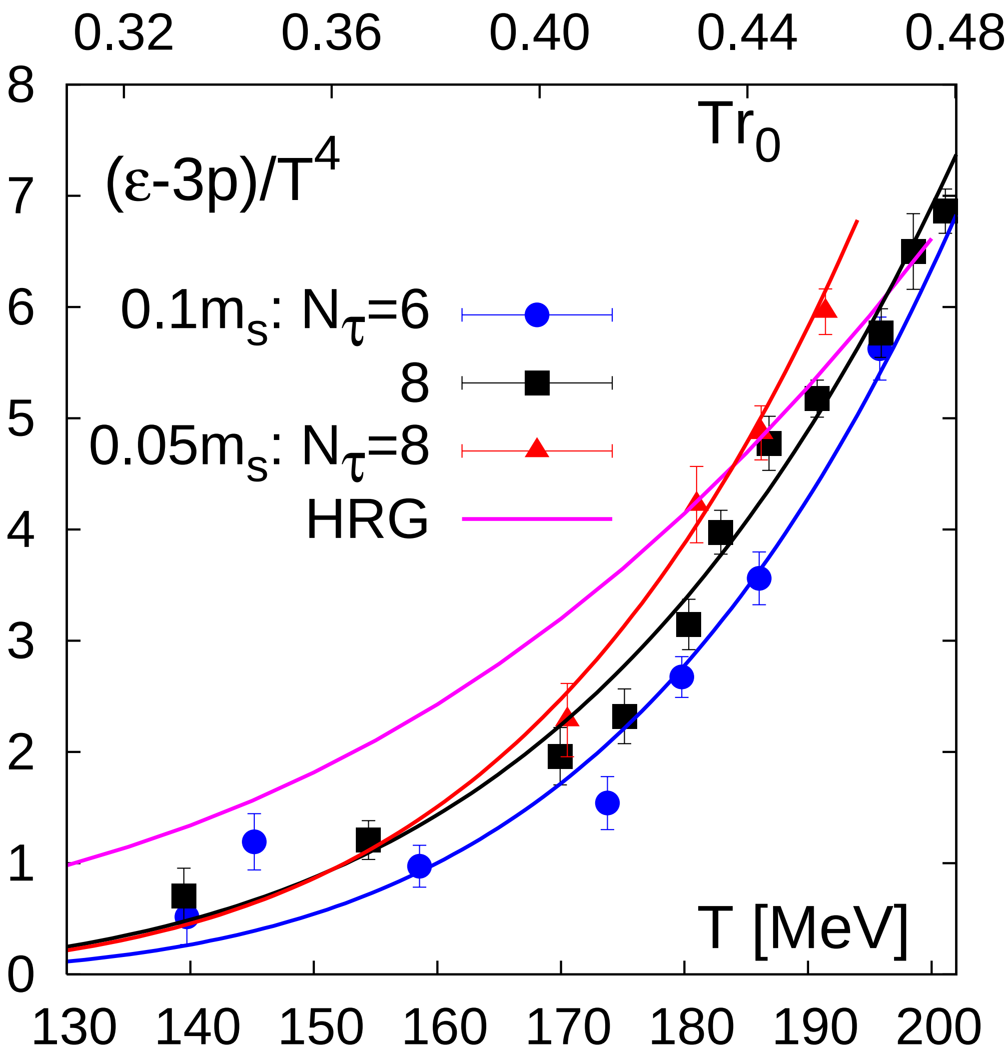
two different fermion discretization schemes agree on shape of $(\epsilon - 3p)/T^4$ AND the temperature scale Tr_0

LCP: $m_q = 0.1 m_s$

$\Rightarrow m_\pi \simeq 220$ MeV

to get from Tr_0 to T [MeV] use $r_0 = 0.469$fm

$(\epsilon - 3p)/T^4$ on LCP



two different fermion discretization schemes agree on shape of $(\epsilon - 3p)/T^4$ AND the temperature scale Tr_0

LCP: $m_q = 0.1 m_s$

$\Rightarrow m_\pi \simeq 220$ MeV

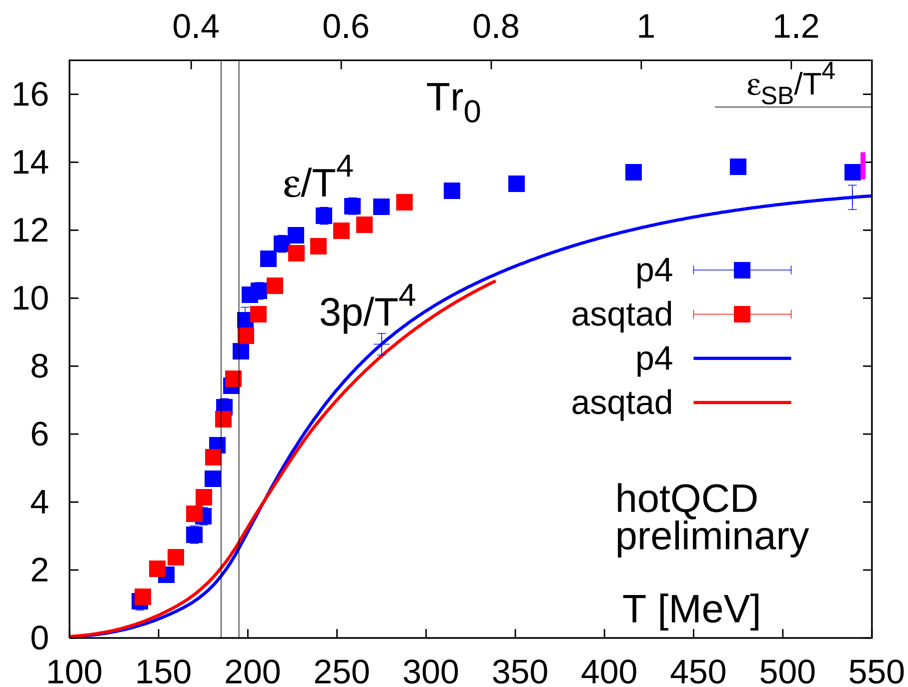
towards the physical LCP: $m_q = 0.05 m_s$

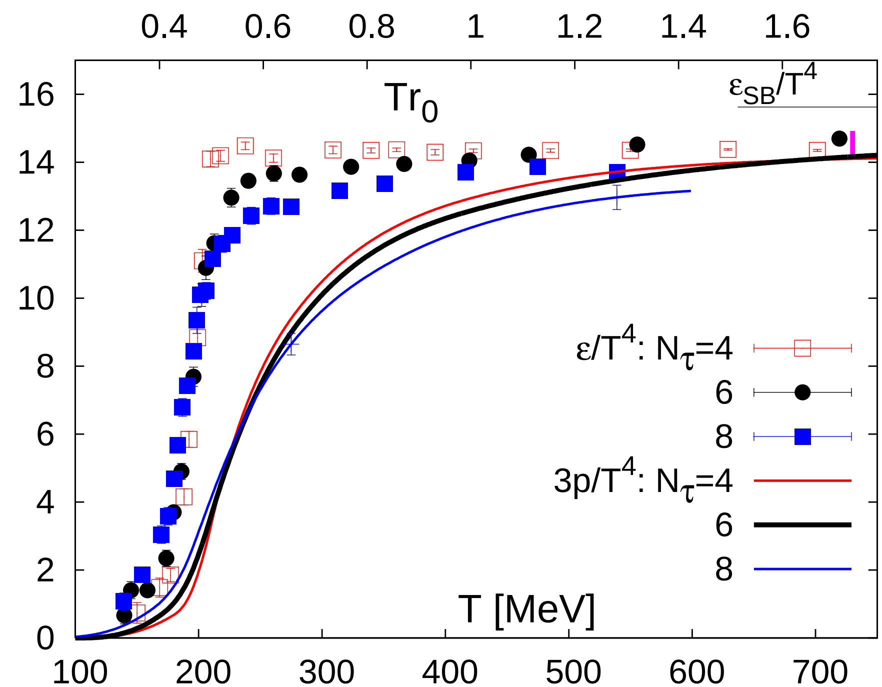
~ 5 MeV shift in T-scale

to get from Tr_0 to T [MeV] use $r_0 = 0.469$fm

Pressure, Energy and Entropy

- p/T^4 from integration over $(\epsilon - 3p)/T^5$;
  $p(T_0) = 0$ at $T_0 = 0$ MeV
  (exponential extrapolation);
- systematic error on $3p/T^4 \simeq 0.33$
- good scaling behavior; good agreement between different discretization schemes





