Explaining LSND and MiniBooNE using altered neutrino dispersion relations.

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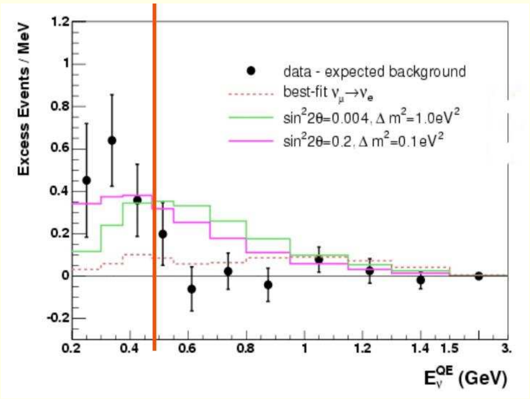
Motivation

- LSND observed a 3.8σ excess of $\bar{\nu}_e$ events in a pure $\bar{\nu}_\mu$ beam
- MiniBooNE sees excess only in low energy regime $\nu_\mu \to \nu_e$



  - No signal on the antineutrino channel!

- $\Delta m^2_\odot$ & $\Delta m^2_{Atm} \Rightarrow 3\Delta m^2$'s!

LSND and MiniBooNE low energy anomaly might hint towards deviations from the usual oscillation mechanism...

- maybe extra dimensions? active-sterile neutrino oscillations?

- CPT- & Lorentz violating terms

Hamiltonian

Neutrino oscillations Hamiltonian (with CPT- & Lorentz violating terms):

$$h_{\text{eff}} = \text{diag}\left(E + \frac{\Sigma m^2}{4E}\right) +$$

$$+\begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta - C(E) & \frac{\Delta m^2}{4E}\sin 2\theta & B(E) & 0 \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta - C(E) & 0 & B(E) \\ B(E) & 0 & -\frac{\Delta m^2}{4E}\cos 2\theta - C(E) & \frac{\Delta m^2}{4E}\sin 2\theta \\ 0 & B(E) & \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta - C(E) \end{pmatrix}$$

- Modified dispersion relations
  - A fourth sterile neutrino $\Rightarrow$ active-sterile neutrino oscillations
    - Hollenberg, Päs, Micu, Weiler; arXiv:0906.0150 [hep-ph]
  - Consider CPT- & Lorentz-violating terms in a 3 neutrino scenario.
    - Kostelecky, Mewes; arXiv:hep-ph/0308300
    - Hollenberg, Päs, Micu; arXiv:0906.5072 [hep-ph].

Active-Sterile Oscillation Probability with Bulk Shortcuts

- Standard Model particles confined to the 3+1 brane.
- Gauge singlet particles as gravitons or sterile neutrinos may travel off the brane into the bulk!
- Virtual gravitons penetrate the bulk $\longrightarrow$ Gauß's law $\longrightarrow$ apparent weak gravity on the brane!

Mechanisms for bulk shortcuts:

- Self-gravity effects in the presence of matter localized on the brane $\Rightarrow$ extrinsic brane curvature.
- Thermal or quantum fluctuations $\Rightarrow$ brane bending.
- The extra dimension can be asymmetrically warped, i.e. warp factors can shrink the space dimensions x parallel to the brane but leave the time and bulk dimension t and u unaffected

$$ds^2 = dt^2 - \sum_{i=1}^{3} a^2(t) e^{-2k|u|} \left(dx^i\right)^2 - du^2,$$

Active-Sterile Oscillation Probability with Bulk Shortcuts

With sterile ν's traveling in the bulk $\Rightarrow$ Effective Hamiltonian:

$$H'_F = \text{Diag. terms} + \frac{\delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} - E\frac{\epsilon}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

with the *shortcut parameter*: $\epsilon \simeq (t_{\text{brane}} - t_{\text{bulk}})/t_{\text{brane}} \simeq \delta t/t$.

