Nuclear EoS and 3-body forces in the T-matrix approach

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Motivation

➧ describe nuclear matter starting from realistic bare nucleon-nucleon potentials (CD-Bonn, Nijmegen, Argonne)

✪ thermodynamic (macroscopic) properties

✪ single-particle (microscopic) properties

$\Downarrow$

thermodynamic Green's functions

❏ (Hot) dense matter $\longrightarrow$ supernovae explosions ($\sim 4\,\rho_0$) and neutron stars ($\sim 8\,\rho_0$)

Real-time Green's functions

- single-particle propagator on the contour $(1 = \mathbf{r}, t)$

$$i\,\mathbf{G}(1,1') = \left\langle \mathcal{T}\,\psi(1)\,\psi^\dagger(1') \right\rangle$$

- matrix representation

$$i\,\mathbf{G}(1,1') = i \begin{pmatrix} G^c(1,1') & G^<(1,1') \\ G^>(1,1') & G^a(1,1') \end{pmatrix}$$

- spectral representation

$$i\,G^>(\mathbf{p},\omega) = [1 - f(\omega)]\,A(\mathbf{p},\omega)$$

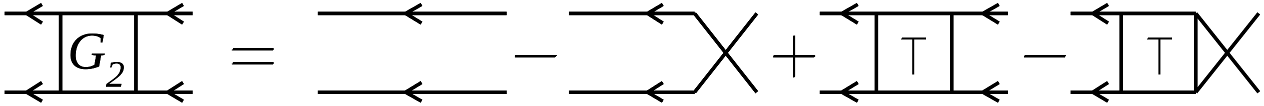
$$-i\,G^<(\mathbf{p},\omega) = f(\omega)\,A(\mathbf{p},\omega)$$

- Dyson equation

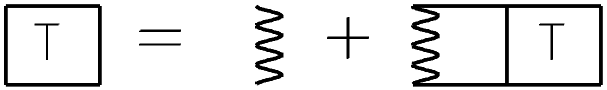
$$\mathbf{G}(1,1')^{-1} = \mathbf{G}_0(1,1')^{-1} - \mathbf{\Sigma}(1,1')$$

T-matrix approximation

Approximation for the two-particle propagator



where the in-medium two-particle scattering matrix T is introduced:



Iterative scheme involving the calculation of

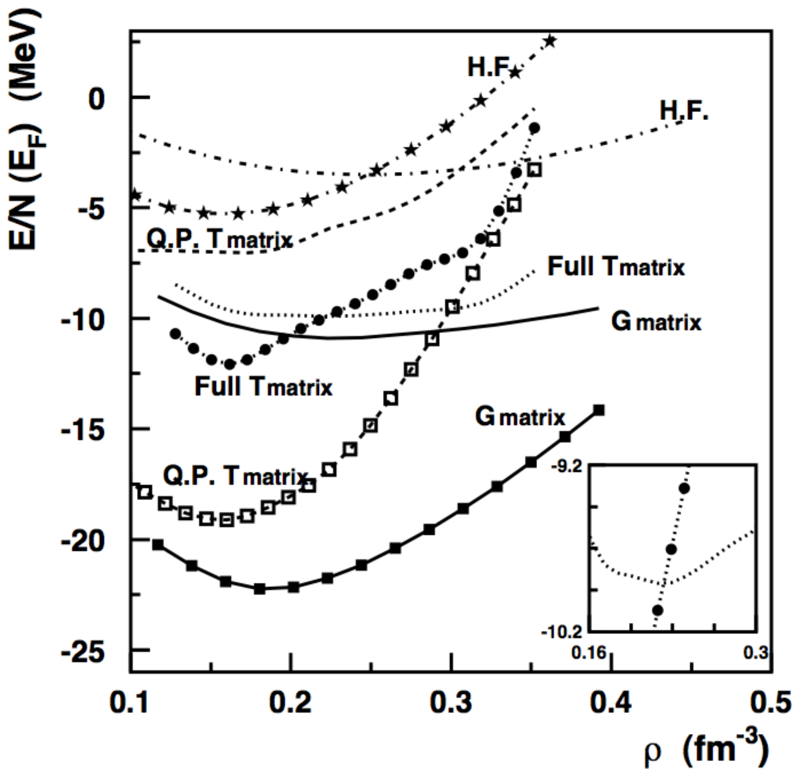
- T-matrix
- Self-energy
- Spectral function (via Dyson equation)

Thermodynamic consistency

A class of approximation can be derived from a *generating functional* $\Phi[G, V]$

$$\Sigma(x,y) = \frac{\delta \Phi}{\delta G(x,y)}$$

(Baym, Kadanoff 1961)