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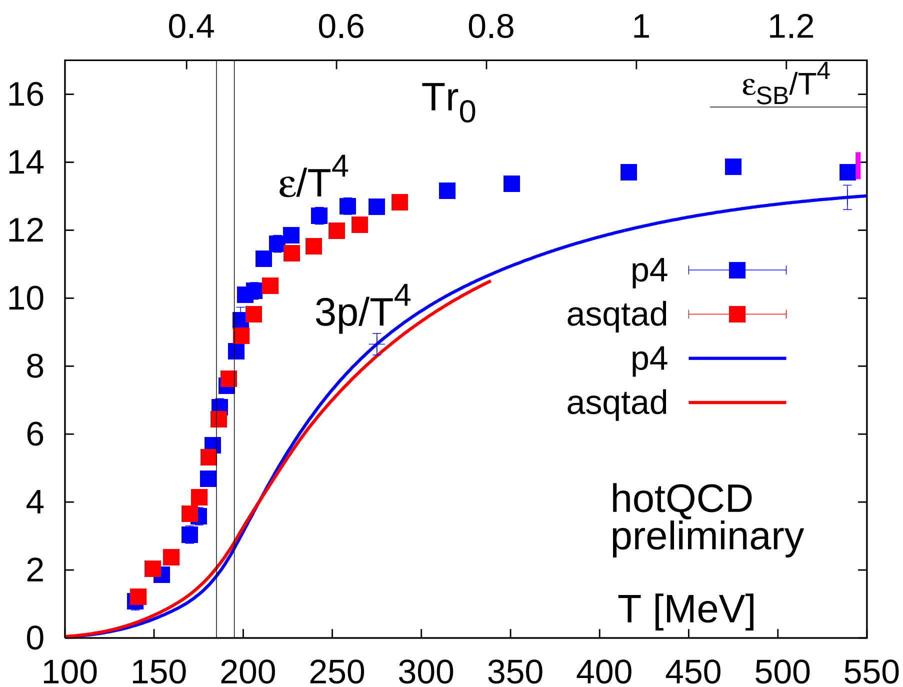
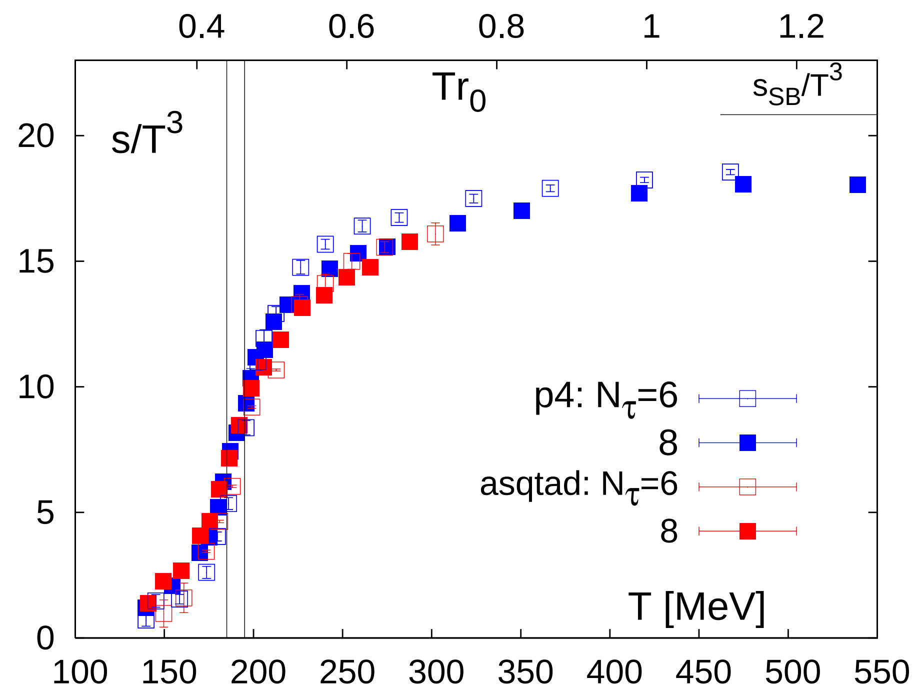
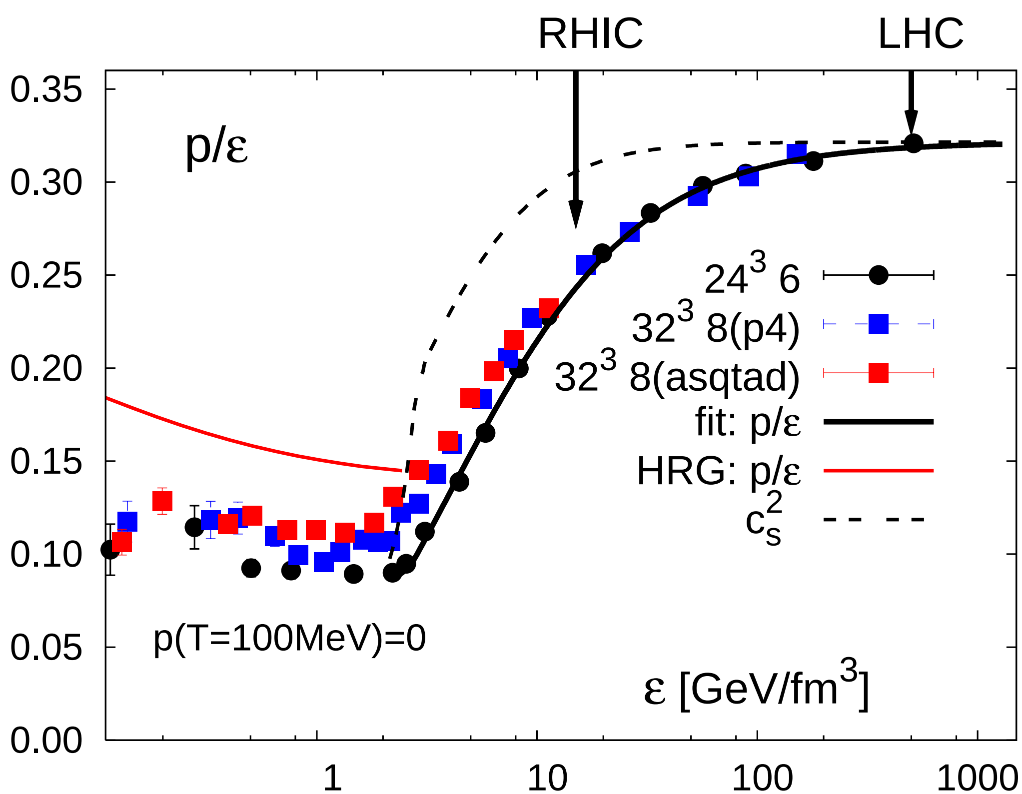




EoS and velocity of sound

- $p/\epsilon \;\Rightarrow$ velocity of sound:

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\epsilon} = \epsilon\frac{\mathrm{d}(p/\epsilon)}{\mathrm{d}\epsilon} + \frac{p}{\epsilon} \equiv \frac{s}{c_V}$$



pressure set to zero

at $T = 100$ MeV

hotQCD preliminary

fit: $p/\epsilon = c - a/(1 + b\epsilon)$ for $\epsilon \gtrsim 4$ GeV/fm^3

Transition temperature

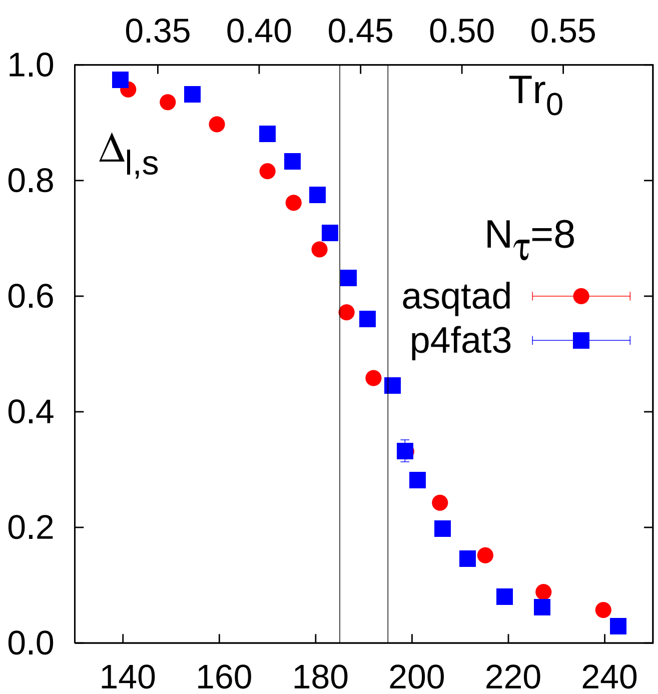
Goal: QCD thermodynamics with realistic quark masses in (2+1)-f QCD and controlled extrapolation to the continuum limit;

- control cut-off dependence: $N_\tau = 4,\ 6,...$
- control volume dependence: $N_\sigma / N_\tau = 2,\ 4...$
- control quark mass dependence, in order to get confidence in stability of results at physical point:
  $150\ \mathrm{MeV} \lesssim m_\pi \lesssim 500\ \mathrm{MeV}$
- establish results in the chiral limit

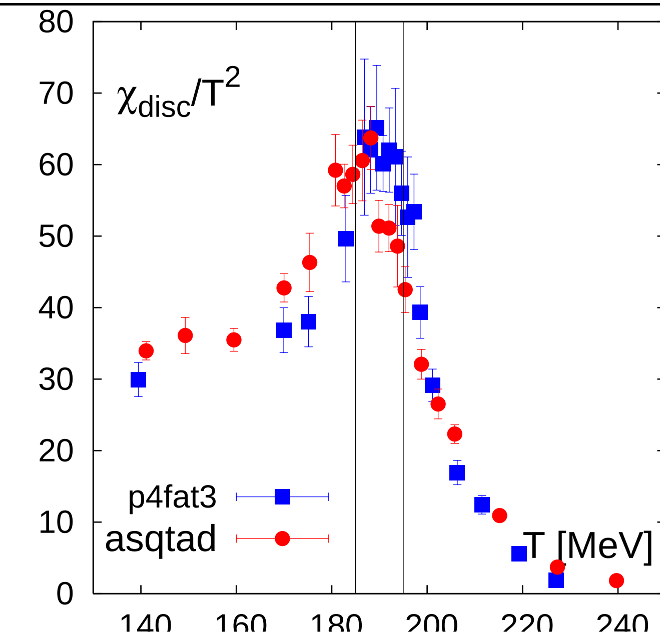
χ-condensate and susceptibility

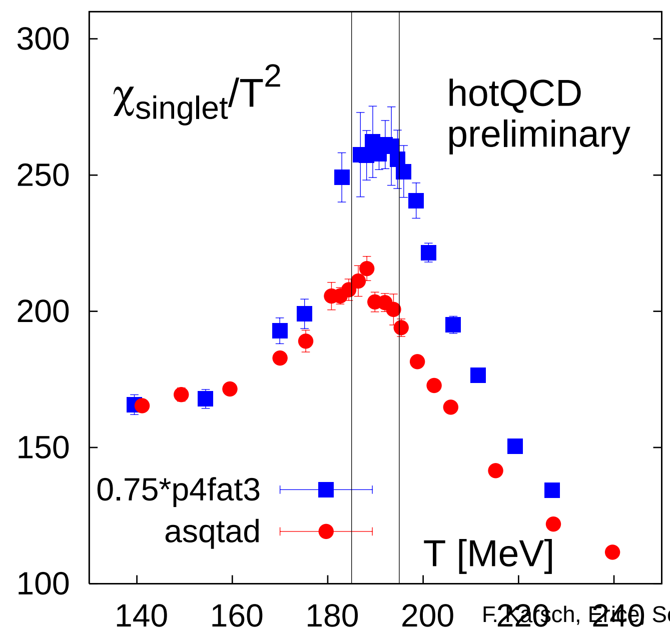
- sudden change in chiral condensate is, of course, related to peaks in the (singlet) chiral susceptibility

$$
\begin{aligned}
\chi_{tot}/T^2 &= 2\chi_{dis}/T^2 + \chi_{con}/T^2 \\
\Delta_{l,s}(T) &= \frac{\langle\bar\psi\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,T}}{\langle\bar\psi\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,0}}
\end{aligned}
$$



