Scaling in the Neutrino Mass Matrix and the See-Saw Mechanism

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$m_\nu = m_L - m_D^T \, M_R^{-1} \, m_D$

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MANITOP
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

- A. S. Joshipura, W.R., Phys. Lett. B **678**, 276 (2009) [arXiv:0905.2126 [hep-ph]]
- A. Blum, R. N. Mohapatra, W.R., Phys. Rev. D **76**, 053003 (2007) [arXiv:0706.3801 [hep-ph]]
- R. N. Mohapatra, W.R., Phys. Lett. B **644**, 59 (2007) [arXiv:hep-ph/0608111]

What is Scaling?

- Ansatz for the neutrino mass matrix m_ν
- obtainable in many scenarios/models
- leads to inverted hierarchy
- alternative to $L_e - L_\mu - L_\tau$
- alternative to tri-bimaximal, trimaximal, μ–τ symmetry etc.
- $|U_{e3}| = 0 \leftrightarrow$ stable under RG
- predictive for low energy phenomenology <u>and</u> for see-saw parameters

Ansatz for m_ν

$$m_\nu = \begin{pmatrix} A & B & B/c \\ B & D & D/c \\ B/c & D/c & D/c^2 \end{pmatrix}$$

rank$(m_\nu) = 2 \Rightarrow$ one eigenvalue zero

$$m_\nu \begin{pmatrix} 0 \\ -1 \\ c \end{pmatrix} = 0$$

so the predictions are **inverted hierarchy** $(m_3 = 0)$ with

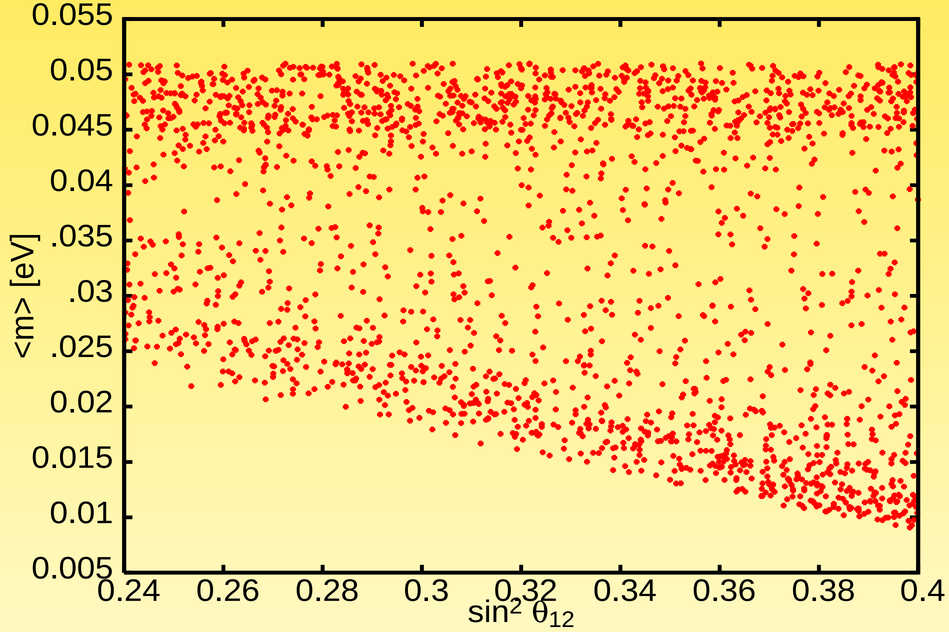
$$U_{e3} = 0 \;\; \text{and} \;\; \tan^2\theta_{23} = 1/c^2$$

in general: θ_{23} non-maximal!

