Effective Field Theory and Electroweak Processes in Nuclei

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Introduction

Low-energy electroweak processes relevant to the present talk

- pp-fusion: $p+p \to d+e^{+}+\bar{\nu}_e$
- ν-d reactions: $\nu_e+d \to e^{-}+p+p$, $\nu+d \to \nu+n+p$
- μ-d capture: $\mu^{-}+d \to \nu_e+n+n$
- ^{3}He$+p \to ^4$He$+\nu_e+e$ (Hep), ^{3}He$+n \to ^4$He$+\gamma$ (Hen)
- radiatice pion capture: $\pi^{-}d \to \gamma nn$

Important for one or more of the following reasons:

(1) Astrophysical processes (pp-fusion and Hep in solar burning, νd reactions in supernova explosion, etc.)

(2) Role in detecting astrophysical neutrinos (νd reactions in the SNO experiments)

(3) Provide information relevant to (1) and (2).

Let's concentrate on pp-fusion

- S factor: For the incident CM energy E

$$\sigma(E) = \frac{S(E)}{E} \exp\{-2\pi\eta(E)\}\,, \quad \eta(E) \equiv \frac{Z_1 Z_2 e^2}{\hbar v}$$

$v = 2E/\mu$, $\mu = mA_1A_2/(A_1 + A_2)$, ($m$; atomic mass unit)

$$S_{pp}(0) = 6\pi^2 m_p\, \alpha\, \ln 2\, \frac{1}{\gamma^3}(g_A/g_V)^2 \frac{f_{pp}^R}{(ft)_{0^+\to 0^+}} |\mathcal{M}|^2$$

$\mathcal{M}$ = nuclear transition matrix element;
$\gamma = (2\mu E_f)^{1/2}$=0.2316 fm^{-1}
f_{pp}^R = phase space factor with radiative corrections

The solar model and stellar evolution theory require
1% precision in S_{pp}.

• Current status [Adelberger et al., RMP70 (1998)]:
$\sim$6 % uncertainty in S_{pp}

• How to go beyond this level ?

• Solar Fusion Workshop at INT (Jan 2009)
— report to appear in RMV

• All the (seemingly unrelated) processes listed above are inter-related in this context.

The latest lattice QCD calculation of the nucleon weak form factors

Yamazaki et al. (RBC-UKQCD Collaborations), Phys. Rev. D, 79 (2009) 114505.

$$g_A/g_V = 1.19(6)_{\text{statistical}}(4)_{\text{systematic}}$$

$\leftrightarrow$ 7% smaller than the experimental value $g_A/g_V = 1,2695(29)$ [PDG08]

• Probably there will be a while before the two-body process can be calculated with required precision by lattice QCD.

Standard nuclear physics approach (SNPA)

The phenomenological potential picture — highly successful.
A-nucleon system described by a Hamiltonian

$$H^{\text{phen}} = \sum_{i=1}^{A} t_i + \sum_{i<j}^{A} V_{ij}^{\text{phen}} + \sum_{i<j<k}^{A} V_{ijk}^{\text{phen}},$$

Short-distance behavior in V_{ij}^{phen} — model-dependent
$\leftrightarrow$ Assume a functional form and adjust parameters to reproduce the two-nucleon data.

- *High-precision phenomenological* N-N potential ($\chi^2 \sim 1$):
  AV18, Nijm, CD-Bonn

Nuclear wave function $|\Psi^{\text{phen}}>$:

$$H^{\text{phen}}|\Psi^{\text{phen}}> = E|\Psi^{\text{phen}}> \ .$$

Finding useful truncation schemes for $|\Psi^{\text{phen}}>$

- An important branch of nuclear physics (shell model, RPA)
- Example: calculation of $\beta\beta$-decay in heavy nuclei

For lightest nuclei — **exact solutions** available !!