p4 and asqtad: hotQCD, preliminary



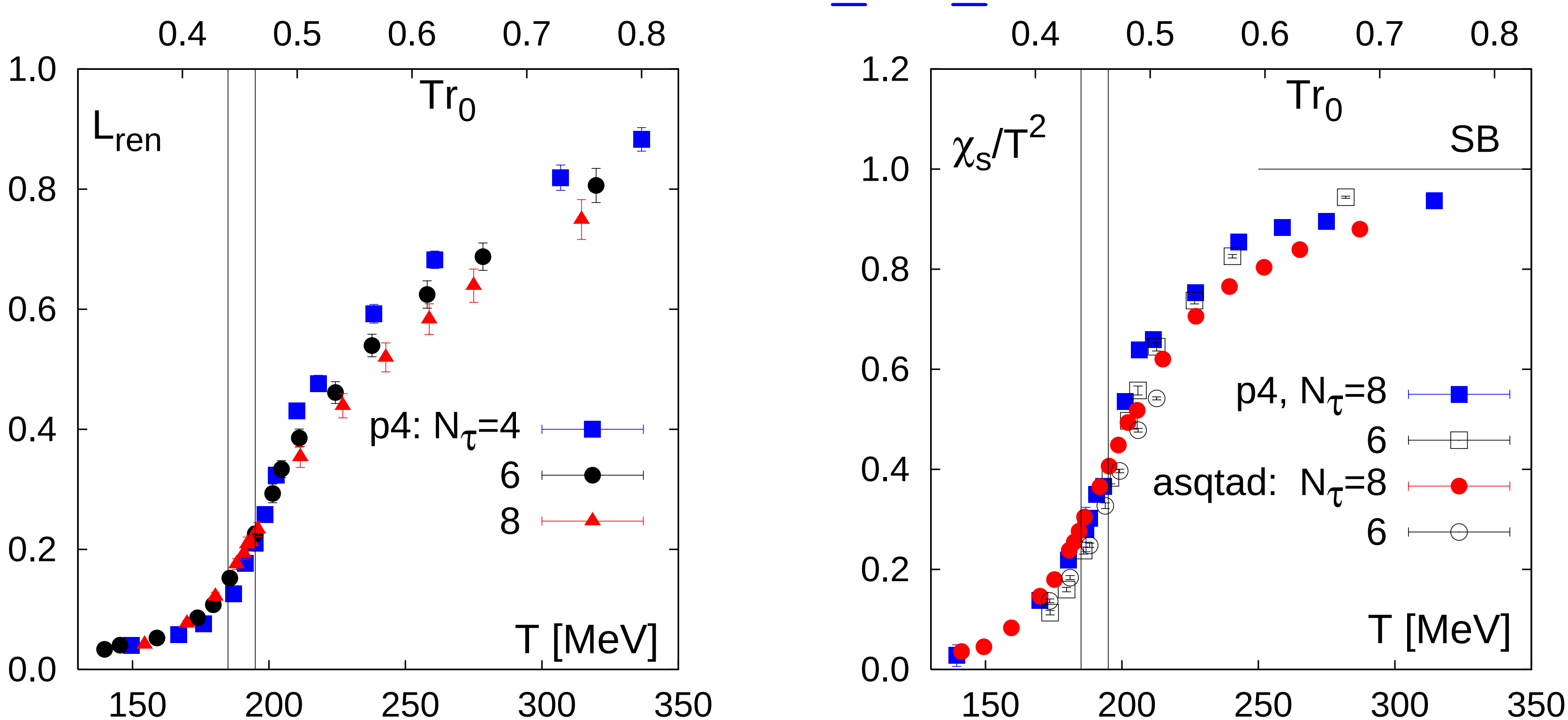


Deconfinement

- renormalized Polyakov loop and strange quark number susceptibility

$$L_{ren} \sim \mathrm{e}^{-F_Q(T)/T}, \qquad \chi_s/T^2 \sim \langle N_s^2 \rangle$$

band: $185\mathrm{MeV} \leq T \leq 195\mathrm{MeV}$

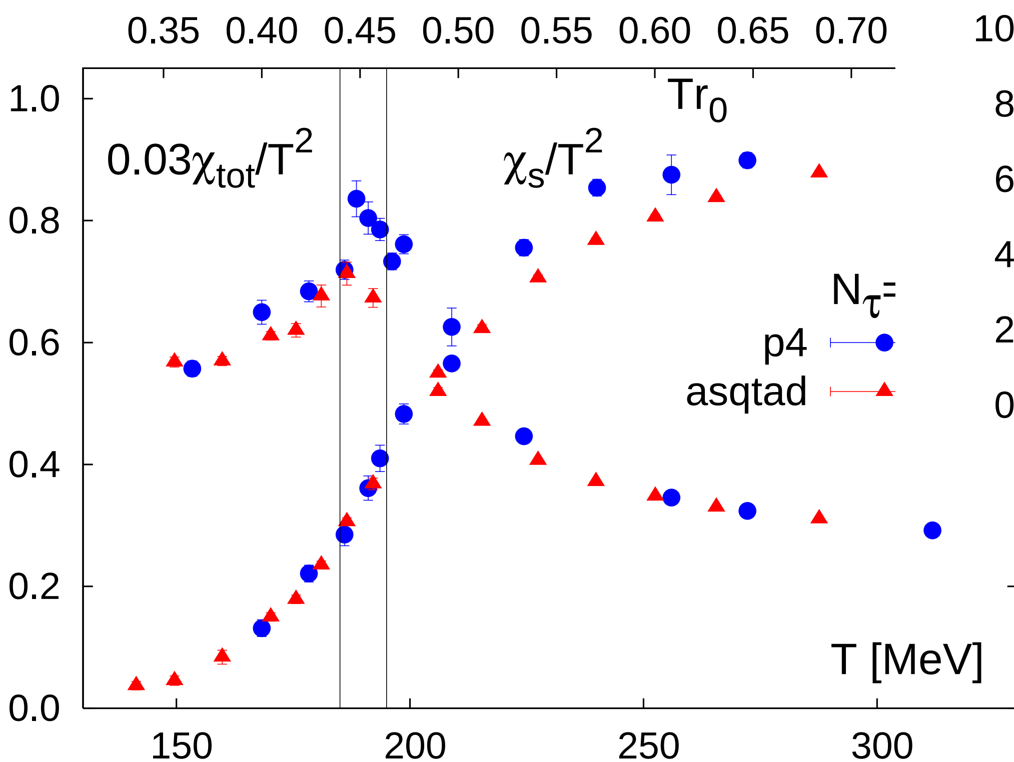


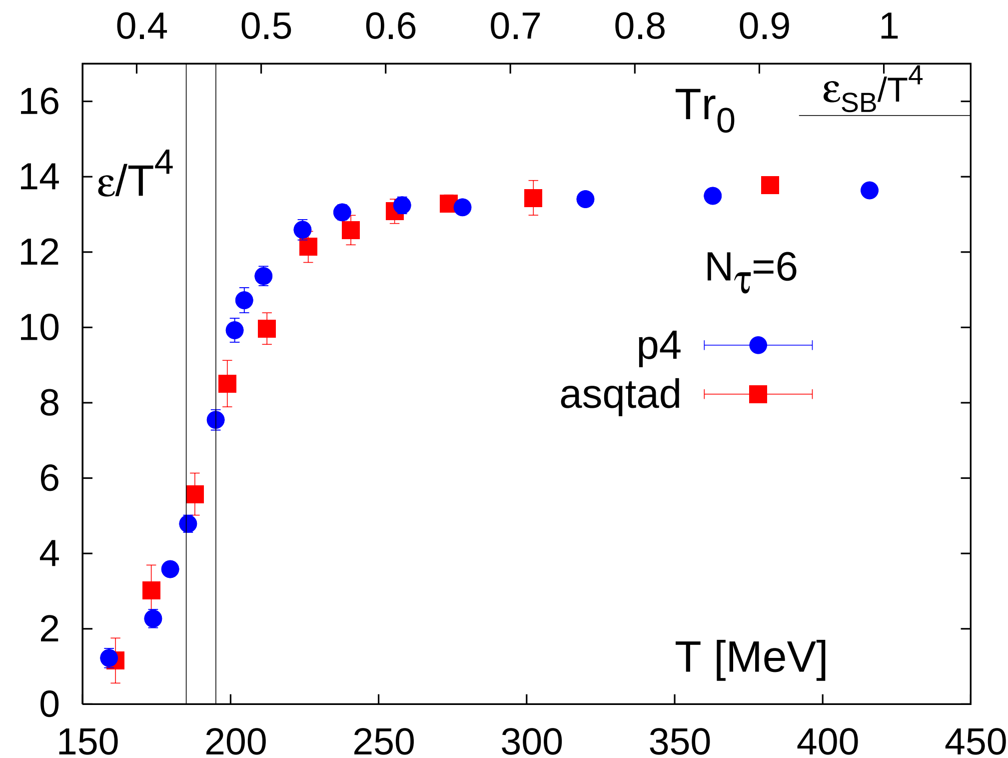
$N_\tau = 4,\ 6$ (p4): RBC-Bielefeld, PRD77, 014511 (2008)

$N_\tau = 8$, and $N_\tau = 6$ (asqtad): hotQCD, preliminary

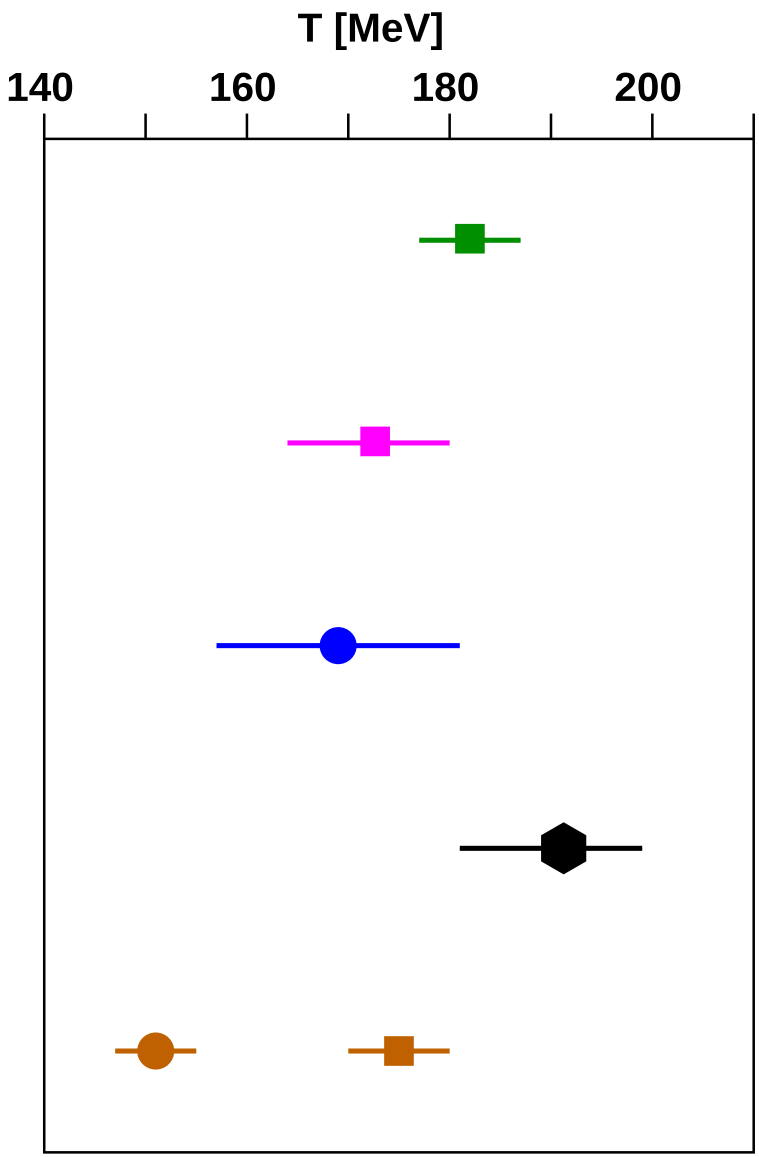
Deconfinement and χ-symmetry and bulk thermodynamics