Predictivity

- 5 physical parameters for θ_{12}, θ_{23}, $\Delta m^2_\odot$, $\Delta m^2_{\rm A}$, $|m_{ee}|$
- only one (Majorana) phase:

$$|m_{ee}| = \sqrt{\Delta m^2_{\rm A}}\,\sqrt{1 - \sin^2 2\theta_{12}\,\sin^2\alpha}$$



Renormalization

take RG into effect by multiplying $(m_\nu)_{\alpha\beta}$ with

$$(1+\delta_\alpha)\,(1+\delta_\beta) \quad \text{with} \quad \delta_\alpha = C\,\frac{m_\alpha^2}{16\pi\, v^2} \ln \frac{M_X}{M_Z}$$

with m_ν obeying scaling:

$$m_\nu = I_K \begin{pmatrix} A & B & B/c\,(1+\delta_\tau) \\ B & D & D/c\,(1+\delta_\tau) \\ B/c\,(1+\delta_\tau) & D/c\,(1+\delta_\tau) & D/c^2\,(1+\delta_\tau)^2 \end{pmatrix}$$

$$= \begin{pmatrix} \tilde{A} & \tilde{B} & \tilde{B}/\tilde{c} \\ \tilde{B} & \tilde{D} & \tilde{D}/\tilde{c} \\ \tilde{B}/\tilde{c} & \tilde{D}/\tilde{c} & \tilde{D}/\tilde{c}^2 \end{pmatrix} \quad \text{with} \quad \tilde{c} = c\,(1+\delta_\tau)$$

$\Rightarrow m_3 = U_{e3} = 0$ not modified!

(Grimus, Lavoura, J. Phys. G 31, 683 (2005))

Comparison with $L_e - L_\mu - L_\tau$

usual Ansatz for inverted hierarchy (Petcov, Phys. Lett. B **110**, 245 (1982))

$$m_\nu = \begin{pmatrix} 0 & A & B \\ A & 0 & 0 \\ B & 0 & 0 \end{pmatrix}$$

- also gives $m_3 = U_{e3} = 0$ and $\theta_{23} \neq \pi/4$
- but predicts $\Delta m^2_\odot = 0$ and $\theta_{12} = \pi/4$
- highly tuned perturbations required: $\Delta m^2_\odot / \Delta m^2_{\rm A} \ll \pi/4 - \theta_{12}$
- perturbations must be of order 30-40 %
- effective mass small: $|m_{ee}| \simeq \cos^2\theta_{13} \sqrt{\Delta m^2_{\rm A}} \cos 2\theta_{12}$

Charged Lepton Corrections

$$U = U_\ell^\dagger \, U_\nu$$

with U_ν from scaling:

- if U_ℓ is only 23-rotation: $c \to \tilde{c} \equiv (c \, \cos\theta_{23}^\ell - \sin\theta_{23}^\ell)/(\cos\theta_{23}^\ell + c \, \sin\theta_{23}^\ell)$
- from μ–τ symmetry in general:

$$|U_{e3}| \simeq \frac{1}{\sqrt{2}} \left| \sin\theta_{12}^\ell - \sin\theta_{13}^\ell \, e^{i\phi_1} \right|$$

$$\sin^2\theta_{23} \simeq \frac{1}{2} + \sin\theta_{23}^\ell \, \cos\phi_2 - \frac{1}{4}\left(\sin^2\theta_{12}^\ell - \sin^2\theta_{13}^\ell\right) + \frac{1}{2}\cos\phi_1 \, \sin\theta_{12}^\ell \, \sin\theta_{13}^\ell$$

rather tuned to have $|U_{e3}| = 0$ and $\theta_{23} \neq \pi/4$

A Model

Field	$D_4 \times Z_2$ quantum number
L_e	1_1^+
$e_R,\ N_e,\ \phi_1$	1_1^-
$N_\mu,\ \phi_2$	1_2^+
N_τ	1_2^-
ϕ_3	1_4^-
$\begin{pmatrix} L_\mu \\ L_\tau \end{pmatrix},\ \begin{pmatrix} \phi_4 \\ \phi_5 \end{pmatrix}$	2^+
$\begin{pmatrix} \mu_R \\ \tau_R \end{pmatrix}$	2^-

Mohapatra, W.R., Phys. Lett. B **644**, 59 (2007)

