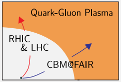


Equation of state for strongly interacting matter within a HTL quasiparticle model

Robert Schulze

collaborators: M. Bluhm, B. Kämpfer
TU Dresden, FZ Dresden-Rossendorf

- QPM with $\mathrm{Im}\Pi_{\mathrm{i}} \neq 0$, plasmons and plasminos from HTL
- extrapolation of lattice QCD to large baryon densities, e.g. CBM@FAIR
- EOS for hydrodynamic phase of heavy-ion collisions (SPS, RHIC, LHC)

From QCD to thermodynamics

QCD: $\mathcal{L}$
$\rightarrow$ propagators
$\rightarrow$ self-energies

Blaizot, Rebhan, Iancu

thermodyn. potential Ω
$\rightarrow$ state variables: p, s, n_q,...
$\rightarrow$ EOS $e = e(p)$
$\rightarrow$ $T^{\mu\nu}$, hydrodynamics

lattice QCD
$\rightarrow$ available @ $\mu \simeq 0$
$\rightarrow$ $\mu \neq 0$: sign problem

CJT formalism

- effective action

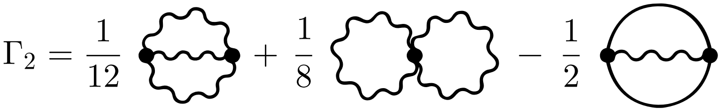
$$\Gamma[D,S] = I - \frac{1}{2}\left\{\mathrm{Tr}\left[\ln D^{-1}\right] + \mathrm{Tr}\left[D_0^{-1}D - 1\right]\right\} + \left\{\mathrm{Tr}\left[\ln S^{-1}\right] + \mathrm{Tr}\left[S_0^{-1}S - 1\right]\right\} + \Gamma_2[D,S]$$

- translation-invariant systems, no broken symmetries

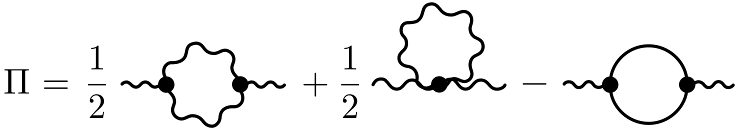
$$\frac{\Omega}{V} = \mathrm{tr}\int\frac{\mathrm{d}^4k}{(2\pi)^4} n_\mathrm{B}(\omega)\,\mathrm{Im}\left(\ln D^{-1} - \Pi D\right) + 2\,\mathrm{tr}\int\frac{\mathrm{d}^4k}{(2\pi)^4} n_\mathrm{F}(\omega)\,\mathrm{Im}\left(\ln S^{-1} - \Sigma S\right) - \frac{T}{V}\Gamma_2$$

2-loop QCD thermodynamics

- truncate Γ_2 at 2-loop order



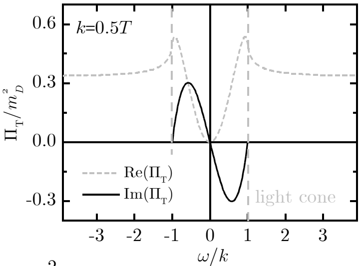
$\longrightarrow$ self-energies of 1-loop order

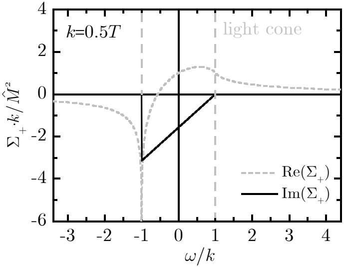


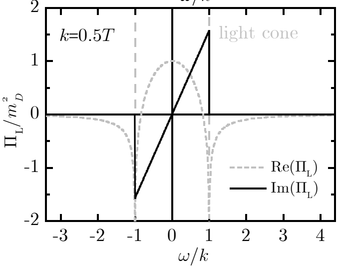
- gauge invariance: HTL self-energies instead

HTL self-energies

- $\mathrm{Im}\,\Pi \neq 0$ below the lightcone (solid lines)







$\longrightarrow$ Landau damping

Entropy

- stationarity of Ω

$$s := -\frac{1}{V}\left.\frac{\partial \Omega}{\partial T}\right|_{\mu} = -\frac{1}{V}\left(\left.\frac{\partial \Omega}{\partial T}\right|_{\text{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D}}_{0}\frac{\partial D}{\partial T}\right)_{\mu}$$

$$= s_{g,\text{T}} + s_{g,\text{L}} + s_{q,\text{Pt}} + s_{q,\text{Pl}} + s' \qquad s' = 0$$