- most prominent features of bulk thermodynamics are related to deconfinement
- χ-symmetry restoration:
  drop in condensate;
  peak in susceptibilities





Where are we now?



use T=0 scale: r0=0.469fm

Nf=2:

V.G. Bornyakov et al, POS Lat2005, 157 (2006)

(improved Wilson, Nt=8, 10; input: r0=0.5 fm)

(added Nt=12, Lattice'07) **(rescaled to r0)**

Y. Maezawa et al., hep−lat/0702005 (QM'2006)

(improved Wilson, Nt=4, 6; input: m−rho)

(no cont. exp. yet)

Nf=2=1:

C. Bernard et al., Phys.Rev. D71, 034504 (2005)

(improved staggered (asqtad), Nt=4,6,8, input r1)

(rescaled to r0)

M. Cheng et al., Phys.Rev D74, 054507 (2006)

(improved staggered (p4), Nt=4,6; input r0)

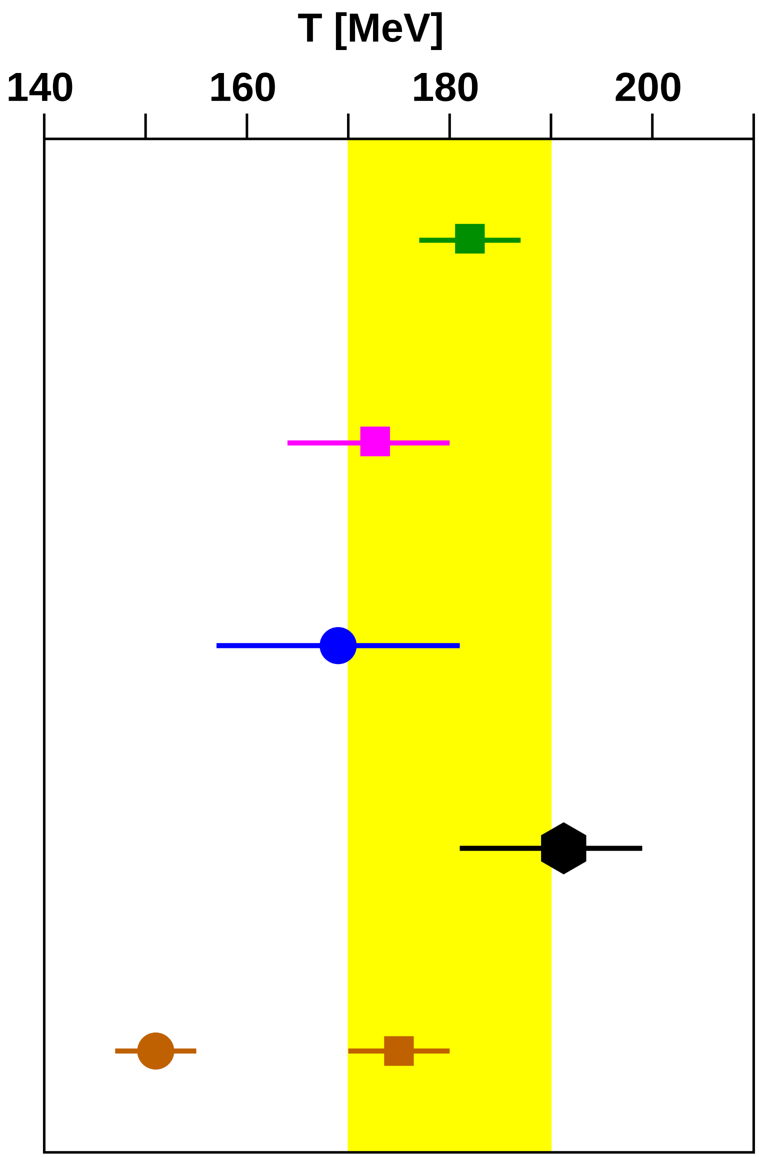
Y. Aoki et al., Phys. Lett. B643, 46 (2006)

(staggered (stout), Nt=4,6,8,10; input fK)

(converted to r0)

chiral **deconfinement** **chiral+deconfinement**

Where are we now?



chiral   deconfinement   chiral+deconfinement

Known shortcomings:

too few data: 3-parameter fit to 4 data points for T_c determined at $m_\pi > 550$ MeV

no continuum extrapolation attempted yet; would like to see results in units of r_0

$N_\tau = 4,\ 6,\ 8$, but small spatial volume for larger N_τ; still large statistical errors on individual data points

only $N_\tau = 4$ and 6

only one quark mass; T_c determination partly based on determination of inflection points

Hadronic fluctuations at $\mu > 0$ from
Taylor expansion coefficients at $\mu = 0$

$n_f = 2,\ m_\pi \simeq 770$ MeV: S. Ejiri, FK, K.Redlich, PLB633 (2006) 275
$n_f = 2+1,\ m_\pi \simeq 220$ MeV: RBC-Bielefeld, preliminary

- Taylor expansion of bulk thermodynamics in terms of $\mu_{u,d,s}$

$$\begin{aligned} \frac{p}{T^4} &\equiv \frac{1}{VT^3} \ln Z(V,T,\mu_u,\mu_d,\mu_s) \\ &= \sum_{i,j,k} c_{i,j,k} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_u}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k \end{aligned}$$

- expansion coefficients evaluated at $\mu_{u,d,s} = 0$ are related to fluctuations of $B,\ S,\ Q$ at $\mu_{B,S,Q} = 0$:

$\Uparrow$ baryon number, strangeness, charge fluctuations

event-by-event fluctuations at RHIC and LHC

Hadronic fluctuations at $\mu > 0$ from Taylor expansion coefficients at $\mu = 0$

$n_f = 2,\ m_\pi \simeq 770$ MeV: S. Ejiri, FK, K.Redlich, PLB633 (2006) 275
$n_f = 2+1,\ m_\pi \simeq 220$ MeV: RBC-Bielefeld, preliminary

- quadratic and quartic fluctuations

$$\chi_2^x = \frac{\partial^2 p/T^4}{\partial(\mu_x/T)^2} = \frac{1}{VT^3}\langle(\delta N_x)^2\rangle_{\mu=0} = \frac{1}{VT^3}\langle N_x^2\rangle_{\mu=0}$$

$$\begin{aligned}\chi_4^x &= \frac{\partial^4 p/T^4}{\partial(\mu_x/T)^4} = \frac{1}{VT^3}\left(\langle(\delta N_x)^4\rangle - 3\langle(\delta N_x)^2\rangle^2\right)_{\mu=0} \\ &= \frac{1}{VT^3}\left(\langle N_x^4\rangle - 3\langle N_x^2\rangle^2\right)_{\mu=0}\end{aligned}$$

- correlations

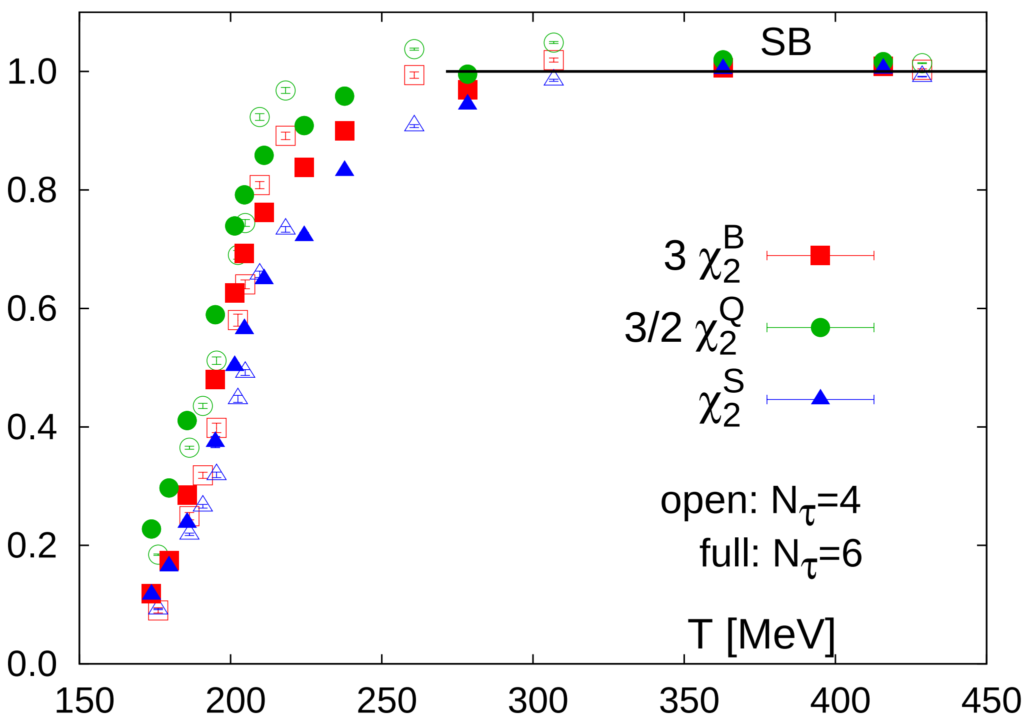
$$\chi_{11}^{x,y} = \frac{\partial^2 p/T^4}{\partial(\mu_x/T)\partial(\mu_y/T)} = \frac{1}{VT^3}\langle(\delta N_x)(\delta N_y)\rangle_{\mu=0}$$

with $x,y = u,\ d,\ s$ or $B,\ Q,\ S$

Quadratic fluctuations of baryon number charge & strangeness in (2+1)-flavor QCD

RBC-Bielefeld, preliminary

vanishing chemical potentials:



