Matter Effects in Active-Sterile Solar Neutrino Oscillations

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Neutrinos in Cosmology, in Astro, in Particle and in Nuclear Physics
Erice, Trapani, Sicily, Italy;
September 16. to 24. 2009.

Based on Carlo Giunti, YuFeng Li, in preparation

Status of Neutrino Oscillations and Sterile Flavors

- The standard scenario of neutrino oscillation is three neutrino mixing with a large mass-square difference hierarchy:

parameter	best fit	2σ	3σ
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\mathrm{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	$\lesssim 0.040$	$\lesssim 0.056$

arXiv:0808.2016[hep-ph]

$\sin^2\theta_{13} \simeq 0.02 \pm 0.01 \quad (1\sigma)$ arXiv:0905.3549 [hep-ph]

- The **LSND** Experimental signal favors a mass-square difference with $\Delta m^2 \gtrsim 0.1\,\mathrm{eV}^2$

  It must be interpreted with mixing of more than three neutrinos due to the large hierarchies of mass-square differences.

- **BUT, LSND** signal is disfavored by **KARMEN** and **MiniBooNE** experiments.

Motivations

Apart from **LSND**, There are also **two anomalies** that may be related to active-sterile oscillations:

- Anomalous ratio in the Gallium radioactive source experiments in GALLEX and SAGE

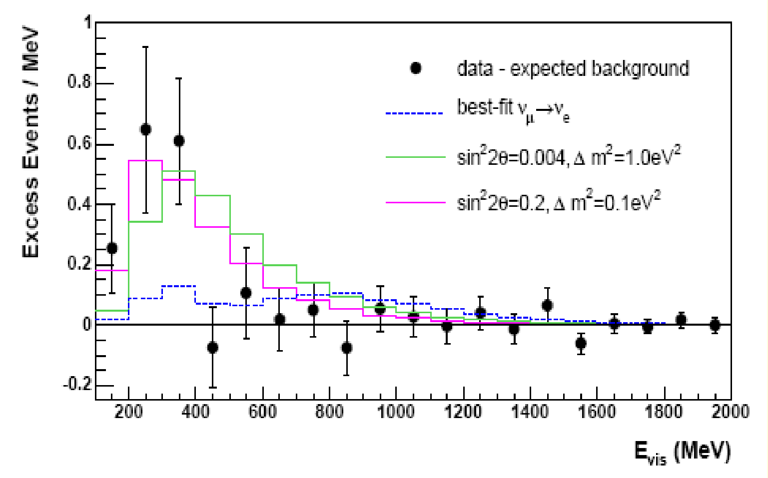
$$^{51}Cr(^{37}Ar)+e^{-}\rightarrow{}^{51}V(^{37}Cl)+\nu_e\Rightarrow\nu_e+{}^{71}Ga\rightarrow{}^{71}Ge+e^{-}$$

$$R=\frac{Ge_{mesured}}{Ge_{predicted}}=0.88\pm0.05$$ (Weighted average)

[SAGE, PRC 73 (2006) 045805**]**

- MiniBooNE low-energy anomaly: $128.8\pm20.4\pm38.3$ excess events within energy region of $200\text{MeV}<E_{QE}<475\text{MeV}$

arXiv:0812.2243[hep-ex]



VSBL Electron Neutrino Disappearance

- The two anomalies can be explained by **Active-Sterile neutrino oscillations** (VSBL) with a normalization factor of the absolute neutrino flux in MiniBooNE

$f = 1.30$ and $P_{\nu_e \to \nu_e} = 0.82$ C. Giunti and M.Laveder,arXiv:0707.4593**.**

- The **Bugey** data present a hint of neutrino oscillations with $0.02 \le \sin^2 2\theta \le 0.08$ and $\Delta \mathrm{m}^2 \approx 1.8\,\mathrm{eV}^2$

M.A. Acero, C. Giunti and M. Laveder, arXiv:0711.4222.

- The weak indication in favor of neutrino oscillations found in the analysis of the **Bugey** data **persists** in the combined analyses of the **Bugey** data with the **Gallium** anomaly and **Chooz** data.

$\sin^2 2\theta \approx 0.054$ and $\Delta \mathrm{m}^2 \approx 1.85\,\mathrm{eV}^2$ M.A. Acero, C. Giunti and M. Laveder,arXiv:0711.4222.

