Neutrino mixing and dark energy

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- Quantum Field Theory of neutrino mixing and oscillations;
- Particle mixing and dark energy.

Motivations

• Dark energy problem

Measurements of the cosmic microwave background (CMBR), gravitational lensing, observations of type Ia supernovae suggested that the expansion of the universe is accelerating.

Possible explanation: exists an hypothetical form of energy which permeates all of space and has strong negative pressure: the dark energy.

Proposed forms for dark energy: cosmological constant, quintessence, extended theories of gravity, braneworld, etc...

Fermion mixing in QFT

Mixing relations for two Dirac fields

$$\begin{aligned}\nu_e(x) &= \nu_1(x)\cos\theta + \nu_2(x)\sin\theta \\ \nu_\mu(x) &= -\nu_1(x)\sin\theta + \nu_2(x)\cos\theta\end{aligned}$$

ν_i $(i = 1, 2)$ **are free field operators with definite masses**

Mixing relations can be written as*

$$\begin{aligned}\nu_e^\alpha(x) &= G_\theta^{-1}(t)\,\nu_1^\alpha(x)\,G_\theta(t) \\ \nu_\mu^\alpha(x) &= G_\theta^{-1}(t)\,\nu_2^\alpha(x)\,G_\theta(t)\end{aligned}$$

with generator given by:

$$G_\theta(t) = \exp\left[\theta \int d^3\mathbf{x}\,\left(\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x)\right)\right]$$

*M.Blasone and G.Vitiello, Annals Phys. (1995)

- $G_\theta(t)$ **is an unitary operator:** $G_\theta^{-1}(t) = G_{-\theta}(t) = G_\theta^\dagger(t)$ **preserving the canonical anticommutation relations**

- $G_\theta^{-1}(t)$ **maps** $\mathcal{H}_{1,2}$ **to** $\mathcal{H}_{e,\mu}$**:** $\quad G_\theta(t) : \mathcal{H}_{1,2} \to \mathcal{H}_{e,\mu}$.

The vacuum $|0\rangle_{1,2}$ is not invariant under the action of $G_\theta(t)$, at finite volume:

$$|0(t)\rangle_{e,\mu} \equiv G_\theta^{-1}(t)\; |0\rangle_{1,2}$$

- **Orthogonality between** $|0\rangle_{1,2}$ **and** $|0(t)\rangle_{e,\mu}$ **for** $V \to \infty$

- **Mass and flavor representations are unitary inequivalent for** $V \to \infty$.

Condensate structure of $|0\rangle_{e,\mu}$ (use $\epsilon^r = (-1)^r$)

$$
\begin{aligned}
|0\rangle_{e,\mu} &= \prod_{\mathbf{k},r} [(1 - \sin^2\theta\, |V_{\mathbf{k}}|^2) - \epsilon^r \sin\theta\, \cos\theta\, |V_{\mathbf{k}}|\, (\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},2} + \alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},1}) \\
&+ \epsilon^r \sin^2\theta\, |V_{\mathbf{k}}||U_{\mathbf{k}}|\, (\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},1} - \alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},2}) \\
&+ \sin^2\theta\, |V_{\mathbf{k}}|^2\, \alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},2}\alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},1}]|0\rangle_{1,2}
\end{aligned}
$$

- 4 kinds of particle-antiparticle pairs with zero momentum and spin.

- Time dependence:

$$
|0\rangle_{e,\mu} \equiv |0(0)\rangle_{e,\mu} = e^{-iHt}|0(t)\rangle_{e,\mu}
$$

Structure of the annihilation operators for $|0(t)\rangle_{e,\mu}$:

$$\alpha^r_{\mathbf{k},\nu_e}(t) \;=\; \cos\theta\,\alpha^r_{\mathbf{k},1}(t) + \sin\theta\left(|U_\mathbf{k}|\,\alpha^r_{\mathbf{k},2}(t) + \epsilon^r|V_\mathbf{k}|\,\beta^{r\dagger}_{-\mathbf{k},2}(t)\right)$$

etc.. with $U_\mathbf{k}\,,\,V_\mathbf{k}$ Bogoliubov coefficients:

$$|U_\mathbf{k}| = u^{r\dagger}_{\mathbf{k},2}u^r_{\mathbf{k},1}\,, \qquad |V_\mathbf{k}| = \epsilon^r\,u^{r\dagger}_{\mathbf{k},1}v^r_{-\mathbf{k},2}\,, \qquad |U_\mathbf{k}|^2 + |V_\mathbf{k}|^2 = 1$$

