Neutrinos and flavour in a brane model

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Outline

References

- Work performed with Rhys Davies, Damien George and Ben Callen.
- DGV, Phys. Rev. D77, 124038 (2008); arXiv:0705.1584 [hep-ph].
- CV, work-in-progress.

Context

Varieties of brane-world models:

- String-theoretic.
- Field-theoretic with brane(s) as δ-fn object(s) put in by hand. Often taken as effective low-energy outcomes of string theory, e.g. RS1.
- Completely field-theoretic, with brane as soliton (domain wall, vortex, domain-wall junctions).

I shall discuss this last class. Specifically, we'll have one topologically-infinite extra dimension (like RS2) and the brane will be a domain wall.

Outcomes

Models in this class may solve the fermion mass hierarchy puzzle without using a horizontal symmetry.

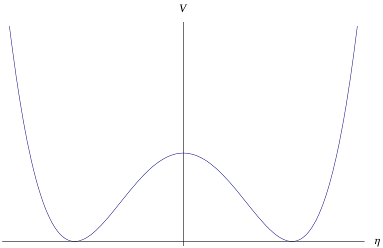
The neutrino mixing angle pattern, however, suggests that horizontal symmetry still has a role to play.

Turns out one is driven to GUT realisations. Two of the usual problems of GUTs,

- unwanted mass relations such as $m_e = m_d$,
- coloured-Higgs induced proton decay,

can be avoided without complicating the theory with epicycles.

Domain walls and kinks



We need a potential with *disconnected* and *degenerate* vacua:

$$V = \lambda(\eta^2 - v^2)^2$$

with η a scalar field.

Domain walls and kinks

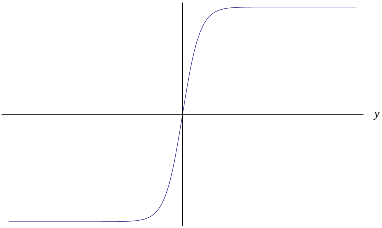
Lagrangian for $\eta(x^\mu, y)$:

$$\mathcal{L} = -\frac{1}{2}\partial_M\eta\,\partial^M\eta - V(\phi)$$

A solution is the *kink*:

$$\eta_{\text{kink}}(y) = v\tanh(\sqrt{2\lambda}vy)$$

It is topologically stable.



Fermion zero-mode localisation

Let $\Psi(x, y)$ be 5d fermion Yukawa-coupled to background scalar field $b(y)$. It obeys 5d Dirac Eq:

$$i\Gamma^M \partial_M \Psi - b(y)\Psi = 0$$

where $\Gamma^M = (\gamma^\mu, -i\gamma_5)$.

A chiral zero-mode starts off the generalised KK decomposition:

$$\Psi(x, y) = f(y)\psi_{L,R}(x) + \dots$$

ψ is a 4d massless fermion, obeying

$$i\gamma^\mu \partial_\mu \psi_{L,R} = 0$$

Fermion localisation

The profile for the chiral zero mode is:

$$f(y) \propto e^{-\int_{y_0}^{y} b(y')dy'}$$

The b's depend on the Yukawas, so the f's are *exponentially sensitive* to them. This is the key to getting fermion mass hierarchies (see later).

Mode functions are normalised as per $\int_{-\infty}^{+\infty} f(y)^2 dy = 1$ to make 4d field kinetic terms conventionally normalised.

Spin-0 boson localisation

This proceeds in a very similar way to fermions. No time to go into the details here.

One important outcome: a localised 4d scalar can have a tachyonic mass-squared. Hence we can have SSB inside the wall. We'll use this for the electroweak Higgs doublet.

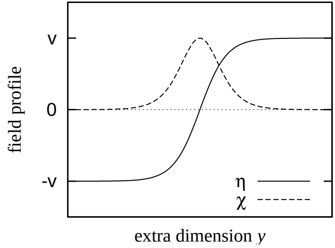
The background DW

The Dvali-Shifman mechanism for gauge boson localisation motivates an $SU(5)$ model that breaks to $SU(3) \times SU(2) \times U(1)$ inside the wall.

We use an $SU(5)$ singlet scalar η to produce a kink, and an $SU(5)$ adjoint χ to break $SU(5)$ to the SM inside the wall.

Write $\chi = \sum_a T^a \chi_a$, where T's are $SU(5)$ generators in the fundamental. If the component χ_1 corresponding to the hypercharge generator Y condenses, then $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)_Y$.

The background DW



You write a Higgs potential, arrange the global minima

$$\langle\eta\rangle = \pm v,\ \langle\chi\rangle = 0,$$

use them as boundary conditions, solve the Euler-Lagrange equations to get, e.g.

$$\eta(y) = v \tanh(ky),$$

$$\chi_1(y) = A\,\mathrm{sech}(ky).$$

This simple analytical solution holds on a certain parameter slice. Off that slice, similar solutions exist but must be obtained numerically.