- **So**, It is also interesting to analyze the Solar Neutrino Active-Sterile oscillations with the VSBL effect (theta_13, theta_14). In this work, we develop the method of calculation of solar neutrino oscillations taking account the matter effect inside the Sun.

Previous Works

- In previous works on this topic, Electron Neutrino has non-negligible mixing with only two massive neutrinos:

$$\nu_e = \cos\theta_{12}\nu_1 + \sin\theta_{12}\nu_2$$

D. Dooling et al., Phys. Rev. D61 (2000) 073011.
C. Giunti et al., Phys. Rev. D62 (2000) 013005.

- So the Survival Probability of Electron Neutrinos has the same expression as two neutrino case

$$P^{\text{Sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_c\right)\cos 2\vartheta_{12}\ \cos 2\vartheta^M_{12}(x_0)$$

- **But** What we need is the information on **VSBL** Electron Neutrino Disappearance

arXiv:0902.1992 [hep-ph] Carlo Giunti, Marco Laveder

- **So** the mixing of Electron Neutrino must be considered as a generic scheme.

$$\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2 + U_{e3}\nu_3 + U_{e4}\nu_4 + \cdots$$

Assumptions

We start with a generic scheme, with an arbitrary number of sterile neutrinos (N_s), and without any constraint on the mixing.

- The first assumed condition is **strong hierarchy** in Δm^2s:

$$\mathrm{A_{CC}} \sim |\mathrm{A_{NC}}| \sim \Delta m_{21}^2 << |\Delta m_{k1}^2| \quad \text{(k>=3)}$$

$$A_{CC} = 2EV_{CC} \text{ and } A_{NC} = 2EV_{NC} \text{ with } V_{CC} = \sqrt{2}G_F N_e \text{ and } V_{NC} = -\frac{1}{2}\sqrt{2}G_F N_n$$

the charge current and neutral current potentials and E is the neutrino energy .

- Another assumption is a **constant** electron fraction inside the Sun:

$$R_{NC} \equiv \frac{A_{NC}}{A_{CC}} = -\frac{1-Y_e}{2Y_e} = \text{Constant}$$

With $Y_e = \frac{N_e}{N_e + N_n}$ the electron fraction in matter.

Parameterizations of the mixing matrix

We Parameterize the mixing matrix as $U = W_{23} S R_{13} R_{12}$

W_{ij} is a complex rotation, and R_{ij} is the corresponding real one with zero phase.

$$W_{23}(\theta_{23}, \eta_{23}) = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & \cos\theta_{23} & \sin\theta_{23} e^{i\eta_{23}} & 0 & \cdots \\ 0 & -\sin\theta_{23} e^{-i\eta_{23}} & \cos\theta_{23} & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$R_{12}(\theta_{12}) = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 & 0 & \cdots \\ -\sin\theta_{12} & \cos\theta_{12} & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

S contains all the rotations with one sterile index.

The numbers of mixing angles and phases are $3N_s + 3$ and $2N_s + 1$.

Evolution of Neutrino Flavors(1): Evolution Equations

- Solar neutrinos are described by the state:

$$|\nu(x)\rangle = \sum_{\alpha=e,\mu,\tau,s_1,\ldots,s_{N_s}} \psi_\alpha(x)|\nu_\alpha\rangle\,,$$

with ψ_α are the amplitudes in the flavor basis which can be written in a vector form:

$$\Psi = \left(\psi_e, \psi_\mu, \psi_\tau, \psi_{s_1}, \ldots, \psi_{s_{N_s}}\right)^T$$

- We will work in the vacuum mass basis defined as:

$$\Psi^{\mathrm{V}} = \left(\psi_1^{\mathrm{V}}, \ldots, \psi_N^{\mathrm{V}}\right)^T = U^\dagger \Psi$$