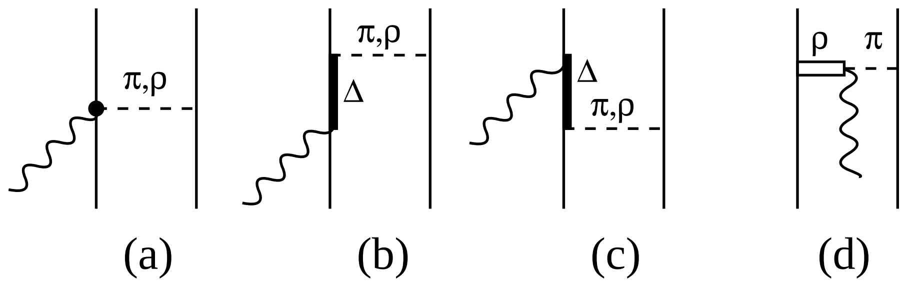
Nuclear Responses to External Electroweak Probes

The transition operators $\mathcal{T}$

$$\mathcal{T} = \mathcal{T}^{(1)} + \mathcal{T}^{(2)} + \ldots = \sum_{i=1}^{A} \mathcal{O}i + \sum_{i<j}^{A} \mathcal{O}_{ij} + \ldots$$

$\mathcal{T}^{(1)}$ – Dominant one-body term (impulse approximation (IA) term)
$\mathcal{T}^{(2)}$ – Exchange-current (EXC) term involving two nucleons.

$\mathcal{T}^{(2)}$ derived from one-boson exchange diagrams – consistent with the nuclear Hamiltonian and satisfy the low-energy theorems



$\Longrightarrow$ Standard Nuclear Physics Approach (SNPA)

- Cornerstone of nuclear physics;

pp-fusion rate in SNPA

The latest calculation in SNPA — Schiavilla et al. (1998).

Similarity of the Gamow-Teller (GT) matrix elements for pp-fusion and tritium β-decay

- **The same applies to ν-d, μ-d and Hep.**

Fine-tune $\mathcal{O}_{ij}$ to reproduce Γ_β^t.

For each of the two-body GT transition operators (with different r-dependences)
• Transition density for pp-fusion $\propto$ that for tritium β-decay

• A single multiplicative constant can match them all.
$\Rightarrow$ Adjust $N\Delta$-axial coupling constant

Nuclear matrix element for $S_{pp}(0)$ calculable with $\sim$ 0.3 % precision
$\leftrightarrow$ The range of variation for the five high-precision phenomenological potentials.

Effective field theory (EFT)

Low energy-momentum phenomena characterized by a scale Q
— Cut-off scale $\Lambda_{\text{cut}} \gg Q$
• Retain only low energy-momentum degrees of freedom (effective fields ϕ_{eff}).

$\Rightarrow$ Effective Lagrangian $\mathcal{L}_{\text{eff}}$, which consists of monomials of ϕ_{eff} and its derivatives consistent with the symmetries.
A term involving n derivatives $\sim (Q/\Lambda_{\text{cut}})^n$
$\Rightarrow$ perturbative series in Q/Λ_{cut}.
The coefficient of each term – low-energy constant (LEC)
  • LECs subsume high energy dynamics
If the LEC's up to a specified order n are known
$\leftrightarrow$ $\mathcal{L}_{\text{eff}}$ serves as a complete (and hence model-independent) Lagrangian.

The results obtained have accuracy of order $(Q/\Lambda_{\rm cut})^{n+1}$.
In our case, $\mathcal{L}_{\rm QCD} \to \mathcal{L}_{\chi \rm PT}$ [Chiral Perturbation Theory (χPT)]
In $\mathcal{L}_{\chi \rm PT}$, nucleons and pions are the effective degrees of freedom
— $\Lambda_{\rm cut} \sim 1$ GeV.

A system involving a nucleon $\leftrightarrow$ heavy-baryon chiral perturbation theory (HBχPT) [Jenkins and Manohar (1991)]

HBχPT cannot be applied in a straightforward manner to nuclei.
$\Leftrightarrow$ The existence of very low-lying excited states in nuclei

Nuclear χPT à la Weinberg (1990)

- Classify Feynman diagrams into two groups.
  Irreducible diagram – Every intermediate state has at least one meson in flight
  Reducible diagrams – Diagrams that are nor irreducible
- Apply the chiral counting rules only to irreducible diagrams.
- Treat irreducible diagrams (up to a specified chiral order) as an effective potential (to be denoted by V_{ij}^{EFT}) acting on nuclear wave functions.

• Incorporate reducible diagrams by solving the Schrödinger equation

$$H^{\mathsf{EFT}}|\Psi^{\mathsf{EFT}}>= E|\Psi^{\mathsf{EFT}}>,$$

$$H^{\mathsf{EFT}} = \sum_{i}^{A} t_i + \sum_{i<j}^{A} V_{ij}^{\mathsf{EFT}} + \sum_{i<j<k}^{A} V_{ijk}^{\mathsf{EFT}}$$

Application of nuclear χPT to a process involving external current(s)

$$\mathcal{T}^{\mathsf{EFT}} = \sum_i^A \mathcal{O}_i^{\mathsf{EFT}} + \sum_{i<j}^A \mathcal{O}_{ij}^{\mathsf{EFT}} + \cdots$$

$\mathcal{T}^{\mathsf{EFT}}$ — all irreducible diagrams (up to a given order ν) involving the relevant external current(s).