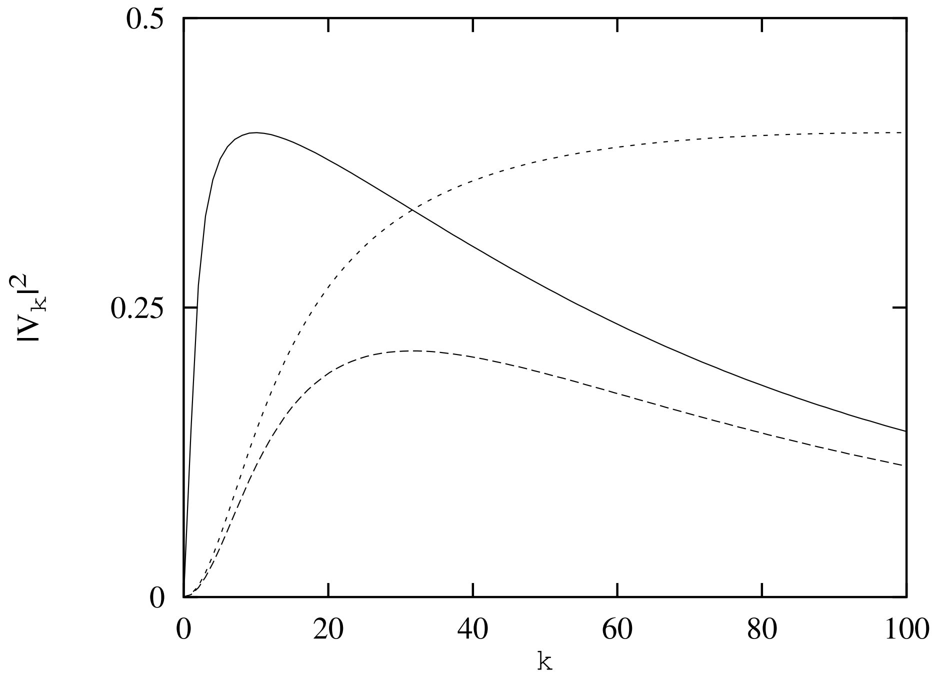
$$|U_\mathbf{k}| = \left(\frac{\omega_{k,1}+m_1}{2\omega_{k,1}}\right)^{\frac{1}{2}}\left(\frac{\omega_{k,2}+m_2}{2\omega_{k,2}}\right)^{\frac{1}{2}}\left(1+\frac{|\mathbf{k}|^2}{(\omega_{k,1}+m_1)(\omega_{k,2}+m_2)}\right)$$

$$|V_\mathbf{k}| = \left(\frac{\omega_{k,1}+m_1}{2\omega_{k,1}}\right)^{\frac{1}{2}}\left(\frac{\omega_{k,2}+m_2}{2\omega_{k,2}}\right)^{\frac{1}{2}}\left(\frac{|\mathbf{k}|}{(\omega_{k,2}+m_2)}-\frac{|\mathbf{k}|}{(\omega_{k,1}+m_1)}\right)$$

Mixing transf. = Rotation $(\cos\theta\,,\,\sin\theta)$ + Bogoliubov transformation

$$\alpha_{\nu_e}(t)|0(t)\rangle_{e,\mu} = G^{-1}_\theta(t)\alpha_1(t)G_\theta(t)\;G^{-1}_\theta(t)|0\rangle_{1,2=0}$$

Condensation density for mixed fermions



Solid line: $m_1 = 1$, $m_2 = 100$; **Long dashed line:** $m_1 = 10$, $m_2 = 100$; **Short dashed line:** $m_1 = 10$, $m_2 = 1000$.