A Model

charged leptons and heavy neutrinos M_R are diagonal

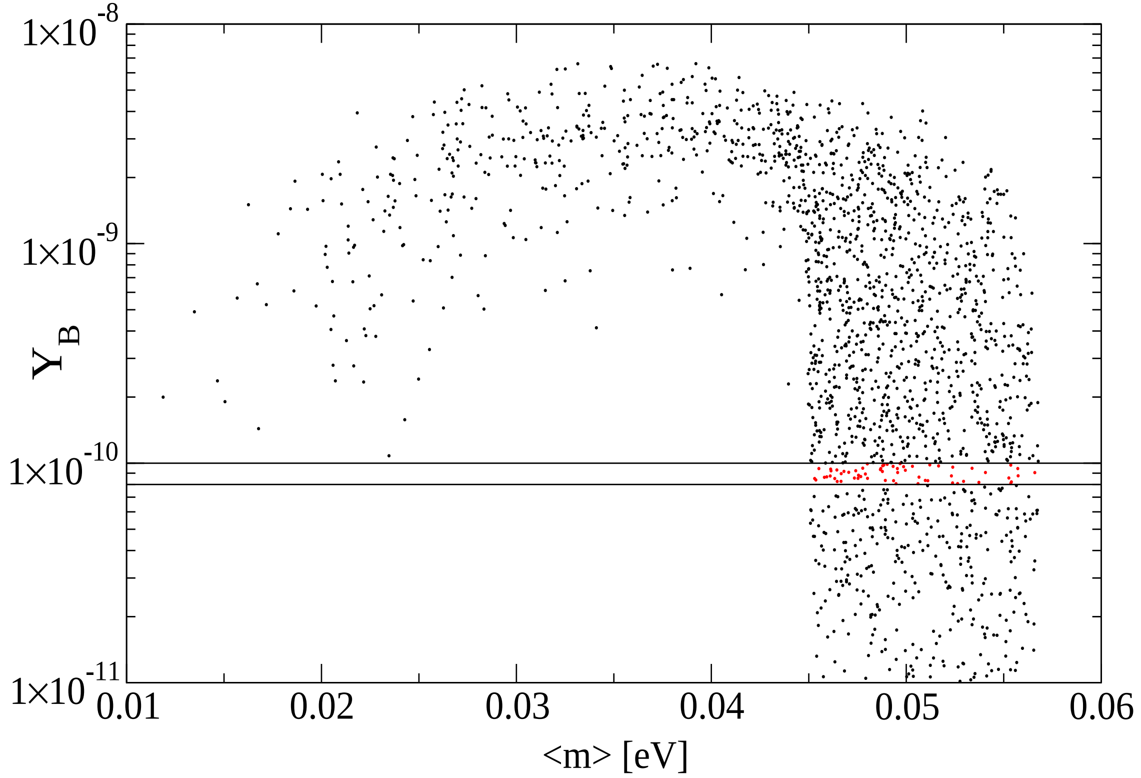
$$m_D = v \begin{pmatrix} a\, e^{i\varphi} & 0 & 0 \\ b & d & e \\ 0 & 0 & 0 \end{pmatrix}$$

and low energy mass matrix

$$m_\nu = -\frac{v^2}{M_2} \begin{pmatrix} \frac{M_2}{M_1}\, a^2\, e^{2i\varphi} + b^2 & b\, d & b\, e \\ b\, d & d^2 & d\, e \\ b\, e & d\, e & e^2 \end{pmatrix}$$

gives scaling with $\tan^2 \theta_{23} = e^2/d^2$

A Model



Blum, Mohapatra, W.R., Phys. Rev. D **76**, 053003 (2007)