(P. Bożek and P. Czerski, Eur.Phys.J. A11 (2001) 271)

Example: pressure at $T = 0$

$$P = \rho^2 \frac{\partial (E/N)}{\partial \rho} = \rho \left(E_F - E/N \right)$$

→ Hugenholz - Van Hove property at saturation

$$E_F = E/N$$

Pressure and entropy

The pressure is related to the thermodynamic potential

$$\Omega(T, \mu, V) = -PV$$

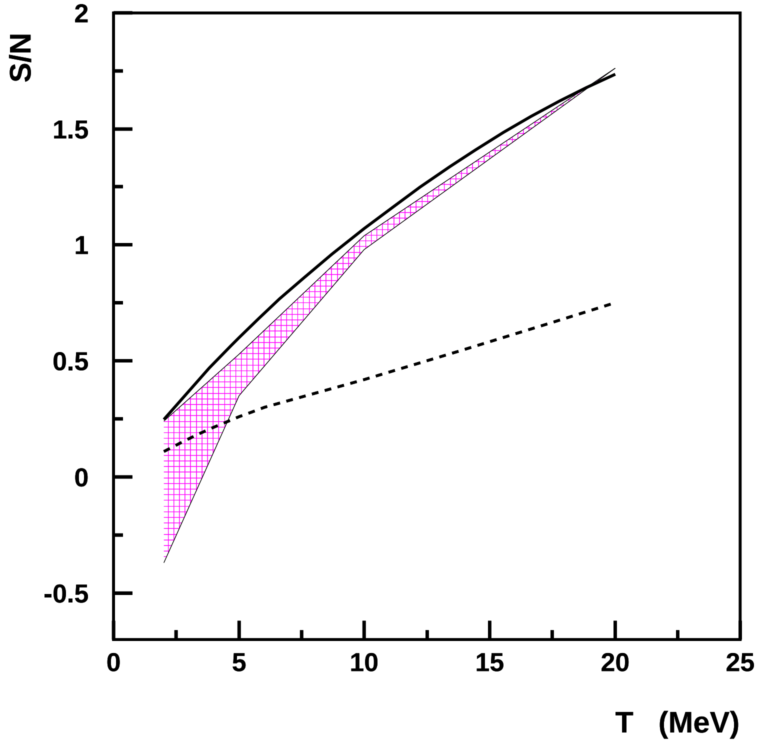
→ in a self-consistent approximation

$$\Omega = -\text{Tr}\{\ln[G^{-1}]\}$$

$$-\text{Tr}\{\Sigma G\} + \Phi$$

✪ entropy

$$\frac{S}{N} = \frac{1}{T}\left[\frac{E}{N} + \frac{P_{tot}}{\rho} - \mu\right]$$



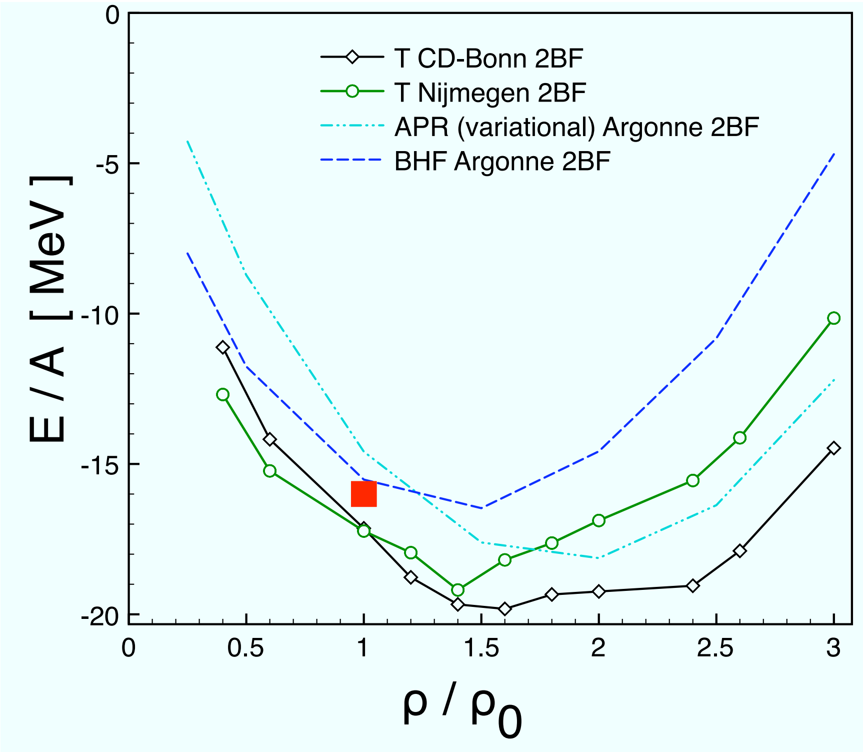
(V. S. and P. Bożek, Phys.Rev. C74 (2006) 045809)