Transition matrix in nuclear EFT

$$\mathcal{M}_{fi}^{\mathsf{EFT}} = <\Psi_f^{\mathsf{EFT}}|\mathcal{T}^{\mathsf{EFT}}|\Psi_i^{\mathsf{EFT}}> = <\Psi_f^{\mathsf{EFT}}|\sum_i^A \mathcal{O}_i^{\mathsf{EFT}} + \sum_{i<j}^A \mathcal{O}_{ij}^{\mathsf{EFT}} + \cdots|\Psi_i^{\mathsf{EFT}}>$$

Hybrid EFT (EFT*) — Park et al., PRC, 67 (2003) 055206

It is a non-trivial task to fully carry out this program:
(i) Difficulties in getting Ψ^{EFT}
(ii) Unknown LECs in EFT-based transition operators

Hybrid EFT called EFT*: $\quad \Psi^{\mathrm{EFT}} \Rightarrow \Psi^{\mathrm{phen}}$

$$\mathcal{M}_{fi}^{\mathrm{EFT}*} = <\Psi_f^{\mathrm{phen}}|\mathcal{T}^{\mathrm{EFT}}|\Psi_i^{\mathrm{EFT}}> = <\Psi_f^{\mathrm{EFT}}|\sum_i^A \mathcal{O}_i^{\mathrm{EFT}} + \sum_{i<j}^A \mathcal{O}_{ij}^{\mathrm{EFT}} + \cdots |\Psi_i^{\mathrm{phen}}>$$

EFT* applicable to complex nuclei (A=3,4 . . .) with essentially the same accuracy and ease as to the A=2 system.
⇒ Determine LEC(s) using observables pertaining to complex nuclei.

Mismatch between Ψ^{EFT} and Ψ^{phen} only affects short-distance behavior.

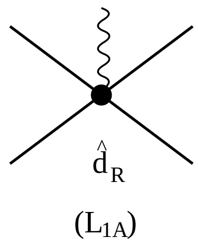
• The use of V^{phenom} introduces high momentum components above Λ_{QCD}.

• Momentum cutoff Λ_{NN} to eliminate high-momentum components

Λ_{NN}-independence $\leftrightarrow$ A measure of model independence of an EFT* calculation

Park *et al.* (2003) – EFT* calculation of the GT transitions in the A=2, 3 and 4 systems

• pp-fusion, tritium β-decay, ν-d reactions, μ-d capture, and Hep are all controlled by the common LEC, $\hat{d}^R$



• $\hat{d}^R$ – strength of contact-type four-nucleon coupling to the axial current
• $\hat{d}_R$ can be determined from Γ_β^t.

EFT* calculation of pp-fusion up to NNLO

$$\Rightarrow S(0) = 3.94 \times (1 \pm 0.004) \times 10^{-24} \text{ MeV b}$$

- The errors dominated by the experimental uncertainties in Γ^t_β
- Variation in $S(0)$ for $\Lambda_{\rm NN} = 500$ MeV $\sim$ 800 MeV less important

MuSun Experiment

- At present, $\hat{d}^R$ is fixed from Γ_β^t
- Desirable to fix $\hat{d}^R$ from an A=2 system.

The **MuSun** experiment at PSI – Andreev et al. (MuSun Collaboration),
URL http://www.npl.uiuc.edu/exp/musun (2008)

- Determination of the μd capture rate with ~1 % accuracy.
  $\Rightarrow$ Determination of $\hat{d}^R$ within A= 2 system

- Calculation of μ-d; EFT* – Ando et al.,(2001); Pionless EFT – Chen et al., (2005)

Hep and Hen [Lazauskas, Song and Park, arXiv: 0905.3119, nucl-th]

$$\text{Hep: } {}^3\text{He}+p \to {}^4\text{He}+\nu_e+e\,, \quad \text{Hen: } {}^3\text{He}+n \to {}^4\text{He}+\gamma$$

Hep produces solar neutrinos of the highest maximum energy ($E_\nu^{\text{max}} = 18.8$ MeV)

Calculation of σ(hep) extremely difficult !!