Higgs Potential

$$
\begin{aligned}
V \quad = \quad & -\sum_{i=1}^{3}\mu_i^2\phi_i^\dagger\phi_i - \mu_4^2(\phi_4^\dagger\phi_4 + \phi_5^\dagger\phi_5) + \sum_{i=1}^{3}\lambda_i(\phi_i^\dagger\phi_i)^2 + \lambda_4(\phi_4^\dagger\phi_4 + \phi_5^\dagger\phi_5)^2 \\
+ \quad & \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_{13}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) \\
+ \quad & \sum_{i=1}^{3}\kappa_i(\phi_i^\dagger\phi_i)(\phi_4^\dagger\phi_4 + \phi_5^\dagger\phi_5) + \alpha_1[(\phi_1^\dagger\phi_2)^2 + h.c.] + \alpha_2|\phi_1^\dagger\phi_2|^2 \\
+ \quad & \alpha_3[(\phi_2^\dagger\phi_3)^2 + h.c.] + \alpha_4|\phi_2^\dagger\phi_3|^2 + \alpha_5(\phi_4^\dagger\phi_5 + \phi_5^\dagger\phi_4)^2 + \alpha_6(\phi_4^\dagger\phi_5 - \phi_5^\dagger\phi_4)^2 \\
+ \quad & \alpha_7(\phi_4^\dagger\phi_4 - \phi_5^\dagger\phi_5)^2 + \alpha_8(\phi_4^\dagger\phi_4 - \phi_5^\dagger\phi_5)(\phi_1^\dagger\phi_3 + h.c.) \\
+ \quad & \alpha_9[(\phi_2^\dagger\phi_4)^2 + (\phi_2^\dagger\phi_5)^2 + h.c.] + \alpha_{10}(|\phi_2^\dagger\phi_4|^2 + |\phi_2^\dagger\phi_5|^2) \\
+ \quad & \alpha_{11}[(\phi_1^\dagger\phi_4)^2 + (\phi_1^\dagger\phi_5)^2 + h.c.] + \alpha_{12}(|\phi_1^\dagger\phi_4|^2 + |\phi_1^\dagger\phi_5|^2) \\
+ \quad & \alpha_{13}[(\phi_3^\dagger\phi_4)^2 + (\phi_3^\dagger\phi_5)^2 + h.c.] + \alpha_{14}(|\phi_3^\dagger\phi_4|^2 + |\phi_3^\dagger\phi_5|^2) \\
+ \quad & \alpha_{15}[(\phi_1^\dagger\phi_4)(\phi_3^\dagger\phi_4) - (\phi_1^\dagger\phi_5)(\phi_3^\dagger\phi_5) + h.c.] \\
+ \quad & \alpha_{16}[(\phi_1^\dagger\phi_4)(\phi_4^\dagger\phi_3) - (\phi_1^\dagger\phi_5)(\phi_5^\dagger\phi_3) + h.c.]
\end{aligned}
$$

Higgs Potential

- 5 Higgs doublets
  - 5 real scalars
  - 4 pseudoscalars
  - 4 charged scalars
- potential can be minimized by choozing

$$\langle \Phi_{1,2,4,5} \rangle = \begin{pmatrix} 0 \\ v/\sqrt{5} \end{pmatrix} \text{ and } \langle \Phi_3 \rangle = \begin{pmatrix} 0 \\ -v/\sqrt{5} \end{pmatrix}$$