Three-body forces

- Necessary to include three-body forces
- Urbana TBF

  $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{R}$

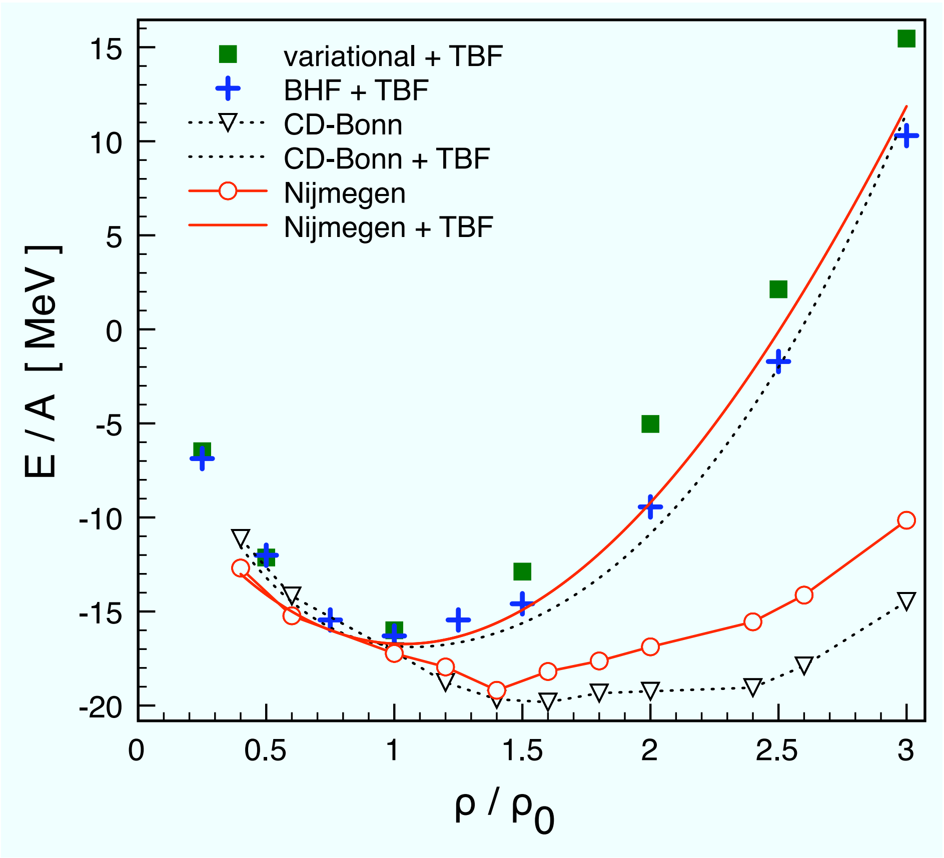
- Two parameters ($A < 0$ and $U > 0$) to be tuned with the saturation properties

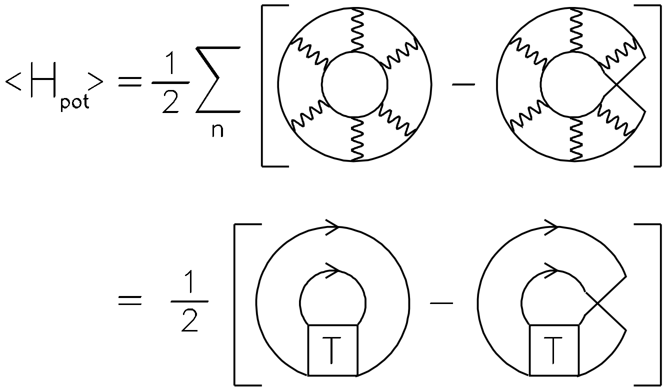


- Derive an effective two-particle potential

$$V^3_{eff}(\mathbf{q},\mathbf{q}') = \sum_{\sigma\,\tau} \int \frac{d\mathbf{k}}{(2\pi)^3}\, n(\mathbf{k})\, V^3(\mathbf{k},\mathbf{q},\mathbf{q}')$$

Energy per particle (symmetric matter)



