Neutrino oscillations beyond the Standard Model

INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS

31st Course

Neutrinos in Cosmology, in Astro-, Particle- and Nuclear Physics

Erice-Sicily: 16 - 24 September 2009

Marek Zralek
University of Silesia,
Katowice,
Poland

Outline

- Introduction
- Neutrino Oscillation in the Standard Model
- What kind of problem we can expect beyond the Standard Model?
- General theory of neutrino oscillation
- Summary

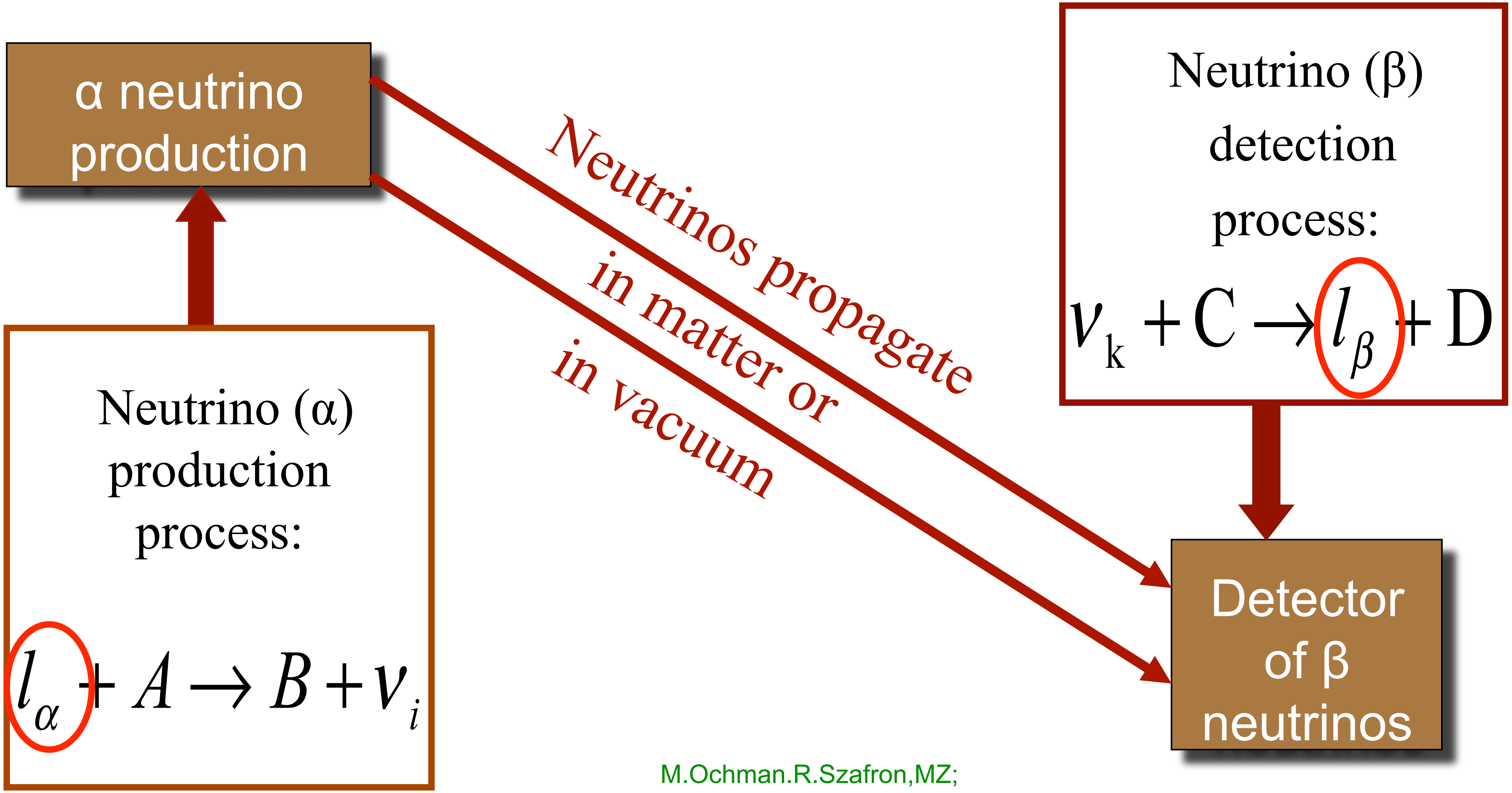
1) Introduction

- In solar, atmospheric, accelerator and reactor neutrino experiment - neutrino oscillation discovered.
- Neutrino oscillation - relativistic quantum mechanical phenomena,
  - *naive description,*
  - *internal wave packet description,*
  - *external wave packet,*
  - *field theoretical approach.*
- In all approaches --SM neutrino interactions and relativistic neutrinos. BSM only for matter oscillation.
- Future experiments will be more precise (super beam, beta beam, neutrino factories) – chance to look for Beyond the Standard Model (BSM) neutrino interaction.
- Theory for neutrino oscillation for any neutrino interaction, for non-relativistic neutrino is welcome.
- Such theory is presented and some numerical examples for neutrino produced in muon decay and detected by inverse muon decay are given.

1956	3
7	8
8	2
9	2
1960	9
1	4
2	6
3	20
4	18
5	12
6	9
7	9
8	29
9	70
1970	91
1	81
2	92
3	132
4	196
5	245
6	311
7	298
8	367
9	311
1980	432
1	412
2	318
3	227
4	375
5	344
6	541
7	598
8	498
9	449
1990	481
1	536
2	693
3	540
4	540
5	563
6	591
7	642
8	876
9	1006
2000	1195
1	1155
2	1119
3	1168
4	1001
5	1031
6	1094
7	829
8	613

2) Neutrino oscillation in the Standard Model

Typical neutrino oscillation experiments



M.Ochman.R.Szafron,MZ;
J.Phys.G35:065003,2008,

J.Syska,S.Zajac,MZ;
Acta Phys.Pol.,B38:3365,2007

(A.1) Produced and detected neutrinos are described by the pure QM states (neutrinos are relativistic particles) :

$$|\nu_\alpha(0)\rangle = \sum_{i=1}^{3} U^*_{\alpha i}|\nu_i\rangle$$

Z. Maki, M. Nakagawa, S. Sakata, Prog.Theor.Phys. 28(1962)870

$$\left|\nu_\alpha \downarrow\right\rangle = \sum_i U^*_{\alpha\, i}\left|\nu_i \downarrow\right\rangle \qquad \left|\bar{\nu}_\alpha \uparrow\right\rangle = \sum_i U_{\alpha\, i}\left|\bar{\nu}_i \uparrow\right\rangle$$

$\left|\nu_\alpha, \uparrow\right\rangle$

$\left|\bar{\nu}_\alpha, \downarrow\right\rangle$

Sterile neutrinos

(A.2) Number of neutrinos in a detector is described by the factorized formula:

$$\sum_{\beta=e,\mu,\tau} P_{\alpha\to\beta}(E,L) = 1$$

Density of the α initial neutrinos

Detection cross section of β neutrinos

$$N(E,L) = \rho_\alpha(E)\; P_{\alpha\to\beta}(E,L)\; \sigma_\beta(E)\; N_T$$

Number of the β neutrinos with energy E, which reach detector in a unit time

Probability of the α to β neutrino conversion

Number of active scattering centres in a detector

(A.3) Dirac and Majorana neutrinos oscillate in the same way, so it is impossible to distinguish both types of neutrinos in any oscillation experiments.

- ✧ Neutrino masses are too small, they are relativistic particles, to give chance to distinguish Dirac from Majorana neutrinos in a production and detection process

- ✧ The CP Majorana phases disappear from the oscillation probabilities

New physics can modify neutrino oscillation

- ❖ at the source (production process)
- ❖ propagation in matter
- ❖ at the detector

3) What kind of problems can we expect beyond the Standard Model?