$$_{e,\mu}\langle 0(t)|\alpha^{r\dagger}_{\mathbf{k},i}\alpha^{r}_{\mathbf{k},i}|0(t)\rangle_{e,\mu} =_{e,\mu} \langle 0(t)|\beta^{r\dagger}_{\mathbf{k},i}\beta^{r}_{\mathbf{k},i}|0(t)\rangle_{e,\mu} = \sin^2\theta\ |V_{\mathbf{k}}|^2, \qquad i = 1, 2.$$

- $V_{\mathbf{k}} = 0$ **when** $m_1 = m_2$ **and/or** $\theta = 0$.
- $|V_{\mathbf{k}}|^2 \simeq \frac{(m_2-m_1)^2}{4k^2}$ **for** $k \gg \sqrt{m_1 m_2}$.
- **Max. at** $k = \sqrt{m_1 m_2}$ **with** $V_{max} \to \frac{1}{2}$ **for** $\frac{(m_2-m_1)^2}{m_1 m_2} \to \infty$.

Neutrino oscillation formulae in QFT*:

$$\mathcal{Q}^{\mathbf{k}}_{\nu_e\rightarrow\nu_e}(t) \;=\; 1 - |U_{\mathbf{k}}|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}\,t\right) - |V_{\mathbf{k}}|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}\,t\right)$$

$$\mathcal{Q}^{\mathbf{k}}_{\nu_e\rightarrow\nu_\mu}(t) \;=\; |U_{\mathbf{k}}|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}\,t\right) + |V_{\mathbf{k}}|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}\,t\right)$$

- **Correction to amplitudes + new oscillating term**

- For $k \gg \sqrt{m_1 m_2}$ we have: $|V_{\mathbf{k}}|^2 \rightarrow 0$ and $|U_{\mathbf{k}}|^2 \rightarrow 1$ the Pontecorvo formulae are reobtained in the relativistic limit.

- Similar results for three flavor neutrino fields and for boson fields†

*M.Blasone, P.Henning and G.Vitiello, Phys. Lett. B (1999)

†M.Blasone, A.Capolupo, O.Romei and G.Vitiello, Phys. Rev. D (2001)
M.Blasone, A.Capolupo and G.Vitiello, Phys. Rev. D (2002)
A.Capolupo, C.R.Ji, Y.Mischenko and G.Vitiello, Phys. Lett.B (2004)

Particle mixing contribution to the dark energy*

Experimental data support the picture that some form of *dark energy*, evolving from early epochs, induces the today observed acceleration of the universe.

There are many proposals to achieve cosmological models justifying such a dark component.

Our result is: The non–perturbative vacuum structure associated with particle mixing leads to a non–zero contribution to the dark energy.

*A.Capolupo, S.Capozziello, G.Vitiello, PLA (2007)
A.Capolupo, S.Capozziello, G.Vitiello, PLA (2009).

We consider the Minkowski metric.

Lorentz invariance of $|0\rangle$ implies that $|0\rangle$ is the zero eigenvalue eigenstate of the normal ordered energy, momentum and angular momentum operators. Therefore $\mathcal{T}^{vac}_{\mu\nu} = \langle 0| : \mathcal{T}_{\mu\nu} : |0\rangle = 0$.

the (0,0) component of $\mathcal{T}_{\mu\nu}(x)$ is

$$: \mathcal{T}_{00}(x) := \frac{i}{2} : \left(\bar{\Psi}_m(x) \gamma_0 \overleftrightarrow{\partial}_0 \Psi_m(x) \right) :$$

In terms of the annihilation and creation operators of fields ν_1 and ν_2, the energy momentum tensor $: T_{00} := \int d^3x : \mathcal{T}_{00}(x) :$ is given by

$$: T^{(i)}_{00} := \sum_r \int d^3\mathbf{k}\, \omega_{k,i} \left(\alpha^{r\dagger}_{\mathbf{k},i} \alpha^r_{\mathbf{k},i} + \beta^{r\dagger}_{-\mathbf{k},i} \beta^r_{-\mathbf{k},i} \right),$$

we note that $T^{(i)}_{00}$ is time independent, moreover

$${}_{e,\mu}\langle 0| : T^{(i)}_{00} : |0\rangle_{e,\mu} = {}_{e,\mu}\langle 0(t)| : T^{(i)}_{00} : |0(t)\rangle_{e,\mu}, \qquad \forall t.$$