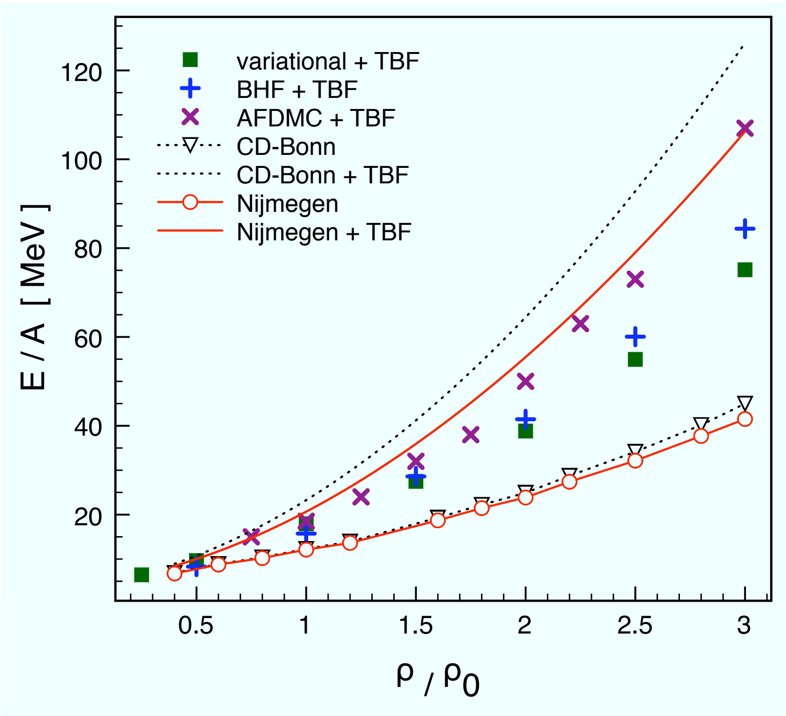


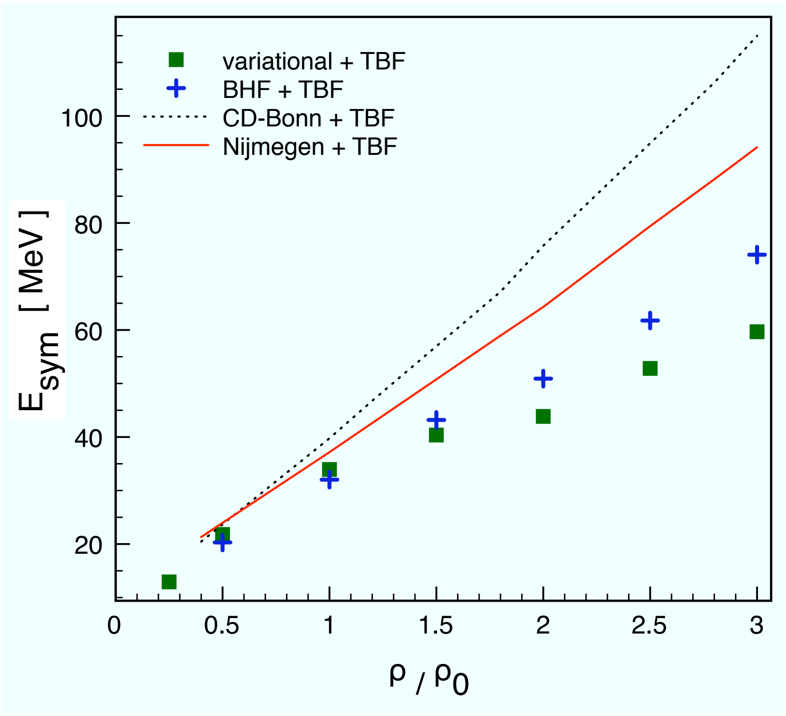
(V. S. and P. Bożek, arXiv:0808.2929)

Alternatively through the Galitskii-Koltun sum rule

$$\frac{E}{N} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} \frac{d\omega}{2\pi} \left[\frac{\mathbf{p}^2}{2m} + \omega \right] A(\mathbf{p}, \omega) f(\omega) \; .$$