Vanderheyden, Baym: JSP'98

- e.g. gluons:

$$s_{g,\text{T}} \sim \int_{\mathrm{d}^4k} \frac{\partial n_{\text{B}}}{\partial T}\left\{\underbrace{\pi\varepsilon(\omega)\Theta\left(-\text{Re}D_{\text{T}}^{-1}\right)}_{\text{qp contribution}} + \underbrace{\text{Re}D_{\text{T}}\text{Im}\Pi_{\text{T}} - \arctan\left(\frac{\text{Im}\Pi_{\text{T}}}{\text{Re}D_{\text{T}}^{-1}}\right)}_{\text{damping terms}}\right\}$$

- HTL QPM model $\Longleftrightarrow$ effective QPM

~~coll. modes~~, ~~damping~~, asympt. disp. rel.

Blaizot, Iancu, Rebhan: PRD'01
Rebhan, Romatschke: PRD'03

Bluhm, Kämpfer, RS, Seipt: EPJC'07

Pressure

$$s \sim \left(\frac{\partial \Omega}{\partial T}\Big|_{\text{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D}}_{0} \frac{\partial D}{\partial T} \right)_\mu$$

$$s_i \sim \int_{\mathrm{d}^4k} \frac{\partial n_{\mathrm{B/F}}}{\partial T} \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

- self-consistent formulation of the pressure

$$p = -\frac{\Omega}{V} := \sum_i p_i - B$$

$$p_i \sim \int_{\mathrm{d}^4k} n_{\mathrm{B/F}} \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

$$\frac{\partial B}{\partial T} := \sum_i \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} \qquad \left(\frac{\partial B}{\partial \mu} = \sum \frac{\cdot\cdot}{\cdot\cdot} \frac{\cdot\cdot}{\partial \mu} \right)$$

- entropy density

$$s = \frac{\partial p}{\partial T} = \sum_i s_i + \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} - \frac{\partial B}{\partial T} = \sum_i s_i$$

- net quark density

$$n \sim \frac{\partial \Omega}{\partial \mu}\Big|_{\text{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D}}_{0} \frac{\partial D}{\partial \mu}$$

$$n_q = \frac{\partial p}{\partial \mu} \sim \int_{\mathrm{d}^4k} \left(\frac{\partial n_{\mathrm{F}}}{\partial \mu} + \frac{\partial n_{\mathrm{F}}^A}{\partial \mu} \right) \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

Effective coupling

- fundamental parameter

$$g^2(x^2) = \frac{16\pi^2}{\beta_0 \ln(x^2)} \left(1 - \frac{4\beta_1}{\beta_0^2} \frac{\ln[\ln(x^2)]}{\ln(x^2)}\right)$$

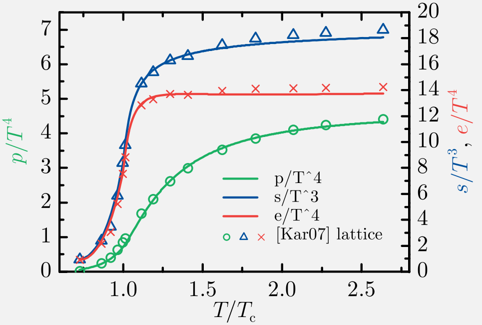
- running coupling g^2 $\quad x = \frac{\bar{\mu}}{\Lambda_{\mathrm{QCD}}}$ $\quad \bar{\mu} \sim T$

$$T > T_c,\ \mu = 0$$

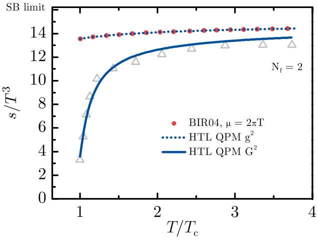
- effective coupling G^2 $\quad x = \frac{\lambda}{T_c}(T - T_s)$

Adjustment @ $\mu = 0$

- $\mu = 0$: adjust to ℓQCD
  - $\rightarrow$ T_s, λ fixed
  - $\rightarrow$ $G^2(T, \mu = 0)$