Fermion localisation

Next, you introduce 5d fermions

$$\Psi_5 \sim 5^*, \quad \Psi_{10} \sim 10, \quad N \sim 1$$

and you Yukawa-couple them to η and χ:

$$\begin{aligned} Y_{DW} &= h_{5\eta}\overline{\Psi}_5\Psi_5\eta + h_{5\chi}\overline{\Psi}_5\chi^T\Psi_5 \\ &+ h_{10\eta}\mathrm{Tr}(\overline{\Psi}_{10}\Psi_{10})\eta - 2h_{10\chi}\mathrm{Tr}(\overline{\Psi}_{10}\chi\Psi_{10}) \\ &+ h_{1\eta}\overline{N}N\eta. \end{aligned}$$

The background fields you use in the 5d Dirac Eq. are:

$$b_{nY}(y) \equiv h_{n\eta}\eta(y) + \sqrt{\frac{3}{5}}\frac{Y}{2}h_{n\chi}\chi_1(y).$$

SM components of different hypercharge Y couple to different linear combinations of $\eta(y)$ and $\chi_1(y)$. Fermions are *split*, but not arbitrarily.

EW symmetry breaking

Now introduce a scalar $\Phi \sim 5^*$ containing the weak doublet Φ_w and a coloured scalar Φ_c. Yukawa-couple it to fermions in the usual way.

You do a mode decomposition, and are interested in the lowest modes:

$$\Phi_{w,c}(x,y) = p_{w,c}(y)\phi_{w,c}(x)$$

You write the Higgs potential, plug the above into the Euler-Lagrange Eqs., solve for the profiles $p(y)$.

Fermion spectra

I now report some preliminary results from work by Ben Callen on fitting the model to the observed quark and lepton masses, including neutrinos.

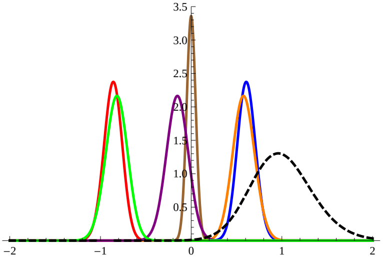
A 4d Yukawa coupling is of the form:

$$h \left[\int dy f_L(y) f_R(y) p(y) \right] \overline{\psi}_L(x) \psi_R(x) \phi(x).$$

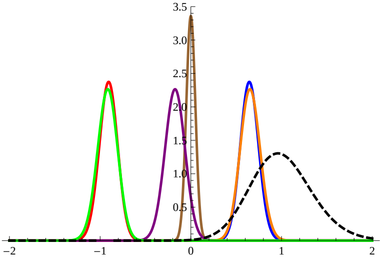
The 4d Yukawa coupling constant is equal to the 5d Yukawa multiplied by an overlap integral of profile functions, which themselves depend on Yukawas in a complicated way.

The profiles are *exponentially sensitive* to the Yukawa coupling constants. Searching the parameter space is numerically intensive. We have been proceeding by trial-and-error. Multiple viable regions exist.

Fermion spectra – no mixing



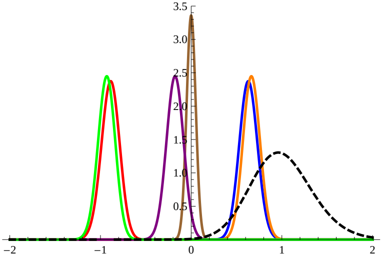
3rd gen profiles



2nd gen profiles

Brown - the right handed neutrinos; Red - the left handed lepton doublets; Blue - right handed down, strange, and bottom quarks; Green - right handed electron, muon and tau; Orange - right handed up, charm, and top quarks; Purple - the left handed quark doublets; Black dashed - the Electroweak Higgs.

Fermion spectra – no mixing



1st gen profiles

All EW Yukawas have been set as equal!

Fermion mass differences *entirely* driven by profiles through coupling to the DW background. The masses can be fitted well, including very light Dirac neutrinos.

Spread used in fermion-DW Yukawas is less than order-of-mag. Fermion mass spread is nevertheless 14 orders-of-mag.

Higgs-induced proton decay suppression

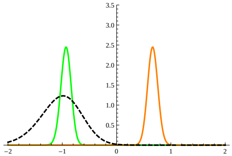
This process proceeds via the Yukawa terms

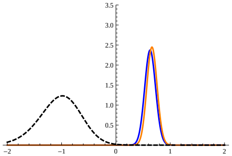
$$\overline{u}_R (e_R)^c \phi_c^* \quad \text{and} \quad \overline{d}_R (u_R)^c \phi_c.$$

For the *same* region of parameter space that fits the masses, the effective 4d Yukawa couplings constants are, respectively, about

$$10^{-31} \quad \text{and} \quad 1.$$

The proton partial lifetime goes below the experimental bound for ϕ_c mass greater than just a few GeV!.





Summary

- The fermion mass spectrum can definitely be fitted, including for Dirac neutrinos, despite the minimal $SU(5)$ structure.
- A very small Yukawa spread gives a 14 order-of-mag fermion mass spread.
- CKM mixing angles can almost certainly be fitted as well.
- Higgs induced proton decay automatically suppressed. Elimination of doublet-triplet splitting problem.
- Neutrino tribimaximal mixing requires a flavour symmetry also.

Open questions:

- Optimisation of the fit, and discovery of all allowed regions of parameter space.
- Detailed fits to mixing angles and phases as well as masses; role of flavour symmetry.
- How *generic* is a strong mass hierarchy in this picture?