$$\chi_2^Q = \frac{1}{VT^3}\langle Q^2\rangle$$

$$\chi_2^B = \frac{1}{VT^3}\langle N_B^2\rangle$$

$$\chi_2^S = \frac{1}{VT^3}\langle N_S^2\rangle$$

rapid approach to SB limit

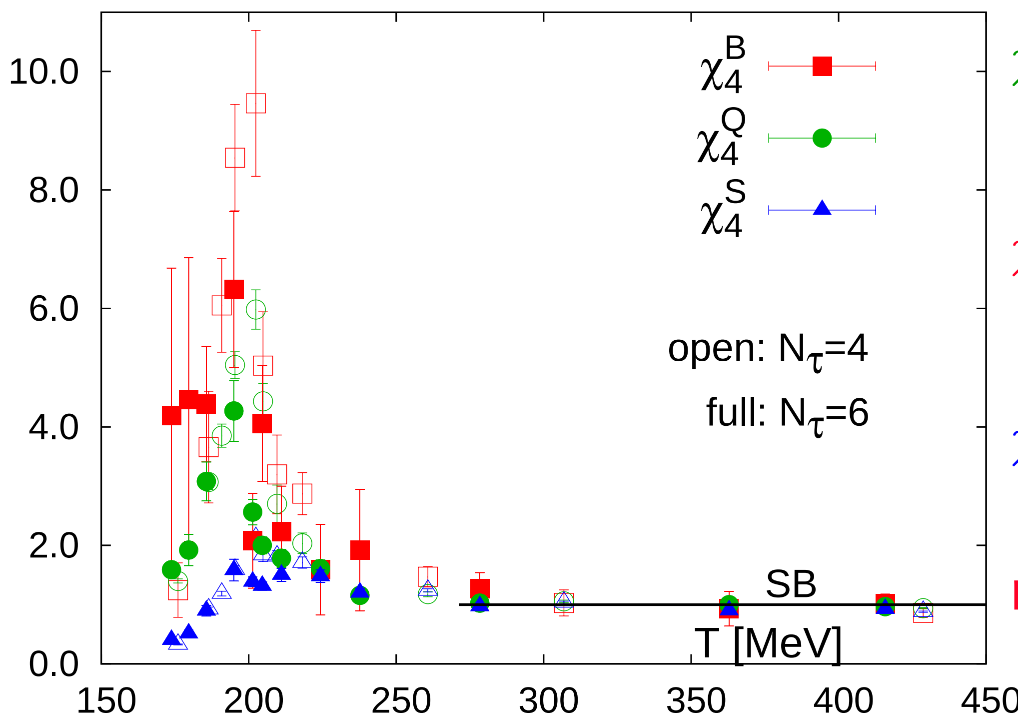
$\Rightarrow$ smooth change of quadratic fluctuations across transition region

chiral limit: $\chi_2^B,\ \chi_2^Q \sim |T - T_c|^{1-\alpha} + \mathbf{regular}$

Quartic fluctuations of baryon number charge & strangeness in (2+1)-flavor QCD

RBC-Bielefeld, preliminary

vanishing chemical potentials:



$$\chi_4^Q = \frac{1}{VT^3}\left(\langle Q^4\rangle - 3\langle Q^2\rangle^2\right)$$

$$\chi_4^B = \frac{1}{VT^3}\left(\langle N_B^4\rangle - 3\langle N_B^2\rangle^2\right)$$

$$\chi_4^S = \frac{1}{VT^3}\left(\langle N_S^4\rangle - 3\langle N_S^2\rangle^2\right)$$

rapid approach to SB limit

$\Rightarrow$ large light quark number & charge fluctuations across transition region

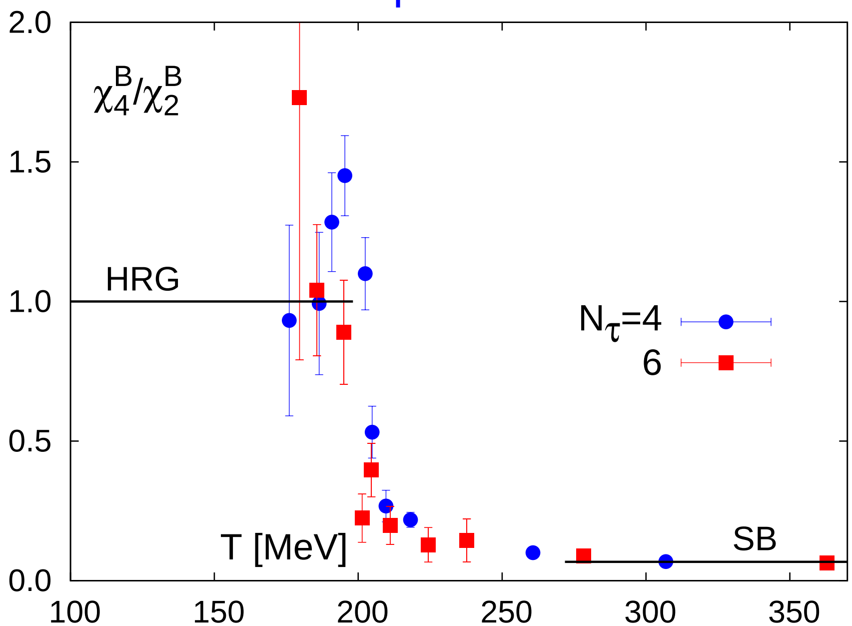
chiral limit: $\chi_4^B,\ \chi_4^Q \sim |T - T_c|^{-\alpha} + \mathrm{regular}$

Ratios of quartic and quadratic fluctuations of charges in (2+1)-flavor QCD

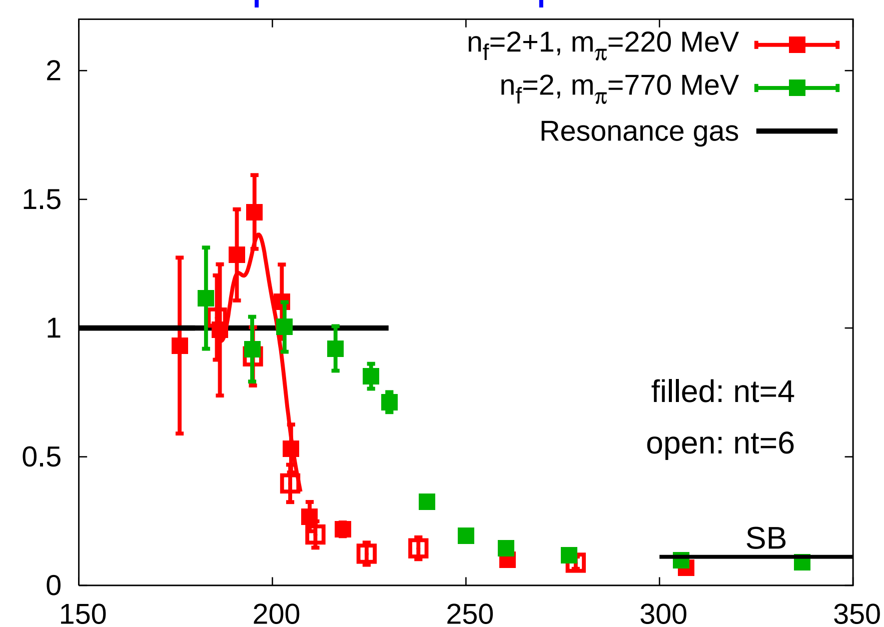
$n_f = 2$: S. Ejiri, FK, K.Redlich, PLB633 (2006) 275

$n_f = 2 + 1$: RBC-Bielefeld, preliminary

baryon number fluctuation
cut-off dependence



quark mass dependence



chiral limit: ratios $\sim |T - T_c|^{-\alpha} + \mathbf{regular}$

$\Rightarrow$ enhancement over resonance gas values? (need to improve $N_\tau = 6$)

$\Rightarrow$ may be observable in event-by-event fluctuations

quark sector quickly ($T \gtrsim 1.5T_c$) behaves perturbative

Quark number in Boltzmann approximation

- baryonic sector of pressure in a hadron resonance gas;

  $m_B \gg T \;\Rightarrow$ Boltzmann approximation: $p_B/T^4 = \sum_{m \le m_{max}} p_m/T^4$

with $\quad p_m/T^4 \;=\; F(T,m,V)\cosh(B_m \mu_B/T)$

$$\chi_2^B \quad : \quad \frac{\partial^2 p_m/T^4}{\partial(\mu_B/T)^2} = B_m^2 F(T,m,V)\cosh(B_m\mu_B/T)$$

$$\chi_4^B \quad : \quad \frac{\partial^4 p_m/T^4}{\partial(\mu_B/T)^4} = B_m^4 F(T,m,V)\cosh(B_m\mu_B/T)$$

ratio of fourth (χ_4^B) and second (χ_2^B) cumulant of quark number fluctuation gives "average unit of charge" carried by all d.o.f (particles):

$$m \gg T \quad \Rightarrow \quad R_{4,2}^B \equiv \frac{\chi_4^B}{\chi_2^B} = 1 \;\; \text{if all } B_m \equiv 0 \text{ or } 1$$

ratio is insensitive to details of the baryon mass spectrum

Charge fluctuations in Boltzmann approximation

- hadronic resonance gas: contributions from isosinglet ($G^{(1)}: \eta, \ldots$) and isotriplet ($G^{(3)}: \pi, \ldots$) mesons as well as isodoublet ($F^{(2)}: p, n, \ldots$) and isoquartet ($F^{(4)}: \Delta, \ldots$) baryons

$$\begin{aligned}\frac{p(T,\mu_q,\mu_I)}{T^4} &\simeq G^{(1)}(T) + G^{(3)}(T)\frac{1}{3}\left(2\cosh\left(\frac{2\mu_I}{T}\right)+1\right) \\ &+F^{(2)}(T)\cosh\left(\frac{3\mu_q}{T}\right)\cosh\left(\frac{\mu_I}{T}\right) \\ &+F^{(4)}(T)\frac{1}{2}\cosh\left(\frac{3\mu_q}{T}\right)\left[\cosh\left(\frac{\mu_I}{T}\right)+\cosh\left(\frac{3\mu_I}{T}\right)\right]\end{aligned}$$

- charge fluctuations at $\mu_q = \mu_I = 0$;
isospin quartet $F^{(4)}$ contains baryons carrying charge 2

$$R^Q_{4,2} \equiv \frac{\chi_4^Q}{\chi_2^Q} = \frac{4G^{(3)} + 3F^{(2)} + 27F^{(4)}}{4G^{(3)} + 3F^{(2)} + 9F^{(4)}} \rightarrow 1 \text{ for } T \rightarrow 0$$