- The amplitudes in this basis are evolved as

$$i\frac{d}{dx}\Psi^{\mathrm{V}} = \frac{1}{2E}\left(M^2 + U^\dagger A U\right)\Psi^{\mathrm{V}}$$

with E is the neutrino energy and

$$M^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & \Delta m_{21}^2 & 0 & 0 & \cdots \\ 0 & 0 & \Delta m_{31}^2 & 0 & \cdots \\ 0 & 0 & 0 & \Delta m_{41}^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad A = \begin{pmatrix} A_{CC} + A_{NC} & 0 & 0 & 0 & \cdots \\ 0 & A_{NC} & 0 & 0 & \cdots \\ 0 & 0 & A_{NC} & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Evolution of Neutrino Flavors(2): Simplifications

Due to the relation $A_{CC} \sim |A_{NC}| \sim \Delta m_{21}^2 << |\Delta m_{k1}^2|$

- All the amplitudes with large indices (k>=3) are decoupled.

$$\psi_k^{\mathrm{V}}(x) \simeq \psi_k^{\mathrm{V}}(0)\exp\left(-i\frac{\Delta m_{k1}^2 x}{2E}\right), \quad \text{for} \quad k \geq 3,$$

- The first two amplitudes can evolve in the truncated effective Hamiltonian:

$$i\frac{d}{dx}\begin{pmatrix}\psi_1^{\mathrm{V}} \\ \psi_2^{\mathrm{V}}\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m_{21}^2 + A\cos 2\xi & A\sin 2\xi \\ A\sin 2\xi & \Delta m_{21}^2 - A\cos 2\xi\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{V}} \\ \psi_2^{\mathrm{V}}\end{pmatrix}$$

where we have take away an identity matrix to make the matrix traceless.

- With Effective Parameters

$$\tan\, 2\xi = \frac{Y}{X}\,, \qquad A = A_{CC}\sqrt{X^2 + Y^2}.$$

$$X = |U_{e1}|^2 - |U_{e2}|^2 + R_{\mathrm{NC}}\sum_{\alpha=e,\mu,\tau}\left(|U_{\alpha 1}|^2 - |U_{\alpha 2}|^2\right),$$

$$Y = 2\left|U_{e1}^* U_{e2} + R_{\mathrm{NC}}\sum_{\alpha=e,\mu,\tau} U_{\alpha 1}^* U_{\alpha 2}\right|.$$

- This evolution equation can be diagonalized by a rotation as:

$$\begin{pmatrix}\psi_1^{\mathrm{V}} \\ \psi_2^{\mathrm{V}}\end{pmatrix} = \begin{pmatrix}\cos\omega & \sin\omega \\ -\sin\omega & \cos\omega\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}} \\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

$$\tan 2\omega = \frac{A\sin 2\xi}{\Delta m_{21}^2 - A\cos 2\xi}.$$

Evolution of Neutrino Flavors(3): Effective Flavor Basis

- Exactly the same evolution equations as two flavor case in vacuum basis.
- We can define **Basis with diagonal potentials**:

$$i\frac{d}{dx}\begin{pmatrix}\psi_1^{\rm F}\\ \psi_2^{\rm F}\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m_{21}^2\cos 2\xi + A & \Delta m_{21}^2 \sin 2\xi \\ \Delta m_{21}^2 \sin 2\xi & \Delta m_{21}^2 \cos 2\xi - A\end{pmatrix}\begin{pmatrix}\psi_1^{\rm F}\\ \psi_2^{\rm F}\end{pmatrix}$$

- The **Effective Flavor Basis** is defined by the rotation with parameter ξ :

$$\begin{pmatrix}\psi_1^{\rm F}\\ \psi_2^{\rm F}\end{pmatrix}=\begin{pmatrix}\cos\xi & \sin\xi \\ -\sin\xi & \cos\xi\end{pmatrix}\begin{pmatrix}\psi_1^{\rm V}\\ \psi_2^{\rm V}\end{pmatrix} \qquad \begin{pmatrix}\psi_1^{\rm F}\\ \psi_2^{\rm F}\end{pmatrix}=\begin{pmatrix}\cos\xi_{\rm M} & \sin\xi_{\rm M} \\ -\sin\xi_{\rm M} & \cos\xi_{\rm M}\end{pmatrix}\begin{pmatrix}\psi_1^{\rm M}\\ \psi_2^{\rm M}\end{pmatrix}$$

with

$$\tan 2\xi_{\rm M} = \frac{\tan 2\xi}{1-(A/\Delta m_{21}^2\cos 2\xi)}\,.$$

- ξ and A play the equivalent role of θ_{12} and A_{CC} :

$$\theta_{12}\rightarrow\xi \quad \text{and} \quad A_{cc}\rightarrow A\equiv A_{CC}\sqrt{X^2+Y^2}$$