Neutron matter and symmetry energy



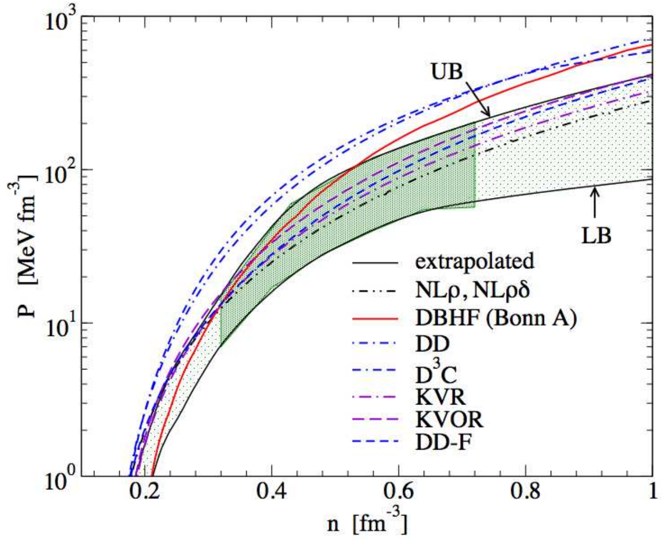


Symmetry energy computed with the parabolic approximation

$$\frac{E}{A}(n,\delta) = \frac{E}{A}(n,\delta=0) + \delta^2\, E_{sym}(n)$$

(Bombaci, Lombardo 1991)

Constraints on the EoS

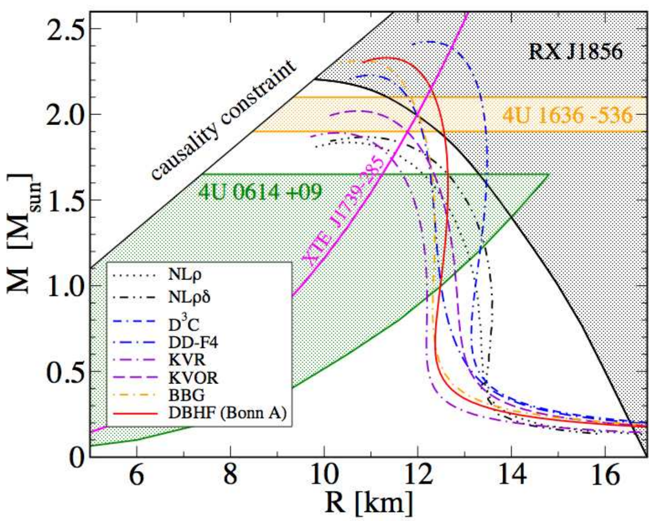


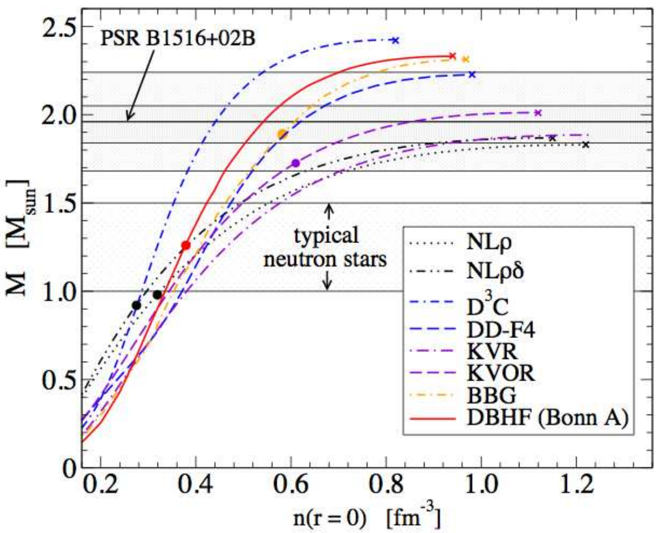
⟵ Heavy-ion collisions

(Blaschke et al.
Phys.Rev. C74 (2006) 035802
arXiv:0808.1279)

