WESTFÄLISCHE WILHELMS-UNIVERSITÄT MÜNSTER

ν oscillations with a polarized laser beam: an analogical demonstration experiment

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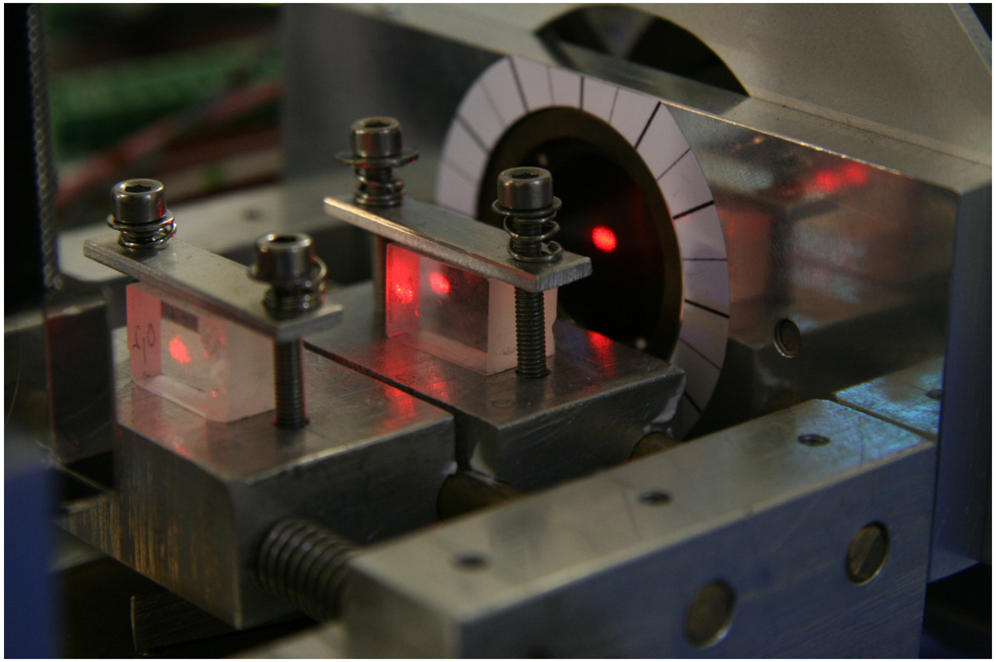
- **Motivation**
- **2 flavour ν vacuum oscillation**
- **Bifringence**
- **Optical analogue**
- **Experiment**



Motivation: neutrino oscillation is

- one of the largest discoveries of the last decade in nuclear/particle/astroparticle physics
- our only evidence for physics beyond the Standard Model
- so well proven, that it turned into textbook knowledge
- based on a general feature of mixed two-state systems, e.g. $K_0 - \overline{K}_0$ ($B_0 - \overline{B}_0$, ..) oscillation
- does not need advanced quantum physics, but is not easy to understand for senior high school or 1st/2nd year university students although they are familiar with the double slit experiment

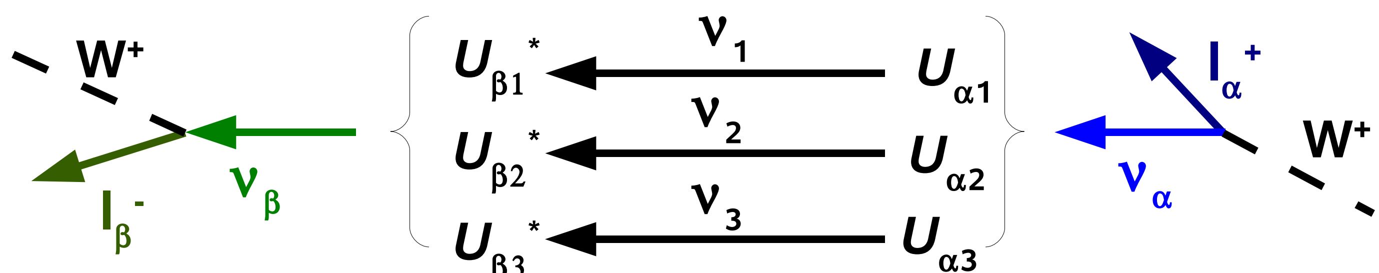
⇒ Try to explain neutrino oscillation without simplifications by an analogue experiment using the two-state system of polarized light, which can be permformed in senior high school classes / with 2nd year students

Neutrino (vacuum) oscillations

Incredients: 1) non-trivial ν mixing matrix *U* between neutrino flavour states (ν_e, ν_μ, ν_τ) and mass states (ν_1, ν_2, ν_3):

