Thermodynamic properties of QED in 1+1 dimensions within light-front quantization

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22. September 2008

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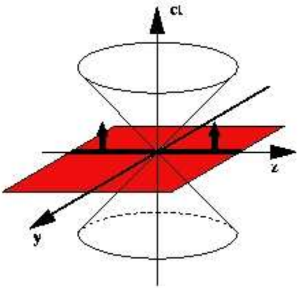
Phys. Rev. Lett. **101**, (2008) 100402

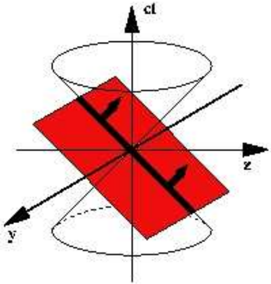
Outline

1. Light Cone Quantization
2. Massive Chiral Schwinger Model
3. Finite Temperatures
4. Perspective

Light Cone Quantization I

Dirac, Rev. Mod. Phys. **21** (1949) 392





instant form

$x^0 = n \cdot x$

$n^2 = 1$, n timelike

$n^\mu = (1, 0, 0, 0)^T$

front form

$x^+ = n \cdot x$

$n^2 = 0$, n lightlike

$n^\mu = (1, 0, 0, -1)^T$

Light Cone Quantization II

- Light cone transform

$$\begin{aligned} x^+ &= x^0 + x^3 \\ x^- &= x^0 - x^3 \\ x_\perp^i &= x^i \,, \qquad i = 1,2 \end{aligned}$$

$$g^{\mu\nu} = \begin{pmatrix} 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Dispersion relation

$$k^- = \frac{m^2 + k_\perp^2}{k^+}$$

- high energy scattering
- Parton model

Light Cone Quantization III

- initial, boundary value problem in light cone coordinates
- Hamiltonian field theory
- Quantization at equal light cone time x^+
- constrained Quantization using Dirac-Bergman or Faddeev-Jackiv
- almost trivial vacuum state
- maximal number of kinematical Poincaré generators

DLCQ I

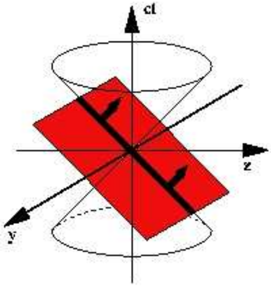
- Discretisation of light cone quantised field theory
- Box volume in x^- direction

$$-L \leq x^- \leq L$$

- boundary conditions

$$\psi(x^- = -L) = \psi(x^- = L)$$

- discreted momenta
  $k_n^+ = \frac{2\pi}{L} n, \quad n = 0, \ldots, \Lambda$
- $P^- = \frac{L}{2\pi} H$, $P^+ = \frac{2\pi}{L} K$



DLCQ II

- invariant mass matrix $M^2 = P^+ P^- = KH$
  and grows exponentially in size with K
- numerically diagonalising M^2
- conservation of momentum $\Rightarrow M^2$ is block diagonal
- continuum limit $K, \Lambda, L \to \infty$
- QED_{1+1}, QCD_{1+1}, supersymmetric Yang-Mills, scalar field theory

T. Eller, H. C. Pauli and S. J. Brodsky, Phys. Rev. D **35** (1987) 1493
K. Hornbostel, S. J. Brodsky and H. C. Pauli, Phys. Rev. D **41** (1990) 3814
Hiller, Pinsky et al.
Vary, Harindranath et al.

Massive Chiral Schwinger Model

- Quantum electro dynamics in $1+1$ dimensions
- shows confinement, theta vacua, axial anomaly, chiral condensate
- dynamical gauge field zero mode A_0^+ in lc gauge
- transition from free fermion $(m/g \to \infty)$ to free Schwinger boson $(m/g = 0)$ model
- here: no zero modes considered $\to$ only one Schwinger boson state
- use spectral information to calculate thermodynamical properties
- first work on light cone thermodynamics:

  S. Elser and A. C. Kalloniatis, Phys. Lett. B **375** (1996) 285

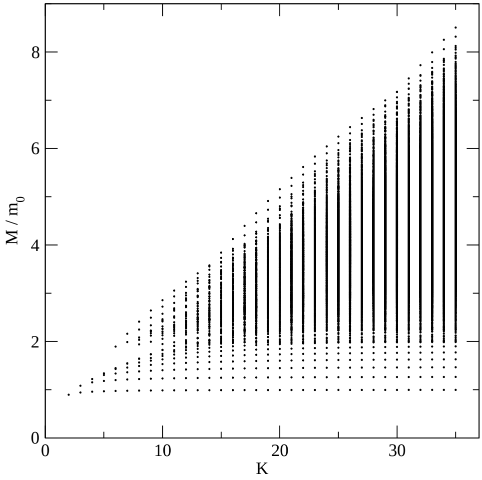
QED in 1+1 Dimensions

- LC-Hamiltonian

$$H = m^2 H_0 + \frac{g^2}{\pi} V = g^2 \left(\frac{m^2}{g^2} H_0 + \frac{1}{\pi} V \right)$$