Neutron stars

↙ ↓





Single-particle properties

- Spectral function

$$G^{<}(\mathbf{p},\omega) = f(\omega)\, A(\mathbf{p},\omega)$$

- Self-energy

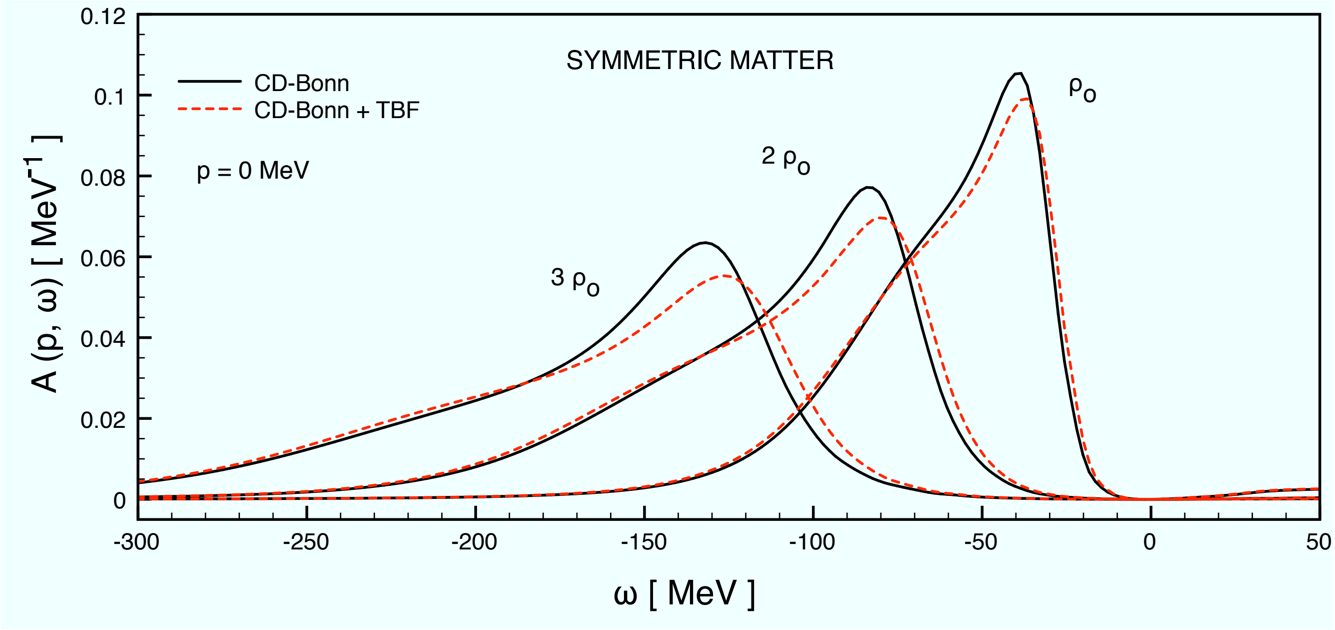
$$\mathrm{Re}\,\Sigma(\mathbf{p},\omega) = \Sigma_{HF}(\mathbf{p}) + \mathcal{P}\int \frac{d\omega'}{\pi}\frac{\mathrm{Im}\,\Sigma(\mathbf{p},\omega')}{\omega-\omega'}$$

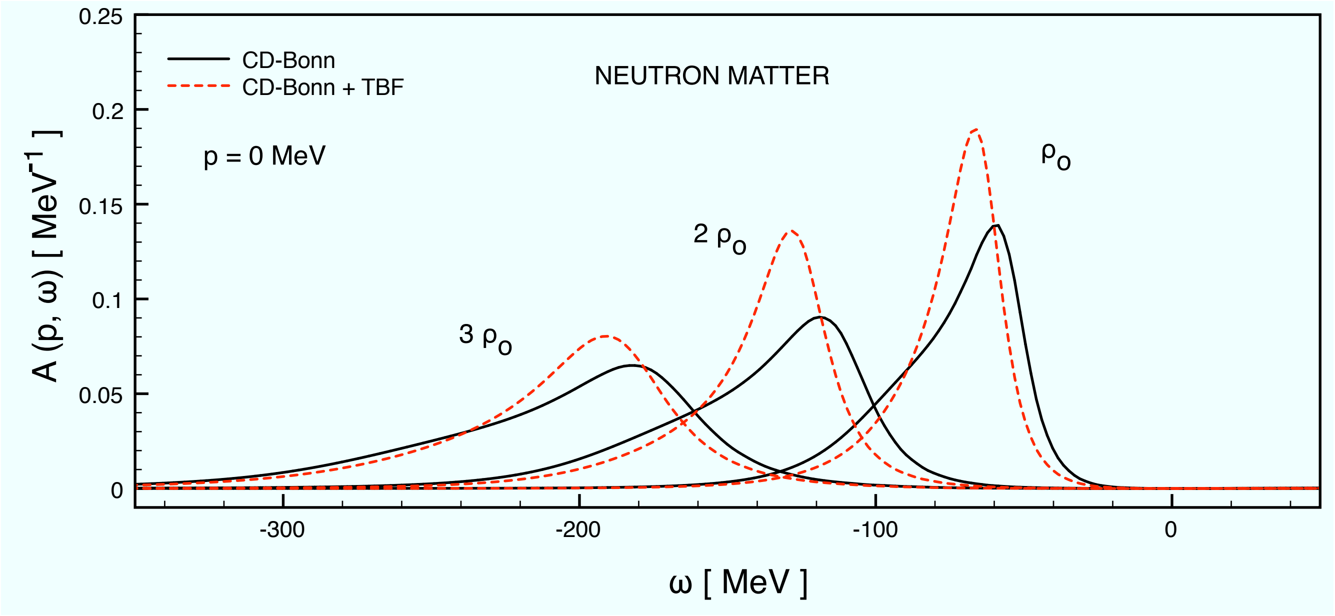
dispersion relation $$\omega_p = \frac{p^2}{2m} + \mathrm{Re}\,\Sigma(\mathbf{p},\omega_p)$$