with masses of scalars above experimental limits

m_D	Leptogenesis	$\tan^2\theta_{23}$
$\begin{pmatrix} 0 & 0 & 0 \\ a_2 & 0 & 0 \\ a_3\, e^{i\alpha_3} & b_3 & c_3 \end{pmatrix}$	$\mathcal{I}^e_{23} = a_2^2\, a_3^2\, \sin 2\alpha_3$	$\frac{c_3^2}{b_3^2}$
$\begin{pmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ a_3\, e^{i\alpha_3} & 0 & 0 \end{pmatrix}$	$\mathcal{I}^e_{23} = a_2^2\, a_3^2\, \sin 2\alpha_3$	$\frac{c_2^2}{b_2^2}$
$\begin{pmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ a_3\, e^{i\alpha_3} & b_3 & c_3 \end{pmatrix}$	$\mathcal{I}^e_{13} = a_1^2\, a_3^2\, \sin 2\alpha_3$	$\frac{c_3^2}{b_3^2}$
$\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ a_3\, e^{i\alpha_3} & 0 & 0 \end{pmatrix}$	$\mathcal{I}^e_{13} = a_1^2\, a_3^2\, \sin 2\alpha_3$	$\frac{c_1^2}{b_1^2}$
$\begin{pmatrix} a_1 & 0 & 0 \\ a_2\, e^{i\alpha_2} & b_2 & c_2 \\ 0 & 0 & 0 \end{pmatrix}$	$\mathcal{I}^e_{12} = a_1^2\, a_2^2\, \sin 2\alpha_2$	$\frac{c_2^2}{b_2^2}$
$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2\, e^{i\alpha_2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\mathcal{I}^e_{12} = a_1^2\, a_2^2\, \sin 2\alpha_2$	$\frac{c_1^2}{b_1^2}$

Goswami, Khan, W.R., Phys. Lett. B **680**, 255 (2009)

Scaling and See-Saw

$$m_\nu = -m_D^T \, M_R^{-1} \, m_D$$

usually: $m_D = i\,\sqrt{M_R}\,R\,\sqrt{m_\nu^{\rm diag}}\,U^\dagger$

infinite number of m_D are allowed and hence no predictions for LFV or leptogenesis possible

In contrast, scaling constraints m_D to have the form:

$$m_D = \left(\begin{array}{ccc} a_1 & b & b/c \\ a_2 & d & d/c \\ a_3 & e & e/c \end{array} \right)$$

no matter what M_R is!!

Proof

$$m_\nu \, |\psi\rangle = 0 \text{ where } |\psi\rangle = \begin{pmatrix} 0 \\ -1 \\ c \end{pmatrix}$$

$\det(m_\nu) = 0$ and therefore (M_R can't be singular)

$$\det(m_D) = 0 \quad \Longrightarrow \quad m_D \, |\chi\rangle = 0$$

for one of its eigenvectors $|\chi\rangle$. With $m_\nu = -m_D^T \, M_R^{-1} \, m_D$ it follows

$$m_\nu \, |\chi\rangle = 0$$

Hence, $|\chi\rangle$ is proportional to $|\psi\rangle$, which means

$$m_D \, |\psi\rangle = 0$$

solution for m_D is as given above

Scaling and Z_2

Suppose Z_2 generated by $\nu_L \to S(\theta)\,\nu_L$

$$S(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

acts on low energy mass matrix from

$$\mathcal{L} = \frac{1}{2}\,\overline{\nu_L^c}\,m_\nu\,\nu_L$$

this implies $\theta_{23} = \theta$ and $U_{e3} = 0$ “generalized μ–τ symmetry”

(Grimus *et al.*, Nucl. Phys. B **713**, 152 (2005))

$\Rightarrow$ scaling can be special case of this $S(\theta)$ when $\cos 2\theta = \frac{c^2-1}{1+c^2}$

Scaling and Z_2

Now assume see-saw

$$\mathcal{L} = \frac{1}{2}\,\overline{N_R}\,M_R\,N_R^c + \overline{N_R}\,m_D\,\nu_L$$

and Z_2 with $\nu_L \to S(\theta)\,\nu_L$ implies that $m_D\,S(\theta) = m_D$ and thus

$$m_D = \begin{pmatrix} a_1 & b & b/c \\ a_2 & d & d/c \\ a_3 & e & e/c \end{pmatrix}$$

$\Rightarrow$ scaling in m_ν!!

