How to measure $M^{0\nu\beta\beta}$

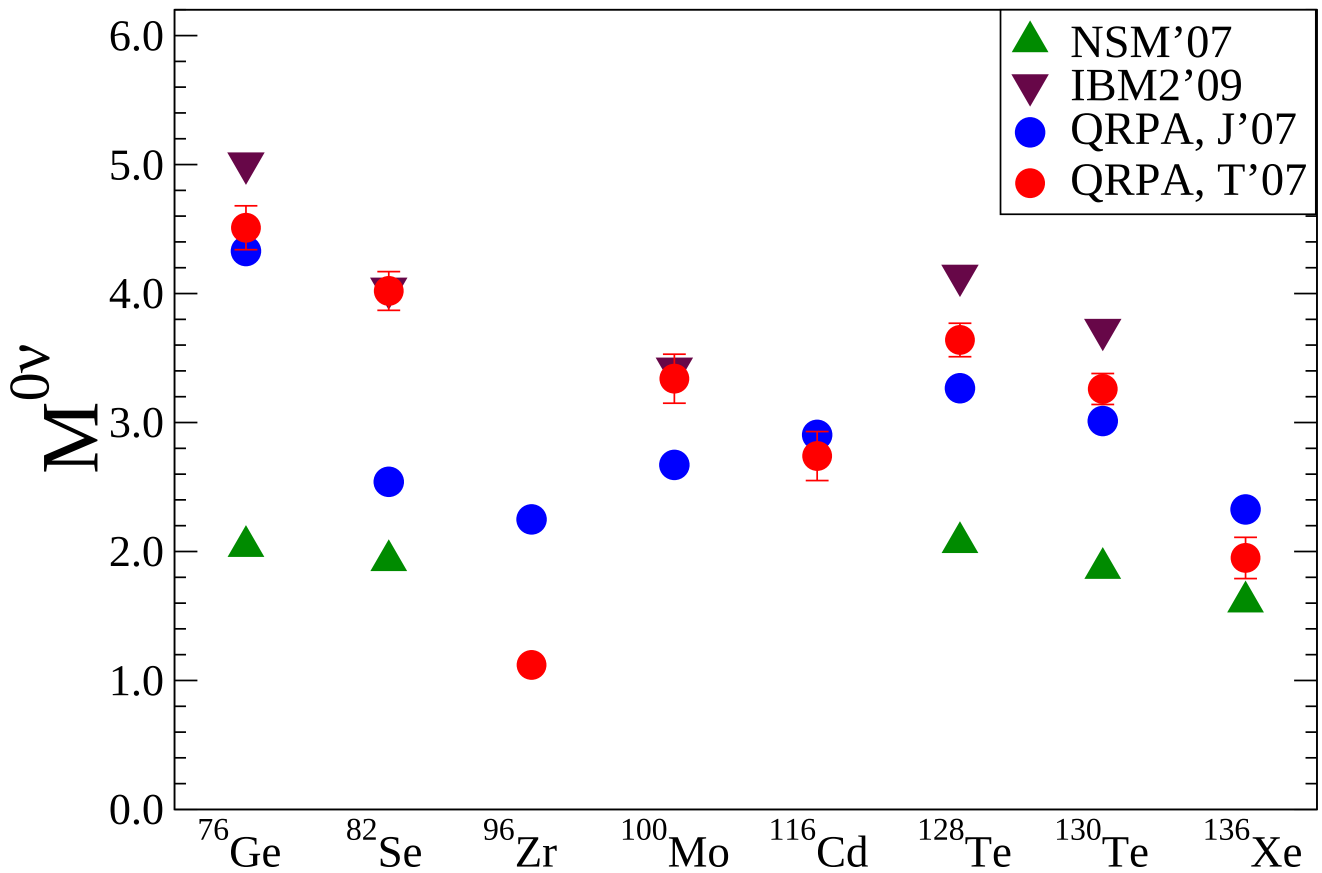
Vadim Rodin

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS

“Neutrinos in Cosmology, in Astro, Particle and Nuclear Physics"

Erice, 18/09/2009



Current status

(g_A = 1.25, Jastrow s.r.c.)

QRPA: **T’07** = V.R., A. Faessler, F. Šimkovic, P. Vogel, NPA 793 (2007) **J’07** = M. Korteleinen, J. Suhonen, PRC **76**, 051303; **76**, 6024315 (2007) **NSM’07** = E. Caurier, J. Menendez, F. Nowacki, A. Poves, PRL **100** (2008) **IBM2’09** = J. Barea and F. Iachello PRC **79** (2009)

Can one measure nuclear matrix elements of neutrinoless double beta decay?