- Effective mass

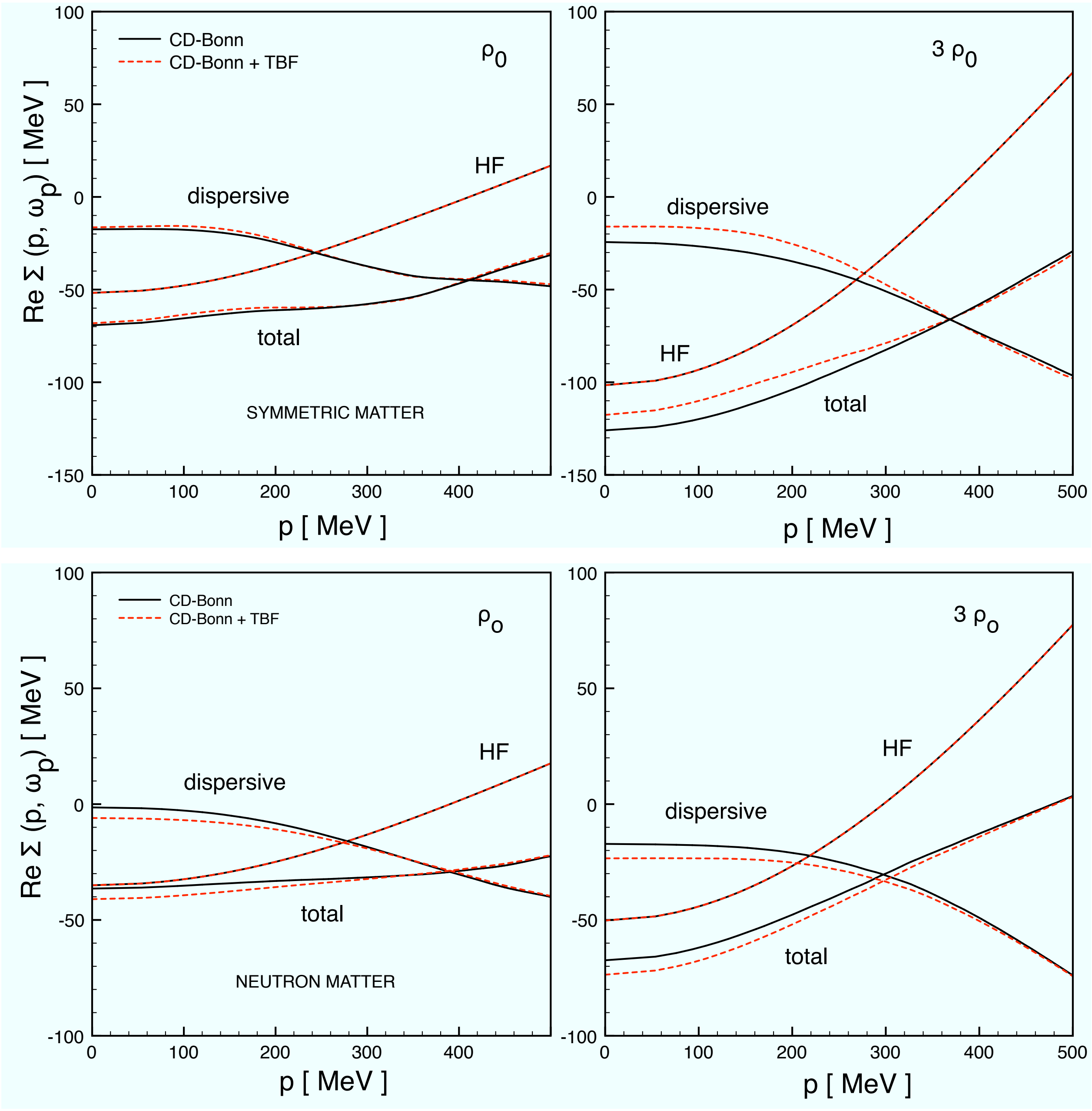
$$\frac{\partial\,\omega_p}{\partial\,p^2} = \frac{1}{2m^\star}$$