As an example for the production process we take: μ^- **decay**

For Dirac neutrinos

$$\mu^-(\lambda_\mu) \to e^-(\lambda_e) + \overline{\nu}_n(\lambda_n) + \nu_m(\lambda_m)$$

In the SM: (+1) (-1)

Beyond the SM: (+1,-1) (-1,+1)

For Majorana neutrinos

$$\mu^-(\lambda_\mu) \to e^-(\lambda_e) + \nu_n(\lambda_n) + \nu_m(\lambda_m)$$

For Dirac nautrinos

In the MS:

$$\bar{\nu}_e = \nu(\lambda = +1) \qquad |\bar{\nu}_e\rangle = \sum_{i=1}^{3} \mathrm{U}_{e\,i} |\bar{\nu}_i\rangle$$

distinguishable

$$\nu_\mu = \nu(\lambda = -1) \qquad |\nu_\mu\rangle = \sum_{i=1}^{3} \mathrm{U}^*_{\mu i} |\nu_i\rangle$$

Pure state

Beyond the SM:

$$\bar{\nu}_e \rightarrow \nu(\lambda = +1)$$
$$\nu_\mu \rightarrow \nu(\lambda = -1)$$

Mixed state

Density Matrix

A.Dolgov, G.Raffelt

For Majorana neutrinos

In the SM or beyond the SM:

$$\nu_e \rightarrow \nu(\lambda = +1)$$
$$\nu_\mu \rightarrow \nu(\lambda = -1)$$

Mixed states

4) General theory of neutrino oscillation

1. Family lepton number is not conserved,
2. Total lepton number is (Dirac neutrinos) or is not (Majorana neutrinos) conserved,
3. As in the SM, neutrino flavour is defined by the accompanying charged lepton,
4. Well defined are only neutrino mass + helicity states,

$$\left| i,\lambda \right\rangle$$

   so neutrino is characterized by two quantum numbers (i,λ),
5. Flavour neutrino states can be defined, but such states are not orthogonal,
6. It can be more then one mixing (=coupling) matrice.

(B) Beyond the Standard Model

(B.1) Initial neutrino states are not pure, they are mixed. State of the neutrinos produced in the process

$$l_\alpha + A \longrightarrow B + \nu_i$$

is described by the density matrix.

If the initial particles **(*l*, A)** are not polarized and polarization of the final particles **(B)** is not measured, then:

$$\rho^\alpha_{\lambda,i;\mu,k} = \frac{1}{N_\alpha} \sum_{\lambda_l,\lambda_A,\lambda_B} \mathrm{A}^\alpha_i(\lambda_A,\lambda_l;\lambda_B,\lambda)\,\mathrm{A}^{\alpha *}_k(\lambda_A,\lambda_l;\lambda_B,\mu)$$

$Tr(\rho^\alpha) = 1$

Initial density matrix

Where the $A^\alpha_i(\lambda_A,\lambda_l;\lambda_B,\lambda)$ are the amplitudes for the production process: $l_\alpha(\lambda_l) + A(\lambda_A) \rightarrow B(\lambda_B) + \nu_i(\lambda)$

(B.2) There is no factorization for the detection rate, and now

$$N(E,L) = \rho_\alpha(E) \quad \sigma_{\alpha\to\beta}(E,L) \quad N_T$$

Where now

$$\sigma_{\alpha\to\beta}(E,L) = \frac{1}{32\pi s}\frac{p_f}{p_i}\frac{1}{2s_C+1}\sum_{spins}\int dLips\ A^\beta \rho^\alpha (T{=}L) A^{\beta *}$$

Generally the $\sigma_{\alpha\to\beta}(E,L)$ cross section does not factorize

$$\sigma_{\alpha\to\beta}(E,L) \neq P_{\alpha\to\beta}(E,L) \ \ \sigma_\beta(E)$$

A^β is the amplitude for the detection process $\nu_k + C \to l_\beta + D$

e.g. $\pi^+ \to \mu^+ \nu_i$ $\quad n \to p e^- \nu_i$

(B3$_1$) If only one type of flavour neutrino is produced - Dirac and Majorana neutrinos propagate in matter in a different way, so in principle both types of neutrinos can be distinguished in future oscillation experiments.

- *Production and detection states are the same (charge currents are responsible for production and detection), but*
- *Propagation in matter distinguishes both types of neutrinos (neutral currents are crucial).*

e.g. $\mu^- \to e^- \overline{\nu}_e \nu_i$

(B3$_2$) If two flavour neutrinos are produced – both production, detection and propagation processes are different for Dirac and Majorana neutrinos, so in principle both types of neutrinos can be distinguished in future oscillation experiments, even in vacuum.

- *For Majorana neutrinos production and detection amplitudes add coherently, giving possible observable effects.*

We have to know the density matrix in the detector frame

$$\rho^{\alpha}_{CM}$$

Calculated in the CM of decaying particle:

$\pi^+ \to \mu^+ \nu_i$ — **Accelerator neutrinos**

$\mu^- \to e^- \bar{\nu}_e \nu_i$ — **Neutrino factory**

$n \to p e^- \nu_i$ — **Beta beam**

Lorentz boost, Wigner spin rotation

$$\rho^{\alpha}_{LAB}(L=0)$$

Neutrinos are almost massles - Lorentz boost does not change the density matrix

Neutrino propagation in vacuum or in matter

$$\rho^{\alpha}_{LAB}(L=T\neq 0)$$

$$\rho^{\alpha}(\mathrm{T{=}L}) = e^{-\mathrm{i\,H\,T}}\rho^{\alpha}(\mathrm{L{=}T}=0)e^{\mathrm{i\,H\,T}}$$