Can one measure nuclear matrix elements of neutrinoless double beta decay?

Yes, we can!

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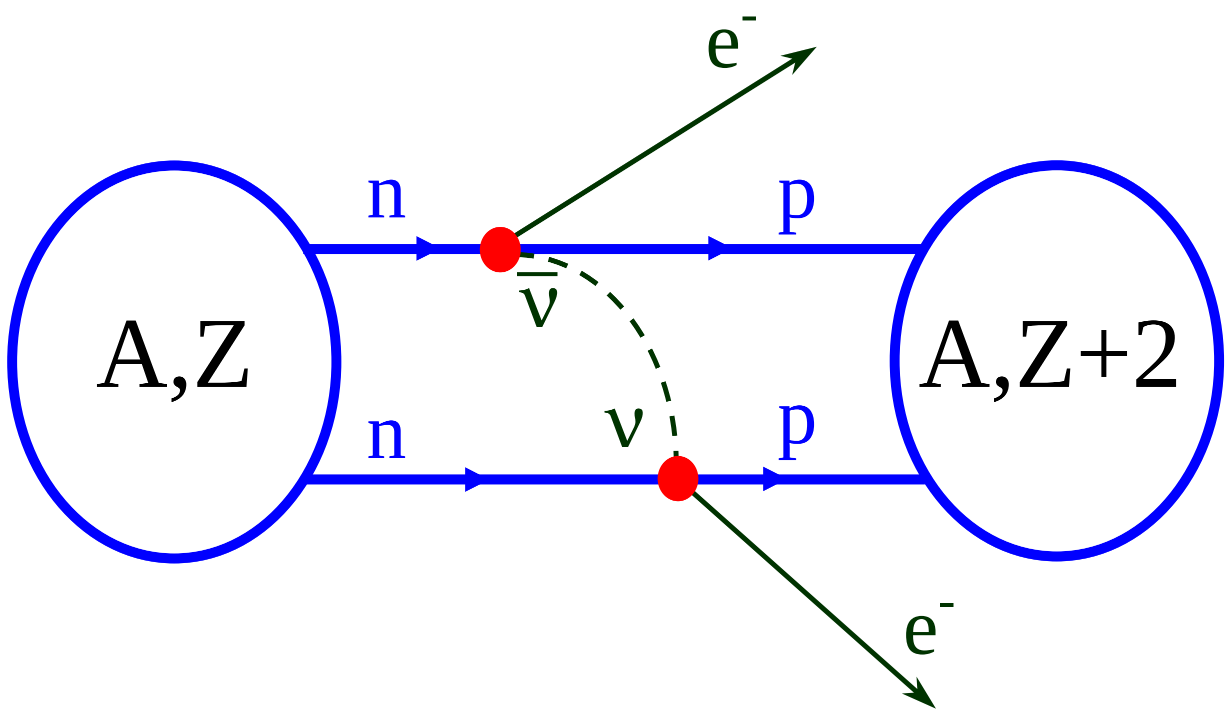
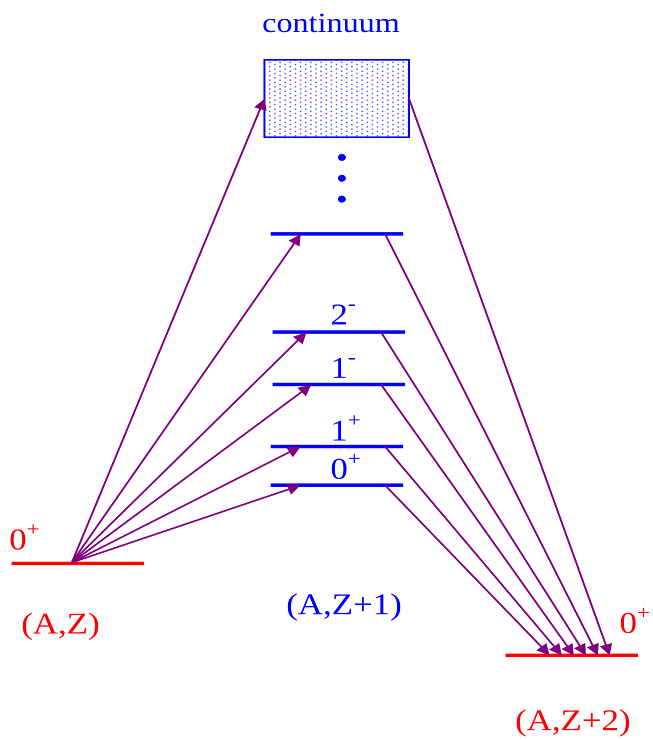
V.R., A. Faessler, arXiv:0906.1759 [nucl-th], to be published in PRC (RC)