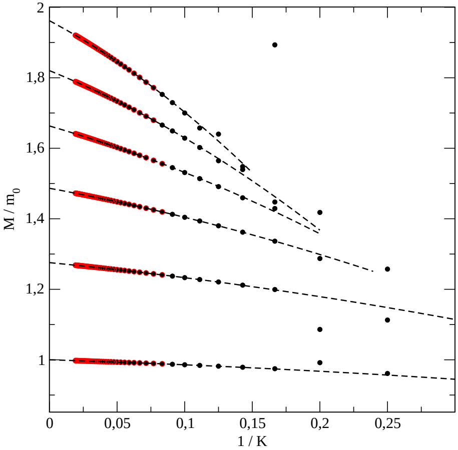
$$\begin{aligned}
H_0 &= \sum_n^{\Lambda} \frac{1}{n} \left(b_n^\dagger b_n + d_n^\dagger d_n \right) \\
V &= \sum_{k,l,m,n} \Bigg\{ \left(b_k^\dagger b_l^\dagger b_m b_n + d_k^\dagger d_l^\dagger d_m d_n \right) \frac{1}{2(k-n)^2} \delta_{k+l,n+m} \\
&\quad + b_k^\dagger b_l d_m^\dagger d_n \left(\frac{1}{(k+m)^2} \delta_{k+m,l+n} - \frac{1}{(k-l)^2} \delta_{k-l,m-n} \right) \\
&\quad + \left(b_k^\dagger d_l^\dagger d_m^\dagger d_n + d_n^\dagger d_m d_l b_k \right) \frac{1}{(k+m)^2} \delta_{k+m,n-l} \\
&\quad + \left(d_k^\dagger b_l^\dagger b_m^\dagger b_n + b_n^\dagger b_m b_l d_k \right) \frac{1}{(k+m)^2} \delta_{k+m,n-l} \Bigg\} \\
&\quad + \sum_n I_n \left(b_n^\dagger b_n + d_n^\dagger d_n \right)
\end{aligned}$$

Mass Spectrum at $m/g = 1$



- find all partitions $\{n_i\}$ with $\#n_i \leq 2$, $K = \sum_i n_i$
- create corresponding fock vectors
- compute H, M^2
- M^2 for fixed K is symmetric, sparse and has structure
- computing full spectrum using HH transformations and QR or parts using Lanzcos methods
- no truncations employed

Mass Spectrum at $m/g = 1$ II



m/g	this work	LC	Lattice
ground state / vector state			
2^5	0.191(3)	0.201	0.194(5)
2^4	0.2366(8)	0.224	0.238(5)
2^3	0.2856(4)	0.288	0.287(8)
2^2	0.33933(5)	0.337	0.340(1)
2^1	0.39355(4)	0.393	0.398(1)
2^0	0.4442(7)	0.444	0.4444(1)
2^{-1}	0.4873(1)	0.488	0.48747(2)
2^{-2}	0.519(1)	0.520	0.51918(5)
2^{-3}	0.538(2)	0.540	0.53950(7)
first excited state / scalar state			
2^5	0.46(7)	0.458	0.45(1)
2^4	0.5623(3)	0.564	0.56(1)
2^3	0.696(4)	0.697	0.68(1)
2^2	0.839(2)	0.838	0.85(2)
2^1	0.9892(1)	0.989	1.00(2)
2^0	1.117(1)	1.119	1.12(3)
2^{-1}	1.2002(2)	1.201	1.20(3)
2^{-2}	1.21(3)	1.230	1.24(3)
2^{-3}	1.27(1)	1.219	1.22(2)

LF Relativistic Statistics I

- relativistic statistical operator

$$\hat{\varrho} = \frac{1}{\mathcal{Z}} \exp\left\{ -\frac{1}{T} \left(u_\nu \hat{P}^\nu - \mu \hat{N} \right) \right\}$$

- medium velocity u, at rest $(u^+, u^-, u^\perp) = (1, 1, 0^\perp)$
- light cone representation

$$\hat{\varrho} = \sum_h \sum_{n,n'} \exp\left\{ -\beta \left(\frac{M_h^2}{2P^+} + \frac{P^+}{2} - \mu N \right) \right\} \phi_{h/n} \phi^*_{h/n'} |n\rangle\langle n'|$$