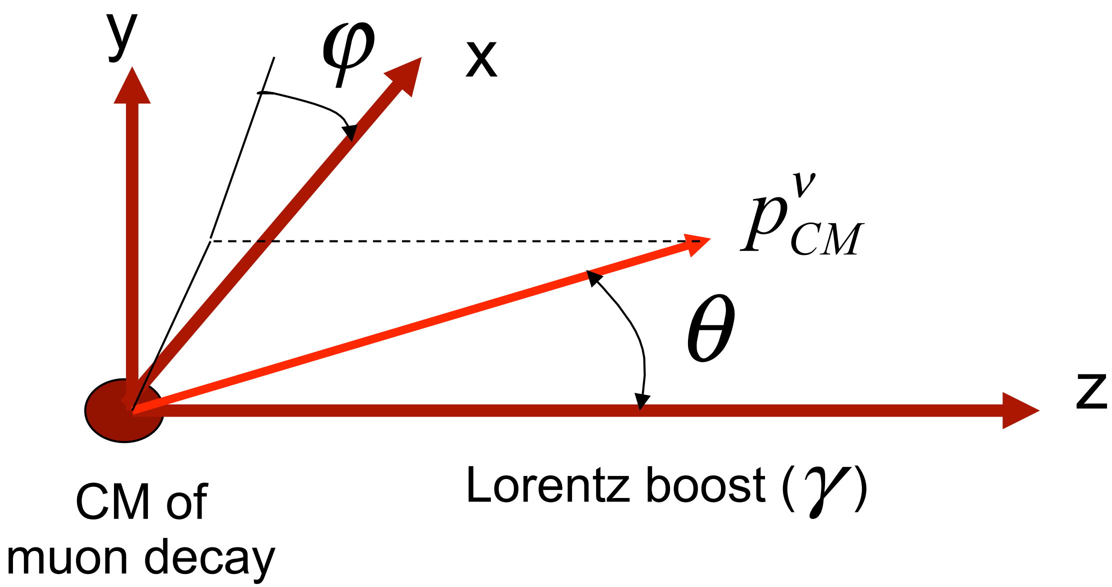
H – vacuum or matter Hamiltonian

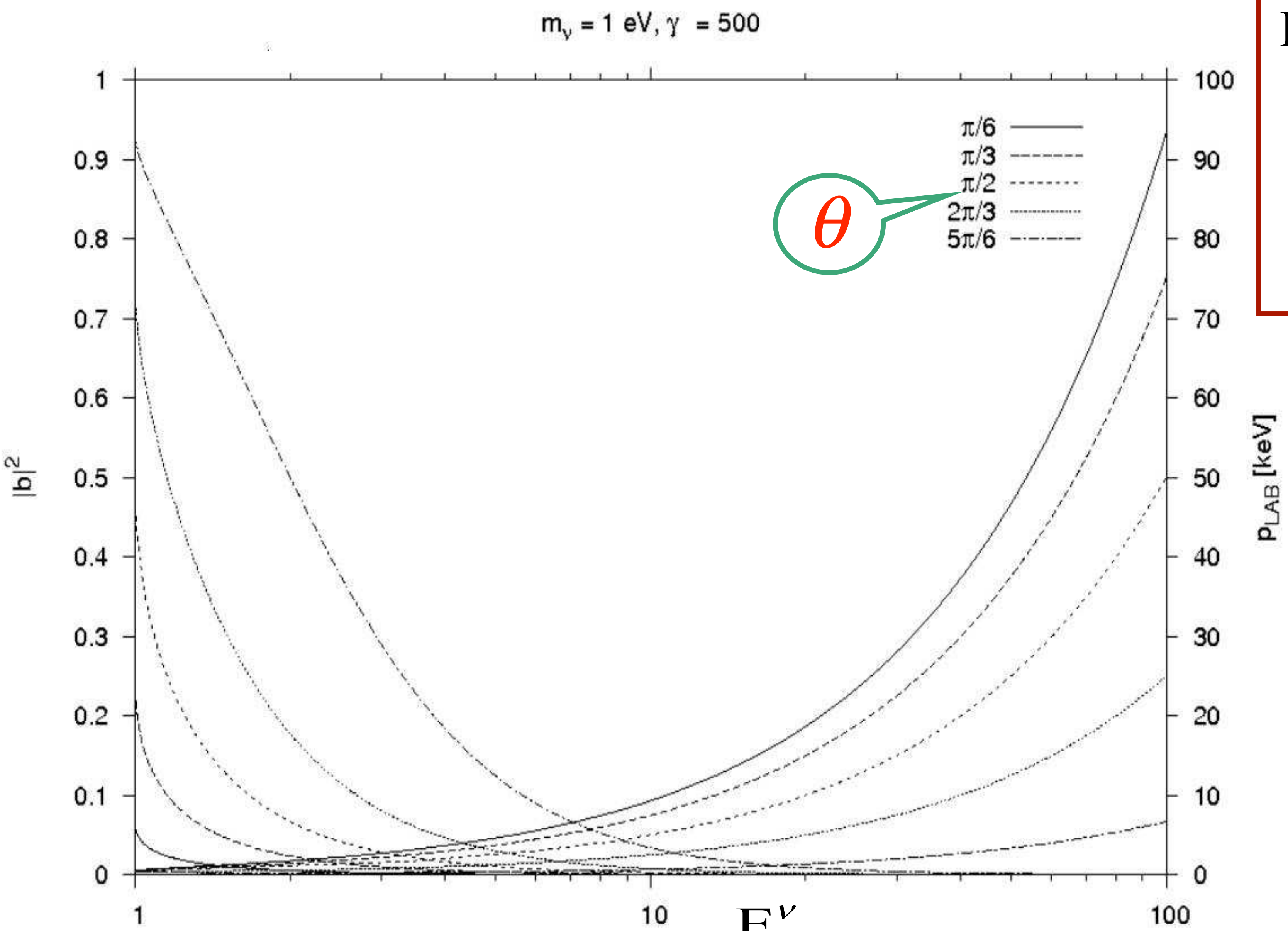
Impact of Lorentz boost $\Longrightarrow$ helicity Wigner rotation

$$\rho^{\alpha}_{CM} \rightarrow \rho^{\alpha}_{LAB} = D^{1/2} \rho^{\alpha}_{CM} D^{1/2*}$$

$$D^{1/2} = \begin{pmatrix} a & b \\ -b^{*} & a^{*} \end{pmatrix}$$

$$a = a(\gamma, \theta, E^{\nu}_{CM}) \qquad b = \left| b(\gamma, \theta, E^{\nu}_{CM}) \right| e^{i\varphi}$$





$$\gamma_\nu = \frac{E^{\nu}_{CM}}{m_\nu}$$

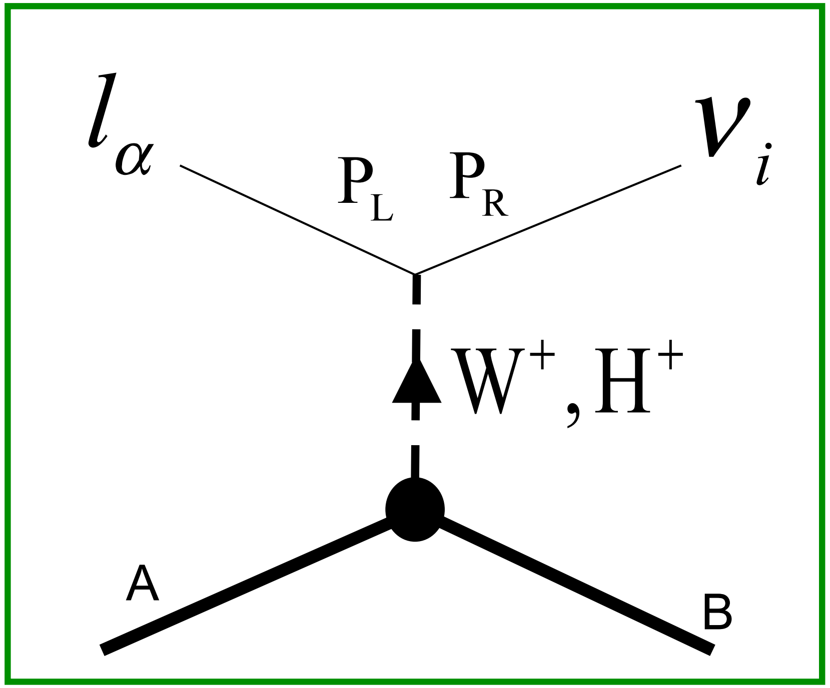
Practically Lorentz boost has no meaning

$$\varrho^{LAB}(\Lambda\vec{p},\ i,\ \lambda;\ \Lambda\vec{p}\,',i',\lambda') = \ \varrho^{CM}(\vec{p},\ i,\ \lambda;\ \vec{p}\,',i',\lambda')$$

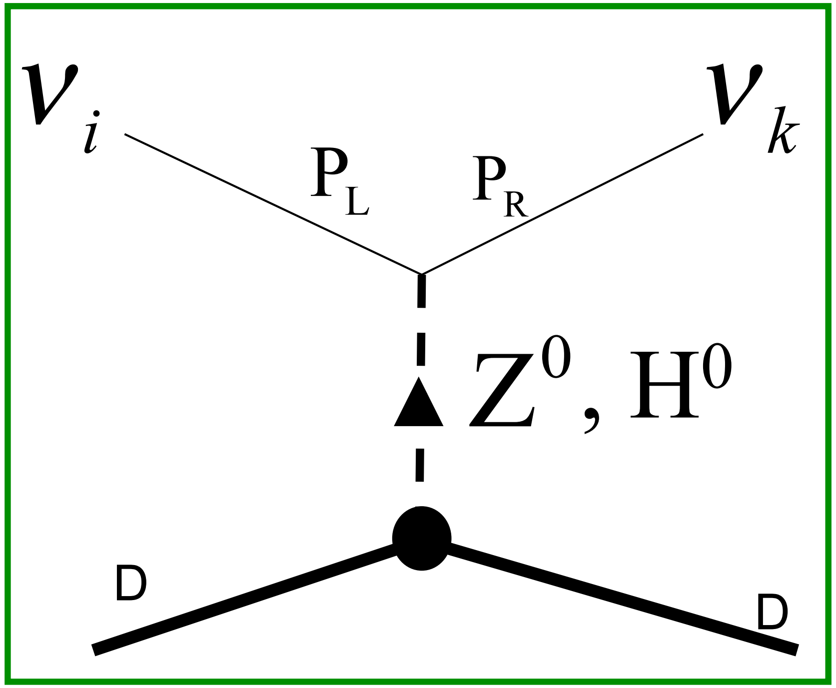
New Physics parameterization

Physics beyond the Standard Model is parameterized by two diagrams with vector ($W^{\pm}$) and scalar ($H^{\pm}$) particles exchange for charge current interactions, and neutral vector (Z^0) and scalar (H^0) exchange for neutral current interactions.

For production and detection



For matter oscillation



Explicit form of the interaction BSM Lagrangian

For production and detection

$$\mathcal{L}_{CC} = - \frac{e}{2\sqrt{2}sin\theta_W}\Big\{\sum_{\alpha,i} \bar{\nu_i}\,[\,\gamma^\mu(1-\gamma_5)\epsilon_L^c U_{\alpha i}^{L*} + \gamma^\mu(1+\gamma_5)\epsilon_R^c U_{\alpha i}^{R*}]\, l_\alpha\, W_\mu^+$$

$$+ \sum_{\alpha,i} \bar{\nu_i}\,[\,(1-\gamma_5)\eta_L V_{\alpha i}^{L*} + (1+\gamma_5)\eta_R V_{\alpha i}^{R*}]\, l_\alpha\, H^+$$

$$+ \sum_{u,d} \bar{u}\,[\,\gamma^\mu(1-\gamma_5)\epsilon_L^q U_{ud}^{*} + \gamma^\mu(1+\gamma_5)\epsilon_R^q U_{ud}^{*}]\, d\, W_\mu^+$$