Nuclear $0\nu\beta\beta$-decay ($\overline{\nu} = \nu$)

Light neutrino
exchange mechanism

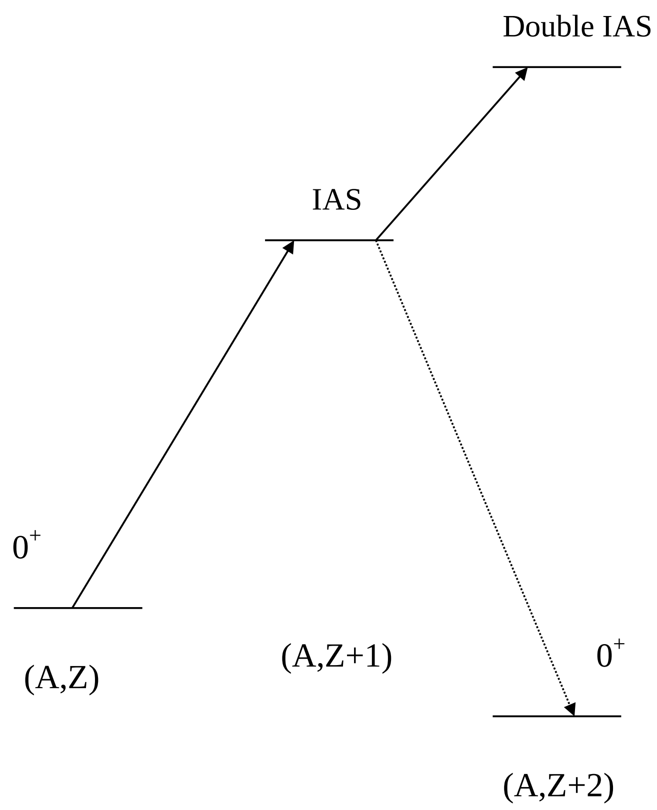




virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

Gamow-Teller amplitudes to 1^+ — from charge-exchange reactions (D. Frekers, Sep 23)

Double Fermi transition ($J^\pi_s = 0^+$)



$M_F^{2\nu} = 0$ if isospin SU(2) symmetry is exact — Violated by Coulomb

$$\hat{W}_F^{0\nu} = \sum_{ab} P_\nu(r_{ab})\tau_a^- \tau_b^- = \frac{1}{e^2}\left[\hat{T}^-, [\hat{T}^-, \hat{V}_C]\right]$$

Isospin lowering operator $\hat{T}^- = \sum_a \tau_a^-$; Coulomb interaction $\hat{V}_C = \frac{e^2}{8}\sum_{a\neq b} \frac{(1-\tau_a^{(3)})(1-\tau_b^{(3)})}{r_{ab}}$

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$$\hat{V}_C = \hat{V}_C^{(0)} + \hat{V}_C^{(1)} + \hat{V}_C^{(2)}$$

$\hat{V}_C^{(0)} = \frac{e^2}{8}\sum_{a\neq b}\frac{1+\frac{\boldsymbol{\tau}_a\boldsymbol{\tau}_b}{3}}{r_{ab}}$ $\hat{V}_C^{(1)} = -\frac{e^2}{8}\sum_{a\neq b}\frac{\tau_a^{(3)}+\tau_b^{(3)}}{r_{ab}}$ $\hat{V}_C^{(2)} = \frac{e^2}{8}\sum_{ab}\frac{T_{ab}^{(2)}}{r_{ab}}$ $(T_{ab}^{(2)} \equiv \tau_a^{(3)}\tau_b^{(3)} - \frac{\boldsymbol{\tau}_a\boldsymbol{\tau}_b}{3})$

Only isotensor $\hat{V}_C^{(2)}$ contributes to $\left[\hat{T}^-, [\hat{T}^-, \hat{V}_C]\right]$

$$\hat{W}_F^{0\nu} = \sum_{ab} P_\nu(r_{ab})\tau_a^- \tau_b^- = \frac{1}{e^2}\left[\hat{T}^-, [\hat{T}^-, \hat{V}_C]\right]$$