- The effective flavor amplitudes are linear combination of the real flavor amplitudes:

$$\begin{pmatrix}\psi_1^{\rm F}\\ \psi_2^{\rm F}\end{pmatrix}=\begin{pmatrix}\cos\xi U_{e1}^* + \sin\xi U_{e2}^* & \cos\xi U_{\mu 1}^* + \sin\xi U_{\mu 2}^* & \cdots \\ -\sin\xi U_{e1}^* + \cos\xi U_{e2}^* & -\sin\xi U_{\mu 1}^* + \cos\xi U_{\mu 2}^* & \cdots\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\\ \vdots\end{pmatrix}$$

Evolution of Neutrino Flavors(4): MSW Effects

- There is a resonance when diagonal elements of evolution Hamiltonian are equal:

$$A_{\mathrm{R}} = \Delta m^2_{21} \cos 2\xi$$

- We get the averaged oscillation probabilities:

$$\overline{P}_{\nu_e \to \nu_\beta} = \left[\frac{1}{2} + \left(\frac{1}{2} - P_{12} \right) \cos 2\vartheta_\beta \cos 2\vartheta^0_e \right] \cos^2 \chi_\beta \cos^2 \chi_e + \sum_{k=3}^{N} |U_{\beta k}|^2 |U_{ek}|^2$$

- The crossing probability:

$$P_{12} = \frac{\exp\left(-\frac{\pi}{2}\gamma_{\mathrm{R}} F\right) - \exp\left(-\frac{\pi}{2}\gamma_{\mathrm{R}} \frac{F}{\sin^2 \xi}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma_{\mathrm{R}} \frac{F}{\sin^2 \xi}\right)}$$

With $\gamma_{\mathrm{R}} = \frac{\Delta m^2 \sin^2 2\xi}{2E \cos 2\xi \, |\mathrm{d} \ln N_e / \mathrm{d}x|_{\mathrm{R}}}$. and $F = 1 - \tan^2 \xi$.

$$|U_{\beta 1}|^2 = \cos^2 \vartheta_\beta \, \cos^2 \chi_\beta \, , \quad |U_{\beta 2}|^2 = \sin^2 \vartheta_\beta \, \cos^2 \chi_\beta \, , \quad \text{with} \quad \sin^2 \chi_\beta = \sum_{k=3}^{N} |U_{\beta k}|^2$$

$\theta^0_e = \theta_e + \omega^0$ is the effective angle at production.

Discussions(1)

- The resonance behavior is controlled by effective parameters ξ and A
- In the extremely non-adiabatic limit, the crossing probability are $P_{12}^{(\gamma_R \ll 1)} \simeq \cos^2 \xi$
- In this case, the oscillation probabilities are **different from** the vacuum values.

$$\overline{P}_{\nu_e \to \nu_\beta}^{(\gamma_R \ll 1)} \simeq \left[\frac{1}{2} + \frac{1}{2} \cos 2\vartheta_\beta \cos 2(\vartheta_e - \xi) \cos 2\xi \right] \cos^2 \chi_\beta \cos^2 \chi_e + \sum_{k=3}^{N} |U_{\beta k}|^2 |U_{ek}|^2 .$$

$$\overline{P}_{\nu_e \to \nu_\beta}^{\mathrm{VAC}} = \sum_{k=1}^{N} |U_{\beta k}|^2 |U_{ek}|^2$$
$$= \left[\frac{1}{2} + \frac{1}{2} \cos 2\vartheta_\beta \cos 2\vartheta_e \right] \cos^2 \chi_\beta \cos^2 \chi_e + \sum_{k=3}^{N} |U_{\beta k}|^2 |U_{ek}|^2 .$$

- Only in the case that $\xi = \theta_e$, they have the same expressions.
- Two possibilities:
  - Neutron free medium $R_{NC} = 0$, or three neutrino mixing .
  - Four mixing with $U_{e3} = U_{e4} = 0$.