J. Raufeisen and S. J. Brodsky, Phys. Rev. D **70** (2004) 085017

H. A. Weldon, Phys. Rev. D **67** (2003) 085027

LF Relativistic Statistics II

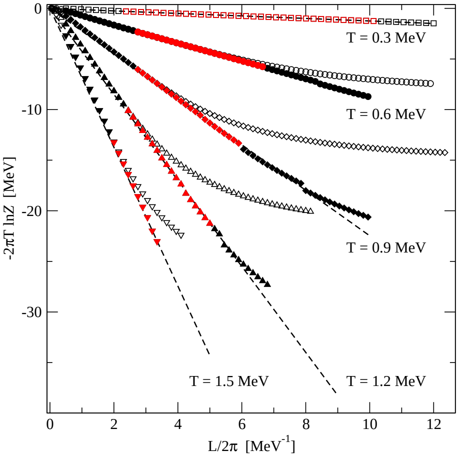
- discretised version

$$\begin{aligned} \ln \mathcal{Z}/L &= \frac{1}{L} \ln \left(\mathrm{Tr}\ \hat{\varrho} \right) \\ &= \frac{1}{L} \ln \left(\sum_{K} \sum_{\{K\}} \exp \left\{ -\frac{\beta}{2} \left(\frac{2\pi}{L} K + m_0^2 \frac{L}{2\pi} \frac{M^2(\{K\})}{K} \right) \right\} \right) \end{aligned}$$

- free fermi system

$$-\frac{T}{L} \ln \mathcal{Z} = -\frac{T}{2\pi} \int dp^+ \ \ln \left(1 + \exp \left\{ -\frac{\beta}{2} \left(p^+ + m^2/p^+ \right) \right\} \right)$$

Free Fermi Gas

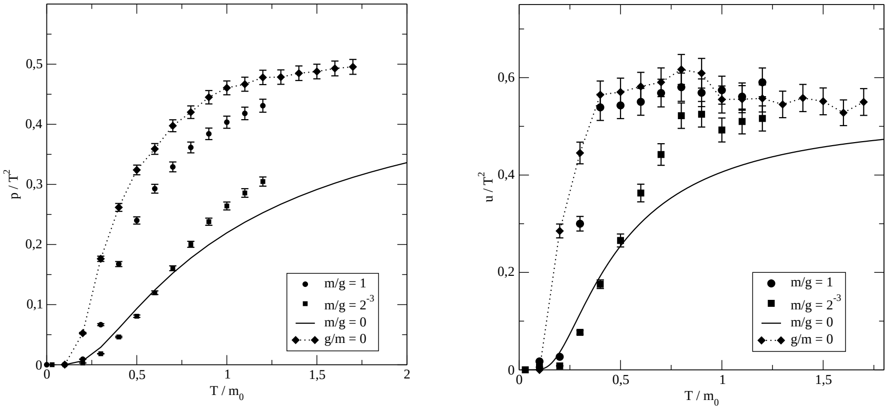


- compute sum over K
- extrapolate to higher K
- linear fit to $\ln \mathcal{Z}$

	$-2\pi T \ln \mathcal{Z}[\mathrm{MeV}]$		
T [MeV]	analytic	numeric	rel.err. [%]
0.3	-0.124156	-0.124263	0.05
0.6	-0.862941	-0.863176	0.03
0.9	-2.24692	-2.24824	0.05
1.2	-4.24937	-4.24566	0.07
1.5	-6.85920	-6.76379	1.39

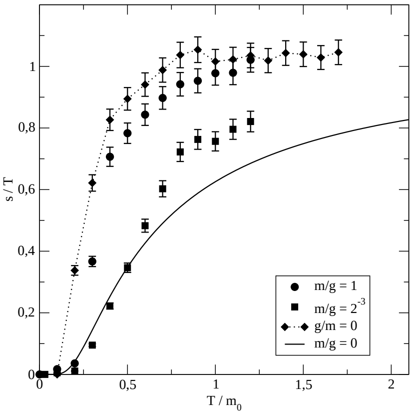
Full calculation I

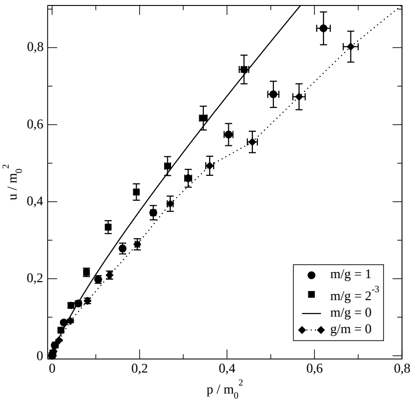
- TD relations: $p = \frac{T}{L} \ln \mathcal{Z}$ and $u = \frac{T^2}{L} \frac{\partial}{\partial T} \ln \mathcal{Z}$



Full calculation II

- entropy density $s = (u + p)/T$ and equation of state





Summary

summary:

- computation of mass spectrum and wavefunctions
- comparison to free fermion gas
- partition function and thermodynamics of QED_{1+1}

further steps:

- correlation functions and spectral function
- apply momentum space DMRG
- QCD in $1+1$ dimensions
- include higher dimensions