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$$\hat{V}_C = \hat{V}_C^{(0)} + \hat{V}_C^{(1)} + \hat{V}_C^{(2)}$$

$$\left[\hat{T}^-, [\hat{T}^-, \hat{V}_C^{(2)}]\right] = \hat{V}_C^{(2)}\left(\hat{T}^-\right)^2 + \left(\hat{T}^-\right)^2 \hat{V}_C^{(2)} - 2\hat{T}^-\hat{V}_C^{(2)}\hat{T}^-$$

$$e^2 M_F^{0\nu} \approx \langle 0_f | V_C^{(2)} \left(\hat{T}^-\right)^2 | 0_i \rangle =$$

$$\langle 0_f | \hat{V}_C^{(2)} | DIAS \rangle \langle DIAS | \left(\hat{T}^-\right)^2 | 0_i \rangle$$

$$\hat{H}_{tot} = \hat{T} + \hat{H}_{str} + \hat{V}_C$$

If $\hat{H}_{str}$ exactly isospin-symmetric: $\left[\hat{T}^-, \hat{H}_{str}\right] = 0$

$$\Downarrow$$

$$\hat{W}_F^{0\nu} = \frac{1}{e^2}\left[\hat{T}^-, [\hat{T}^-, \hat{H}_{tot}]\right]$$

$$M_F^{0\nu} =$$

$$-\frac{2}{e^2}\sum_s \bar{\omega}_s \langle 0_f|\hat{T}^-|0_s^+\rangle\langle 0_s^+|\hat{T}^-|0_i\rangle$$

$$\bar{\omega}_s = E_s - (E_{0_i} + E_{0_f})/2$$

Just equivalent representation of

$$M_F^{0\nu} = \frac{1}{e^2}\langle 0_f|\left[\hat{T}^-,[\hat{T}^-,\hat{V}_C^{(2)}]\right]|0_i\rangle$$

$$M_F^{0\nu} \approx -\frac{2}{e^2}\,\overline{\omega}_{IAS} \langle 0_f | \hat{T}^- | IAS \rangle \langle IAS | \hat{T}^- | 0_i \rangle$$

$$\langle 0_f | \hat{T}^- | IAS \rangle \langle IAS | \hat{T}^- | 0_i \rangle = \langle 0_f | DIAS \rangle \langle DIAS | \hat{T}^- | IAS \rangle \langle IAS | \hat{T}^- | 0_i \rangle$$

$$\langle 0_f | DIAS \rangle = -\frac{\langle 0_f | \hat{V}_C^{(2)} | DIAS \rangle}{E_{DIAS} - E_{0_f}}, \quad \text{with } E_{DIAS} - E_{0_f} \approx 2\overline{\omega}_{IAS}.$$

$$M_F^{0\nu} \approx \frac{1}{e^2} \langle 0_f | \hat{V}_C^{(2)} | DIAS \rangle \langle DIAS | \left(\hat{T}^-\right)^2 | 0_i \rangle$$

Measure the $\Delta T = 2$ isospin-forbidden matrix element $\langle 0_f | \hat{T}^- | IAS \rangle$