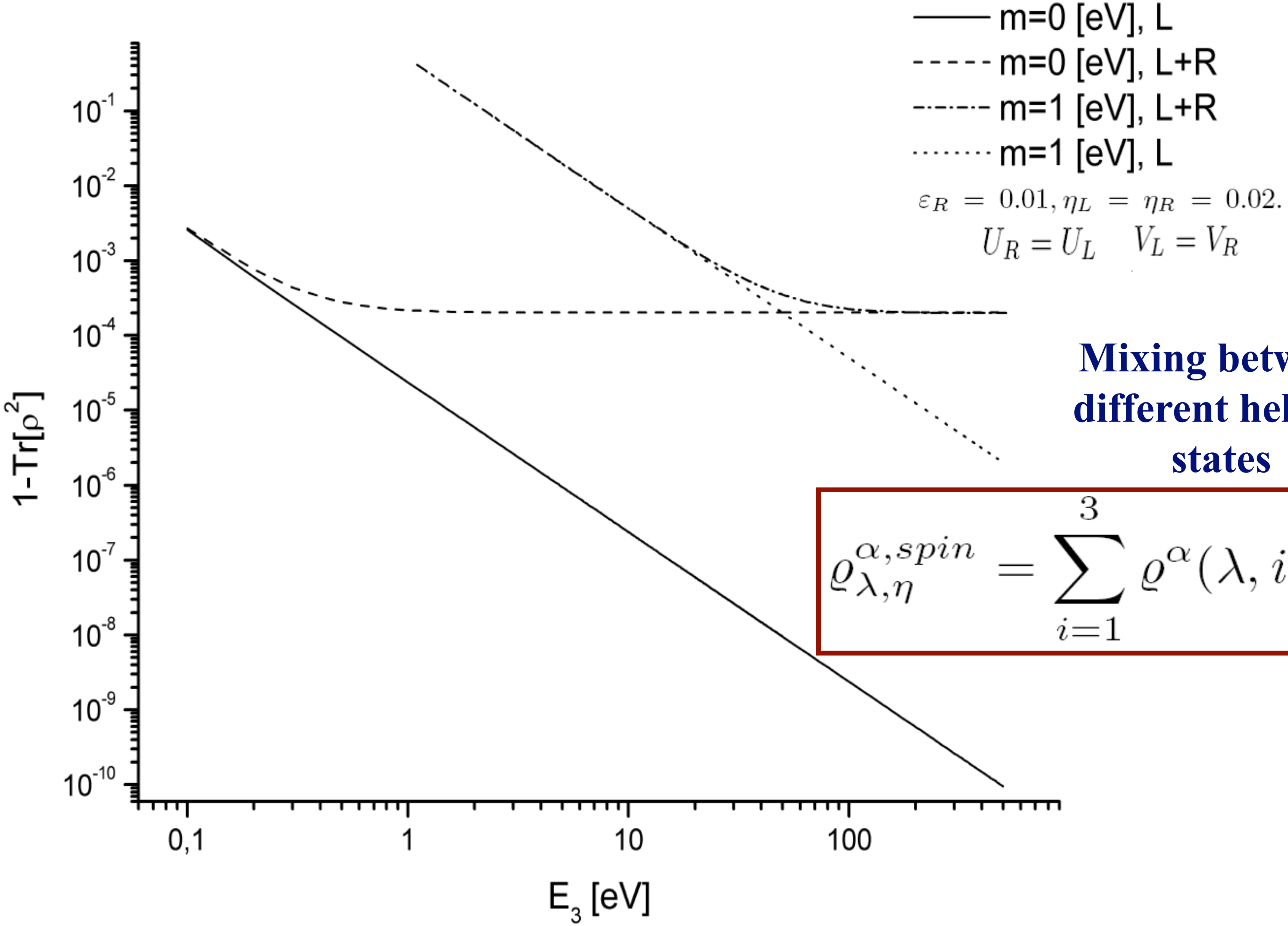
$$+ \sum_{u,d} \bar{u}\,[\,(1-\gamma_5)\tau_L W_{ud}^{L*} + (1+\gamma_5)\tau_R W_{ud}^{R*}]\, d\, H^+ \Big\} + h.c$$

For matter oscillation

$$\mathcal{L}_{NC} = \frac{-\,e}{4sin\theta_W cos\theta_W}\Big\{\sum_{i,j} \bar{\nu_i}\,[\,\gamma^\mu(1-\gamma_5)\varepsilon_L^{N\nu}\delta_{ij} + \gamma^\mu(1+\gamma_5)\varepsilon_R^{N\nu}\Omega_{ij}^R]\,\nu_j$$

$$+ \sum_{f=e,u,d} \bar{f}\,[\,\gamma^\mu(1-\gamma_5)\varepsilon_L^{Nf} + \gamma^\mu(1+\gamma_5)\varepsilon_R^{Nf}]\, f\,\Big\}\, Z_\mu$$

Because of the New Physics the production and detection neutrino states are not pure quantum mechanical states



$\varepsilon_R = 0.01, \eta_L = \eta_R = 0.02.$

$U_R = U_L \quad V_L = V_R$

Mixing between different helicity states

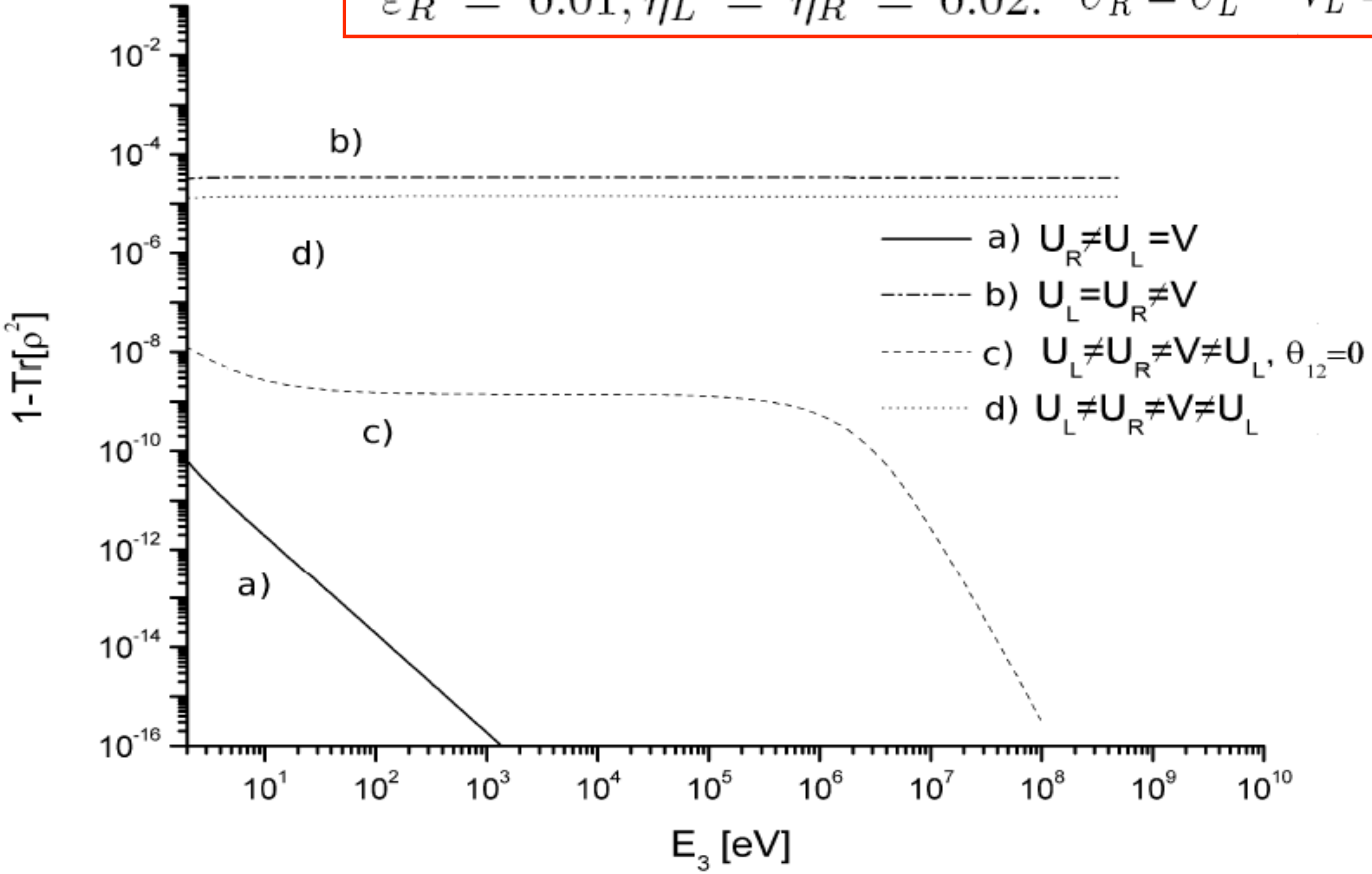
$$\varrho^{\alpha,spin}_{\lambda,\eta} = \sum_{i=1}^{3} \varrho^{\alpha}(\lambda, i; \eta, i),$$