Modification of the spectral function

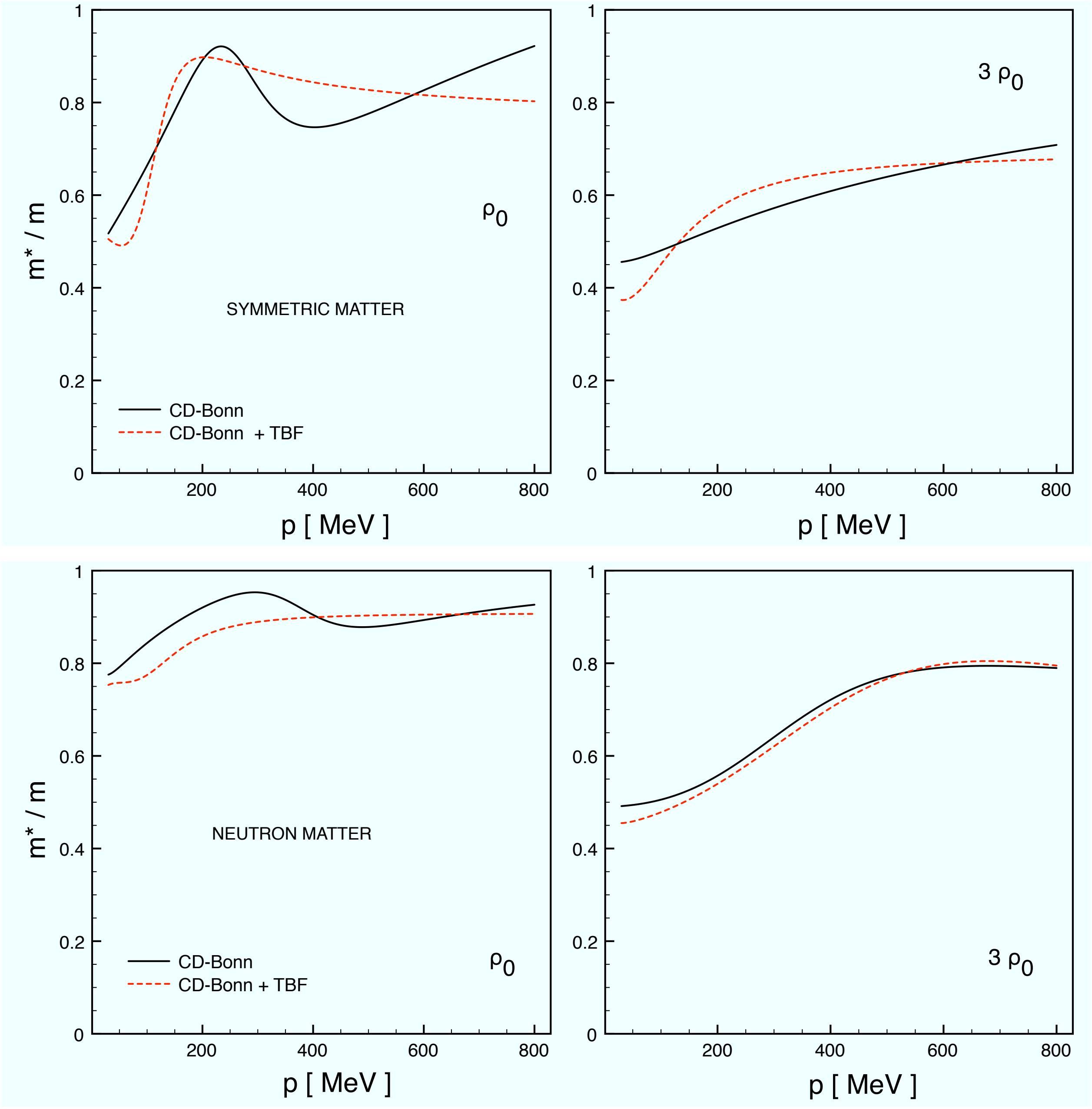




Self-energies



Effective mass



Summary

	$\rho_0[\text{fm}^{-3}]$	E_0 [MeV]	K_0 [MeV]	S_0 [MeV]	m^*/m
exp	0.16 ± 0.01	-16 ± 1	210 ± 30	32 ± 6	≈ 0.8
CD-B 2	0.287	-19.9	70	32.2	0.90
CD-B 3	0.171	-16.3	148	39.7	0.87
Nijm 2	0.235	-18.4	76	30.5	0.87
Nijm 3	0.164	-16.4	158	37.1	0.90

✪ on progress: complete EoS $\rightarrow \left\{ \frac{E}{N}, P, \frac{S}{N} \right\} (T, \rho)$

➥ (proto-) neutron star matter (T, ρ, x_p)

✪ application of the Green's function formalism to the dynamics of central nuclear reactions (A. Rios, P. Danielewicz, arXiv:0801.4171)