• **Early universe epochs: Lorentz invariance is broken. Neutrino mixing contributes**[†] **to the vacuum energy density** ρ_{vac}^{mix}**:**

$$\rho_{vac}^{mix} = \frac{1}{V}\eta_{00\;e,\mu}\langle 0|\sum_i : T_{00}^{(i)} : |0\rangle_{e,\mu} \Rightarrow$$

$$\Rightarrow \rho_{vac}^{mix} = \sum_{i,r}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\omega_{k,i}\left({}_{e,\mu}\langle 0|\alpha_{\mathbf{k},i}^{r\dagger}\alpha_{\mathbf{k},i}^{r}|0\rangle_{e,\mu} + {}_{e,\mu}\langle 0|\beta_{\mathbf{k},i}^{r\dagger}\beta_{\mathbf{k},i}^{r}|0\rangle_{e,\mu}\right).$$

Since

$${}_{e,\mu}\langle 0|\alpha_{\mathbf{k},i}^{r\dagger}\alpha_{\mathbf{k},i}^{r}|0\rangle_{e,\mu} = {}_{e,\mu}\langle 0|\beta_{\mathbf{k},i}^{r\dagger}\beta_{\mathbf{k},i}^{r}|0\rangle_{e,\mu} = \sin^2\theta|V_{\mathbf{k}}|^2 \Rightarrow$$

$$\Rightarrow \rho_{vac}^{mix} = 4\sin^2\theta\int\frac{d^3\mathbf{k}}{(2\pi)^3}\left(\omega_{k,1}+\omega_{k,2}\right)|V_{\mathbf{k}}|^2,$$

$$\rho_{vac}^{mix} = \frac{2}{\pi}\sin^2\theta\int_0^K dk\,k^2(\omega_{k,1}+\omega_{k,2})|V_{\mathbf{k}}|^2, \quad \mathrm{K\ is\ the\ cut-off}$$

[†]**M.Blasone, A.Capolupo, S.Capozziello, S.Carloni G.Vitiello, PLA (2004).**

- In a similar way, the contribution p_{vac}^{mix} of the neutrino mixing to the vacuum pressure is:

$$p_{vac}^{mix} \quad = \frac{1}{V} \eta_{jj} \; {}_{e,\mu}\langle 0| \sum_i : T_{jj}^{(i)}(0) : |0\rangle_{e,\mu}$$

(no summation on the index j is intended). Being

$$: T_{(i)}^{jj} := \sum_r \int d^3\mathbf{k} \, \frac{k^j k^j}{\omega_{k,i}} \left(\alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^{r} + \beta_{-\mathbf{k},i}^{r\dagger} \beta_{-\mathbf{k},i}^{r} \right),$$

in the case of the isotropy of the momenta: $T^{11} = T^{22} = T^{33}$

$$p_{vac}^{mix} = \frac{2}{3\,\pi} \sin^2\theta \int_0^K dk\, k^4 \left[\frac{1}{\omega_{k,1}} + \frac{1}{\omega_{k,2}} \right] |V_{\mathbf{k}}|^2$$

- $\rho_{vac}^{mix} \neq -p_{vac}^{mix}$

- $w = p_{vac}^{mix} / \rho_{vac}^{mix} \simeq 1/3$ when the cut-off is chosen to be $K \gg m_1, m_2$.

- no mixing: $|V_{\mathbf{k}}|^2 = 0 \Rightarrow \rho_{vac}^{mix} = p_{vac}^{mix} = 0$

ρ_{vac}^{mix} is time-independent since, for simplicity, we consider Minkowski metric. When the curved background metric is considered, ρ_{vac}^{mix} is time-dependent, but the essence of the result is the same.[‡]

At the present epoch, the breaking of Lorentz invariance is negligible $\Rightarrow$ ρ_{vac}^{mix} comes from space-time independent condensate contributions (i.e. contributions carrying non-vanishing $\partial_\mu \sim k_\mu = (\omega_k, k_j)$ are missing).