Resonant energy condition

$$E_{\text{res}} = \sqrt{\frac{\delta m^2 \cos 2\theta}{2\epsilon}}$$

Flavor oscillation probability:

$$P_{as} = \sin^2(2\tilde{\theta})\sin^2(\delta H L/2)$$

$$\sin^2(2\tilde{\theta}) = \frac{\sin^2(2\theta)}{\sin^2(2\theta) + \cos^2(2\theta)\left[1 - (E/E_{res})^2\right]^2}$$

$$\delta H = \frac{\delta m^2}{2E}\sqrt{\sin^2(2\theta) + \cos^2(2\theta)\left[1 - (E/E_{res})^2\right]^2}$$

Resonance plot

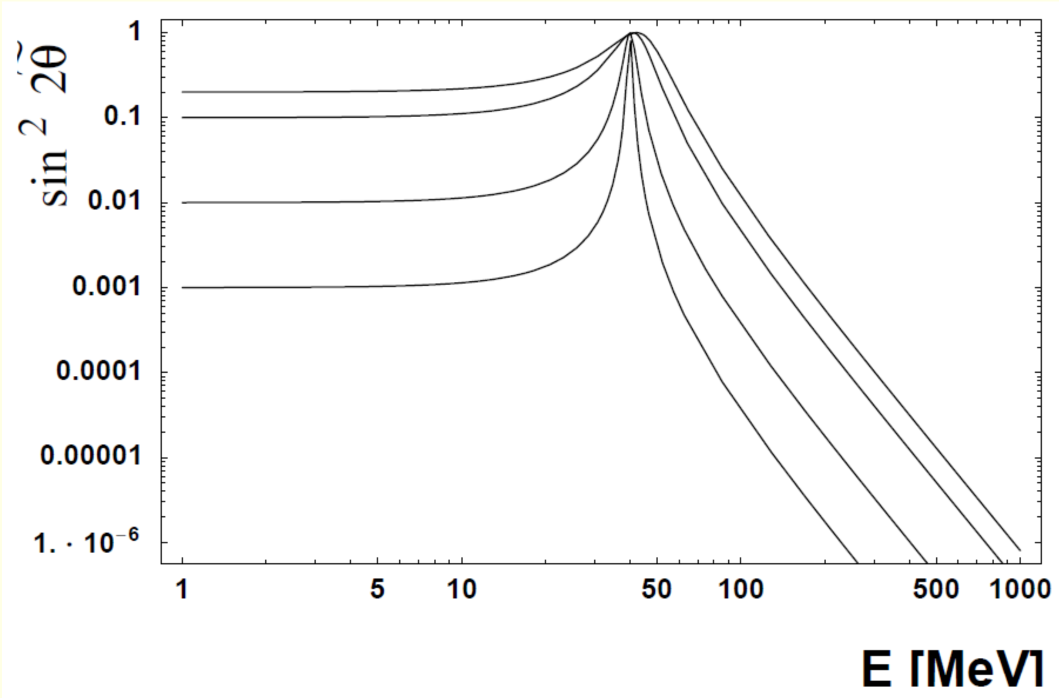
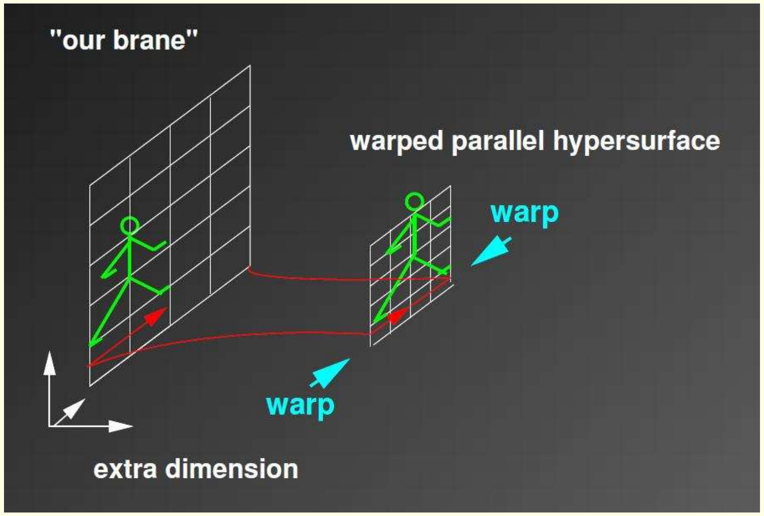


Figure 1: Oscillation amplitude $sin^2 2\tilde{\theta}$ as a function of the neutrino energy E_ν, for a resonance energy of $E_{\text{res}} = 40MeV$. The different values correspond to different values for the standard angle $sin^2 2\theta = 0.2, 0.1, 0.01, 0.001$ (from above).