Mixing between different mass states

$$\varrho_{i,k}^{\alpha,mass} = \sum_{\lambda=\pm 1} \varrho^{\alpha}(\lambda, i; \lambda, k).$$

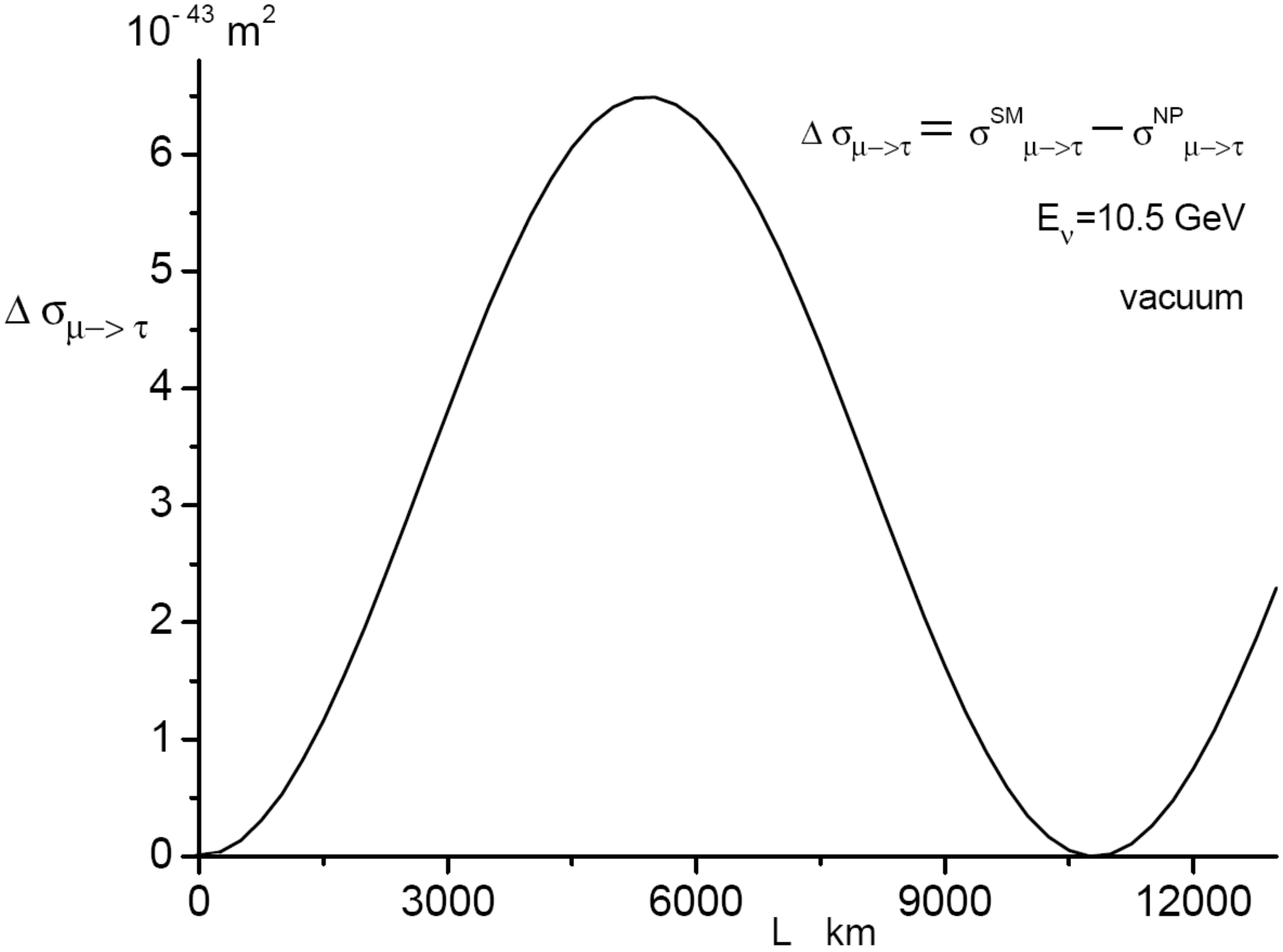
$sin^2\theta_{12} = 0.87, sin^2\theta_{23} = 0.09, sin^2\theta_{13} = 0.23,\ \delta_{CP} = 0.5$

$\varepsilon_R = 0.01, \eta_L = \eta_R = 0.02.\ U_R = U_L \quad V_L = V_R$



New Physics can change the production and detection neutrino states and has the impact on the full oscillation process

$$\varepsilon_R = 0.01, \eta_L = \eta_R = 0.02.$$



We assume that neutrino interactions are described by the most general 4-fermion interaction

W.Fetscher, H,-J.Gerber and K.J.Johnson, Phys. Lett. B173(1986)102;

$$\boldsymbol{H} = \frac{4G_F}{\sqrt{2}} \sum_{\delta,\varepsilon,\varepsilon'} \sum_{i,k=1}^{3} \left[(g^{\delta}_{\varepsilon,\varepsilon'})_{i,k} (\bar{l}_{\varepsilon,e} \Gamma^{\delta} \nu_i)(\overline{\nu}_k \Gamma_{\delta} l_{\varepsilon',\mu}) + (g^{\delta\,*}_{\overline{\varepsilon},\overline{\varepsilon}'})_{i,k} (\overline{\nu}_i \Gamma^{\delta}\, l_{\overline{\varepsilon},e})(\bar{l}_{\overline{\varepsilon}',\mu} \Gamma_{\delta} \nu_k) \right]$$

$$\delta = (S(=1), V(=\gamma^{\mu}), T(= \tfrac{i}{2\sqrt{2}}[\gamma^{\mu},\gamma^{\nu}] \equiv t_{\mu\nu}))$$

$$\varepsilon, \varepsilon' = L, R(P_{R,L} = \tfrac{1}{2}(1 \pm \gamma_5))$$

$$\overline{\varepsilon} = \varepsilon \text{ for } V,\ \overline{\varepsilon} = -\varepsilon \text{ for } S \text{ and } T \text{ and the same for } \overline{\varepsilon}'.$$

Standard Model is recovered for

$$(g^{V}_{LL})_{i,k} = g^{V}_{LL} U_{e,i} U^{*}_{\mu,k},$$

MNSP matrix (U)

g^{V}_{LL} =1

All other couplings equal zaro

In the same way we parameterize:

$$(g^{\delta}_{\varepsilon,\varepsilon'})_{i,k} = g^{\delta}_{\varepsilon,\varepsilon''}(U^{\delta}_{\varepsilon})_{e,i}(U^{\delta}_{\varepsilon'})^{*}_{\mu,k}$$

$$(S_{LL}, S_{LR}, S_{RL}, S_{RR}) \quad (U^S_L, U^S_R) \quad (= V)$$

$$(V_{LL}, V_{LR}, V_{RL}, V_{RR}) \quad (U^V_L, U^V_R) \quad (= U_{MNSP} \equiv U)$$

$$(T_{LR}, T_{RL}) \quad (U^T_L, U^T_R)$$

Neutrino masses are not measured,

- sumed uncoherently over final neutrino mass states,
- averaged over initial neutrino states.

$$\Gamma_{total} = \sum_{n,m=1}^{3} \Gamma_{\overline{n},m}$$

$$\boldsymbol{H} = \frac{4G_F}{\sqrt{2}} \sum_{\delta,\varepsilon,\varepsilon'} \sum_{i,k=1}^{3} [(g^{\delta}_{\varepsilon,\varepsilon'})_{i,k}(\bar{l}_{\varepsilon,e}\Gamma^{\delta}\nu_i)(\bar{\nu}_k\Gamma_{\delta}l_{\varepsilon',\mu})$$

$$(g^{V}_{LL})\Big[(\bar{e}_L\gamma^{\mu}\nu_{iL})(\bar{\nu}_{kL}\gamma_{\mu}\mu_L)\Big] + (g^{S}_{LL})\Big[(\bar{e}_L\nu_{iR})(\bar{\nu}_{kR}\mu_L)\Big]$$