$\Rightarrow$ the stress energy tensor of the vacuum condensate is

$$T_{\mu\nu}^{cond} = V^{cond}\, \eta_{\mu\nu}$$

that is

$$_{e,\mu}\langle 0| : T_{\mu\nu} : |0\rangle_{e,\mu} = \eta_{\mu\nu} \sum_i m_i \int \frac{d^3x}{(2\pi)^3}\, _{e,\mu}\langle 0| : \bar{\nu}_i(x)\nu_i(x) : |0\rangle_{e,\mu} = \eta_{\mu\nu}\, \rho_\Lambda^{mix}.$$

[‡]A.Capolupo, S.Capozziello, G.Vitiello, work in progress.

Since

$$\eta_{\mu\nu} = diag(1, -1, -1, -1)$$

and, in a homogeneous and isotropic universe, $T_{\mu\nu}$ is

$$T_{\mu\nu} = diag(\rho\,, p\,, p\,, p\,),$$

$\Rightarrow$ **The state equation** is now $\rho_\Lambda^{mix} \sim -p_\Lambda^{mix}$, where[§]

$$\rho_\Lambda^{mix} = \frac{2}{\pi} \sin^2\theta \int_0^K dk\, k^2 \left[\frac{m_1^2}{\omega_{k,1}} + \frac{m_2^2}{\omega_{k,2}}\right] |V_{\mathbf{k}}|^2.$$

Present epoch: ρ_Λ^{mix} has a behavior similar to that of Λ.

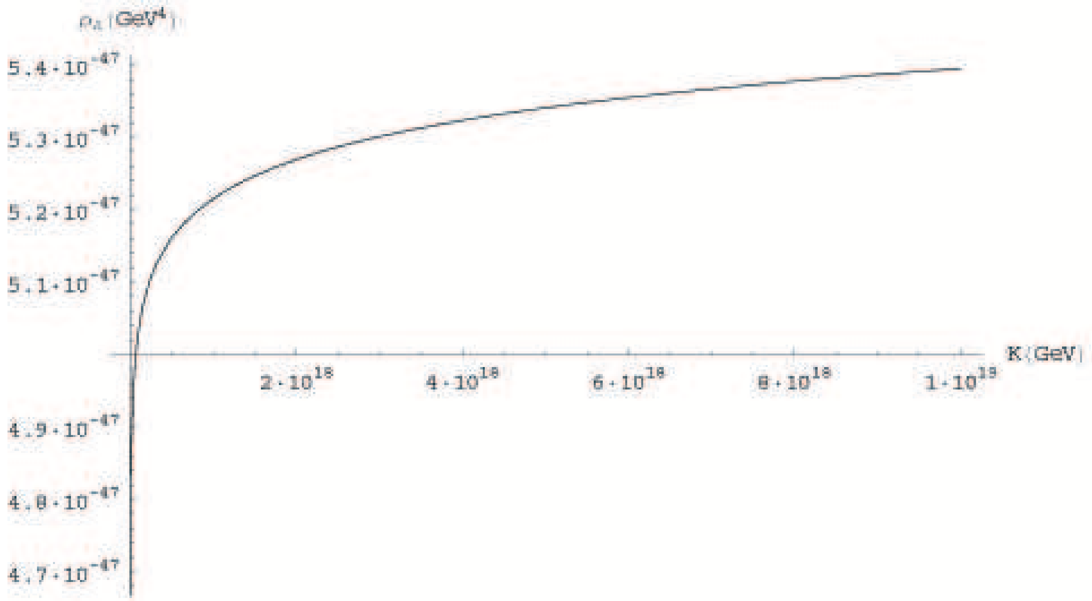
- neutrino oscillation length $\ll$ radius of curvature of the universe

$\Rightarrow$ mixing treatment in the flat space-time is a good approximation of that in FRW space-time.

[§] A.Capolupo, S.Capozziello, G.Vitiello, PLA(2007).

For m_i of order of $10^{-3}eV$ we have $\rho_\Lambda^{mix} = 5.4 \times 10^{-47} GeV^4$ for a value of the cut-off of order of the Planck scale $K = 10^{19} GeV$.: agreement with the observed value of cosmological constant.