Scaling and Lepton Flavor Violation

SUSY see-saw

$$\text{BR}(\ell_i \to \ell_j\,\gamma) \propto \left|(\tilde{m}_D^\dagger\, L\, \tilde{m}_D)_{ij}\right|^2 \quad \text{with} \quad L_{ij} = \delta_{ij}\,\log M_i/M_X$$

Note: basis in which M_R diagonal: $\tilde{m}_D = V_R^T\, m_D$

If m_D obeys scaling, then

$$\frac{\left|(\tilde{m}_D^\dagger\, L\, \tilde{m}_D)_{12}\right|^2}{\left|(\tilde{m}_D^\dagger\, L\, \tilde{m}_D)_{13}\right|^2} = c^2 = \cot^2\theta_{23}$$

and $\tau \to e\gamma$ too rare to be observable

Lepton Flavor Violation

Note: if μ–τ symmetric see-saw

$$m_D = \begin{pmatrix} a_1 & d_1 & d_1 \\ a_2 & d_2 & d_3 \\ a_2 & d_3 & d_2 \end{pmatrix} \text{ and } M_R = \begin{pmatrix} W & X & X \\ X & Y & Z \\ X & Y & Y \end{pmatrix}$$

then LFV prediction is

$$\frac{\left|(\tilde{m}_D^\dagger \, \tilde{m}_D)_{12}\right|^2}{\left|(\tilde{m}_D^\dagger \, \tilde{m}_D)_{13}\right|^2} = 1$$

and logarithmic corrections due to L

Scaling and Leptogenesis

if μ–τ symmetric see-saw

$$m_D = \begin{pmatrix} a_1 & d_1 & d_1 \\ a_2 & d_2 & d_3 \\ a_2 & d_3 & d_2 \end{pmatrix} \text{ and } M_R = \begin{pmatrix} W & X & X \\ X & Y & Z \\ X & Y & Y \end{pmatrix}$$

then

- 2 RH neutrinos: no leptogenesis, neither flavored nor unflavored
- 3 RH neutrinos: unflavored $Y_B \propto \Delta m^2_\odot$

Mohapatra, Nasri, Phys. Rev. D **71**, 033001 (2005); Mohapatra, Nasri, Yu, Phys. Lett. B **615**, 231 (2005)

Scaling and Leptogenesis

2 RH neutrinos and scaling

$$m_D = \begin{pmatrix} A_1 & B & B/c \\ A_2 & D & D/c \end{pmatrix} \text{ and } M_R = V_R^* \, D_R \, V_R^\dagger$$

gives decay asymmetry

$$\boxed{\varepsilon_1 = -\frac{3}{16\pi \, v^2} \frac{M_1}{\tilde{m}_1} \, r \, \frac{\Delta m_\odot^2}{|1 + r \, e^{i\rho}|^2} \, \sin\rho}$$

just as for μ–τ symmetry and 3 RH neutrinos!

wash-out parameter:

$$\tilde{m}_1 = \frac{m_1 + r \, m_2}{|1 + r \, e^{i\rho}|} = \mathcal{O}\left(\sqrt{\Delta m_{\rm A}^2}\right) \Rightarrow \text{ strong wash-out}$$

(flavored leptogenesis: ε_i^α determined by $\Delta m_{\rm A}^2$)

Scaling and Leptogenesis

add additional Z_2

$$N_R \to S(\theta)\, N_R$$

which implies

$$S(\theta)\, m_D = m_D \text{ and } S(\theta)\, M_R\, S(\theta) = M_R$$

This $Z_{2L} \times Z_{2R}$ gives

$$m_D = \begin{pmatrix} A_1 & B & B\, s_{23}/c_{23} \\ A_2\, c_{23} & D\, c_{23} & D\, s_{23} \\ A_2\, s_{23} & D\, s_{23} & D\, s_{23}^2/c_{23} \end{pmatrix} \text{ and } M_R = \begin{pmatrix} A & B & B/c \\ B & F\,(c - 1/c) + G & F \\ B/c & F & G \end{pmatrix}$$

One heavy RH neutrino decouples and formulae are as above

Scaling and Non-Standard Neutrino Physics

- Dirac neutrinos:

$$m_\nu = \begin{pmatrix} a_1 & b & b/c \\ a_2 & d & d/c \\ a_3 & e & e/c \end{pmatrix}$$