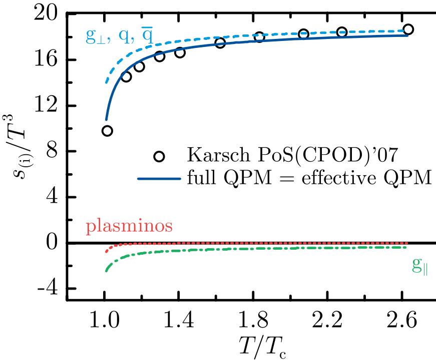


- $T_s = 0$:



Influence of coll. modes + LD @ $\mu = 0$

- individual entropy contributions



- Landau damping large close to T_c, decreases for higher temperatures state variables s, n, p, ...

↑

effective coupling G^2

↗ ↖

@ $\mu = 0$:
lattice data

@ $\mu \neq 0$:
??

Into the T-μ-plane

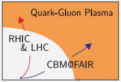
- $\mu > 0$: stationary potential, self-consistent model
  $\rightarrow$ impose Maxwell's relation

$$\frac{\partial s}{\partial \mu} = \frac{\partial n}{\partial T} \qquad \longrightarrow \qquad a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

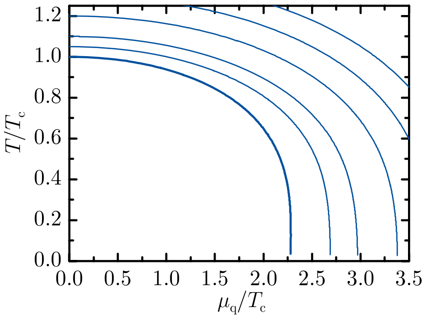
Peshier, Kämpfer, Soff: PRC'00, PRD'02

- solve quasilinear PDE for $G^2(T,\mu\neq 0)$ using method of characteristics
- test with lattice data for $\mu \simeq 0$: successful (eQPM)

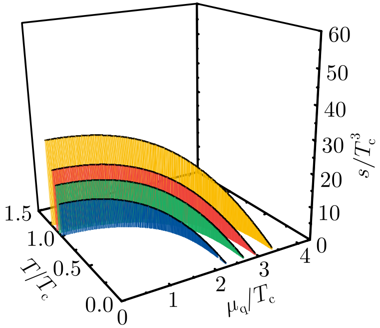
Larger chemical potential



- full HTL QPM:
  stable characteristics



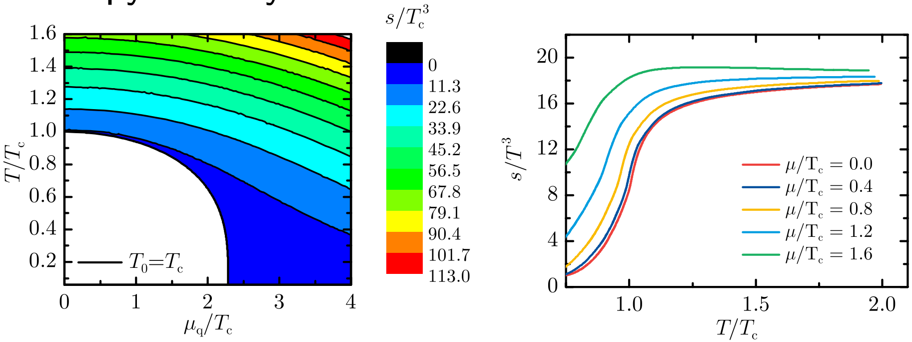
- entropy density



RS, Bluhm, Kämpfer: EPJ ST'08

Results (1)

- entropy density

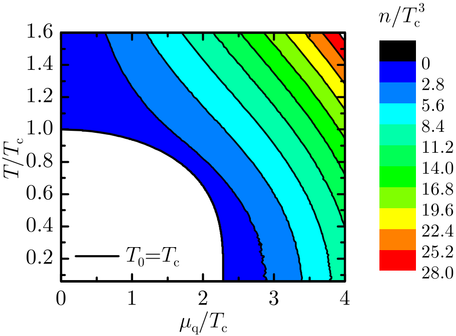


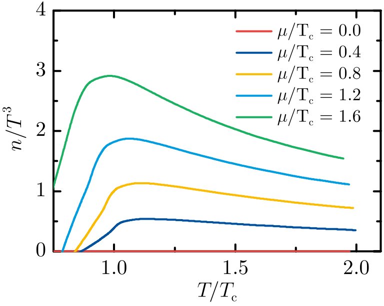
RS, Bluhm, Kämpfer: arXiv:0803.1571

- increases with temperature and chemical potential

Results (2)