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

2) a flavour state propagates as a coherent sum of mass states $m(\nu_i)$ if the $m(\nu_i)$ differ ⇒ neutrino oscillation



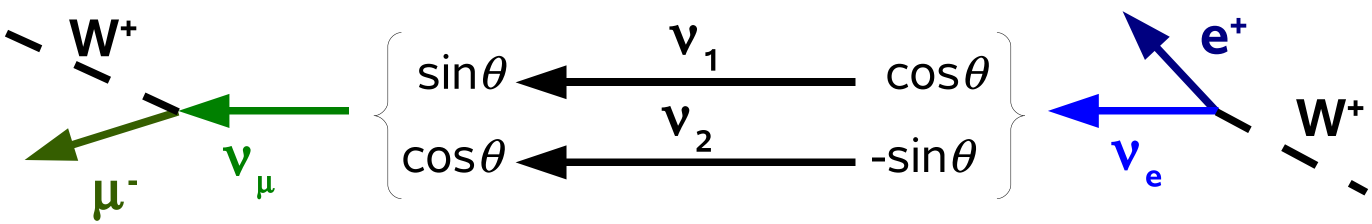
detection of a ν_β via weak interaction | propagation as coherent superposition of mass states | creation of a ν_α via weak interaction

$$\mathbf{P}(\nu_\alpha \rightarrow \nu_\beta) = \left| \sum_i U_{\alpha i}\, e^{-iE_i t}\, U_{\beta i}^* \right|^2 = \underbrace{\sin^2(2\theta) \cdot \sin^2 \frac{|m_2^2 - m_1^2| \cdot L}{4E}}_{\text{2 flavor mixing}}$$

2 flavour neutrino (vacuum) oscillations: the two essential incredients

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{U} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{U^{-1}} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

2 states mixing



pure flavor | rotating back | propagation | rotating | pure flavour

Double slit experiment

$$P(\nu_e \to \nu_\mu) = \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} U \begin{pmatrix} e^{-iE_1 t} & 0 \\ 0 & e^{-iE_2 t} \end{pmatrix} U^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2$$

$$= \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-iE_1 t} & 0 \\ 0 & e^{-iE_2 t} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2$$

$$= \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} e^{-iE_1 t} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i(E_2-E_1)t} \end{pmatrix} \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} \right|^2$$

$$= \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\frac{\Delta m^2 L}{2E}} \end{pmatrix} \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} \right|^2 = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

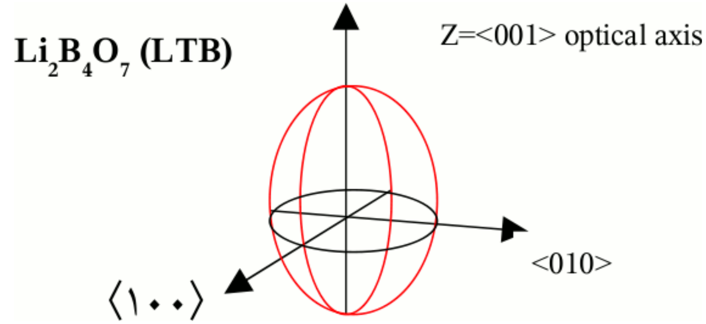
$$= \sin^2(2\theta)\sin^2\left(\pi \frac{L}{\lambda_{osc}}\right) \quad \text{with} \quad \lambda_{osc} = \frac{4\pi E}{\Delta m^2}$$

Reminder: birefringence

If an optical medium is anisotropic

(intrinsic, by a tension, ...)

it exhibits different refraction indices



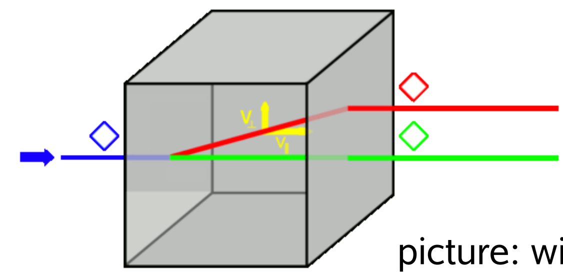
picture: A. Peter

A birefringent crystal has one „optical axis“

ordinary beam:

pol. vector E ⊥ to optical axis

with refraction index n_o



picture: wikipedia

extraordinary beam:

pol. vector E (partly) || to optical axis

with refraction index n_e

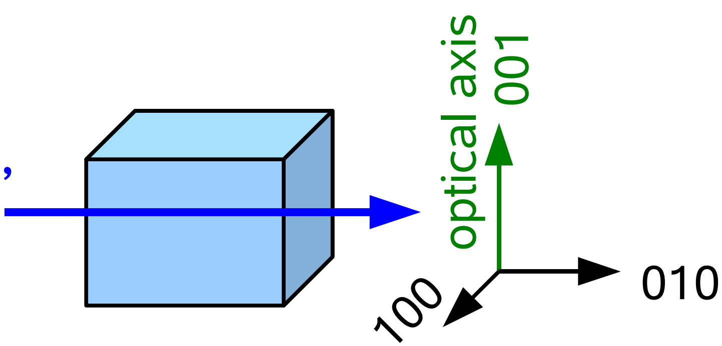
calcite

picture: wikipedia

No „double image“ but different phase propagation,

if the entrance surface of the crystal

contains the optical axis



2 polarized photon state oscillations: the same two incredients

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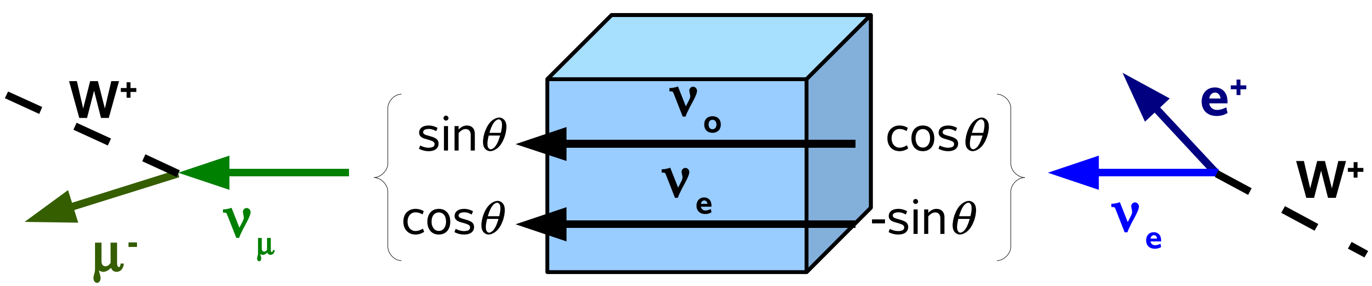
$$\begin{pmatrix} \text{“}\nu_e\text{”} \\ \text{“}\nu_\mu\text{“} \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{U} \begin{pmatrix} \nu_o \\ \nu_e \end{pmatrix} \qquad \begin{pmatrix} \nu_o \\ \nu_e \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{U^{-1}} \begin{pmatrix} \text{“}\nu_e\text{”} \\ \text{“}\nu_\mu\text{”} \end{pmatrix}$$

2 states mixing



Double slit experiment

pure flavor | rotating back | propagation | rotating | pure flavour

$$P(\text{“}\nu_e\text{“} \to \text{“}\nu_\mu\text{“}) = \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \; U \; \begin{pmatrix} e^{-2\pi i L n_o/\lambda} & 0 \\ 0 & e^{-2\pi i L n_e/\lambda} \end{pmatrix} \; U^{-1} \; \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2$$

$$= \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-2\pi i n_o L/\lambda} & 0 \\ 0 & e^{-2\pi i n_e L/\lambda} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2$$

$$= \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} e^{-2\pi i n_o L/\lambda} \begin{pmatrix} 1 & 0 \\ 0 & e^{-2\pi i (n_e - n_o) L/\lambda} \end{pmatrix} \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} \right|^2$$

$$= \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-2\pi i \Delta n L/\lambda} \end{pmatrix} \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} \right|^2 = \sin^2(2\theta)\sin^2\left(\frac{\Delta n L}{\lambda/\pi}\right)$$

$$= \sin^2(2\theta)\sin^2\left(\pi \frac{L}{\lambda_{osc}}\right) \quad \text{with} \quad \lambda_{osc} = \frac{\lambda}{\Delta n}$$