D. Dooling et al., Phys. Rev. D61 (2000) 073011.
C. Giunti et al., Phys. Rev. D62 (2000) 013005.

Discussions(2)

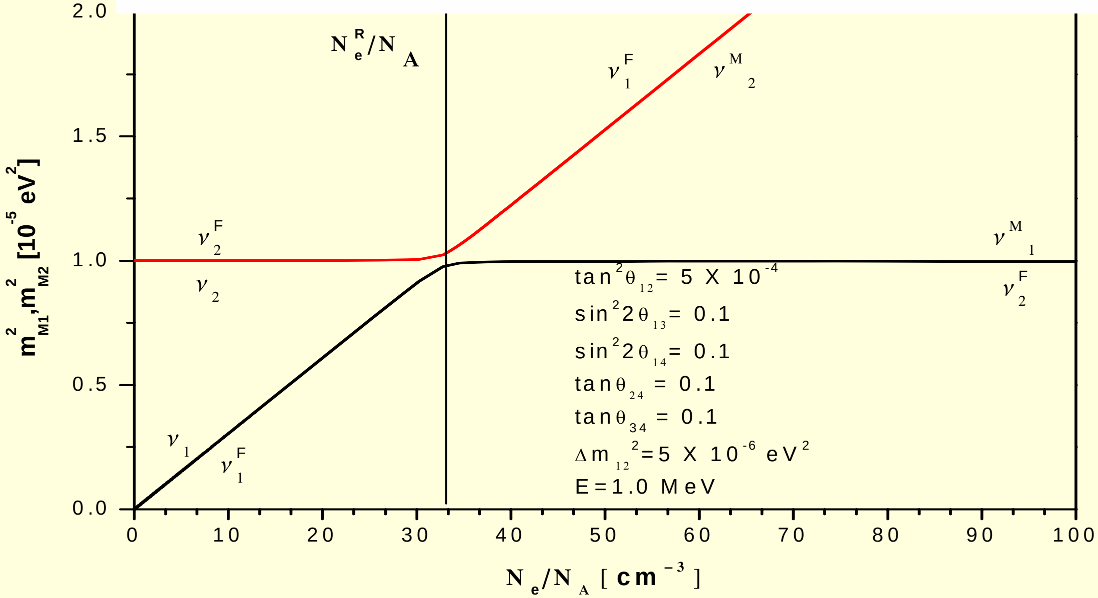
- In two neutrino case, the flavor states are conserved in the resonance. After that, neutrino states evolve practically as vacuum oscillation.
- In four neutrino case, it is the effective flavor states that are conserved in the resonance.

$$|\nu_e\rangle = \left[\cos(\vartheta_e+\omega)|\nu_1^{\rm M}\rangle + \sin(\vartheta_e+\omega)|\nu_2^{\rm M}\rangle\right]\cos\chi_e + \sum_{k=3}^{N} U^*_{\alpha k}|\nu_k^{\rm V}\rangle$$

$$= \left[\cos(\vartheta_e-\xi)|\nu_1^{\rm F}\rangle + \sin(\vartheta_e-\xi)|\nu_2^{\rm F}\rangle\right]\cos\chi_e + \sum_{k=3}^{N} U^*_{\alpha k}|\nu_k^{\rm V}\rangle$$

$$\xi_{\rm M} \simeq \frac{\pi}{2}\,, \quad \omega \simeq \frac{\pi}{2} - \xi\,, \quad \text{for} \quad A \gg A_{\rm R}$$