charge-exchange (n, p)-type reaction

Challenge: $\langle 0_f | \hat{T}^- | IAS \rangle \sim 0.005$

$$\langle IAS | \hat{T}^- | 0_i \rangle \approx \sqrt{N - Z} \sim 5$$

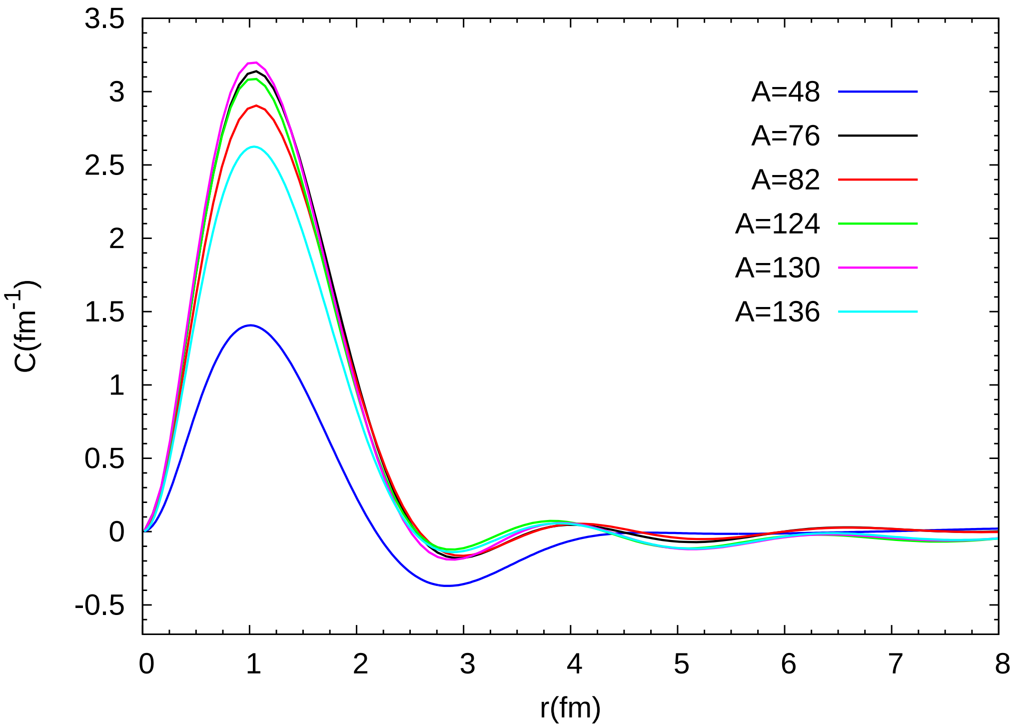
$$M_F^{0\nu}(QRPA)/M_F^{0\nu}(SM) \approx 3 \div 5$$

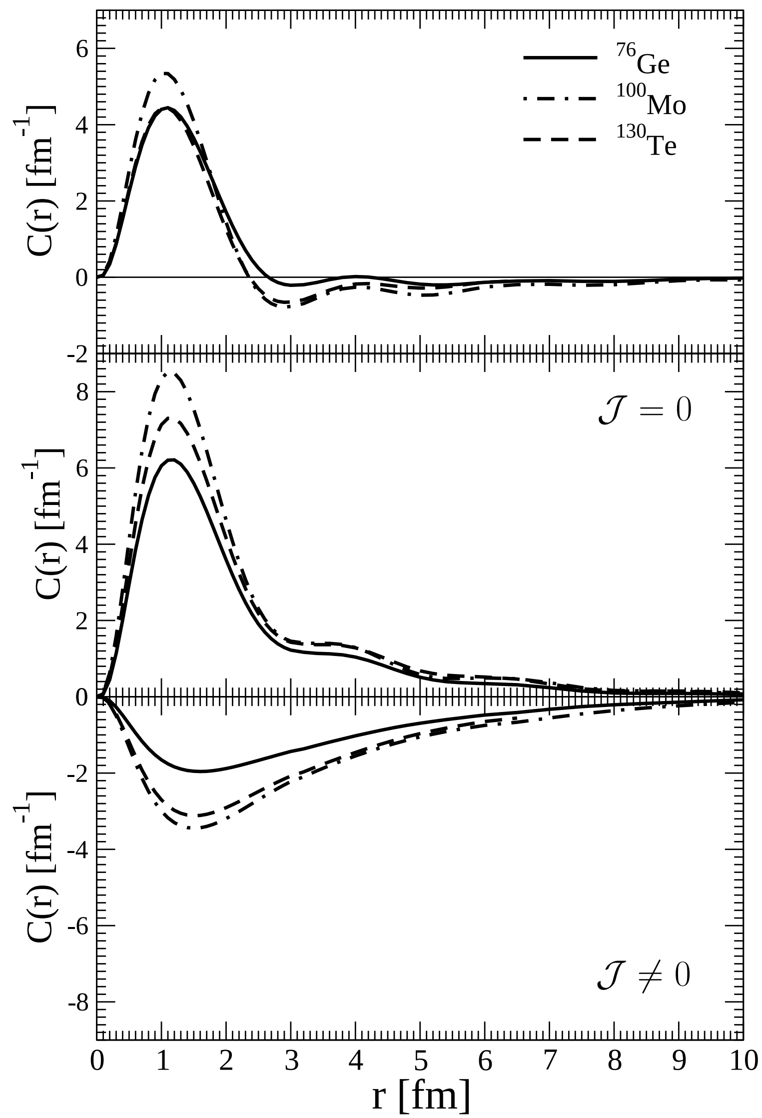
But $M_F^{0\nu}/M_{GT}^{0\nu} \approx 0.3$

Ratio $M_F^{0\nu}/M_{GT}^{0\nu}$

may be more reliably calculable than $M_F^{0\nu}$ and $M_{GT}^{0\nu}$ separately

$$\int_0^{\infty} C(r)dr = M^{0\nu}$$





Only small r_{ab} ~1–2 fm determine $M^{0\nu}$

$\Rightarrow$ nucleon pairs in the relative s-wave contribute $\Rightarrow$ $T = 1, S = 0$ pairs