• Leading-order 1B term ≈ 0

• Destructive interference between suppressed 1B term and 2B term

- $S_{\mathsf{Hep}}(0)_{\mathsf{Salpeter}} = 630 \times 10^{-20}$ keV-b $\quad \Rightarrow$

  $S_{\mathsf{Hep}}(0)_{\mathsf{SNPA}} = 9.6 \times 10^{-20}$ keV-b ,
  $S_{\mathsf{Hep}}(0)_{\mathsf{EFT}^*} = (8.6 \pm 1.3) \times 10^{-20}$ keV-b

Testing the reliability of EFT* $\leftrightarrow$ Use the Hen reaction

• Hen is very similar to Hep except: EM interaction $\rightarrow$ Weak interaction

• $\sigma_{\text{Hen}}^{\text{exp}} = (54 \pm 6)\ \mu b$
• Two LECs can be determined from $\mu(^3\text{H})$ and $\mu(^3\text{He})$

• N^3LO calculation gives $\sigma_{\text{Hen}} = (38 \sim 58)\ \mu b\,$,
• High stability against changes: $\Lambda_{NN} = 500 \sim 900$ MeV

Points to be considered for completeness

(i) Neutron-neutron scattering length a_{nn}

- Experimental errors in a_{nn} neglected in the construction of the potential models.

- The resulting error on S_{pp} estimated to be 0.5%.

Concentrate on $\pi^- d \to \gamma nn$.

The LAMPF experiment [Howell et al., 1998]:

$$a_{nn} = -18.63 \pm 0.10(\text{stat}) \pm 0.44(\text{syst}) \pm 0.30(\text{theor})\ \text{fm}$$

• The theory essentially based on SNPA $\leftrightarrow$ Uncertainties in short-distance physics

$\Longrightarrow$ Use of EFT [see, e.g., Gardestig, J. Phys. G, 36 (2009) 053001]

• Currently accepted accuracy in $\hat{d}_R \longrightarrow \Delta a_{nn}$(theor) = 0.05 fm

• Combination of this consideration with MuSun – important

(ii) Radiative corrections

Towner [Phys. Rev. C, 58 (1998) 1288]
Kurylov et al. [Phys. Rev. C, 67 (2003) 035502]
Fukugita and Kubota [Phys. Rev. D, 72 (2005) 071301] .

- G_F from μ-decay
- "effective" g_A obtained from neutron β-decay

$\Longrightarrow$ Radiative corrections specific to pp-fusion
$\sim$ 3-4 % effects (with 0.1 % uncertainty)

Pionless EFT

Bedaque, Hammer and van Kolck (1998); Chen, Rupak and Savage (1999)

- Low energy processes with $p \ll m_\pi$
- Pions "integrated out" from $\mathcal{L}_{\text{eff}}$
- $\Lambda_{\text{cut}} \approx m_\pi$ — systematic expansion in powers of p/m_π
- N-N interactions and electroweak currents described by point-like contact terms
- Up to NNLO, pp fusion and other GT transitions invlove only one LEC, $L_{1,A}$, analogous to $\hat{d}^R$
- KSW scheme [Kaplan, Savage and Wise (1996)] – does not involve the explicit calculation of wave functions.

- At present, no direct determination of $L_{1,A}$.
  (relies on the theoretical values calculated in SNPA or EFT*)
- $S(0)$ calculated in pionless EFT
  Up to second order: Kong and Ravndal (2001); Ando et al. (2008)
  Up to fifth order: Butler and Chen (2001)
- MuSun important in this connection also

ν-d **Reactions**

• Precision calculation of ν-d reactions (both CC and NC) – similar to pp-fusion

• SNPA calculation – Nakamura *et al.* (2002) (use of Γ^t_β) – Reliable at 1 % level

• EFT* calculation – Ando *et al.* (2003) (agrees with SNPA calculation)

• Pionless EFT – Butler, Chen and Kong (2001)
  Consistent with those of SNPA after one LEC is adjusted

Conclusions

• $S_{pp}(0)$ can be calculated with $\sim$ 1% precision.

• The results are (in all likelihood) already available from the combination SNPA and EFT*

• MuSun experiment is very important in this context

• Full EFT calculations that use Ψ^{EFT} instead of Ψ^{phen} are eagerly awaited.
For the construction of EFT-based nuclear interactions, see *e.g.*, [Epelbaum, Prog. Part. Nucl. Phys. **57**, 654 (2006)].