⇒ complete equivalence of formalism for

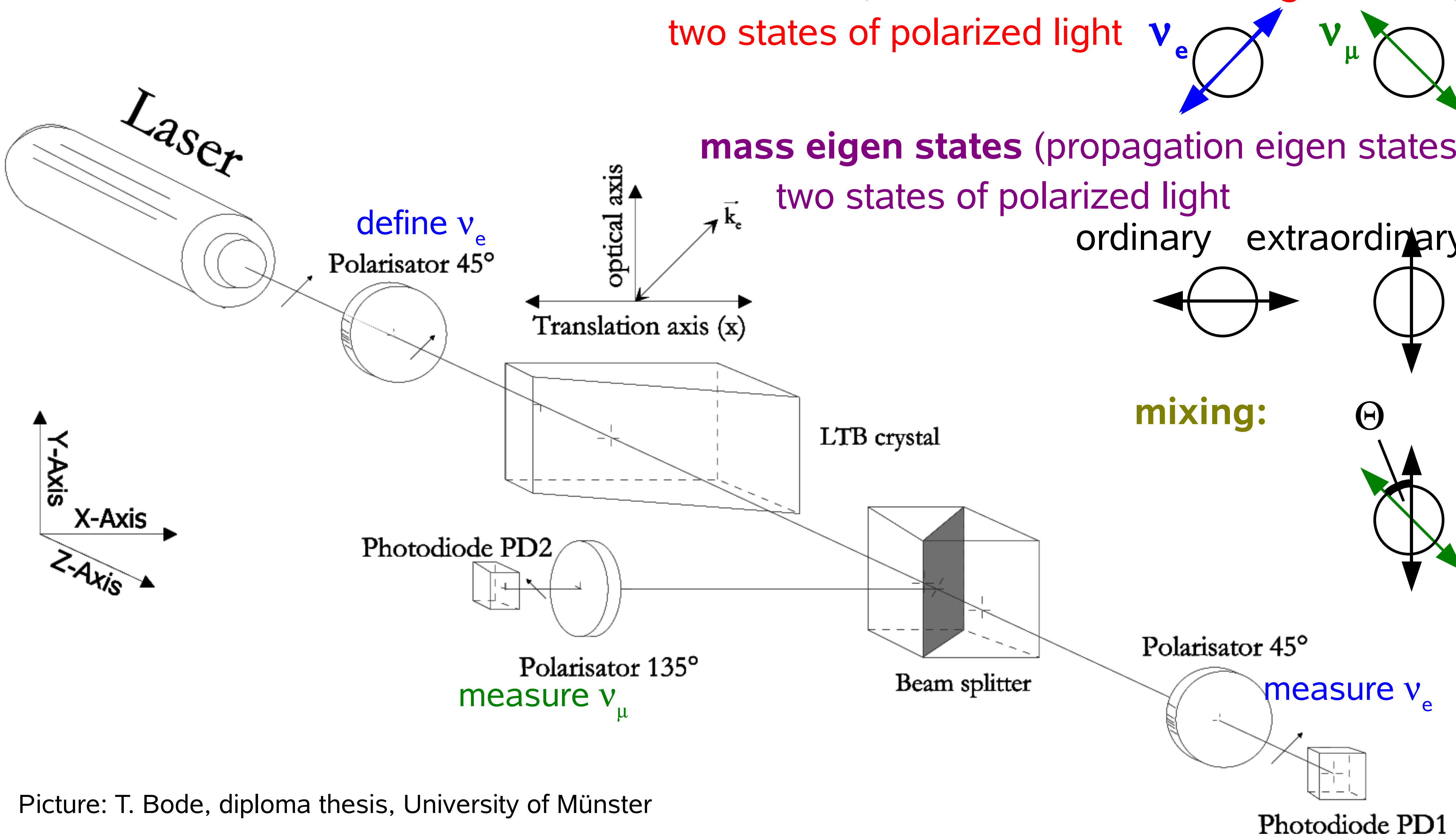
$\theta \equiv \theta$ $\quad \Delta m^2 \equiv \Delta n$ $\quad E \equiv \lambda/4\pi$

Idea of the demonstration experiment

flavour states (creation and detection eigen states):
two states of polarized light ν_e ν_μ

mass eigen states (propagation eigen states):
two states of polarized light
ordinary extraordinary