- net quark density

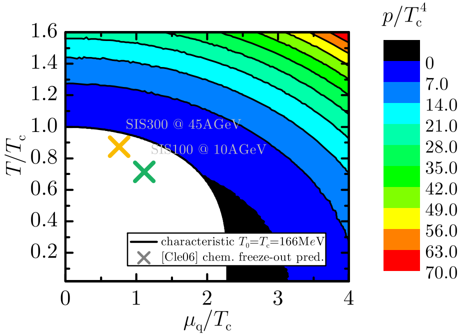


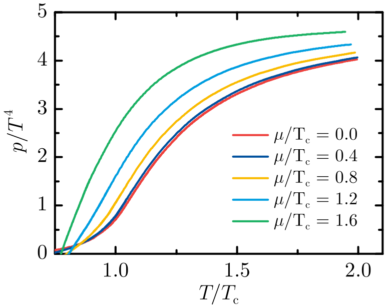


- small area of negative net quark density below transition line

Results (3)

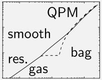
- pressure also increases with T and μ





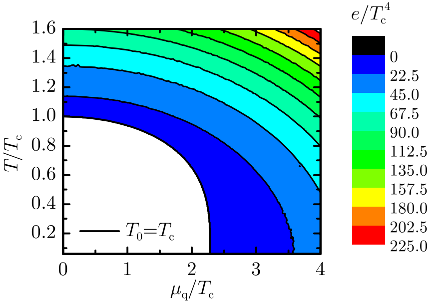
- small area of negative pressure also above transition line
  $\rightarrow$ no problems for EoS @ RHIC, LHC, SPS, FAIR

EOS for RHIC and LHC



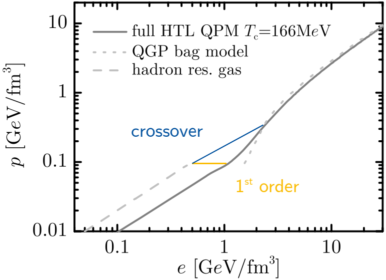
- energy density

$$e = -p + sT + \mu n$$



- EOS for LHC, RHIC

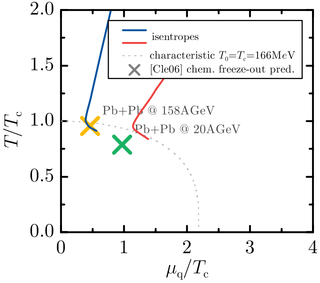
$$n_b/s \approx 0$$

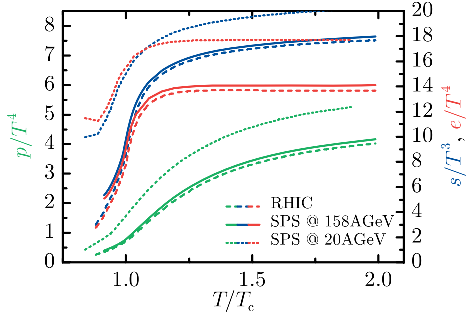


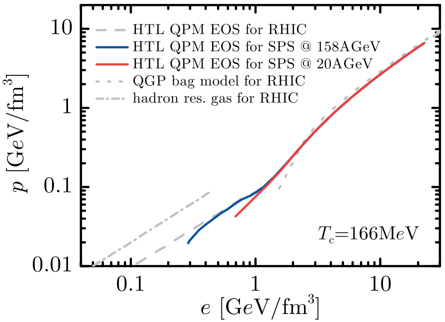
EOS for SPS

PRELIMINARY

- SPS







Summary & Outlook

- 2-loop Γ_2 + eff. coupling $G^2 \rightarrow$ HTL QPM
- Landau damping + collective modes
  $\rightarrow$ large μ accessible
- limitation: negative pressure for small T and μ
  $\rightarrow$ however deconfinement region @ LHC, RHIC, FAIR, SPS accessible
- outlook: EOS for FAIR, critical endpoint

Kämpfer, Bluhm, RS, Seipt: NPA'06