- Päs, Pakvasa, Weiler, Phys.Rev.D72:095017,2005.

Asymmetrically warped space-times geodesics



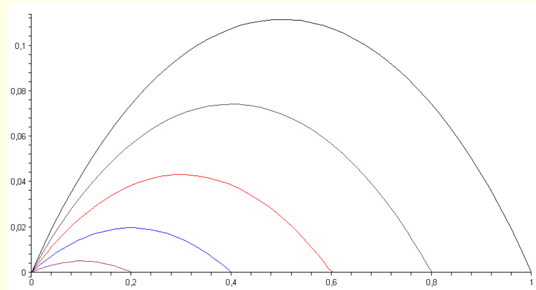
$\longrightarrow$ how do geodesics in the bulk alter the oscillation probabilities?

$$ds^2 = dt^2 - \sum_{i=1}^{3} a^2(t) e^{-2k|u|} \left(dx^i\right)^2 - du^2$$

$\mapsto$ Geodesic equations $\Rightarrow$ shortcut parameter $\Rightarrow$ resonances...

Geodesics

Geodesic: $$u(x) = \pm \frac{1}{2k} \ln \left[1 + k^2 x \left(L - x \right) \right].$$



Shortcut parameter: $$\epsilon(L) = 1 - \frac{2}{\beta k L} \,\text{arcsinh}\, \frac{kL}{2}$$

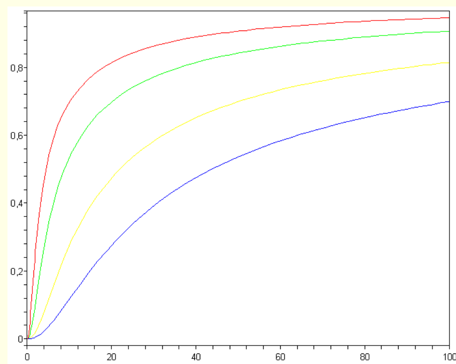


Figure 2: The relative difference between the travel time for SM neutrinos and sterile neutrinos. Curves are parametrized by geodesic mode number $n = 1, 2, 5, 10$ (from top to bottom)

Oscillation Probability

$$A_{as} = \underbrace{\sum_{n=1}^{\infty}}_{\text{Modes } n} \Delta n \, \overbrace{e^{iS_{cl}(n)}}^{\text{Path integral weight}} \underbrace{\frac{4nkL}{(4n^2+(kL)^2)^{3/2}} \left[\frac{\sqrt{2}\beta E}{\sqrt{\pi}\sigma} \; e^{-\frac{(\beta E k L)^2}{2\sigma^2(4n^2+(kL)^2)}} \right]}_{\text{Distribution of initial } \dot{u}_0} \overbrace{\sin 2\tilde{\theta}_n}^{\sin \Theta_{\text{eff}}} \sin \frac{L\delta \tilde{H}_n}{2}$$

Path-integral weight: $S_{cl}(n) = m \int d\tau = \left(\frac{m^2 L}{\beta E} \right) (1 - \epsilon_n)$

Distribution of initial $\dot{u}_0$ (normalized)$\mapsto$ Gaußian:

$$dN_G(p_u) = \frac{1}{\sqrt{2\pi}\sigma} \; e^{-\left\{ \frac{p_u^2}{2\sigma^2} \right\}} \; dp_u$$