Moreover $\frac{d\rho_\Lambda^{mix}(K)}{dK} \propto \frac{1}{K} \to 0$ for large K.



The neutrino mixing dark energy as a function of cut-off K.

The vacuum condensate from neutrino mixing contributes to the *observed* value of the cosmological constant. Exotic components to dark energy are not necessary in this approach.

- **Dark energy gets non-zero contribution induced from the neutrino mixing.**

- **Such a contribution is zero in the no-mixing limit: $\theta = 0$ and/or $m_1 = m_2$.**

- **The contribution is absent in the QM mixing treatment.**

Contributions of the mixing condensate at the present epoch

The very small breaking of Lorentz invariance of the flavor vacuum at the present epoch constrains the value of the cut-off on the momenta and consequently the value of the dark energy contributions due to the particle mixing.

Since $\omega_{k,i} = \frac{k^2}{\omega_{k,i}} + \frac{m_i^2}{\omega_{k,i}}$, we can write

$$\rho_{mix} = \Sigma_{mix} + V_{mix}$$

Σ_{mix}: kinematic term; V_{mix}: potential term;

$$\Sigma_{mix} = \frac{2}{\pi}\sum_i \int_0^K dk\, k^2 \,\frac{k^2}{\omega_{k,i}}\, \mathcal{N}_i^{\mathbf{k}}\,,$$

$$V_{mix} = \frac{2}{\pi}\sum_i \int_0^K dk\, k^2 \,\frac{m_i^2}{\omega_{k,i}}\, \mathcal{N}_i^{\mathbf{k}}\,.$$

We have seen that, at the present epoch, $\rho_{mix} \simeq V_{mix}$; i.e. $\Sigma_{mix} \ll V_{mix}$.

By solving numerically the above equations:
the condition $\Sigma_{mix} \ll V_{mix}$, $\Rightarrow$

$$K \ll \sqrt[3]{m_1 m_2 m_3}\,.$$

How much K has to be be smaller than $\sqrt[3]{m_1 m_2 m_3}$?

Consider adiabatic expansion of a sphere of volume V.

Let p denote the pressure at which the sphere expands.

The total energy, $E = \rho\mathbf{V}$, is not conserved since the pressure does work: $dE = \rho\, d\mathbf{V} + \mathbf{V}\, d\rho - p\, d\mathbf{V}$, i.e. $d[(\rho + p)\mathbf{V}] = 0$, thus:

$$\rho + p = \frac{const}{\mathbf{V}}\,.$$

For $\mathbf{V} \to \infty$ (in the bulk of the Universe, i.e. far from the Universe "boundaries"):

$$\rho \simeq -p \quad \text{and the adiabatic index is} \quad w = p/\rho \simeq -1.$$

Thus, $\rho = \frac{const}{\mathbf{V}} - p$ and, since $\Sigma \ll V$, $\rho = \Sigma + V \simeq V \simeq -p$.

In the limit $\mathbf{V} \to \infty$, we thus have

$$\Sigma \simeq \frac{const}{\mathbf{V}} \simeq 0\ .$$

Note consistency between the condition $\Sigma \ll V$ and $\mathbf{V} \to \infty$ limit.

Conclusion: at the present epoch, $\rho = \Sigma - p \simeq -p$.

Then, state equation for the flavor vacuum condensate at the present epoch:

$$w_{mix} = \frac{p_{mix}}{\Sigma_{mix} - p_{mix}} \rightarrow -1 \qquad \text{since} \qquad \Sigma_{mix} \rightarrow 0$$

Note that $\Sigma_{mix} - p_{mix} \neq 0$, **since** $\Sigma_{mix} \ll p_{mix}$.

Since Σ_{mix} **and** p_{mix} **are function of the cut-off** K, **then** $w_{mix} = w_{mix}(K)$.