Produced helicity

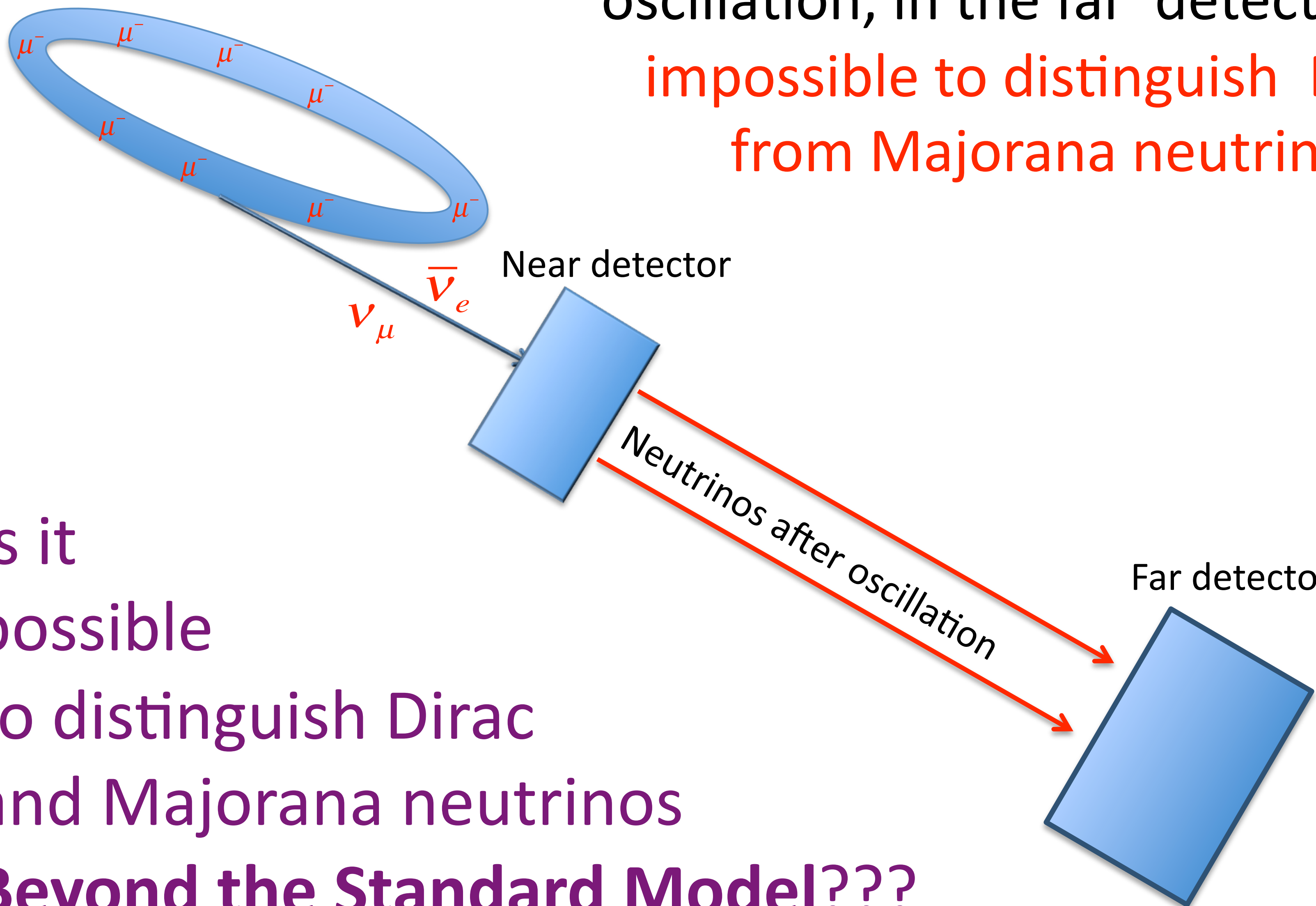
-1	for neutrino	+1
+1	for antyneutrino	-1

Dirac neutrino: $A_N[-,-,-,-] = (g^{V}_{LL})A^{N}_{-,-,-,-}$

Dirac antineutrino: $A_{AN}[-,-,-,-] = (g^{S}_{LL})A^{AN}_{-,-,-,-}$

Majorana neutrino: $A_M[-,-,-,-] = \{(g^{V}_{LL})A^{N}_{-,-,-,-}\} - \{(g^{S}_{LL})A^{AN}_{-,-,-,-}\}$

In the **Standard Model**
whether in a near detector or, after oscillation, in the far detector, it is impossible to distinguish Dirac from Majorana neutrinos.



Is it
possible
to distinguish Dirac
and Majorana neutrinos
Beyond the Standard Model???

Using general interaction we calculate for neutrinos from muon decay

1. Density matrix for final Dirac neutrinos

$$\varrho^{\nu}_{i,\lambda;k,\eta}(E,\theta,\varphi) =$$
$$\frac{1}{N_\mu}\int_0^{2\pi} d\overline{\psi}\int_{\overline{E}_{min}}^{\overline{E}_{max}} d\overline{E}_\nu \sum_{\lambda_\mu,\lambda'_\mu,\lambda_e,\overline{\lambda}_\nu,\overline{j}} A^{\overline{j},i}_{(\lambda_\mu,\lambda_e,\overline{\lambda}_\nu,\lambda)}(E,\theta,\varphi;\overline{E}_\nu,\overline{\psi})\varrho_{\lambda_\mu,\lambda'_\mu} A^{\overline{j},k\ *}_{(\lambda'_\mu,\lambda_e,\overline{\lambda}_\nu,\eta)}(E,\theta,\varphi;\overline{E}_\nu,\overline{\psi})$$

2. In the same way final density matric for Dirac antineutrinos

3. Density matrix for final Majorana neutrinos

Then we calclate the cross sections for muon production processes with Dirac neutrinos

$$\nu_m(\lambda_m) + e^-(\lambda_e) \to \mu^-(\lambda_\mu) + \nu_n(\lambda_n)$$

$$\sigma^\nu = \frac{p_f}{264\pi^2 s p_i}\sum_{\lambda_e,n,\lambda_n;\lambda_\mu,i,\lambda_i,k,\lambda_k}\int d\vartheta d\phi f^{\nu}{}_{\lambda_e,n,\lambda_n;\lambda_\mu,i,\lambda_i}(E,\vartheta,\phi)\varrho^{\nu}_{i,\lambda;k,\eta} f^{\nu *}{}_{\lambda_e,n,\lambda_n;\lambda_\mu,k,\lambda_k}(E,\vartheta,\phi)$$

And similarly for Dirac antineutrin $\longrightarrow$ $\sigma^{\bar{\nu}}$

and for Majorana neutrino $\longrightarrow$ σ^{ν_M}

We have to know the number of Dirac neutrinos and Dirac antineutrinos flying in direction (θ,φ) ,

$\longrightarrow$ we caculate angular distribution:

$$N^{\nu}(E,\theta,\varphi) = \frac{d^3\Gamma^{\nu}}{dEd\theta d\varphi} \qquad N^{\bar{\nu}}(E,\theta,\varphi) = \frac{d^3\Gamma^{\bar{\nu}}}{dEd\theta d\varphi}$$

So number of neutrino and antineutrino in the beam is proportional respectively to:

$$\alpha(E,\theta,\varphi) = \frac{N^{\nu}}{N^{\nu}+N^{\bar{\nu}}} \qquad \beta(E,\theta,\varphi) = \frac{N^{\bar{\nu}}}{N^{\nu}+N^{\bar{\nu}}} \qquad \alpha+\beta=1$$

$$\bar{\sigma}_D = \alpha\ \sigma^{\nu} + \beta\ \sigma^{\bar{\nu}}$$

For Majorana neutrinos such weight factors are automaticaly included in the density matrix.