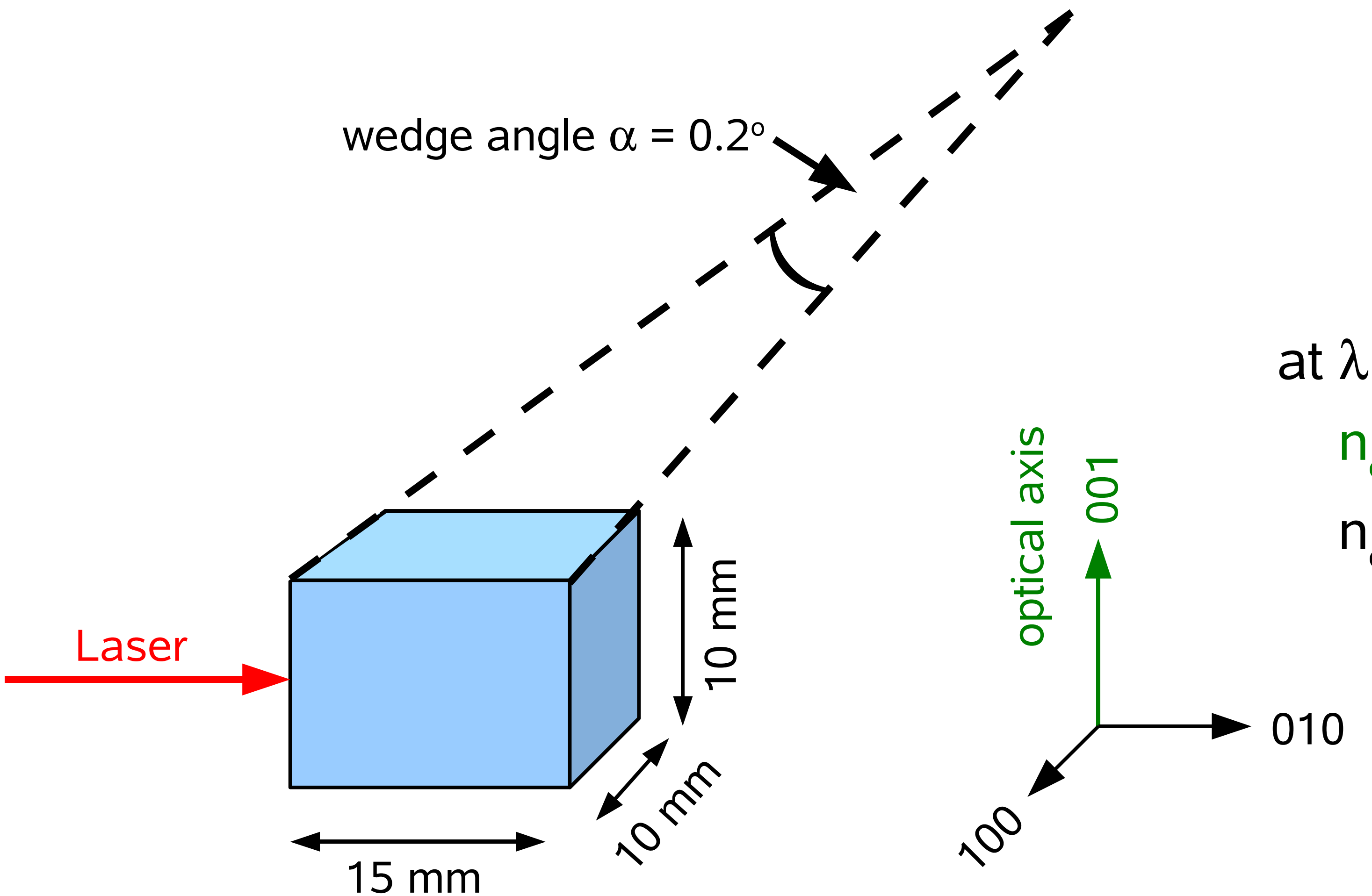
mixing: Θ



Picture: T. Bode, diploma thesis, University of Münster