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 |S = 0, T = 1\rangle = -3|S = 0, T = 1\rangle$$

$$\Downarrow$$

$$M^{0\nu}_{GT} = -3M^{0\nu}_{F}$$

provided the neutrino potential is the same in both F and GT cases

High-order terms of nucleon weak current $\Rightarrow$ $M^{0\nu}_{GT}/M^{0\nu}_{F} \approx -2.5$

Conclusions

- $M_F^{0\nu}$ can be related to Coulomb m.e. determining $\Delta T = 2$ isospin admixture of the DIAS in the final g.s.

- $M_F^{0\nu}$ can be reconstructed if one is able to measure Fermi m.e. $\langle 0_f | \hat{T}^- | IAS \rangle$ (e.g. charge-exchange (n, p)-type reactions)

- can help to discriminate between nuclear structure models (difference in $M_F^{0\nu}$ as much as the factor of 5)

- Estimate $M_{GT}^{0\nu} / M_F^{0\nu} \approx -2.5$ must hold

Supported by: DFG SFB TR27 “Neutrinos and beyond”

Backup

Switching off Coulomb

$$\hat{H}_{tot}(\lambda) = \hat{T} + \hat{H}_{str} + \lambda \hat{V}_C$$

$$\hat{W}_F^{0\nu} = \frac{1}{e^2 \lambda} \left[\hat{T}^-, [\hat{T}^-, \hat{H}_{tot}] \right]$$

$$M_F^{0\nu} = -\frac{2}{e^2 \lambda} \sum_s \bar{\omega}_s \langle 0_f | \hat{T}^- | 0_s^+ \rangle \langle 0_s^+ | \hat{T}^- | 0_i \rangle$$

Backup

$$\lambda \rightarrow 0$$

$$e^2 M_F^{0\nu} = \langle 0_f | \left[\hat{T}^-, [\hat{T}^-, \hat{V}_C] \right] | 0_i \rangle$$

$$= \langle 0_f | \hat{V}_C \hat{T}^- \hat{T}^- | 0_i \rangle$$

$$= \langle 0_f | \hat{V}_C | DIAS \rangle \langle DIAS | \hat{T}^- \hat{T}^- | 0_i \rangle$$

Backup

$$\hat{V}_C = \overline{V}_C + \Delta\hat{V}_C$$

$$\overline{V}_C = \hat{V}_C^{(0)} + \overline{V}_C^{(1)} + \overline{V}_C^{(2)}$$

$$\overline{V}_C^{(1)} = -\frac{e^2}{8}\sum_{a\neq b}\frac{\tau_a^{(3)} + \tau_b^{(3)}}{R_1} = -\frac{e^2 A}{2R_1}\hat{T}^{(3)}$$

$$\Delta\hat{V}_C^{(1)} = -\frac{e^2}{8}\sum_{a\neq b}(\tau_a^{(3)} + \tau_b^{(3)})(\frac{1}{r_{ab}} - \frac{1}{R_1})$$

$$\overline{V}_C^{(2)} = \frac{e^2}{8}\sum_{ab}\frac{T_{ab}^{(2)}}{R_2} = \frac{e^2}{2R_2}(\hat{T}^{(3)}\hat{T}^{(3)} - \frac{\mathbf{T}^2}{3})$$

$$\Delta\hat{V}_C^{(2)} = \frac{e^2}{8}\sum_{ab} T_{ab}^{(2)}(\frac{1}{r_{ab}} - \frac{1}{R_2})$$

$\overline{V}_C$ does not mix $|T\ T_z\rangle$

Backup

$$\langle 0_f | \overline{V}_C^{(2)} \left(\hat{T}^-\right)^2 |0_i\rangle =$$

$$\frac{e^2}{2R_2}\langle 0_f | \left(\hat{T}^-\right)^2 |0_i\rangle = \frac{e^2}{2R_2}\sum_s \langle 0_f|\hat{T}^-|0_s^+\rangle\langle 0_s^+|\hat{T}^-|0_i\rangle$$

Suppression by $\frac{e^2}{4R_2\overline{\omega}_{IAS}} \ll 1$

$$\langle DIAS|\hat{V}_C^{(2)}|0_f^+\rangle = \langle DIAS|\Delta\hat{V}_C^{(2)}|0_f^+\rangle.$$