For Majorana neutrino dominant term – pure states

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} (a\mathrm{U}^*_{\alpha,i} + b\mathrm{V}^*_{\alpha,i})|\nu_i\rangle$$

MNSP matrix

Mixing for S_{LL}

$$\rho^M_\alpha \approx |\nu_\alpha\rangle\langle\nu_\alpha| + \ldots.$$

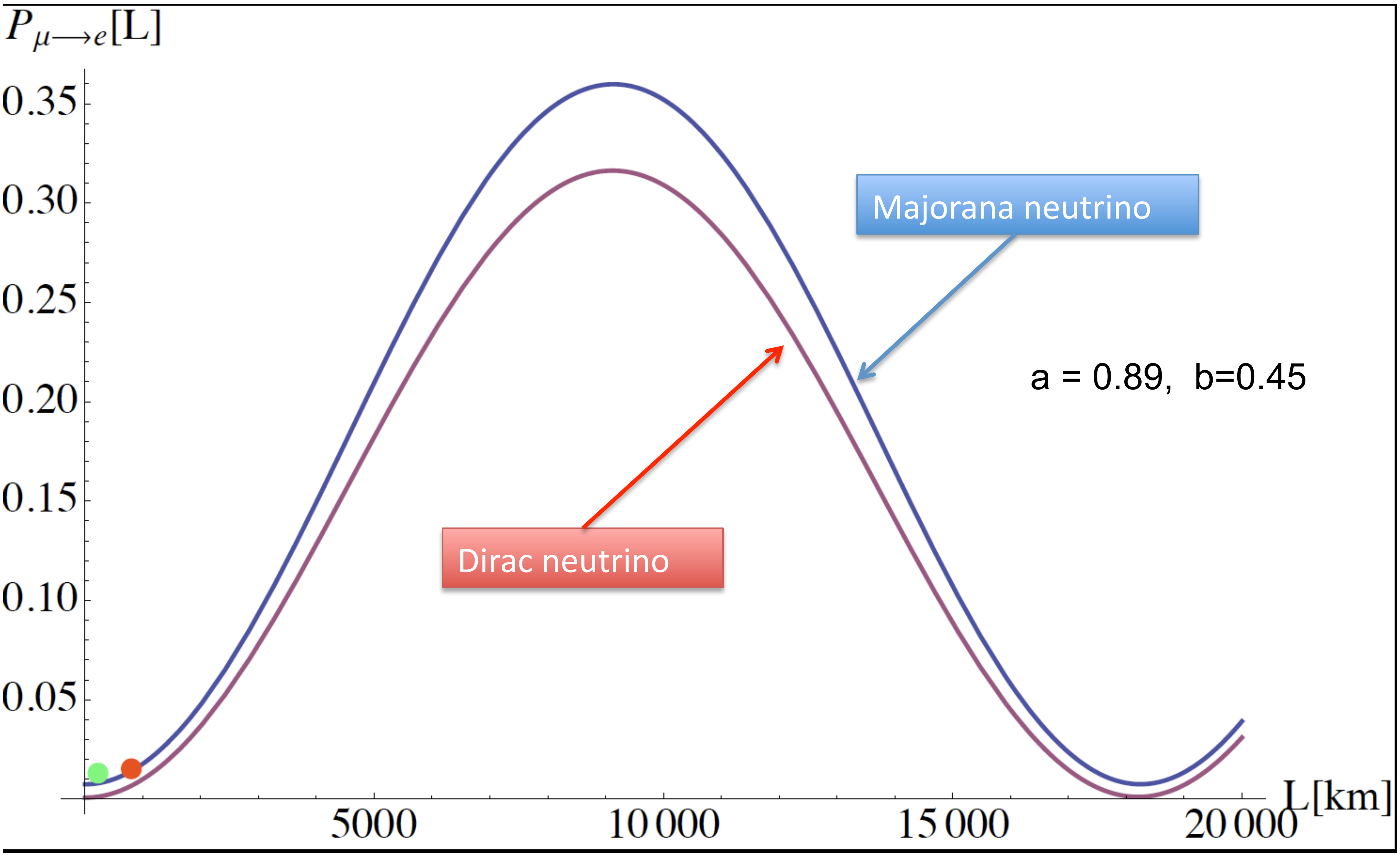
has interference terms U x V

Coherent oscillations

$$(\rho^D_\alpha)_{i,k} = |a|^2\, \mathrm{U}^*_{\alpha,i}\mathrm{U}_{\alpha,k} + |b|^2\, \mathrm{V}^*_{\alpha,i}\mathrm{V}_{\alpha,k} + \ldots..$$

No interfernce terms

Incoherent oscillations


$P_{\mu\longrightarrow e}[L]$
0.35
0.30
0.25
0.20
0.15
0.10
0.05
5000
10 000
15 000
20 000
L[km]
Majorana neutrino
Dirac neutrino
a = 0.89, b=0.45

Density matrix for Dirac neutrino:

$$\varrho_\nu^D = \begin{pmatrix} (g_{LL}^S)^2 f^+ V_{\mu j} V_{\mu i}^* & 0 \\ 0 & (g_{LL}^V)^2 f^- U_{\mu j} U_{\mu i}^* \end{pmatrix}$$

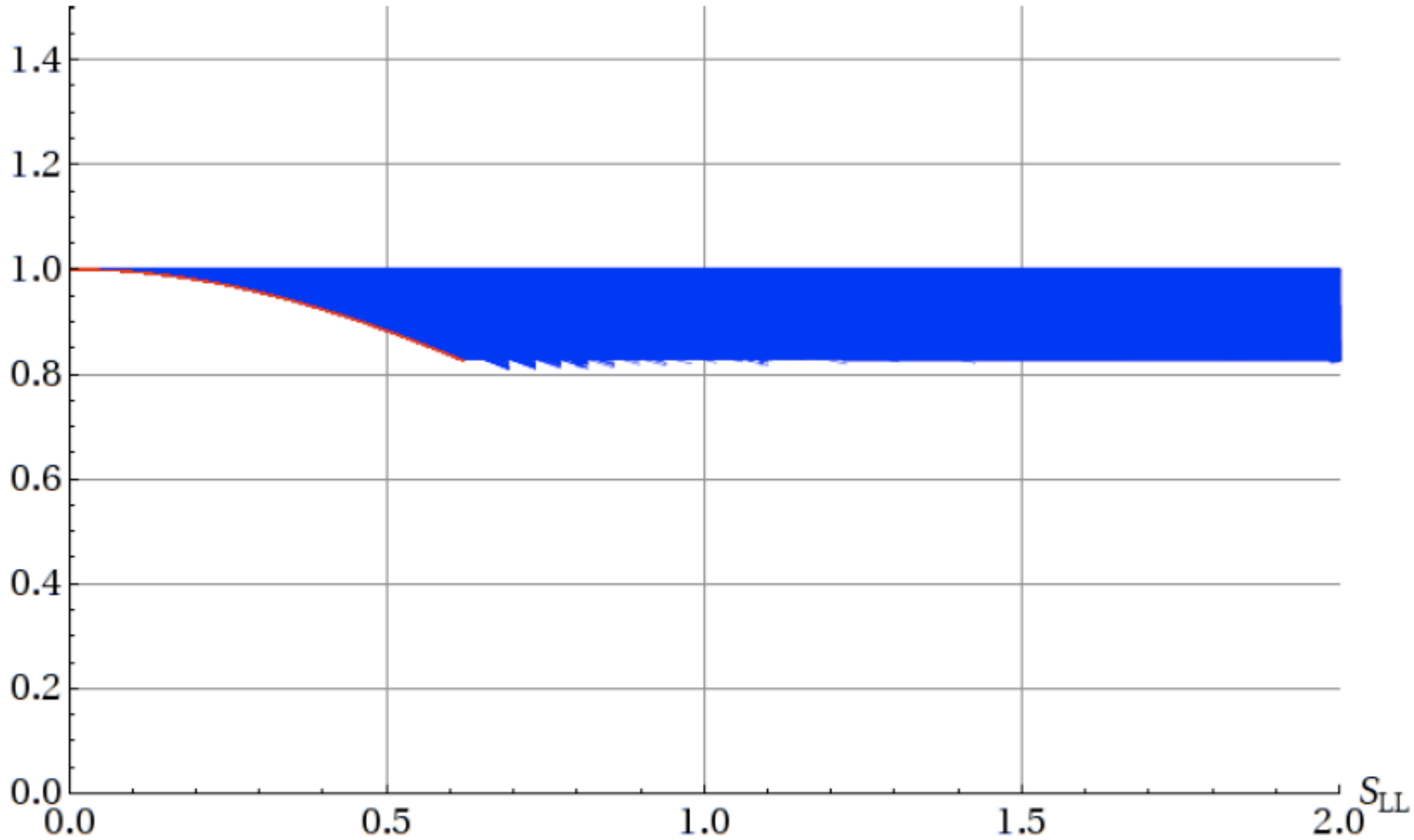
and for Majorana neutrino:

$$(\varrho_\nu^M)_{--} = [6(2U_{\mu i}^* g_{LL}^V (a^* V_{ej}^* g_{LL}^S + 2g_{LL}^V U_{\mu j}) + g_{LL}^S (V_{ej}^* g_{LL}^S + 2a g_{LL}^V U_{\mu j}) V_{ei}] f_M^- ,$$