Our birefringent crystals

LTB: lithium tetra borate $Li_2B_4O_7$



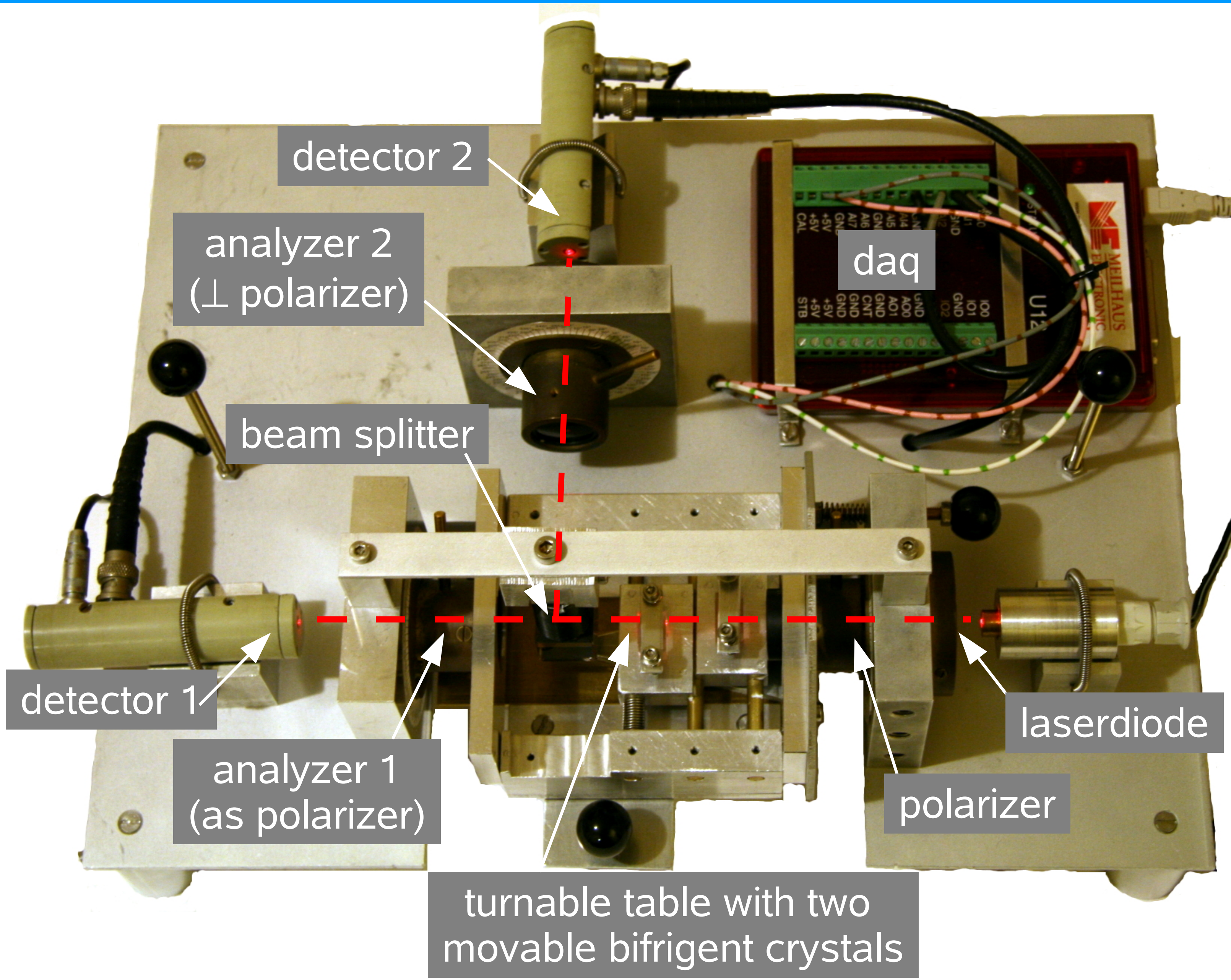
at λ = 632.8 nm:

$n_e = 1.609$

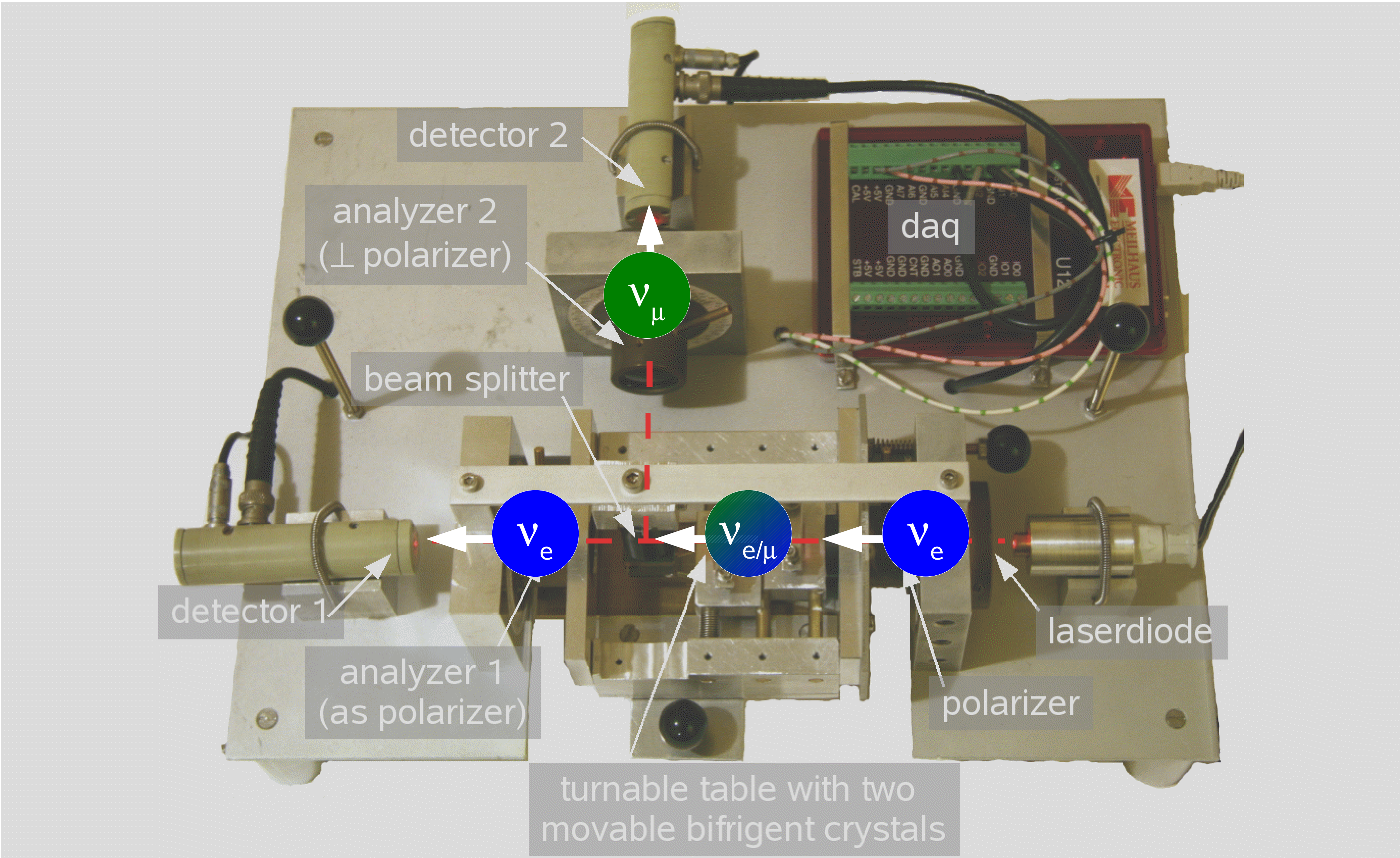
$n_o = 1.552$

from Dr. A. Peter (Research Institute for Solid State Physics and Optics Hungarian Academy of Sciences, Budapest)