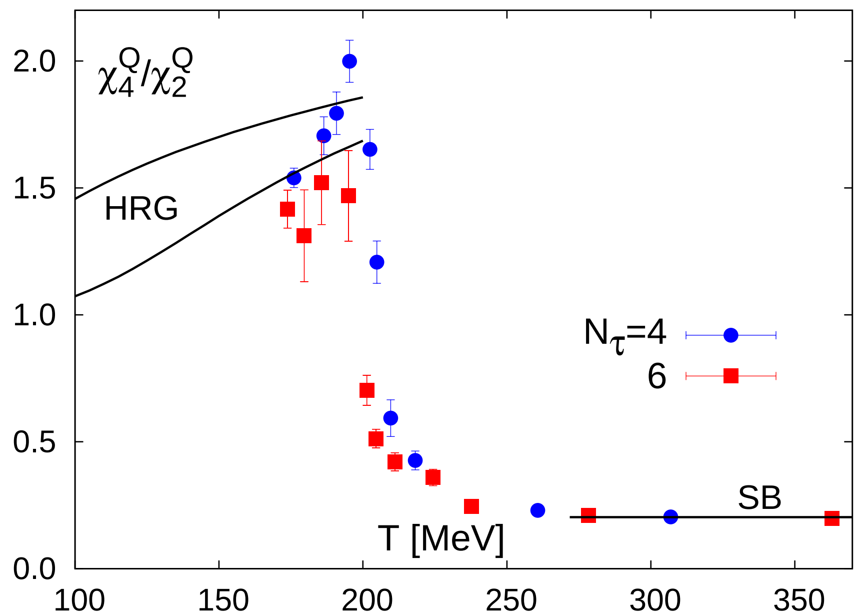
contribution of doubly charged baryons increases quartic relative to quadratic fluctuations

Ratios of quartic and quadratic fluctuations of charges in (2+1)-flavor QCD

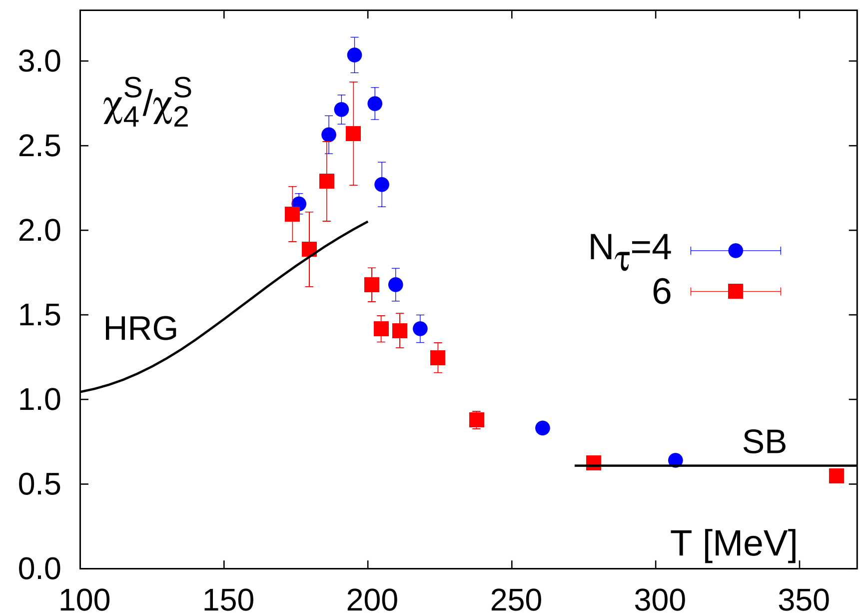
Kurtosis

$n_f = 2 + 1$: RBC-Bielefeld, preliminary

electric charge fluctuations



strangeness fluctuations



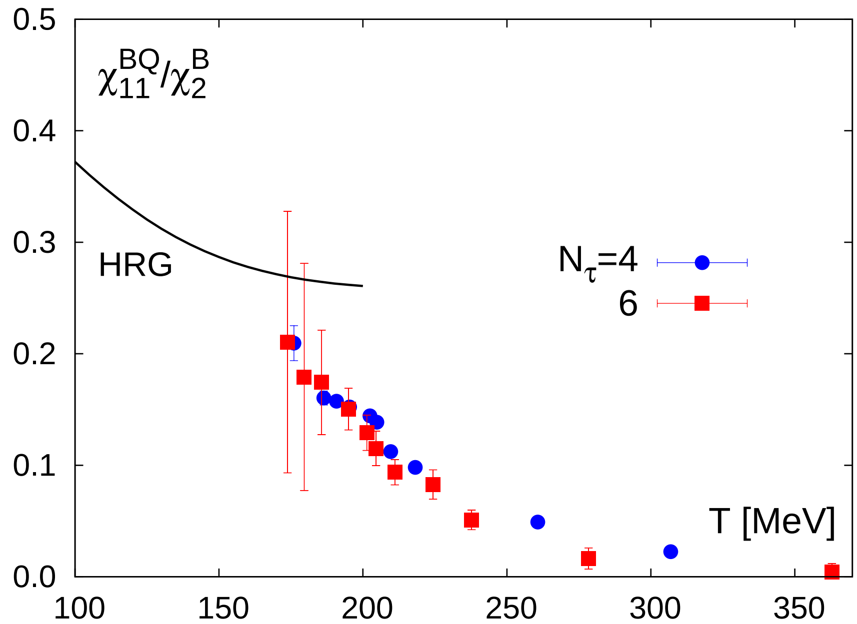
ratios are sensitive to multiple charged hadronic sectors

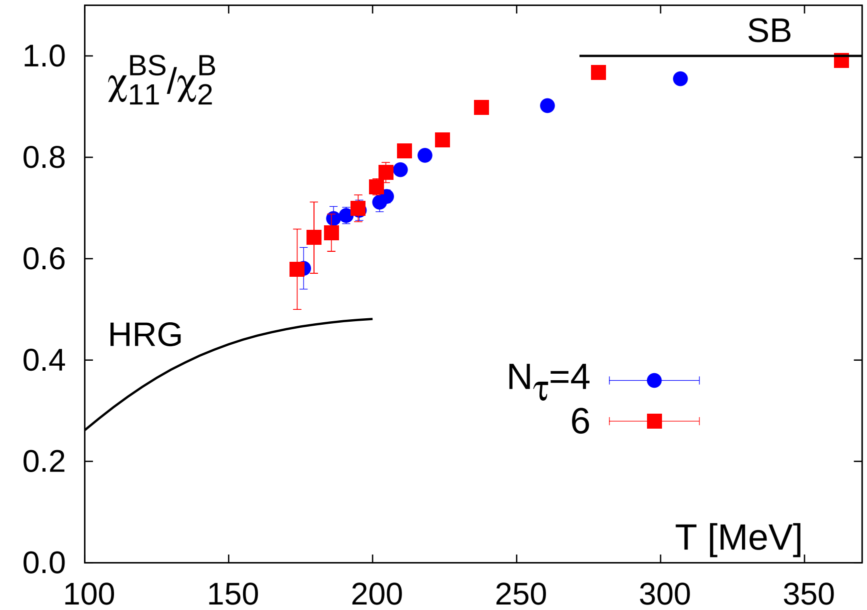
electric charge fluctuations are sensitive to light pion masses (Bose statistics)

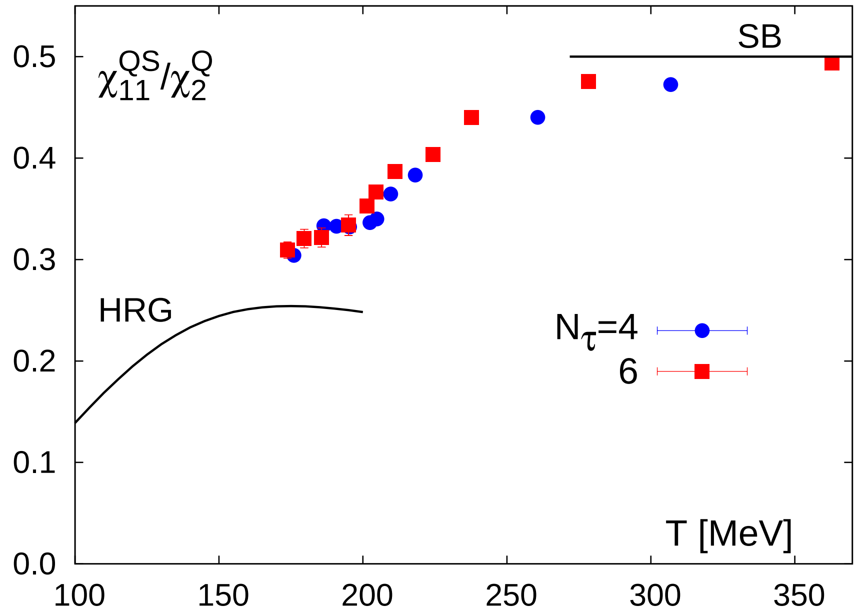
quark sector quickly ($T \gtrsim 1.5 T_c$) behaves perturbative

Correlations among conserved charges

$n_f = 2 + 1$: RBC-Bielefeld, preliminary







$$\frac{\chi_{11}^{XY}}{\chi_2^{X}} \equiv \frac{\langle XY \rangle}{\langle X^2 \rangle}$$

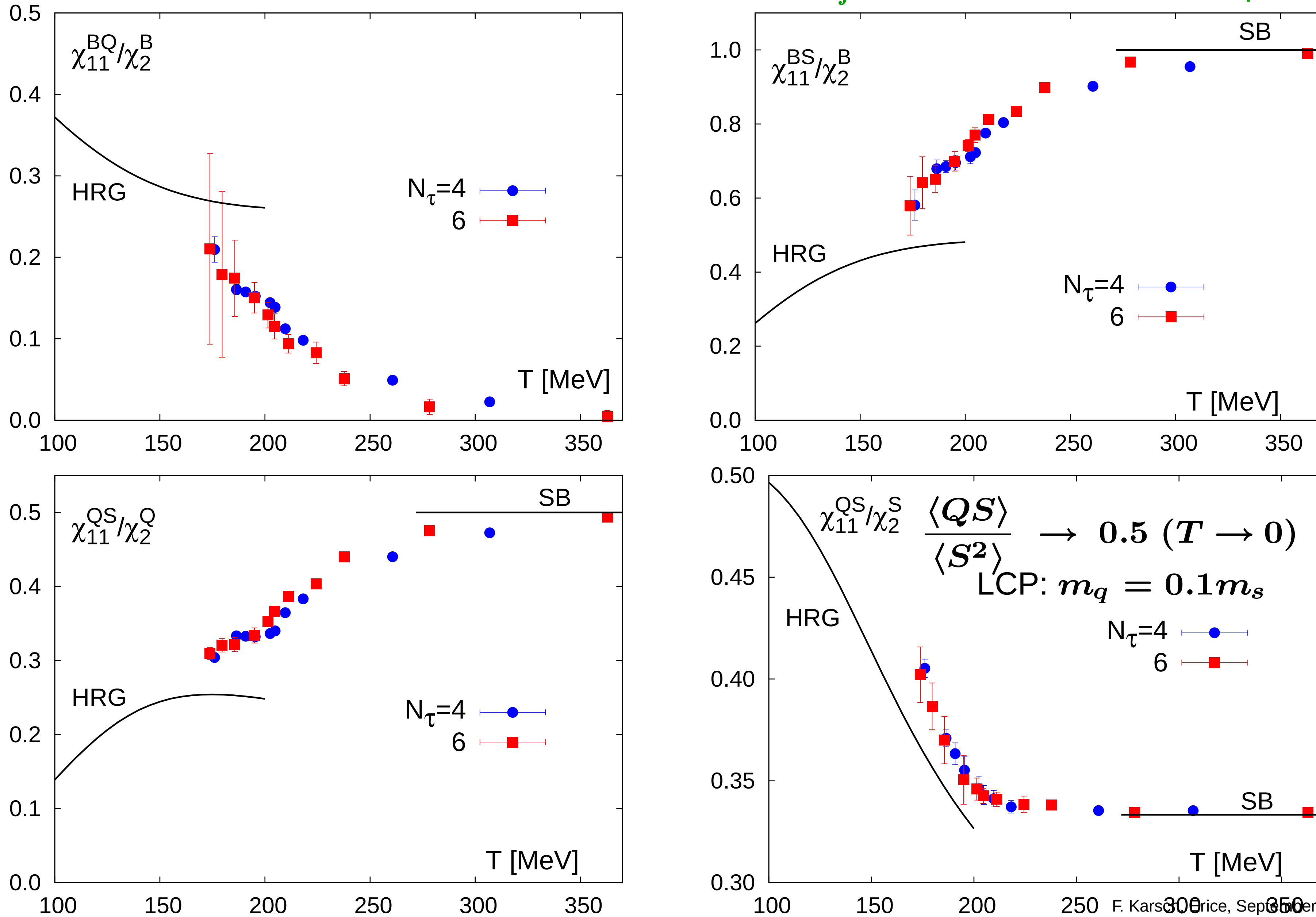
HRG:

ratios $\longrightarrow$ $\mathbf{0.5}$ (BQ)

$\mathbf{0}$ (BS, QS) for $T \longrightarrow 0$

Correlations among conserved charges

$n_f = 2 + 1$: RBC-Bielefeld, preliminary



Hadronic fluctuations at $\mu_q = 0$

- expect 2^{nd} order transition in 3-d, O(4) symmetry class;

  scaling field: $t = \left|\frac{T-T_c}{T_c}\right| + A\left(\frac{\mu_q}{T_c}\right)^2$, $\mu_{crit} = 0$

  singular part: $f_s(T,\mu_u,\mu_d) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{2-\alpha}$

$$c_2 \sim \frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_q^2} \sim t^{1-\alpha} \quad , \quad c_4 \sim \frac{\partial^4 \ln \mathcal{Z}}{\partial \mu_q^4} \sim t^{-\alpha} \qquad (\mu = 0)$$

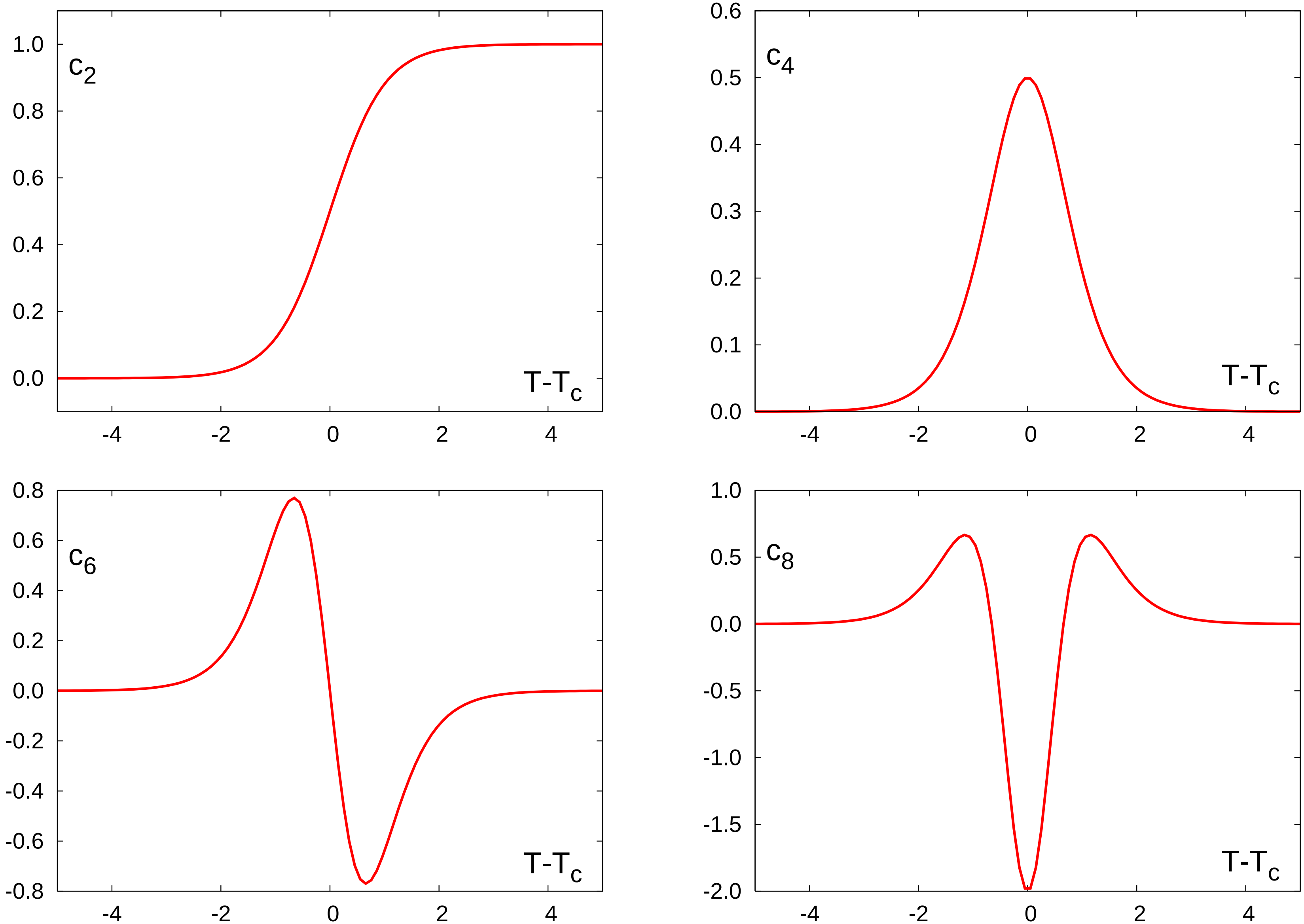
- O(4)/O(2): $\alpha < 0$, small $\Rightarrow$

  $c_2 \sim \langle (\delta N_q)^2 \rangle$ dominated by T-dependence of regular part

  $c_4 \sim \langle (\delta N_q)^4 \rangle - 3\langle (\delta N_q)^2 \rangle^2$ develops a cusp

  Y. Hatta, T. Ikeda, PRD67 (2003) 014028

Generic expansion coefficients



similar in PNJL model: S. Roessner et al, PR D75 (2007) 034007

Estimating $T_c(\mu_c)$ and μ_c/T

How can we estimate the location of the critical endpoint from expansion coefficients of the Taylor series?

- need $c_n(T) > 0$ to have a singularity on the real axis
- expect hadron resonance gas to be a good approximation at low T:

  $c_n^{HRG} > 0$ for all n, but $r_n^{HRG} = \sqrt{1/(n+2)/(n+1)} \rightarrow 0$

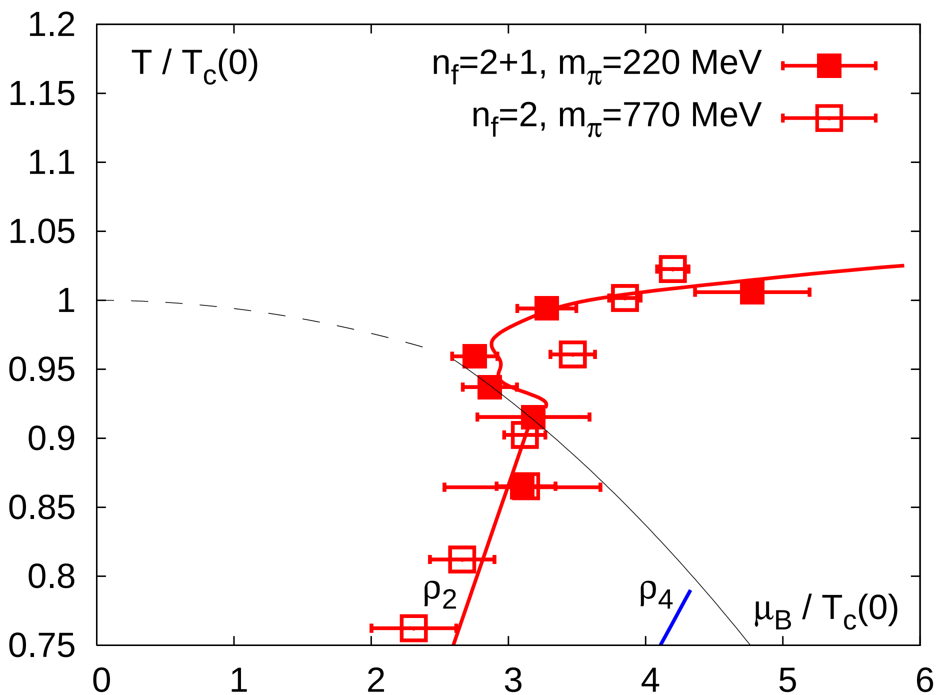
$\Rightarrow$ conjecture:

the position of the first maximum of $c_n(T)$, e.g. at $T_n < T_c(0)$, gives an upper bound on $T_c(\mu_c)$ as one will find $c_{n+2}(T) < 0$ for $T > T_n$

Estimating $T_c(\mu_c)$ and μ_c/T

Status of the RBC-BI project

- calculations for $N_\tau = 4$ and 6; $N_\sigma = 4N_\tau$
- uses an $\mathcal{O}(a^2)$ improved staggered action (p4fat3)
- estimator for μ_c: $$\left(\frac{\mu_c(T)}{T_c(0)}\right)_n \equiv \rho_n = \frac{T}{T_c(0)}\sqrt{\frac{c_n}{c_{n+2}}}$$