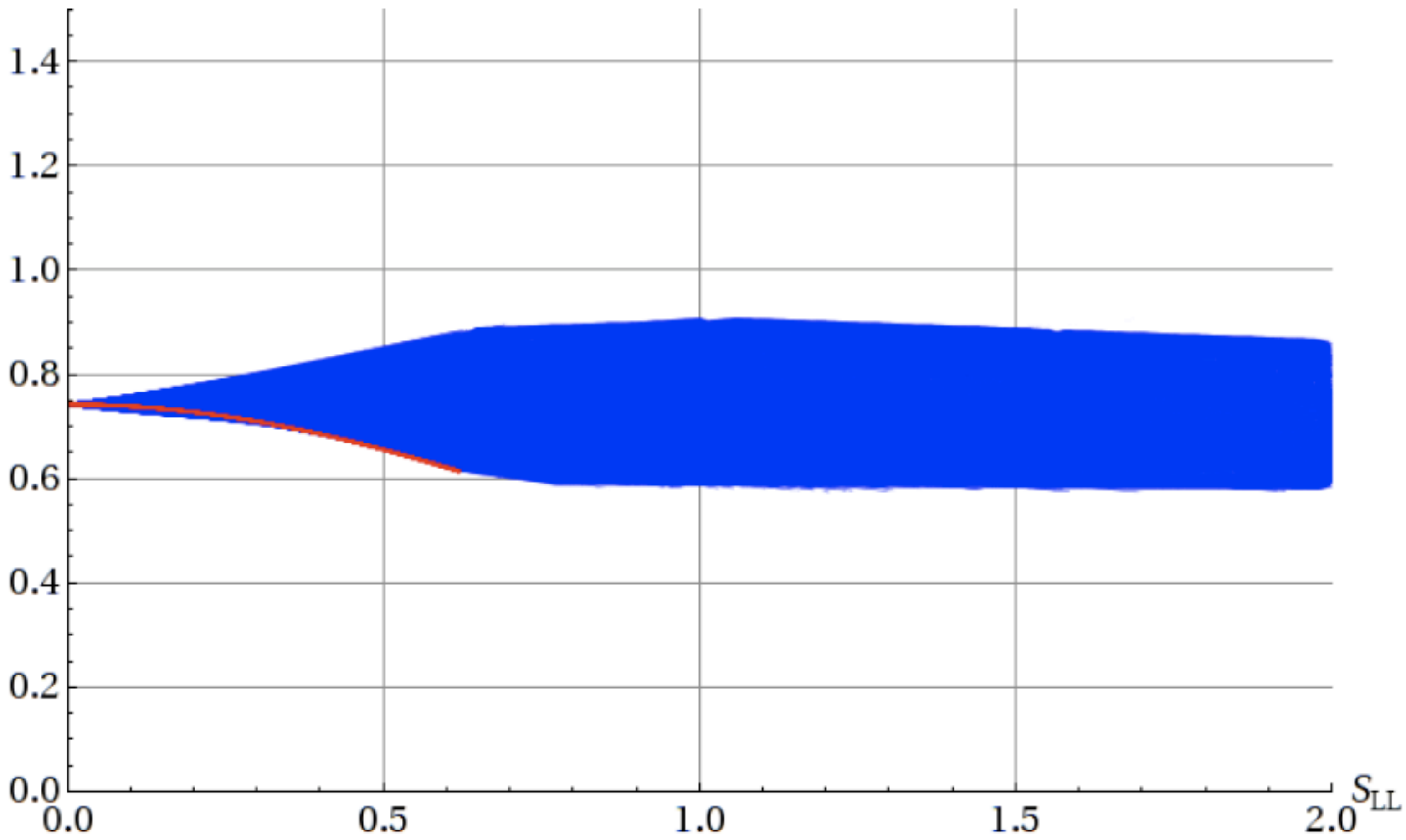
$$(\varrho_\nu^M)_{++} = [(2U_{ej}^* g_{LL}^V (b V_{\mu i}^* g_{LL}^S + 2g_{LL}^V U_{ei}) + g_{LL}^S (V_{\mu i}^* g_{LL}^S + 2b^* g_{LL}^V U_{ei}) V_{\mu j}] f_M^+ ,$$

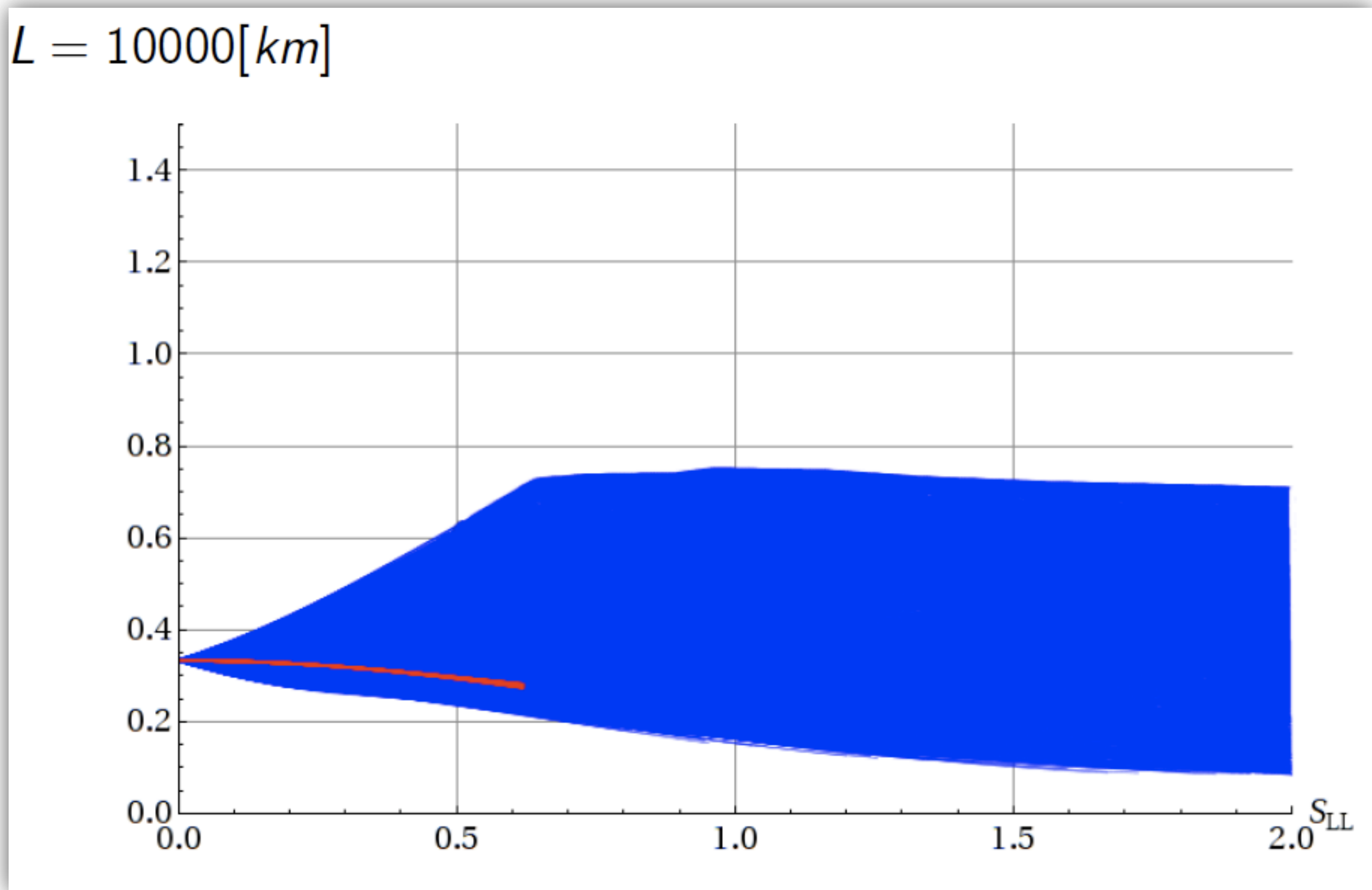
$$a = \sum_k U_{ek}^* V_{\mu k}^* \qquad b = \sum_k U_{\mu k} V_{ek}$$

We calculate cross section and normalise it, so that $\sigma_{SM}(L=0)=1$, Dirac - red region, Majorana - blue region, $L=0$, neutrino energy: $E = 12 GeV$.



$L = 5000[km]$




$L = 10000[km]$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
S_{LL}

4) Summary

- For beyond the SM interaction neutrino production states may not be pure QM states – density matrix.
- For BSM final detection rates generally do not factorize.
- It is possible (in principle) for neutrino produced in muon decay to distinguish Dirac from Majorana neutrinos in a near (problematic) and (better) in a far detectors.
- Depending on the neutrino production process and BSM interaction coherent and incoherent oscillation can be distinguished,
- Density matrix is useful even for the νSM neutrino oscillation (e.g. neutrinos from muon decay).