- sterile neutrinos:

$$m_\nu = \begin{pmatrix} a_1 & b_1 & b_1/c & d_1 & e_1 \\ b_1 & b_2 & b_2/c & d_2 & e_2 \\ b_1/c & b_2/c & b_2/c^2 & d_2/c & e_2/c \\ d_1 & d_2 & d_2/c & d_2/c^2 & e_2/c^2 \\ e_1 & e_2 & e_2/c & e_2/c^2 & e_5 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ \frac{-1}{\sqrt{1+c^2}} \\ \frac{c}{\sqrt{1+c^2}} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} U_{e3} \\ U_{\mu 3} \\ U_{\tau 3} \\ U_{s_1 3} \\ U_{s_2 3} \end{pmatrix}$$

Summary

$$m_\nu = \begin{pmatrix} A & B & B/c \\ B & D & D/c \\ B/c & D/c & D/c^2 \end{pmatrix}$$

- highly predictive and reconstructible scenario
- inverted hierarchy and $U_{e3} = 0$, stable under RG
- obtainable in flavor models, see-saw texture analyses,...
- determines m_D irrespective of M_R
- interesting differences to μ–τ symmetry


Thanks!!

Scaling and Leptogenesis

$$\tilde{m}_D \tilde{m}_D^\dagger = \begin{pmatrix} Z Z^\dagger & 0 \\ 0 & 0 \end{pmatrix}$$

and

$$m_1\, e^{i\alpha_1} = -\frac{Z_{11}^2}{M_1}\left(1 + \frac{Z_{21}^2}{Z_{11}^2}\,\frac{M_1}{M_2}\right) \equiv -\frac{Z_{11}^2}{M_1}\,(1 + r\, e^{i\rho})$$

$$m_2\, e^{i\alpha_2} = -\frac{Z_{22}^2}{M_2}\left(1 + \frac{Z_{21}^2}{Z_{11}^2}\,\frac{M_1}{M_2}\right) \equiv -\frac{Z_{22}^2}{M_2}\,(1 + r\, e^{i\rho})\,,$$

$$\frac{Z_{12}}{Z_{22}} = -\frac{Z_{21}}{Z_{11}}\,\frac{M_1}{M_2} = -\sqrt{r}\,\sqrt{\frac{M_1}{M_2}}\, e^{i\rho/2}$$