- slight quark mass dependence

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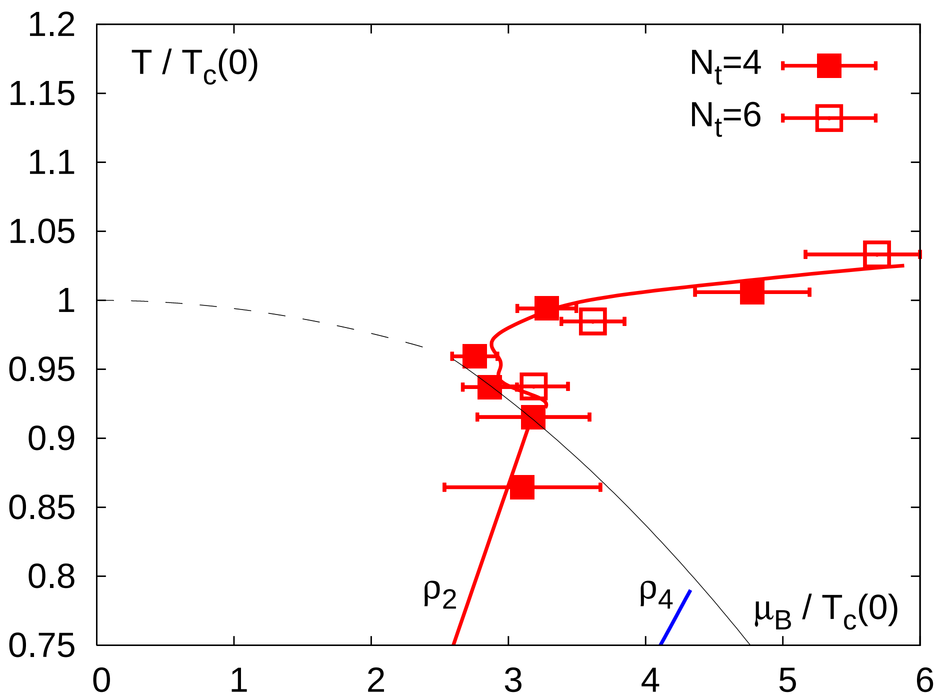

- slight quark mass dependence
- weak cut-off dependence

Estimating $T_c(\mu_c)$ and μ_c/T

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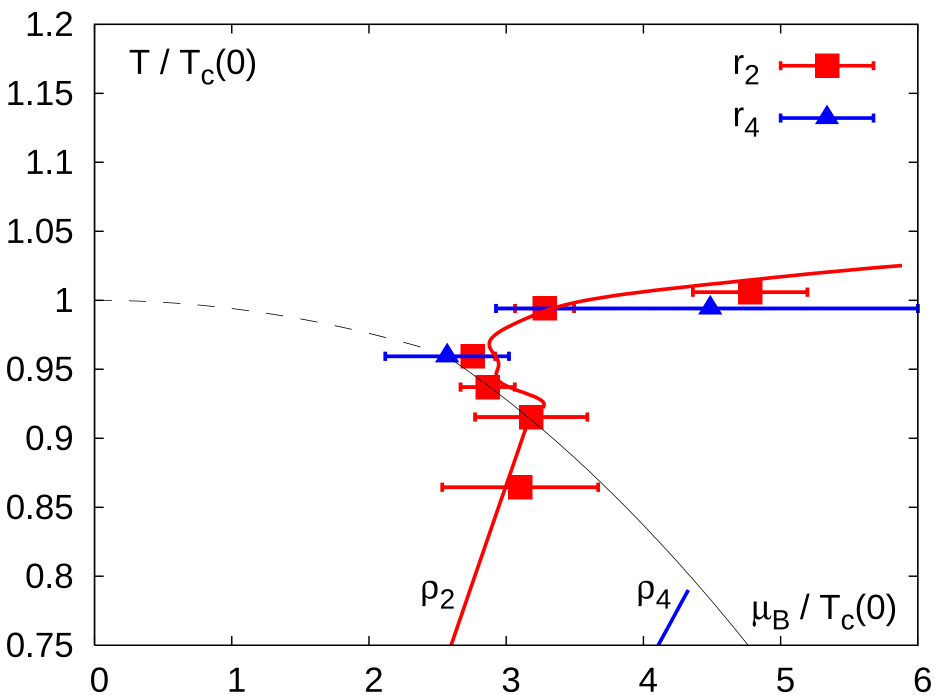

- slight quark mass dependence
- weak cut-off dependence
- $\mathcal{O}(\mu^6)$ requires more statistics

Estimating $T_c(\mu_c)$ and μ_c/T

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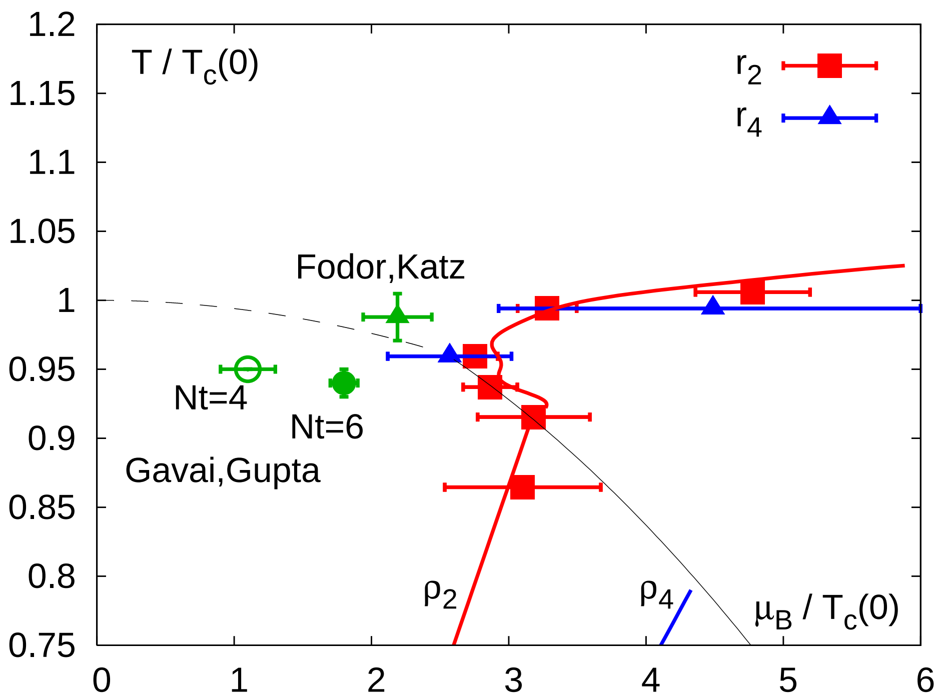
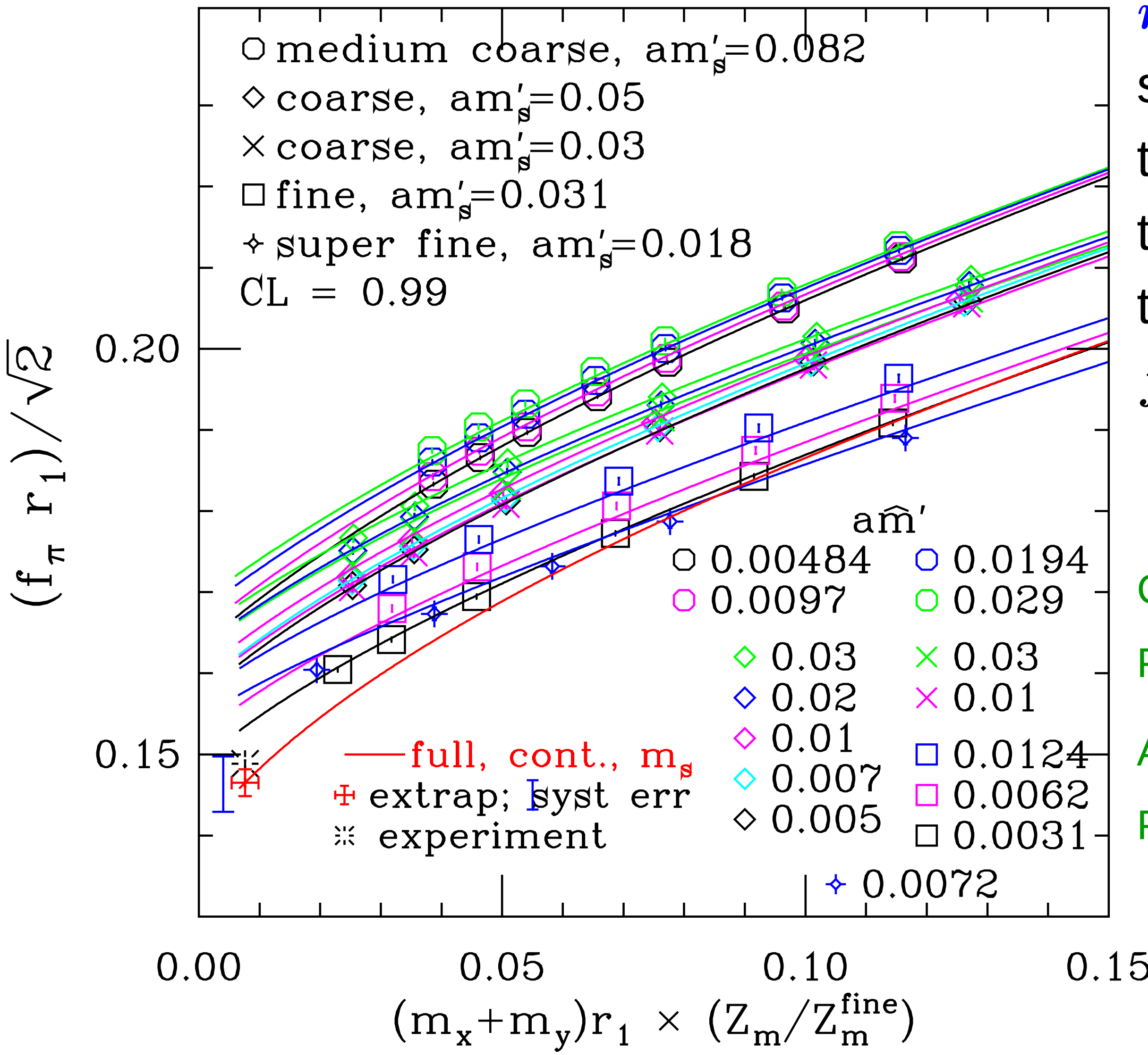


- slight quark mass dependence
- weak cut-off dependence
- $\mathcal{O}(\mu^6)$ requires more statistics
- controled calculations of higher order coefficients with improved actions are still needed
  $\longrightarrow$ computationally demanding
  Teraflops projects

Conclusions

- LGT calculations with (almost) physical quark masses and (reasonably) good control over the continuum extrapolation are now possible
- these calculations provide important input to the quantitative modelling of HIC and the analysis of signatures for the formation of a quark gluon plasma
- a major effort is still needed to provide results from LGT calculations with non-vanishing chemical potential to explore the entire phase diagram of QCD and verify or falsify the existence of a critical point in this phase diagram

f_π and f_K using staggered fermions $\Rightarrow$ r_0, r_1



r_0 can be fixed using high statistics, chiral and continuum extrapolated calculations of, e.g. Υ 2S-1S splitting and/or decay constants f_π, f_K

C. Aubin et al. (MILC), PRD70 (2004) 114501

A. Gray et al, Phys. Rev. D72 (2005) 094507