We now estimate the contributions given to the dark energy by the particle mixing condensates for different values of w_{mix} **close to** -1, **both for neutrino and for quark mixing condensates.**

Neutrino mixing condensate contribution to dark energy

Consider experimental values: $\Delta m^2_{12} = 7.9 \times 10^{-5} eV^2$, $\Delta m^2_{23} = 2.3 \times 10^{-3} eV^2$, $s^2_{12} = 0.31$, $s^2_{23} = 0.44$, $s^2_{13} = 0.009$.

In the normal hierarchy case: $|m_3| \gg |m_{1,2}|$, **consider neutrino masses compatible with such values, e.g.:** $m_1 = 4.6 \times 10^{-3} eV$, $m_2 = 1 \times 10^{-2} eV$, $m_3 = 5 \times 10^{-2} eV$.

Then the condition $K \ll \sqrt[3]{m_1 m_2 m_3}$ **for neutrinos reads**

$$K \ll 1.2 \times 10^{-2} eV\,,$$

namely:

K	$\rho_{\nu-mix}(GeV^4)$	$\Sigma_{\nu-mix}(GeV^4)$	$w_{\nu-mix}$
$1.2 \times 10^{-2} eV$	1.1×10^{-45}	1.6×10^{-46}	-0.85
$4 \times 10^{-3} eV$	1.2×10^{-47}	3.5×10^{-49}	-0.97
$3 \times 10^{-3} eV$	0.3×10^{-47}	5.8×10^{-50}	-0.98
$4 \times 10^{-4} eV$	1.6×10^{-52}	6.1×10^{-56}	-0.99
$4 \times 10^{-5} eV$	1.6×10^{-57}	6.2×10^{-63}	$-0.\overline{99}$

Table 1: Values of $\rho_{\nu-mix}$ and $w_{\nu-mix}$ for for different cut-offs.

The result we find is that contributions to the dark energy compatible with its estimated upper bound, $\rho_{\nu-mix} \sim 10^{-47} GeV^4$, are obtained for values of the adiabatic index $w_{\nu-mix}$ of the neutrino mixing dark energy component:

$$-0.98 \leq w_{\nu-mix} \leq -0.97\,.$$

This is in agreement with the constraint on the equation of state of the dark energy given by the combination of WMAP and Supernova Legacy Survey (SNLS) data:

$$w = -0.967^{+0.073}_{-0.072}$$

and with the constraint given by combining WMAP, large-scale structure and supernova data:

$$w = -1.08 \pm 0.12.$$

A value of $w_{\nu-mix} < -0.98$ leads to negligible contributions of $\rho_{\nu-mix}$.

The results we found are dependent on the neutrino mass values one uses.

Quark mixing condensate contribution to dark energy

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} ,$$

where V is is the CKM matrix.

For the values of the quark masses given in PDG (2006), we have

$$K \ll 120 MeV ,$$

namely,

K	$\rho_{q-mix}(GeV^4)$	$\Sigma_{q-mix}(GeV^4)$	w_{q-mix}
$120MeV$	5.1×10^{-7}	3.5×10^{-7}	-0.3
$10MeV$	2×10^{-10}	1.4×10^{-11}	-0.93
$300KeV$	1.5×10^{-17}	1.8×10^{-21}	-0.99
$30KeV$	1.5×10^{-22}	1.8×10^{-28}	$-0.\overline{99}$
$0.3eV$	1.5×10^{-47}	1.8×10^{-63}	-1

Table 2: Values of ρ_{q-mix} and w_{q-mix} for for different cut-offs.

The exact Lorentz invariance of the quark mixing condensate $w_{q-mix} = -1$ (Σ_{q-mix} is 16 orders less than V_{q-mix}), at the present epoch, leads to a dark energy contribution compatible with its estimated upper bound: $\rho_{q-mix} = 1.5 \times 10^{-47} GeV^4$.

We stress that very small deviations from the value $w_{q-mix} = -1$ give rise to contributions of ρ_{q-mix} that are beyond the accepted upper bound of the dark energy.

Our results are dependent on the mass values one uses.

Conclusions

The vacuum condensate from particle mixing can contribute to the observed value of the cosmological constant. Exotic components to dark energy are not necessary in this approach.

- Such a contribution is zero in the no-mixing limit: $\theta = 0$ and/or $m_1 = m_2$.

- The contribution is absent in the QM mixing treatment.