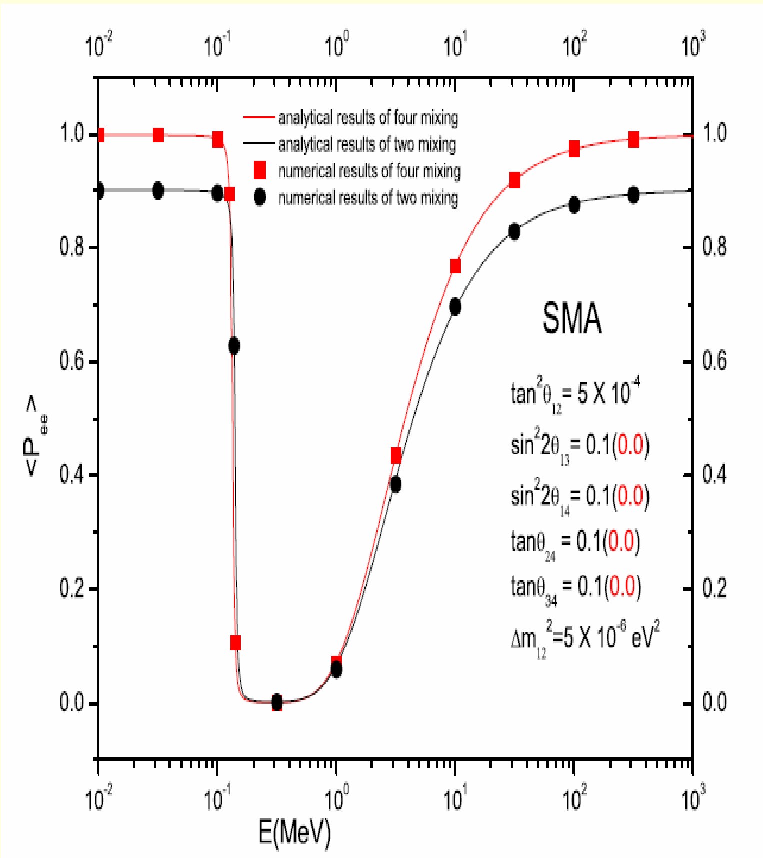
$$\vartheta_e^0 \simeq \frac{\pi}{2} + \vartheta_e - \xi$$

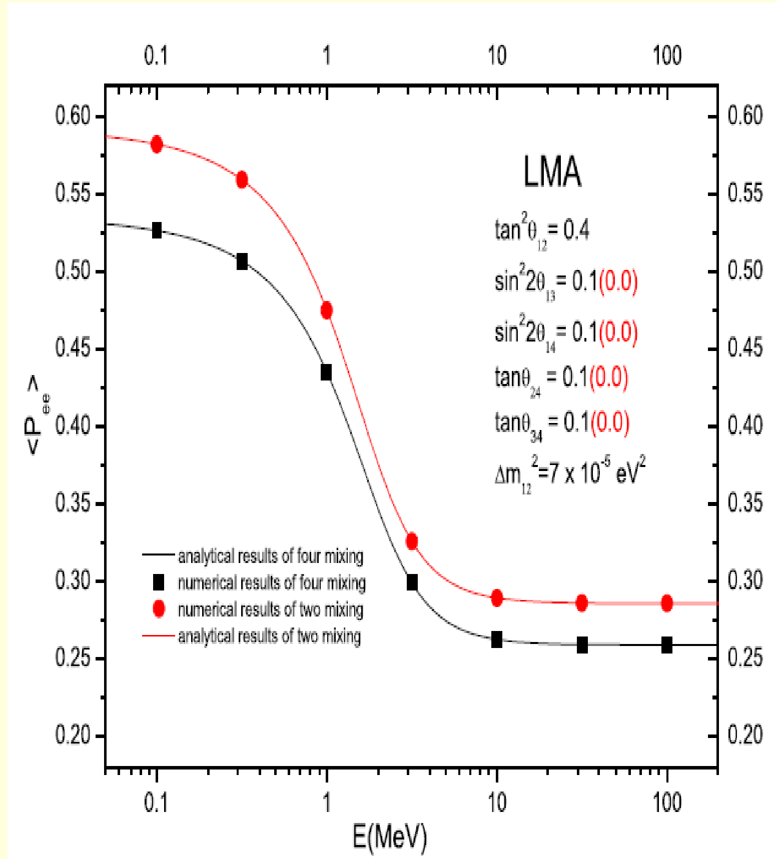


- So, only in the case of $\xi = \theta_e$, the electron neutrino states are identical to effective flavor states, leading to averaged vacuum results

Numerical Examples

We illustrate the validity of our analytical expression within four neutrino mixing with different oscillation parameters.





Conclusions

- We give some simple formulas on Active-Sterile Solar Neutrino Oscillations, without constraints on the mixing and number of sterile flavors.
- The new sterile flavors contribute to the results both in the form of effective mixing angle and potential and in the form of a constant term.
- The distinct property of our results shows up in the extremely non-adiabatic limit.
- We verify our results with numerical calculations with different parameters in four neutrino mixing case.

Thank you
Grazie