Experimental setup



Experimental setup



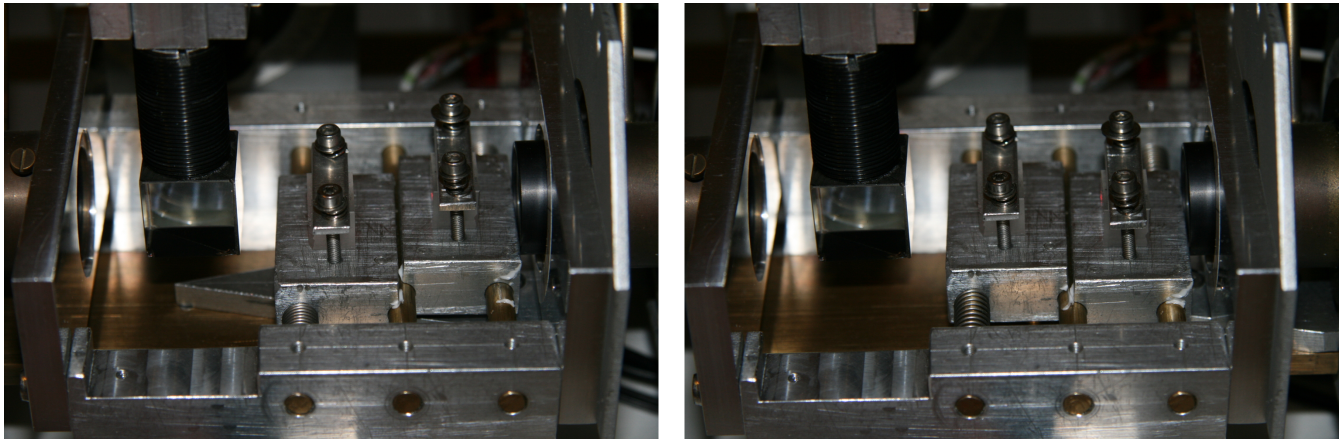
Changing the „oscillations length“ L

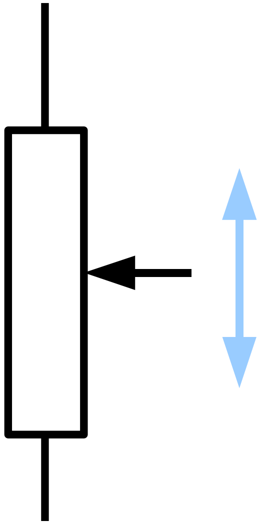
wedge angle: 0.2°

each crystal moves ≈5 mm

$\Delta L \approx 38\ \mu m$

$\lambda_{osc} = 11\ \mu m$

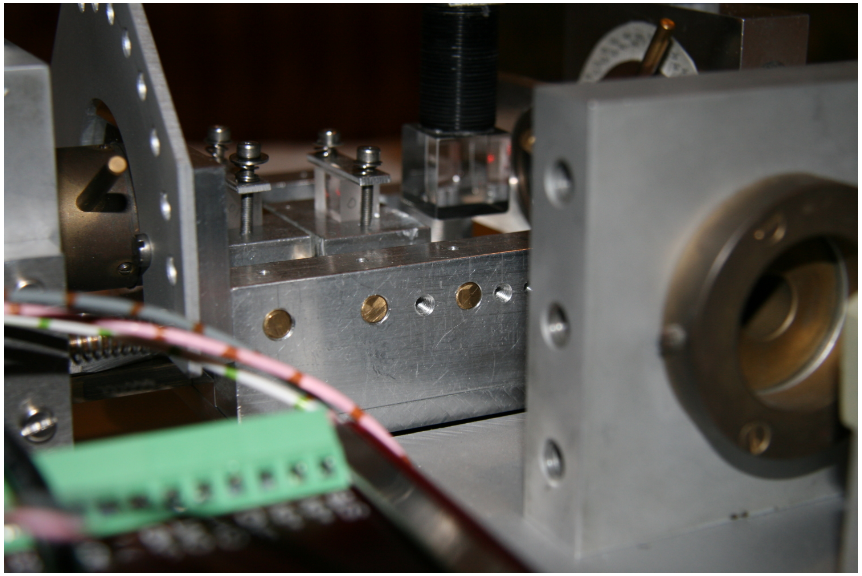


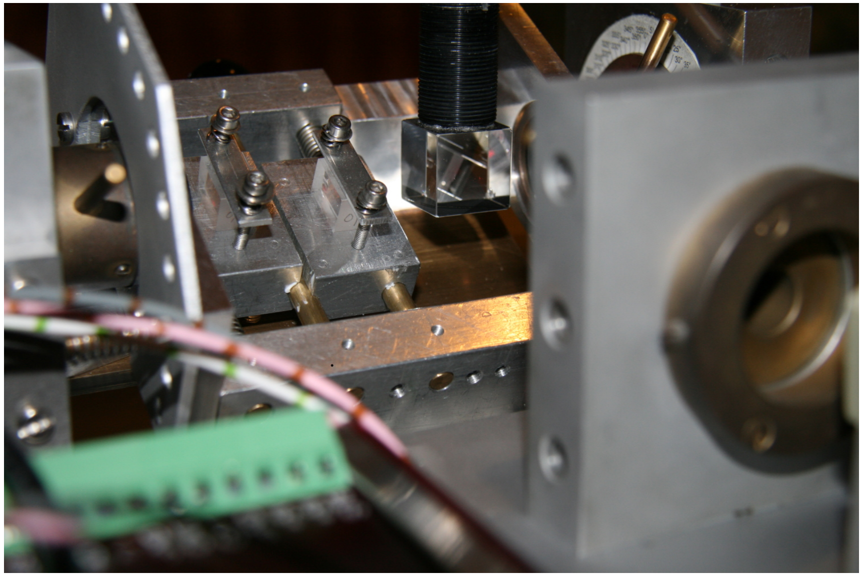


measure position by linear potentiometer

Changing the „mixing angle“ θ

by turning the crystal table with respect to the polarization directions

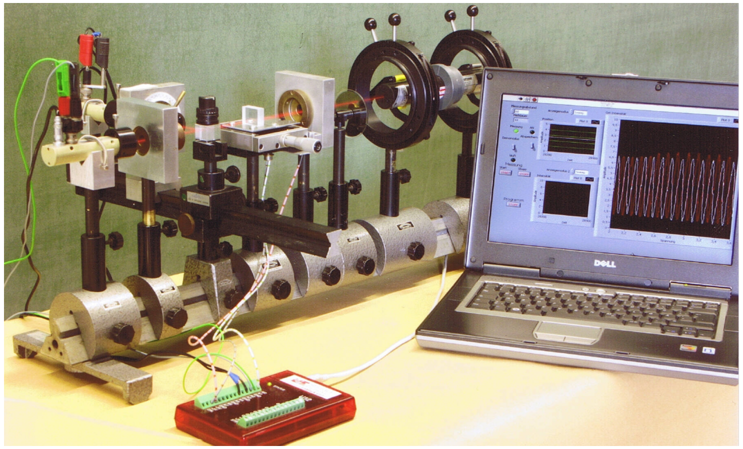




Conclusions

- Optical demonstration experiment for 2 flavour neutrino vacuum oscillations
  with both essential features: - mixing of two microscopic states
  - coherent propagation (double slit exp.)
  and with: - fully analogical calculation to ν oscillation
  - easy variation of mixing angle θ

- Experiment can be build with
  high school or lab class equipment
  (excluding bifrigent crystals: ≈100 Euro each)



- Another demonstration experiment with coupled pendula
  (with a macroscopic 2 state system):
  Neutrino pendulum by Michael Kobel et al., TU Dresden
  (http://neutrinopendel.tu-dresden.de/animation.html)