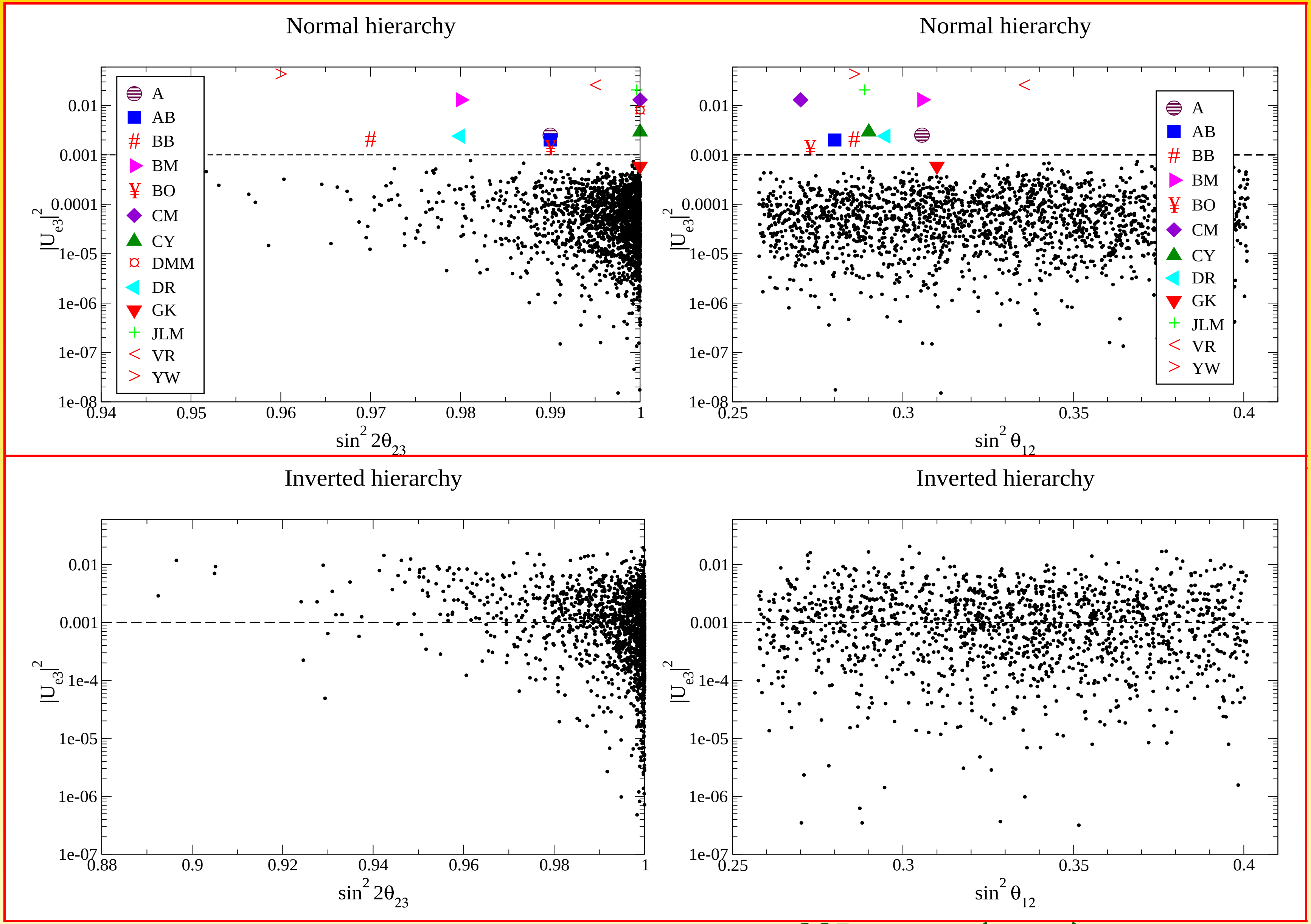
Scaling and Leptogenesis

The individual flavored decay asymmetries read

$$\varepsilon_1^\mu = c_{23}^2\,(\varepsilon_1 - \varepsilon_1^e)$$
$$\varepsilon_1^\tau = s_{23}^2\,(\varepsilon_1 - \varepsilon_1^e)$$

and

$$\begin{aligned}\varepsilon_1^e \quad = \quad & -\frac{3\,M_1}{16\pi\,v^2\,\tilde{m}_1\,|1+r\,e^{i\rho}|^2}\left(r\left(m_2^2\,s_{12}^2 - m_1^2\,c_{12}^2\right)\sin\rho + \right.\\ & c_{12}\,s_{12}\sqrt{m_1\,m_2\,r}\,((m_1 - m_2\,r)\,\sin(\alpha_1-\alpha_2-4\beta-\rho)/2 + \\ & \left.(m_1\,r - m_2)\,\sin(\alpha_1-\alpha_2-4\beta+\rho)/2)\right)\end{aligned}$$



Albright, W.R., Phys. Lett. B **665**, 378 (2008)

Comparison with μ–τ Symmetry

$$m_\nu^{\mu-\tau} = \begin{pmatrix} A & B & B \\ B & D & E \\ B & E & D \end{pmatrix}$$

obtained from Z_2 invariance $\nu_L \to S_{\mu\tau}\,\nu_L$ leading to $S_{\mu\tau}^{-1}\,m_\nu\,S_{\mu\tau} = m_\nu$ with

$$S_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(cf. with scaling and $c = 1$)

Broken μ–τ symmetry gives in general $\mathcal{O}(|\theta_{23} - \pi/4|) = \mathcal{O}(|U_{e3}|)$