The weight Δn: $\Delta n = \frac{dn}{dS} \Delta S \sim \frac{1}{dS/dn}$

The probability of oscillation:

$$P_{as} = |A_{as}|^2 = \left| \sum_{n=1}^{\infty} A(n) \right|^2 .$$

Oscillation Probability

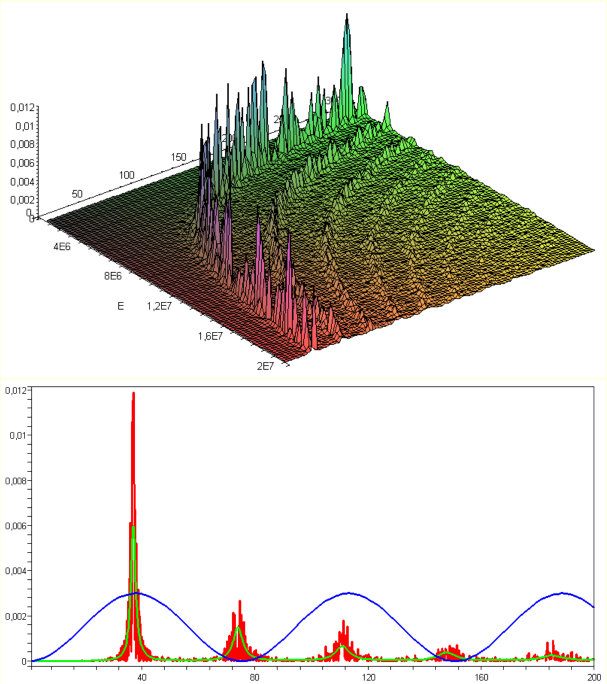


Figure 3: Oscillation probability as a function of the experimental baseline (red and green curves). The green curve presents the phase-averaged oscillation probability, and the sinusoidal blue curve presents the standard 4D vacuum oscillation probability between sterile and active neutrinos. Parameter choices are $\sin^2 2\theta = 0.003$, $k = 5/(10^8$ m), $E = 15$ MeV, $\Delta m^2 = 64$ eV^2, and $\sigma = 100$ eV. .

Summary

⋆ Bulk shortcuts

- Can arise naturally in extra dimensional theories
- **Shortcut parameter** is **baseline dependent**!
- **Affect neutrino mixings** and imply **new resonances**!
- The resonances depend on the product LE rather than E!
- **LSND** data and the **MiniBooNE** null result may be explained.
- It can also help solve the problems with the long baseline experiments.
- Disappearance into sterile neutrinos?
- No explanation of MiniBooNE excess only for neutrinos.

Model II. CPT-& Lorentz violation

•Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_e^c \\ \nu_\mu^c \end{pmatrix} = h_{\mathsf{eff}} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_e^c \\ \nu_\mu^c \end{pmatrix}$$

•Effective hamiltonian

$$h_{\mathsf{eff}} = \text{Diagonal part} +$$

$$+\begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta - C(E) & \frac{\Delta m^2}{4E}\sin 2\theta & B(E) & 0 \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta - C(E) & 0 & B(E) \\ B(E) & 0 & -\frac{\Delta m^2}{4E}\cos 2\theta - C(E) & \frac{\Delta m^2}{4E}\sin 2\theta \\ 0 & B(E) & \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta - C(E) \end{pmatrix}$$

• Block diagonalize it so that $h_{\mathsf{eff}} = U\ \tilde{h}_{\mathsf{eff}}\ U^\dagger$ with the unitary matrix:

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

Model II. CPT-& Lorentz violation

- Change of basis

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_e^c \\ \nu_\mu^c \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_e^c \\ \nu_\mu^c \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \nu_e - \nu_e^c \\ \nu_\mu - \nu_\mu^c \\ \nu_e + \nu_e^c \\ \nu_\mu + \nu_\mu^c \end{pmatrix} = \begin{pmatrix} \nu_e^- \\ \nu_\mu^- \\ \nu_e^+ \\ \nu_\mu^+ \end{pmatrix}$$

- Charge conjugation eigenstates:

$$\begin{aligned} C\,\nu^- &= -\nu^-, \\ C\,\nu^+ &= +\nu^+ \end{aligned}$$

- C -eigenstates basis:

$$i\frac{d}{dt}\begin{pmatrix} \nu^- \\ \nu^+ \end{pmatrix} = \begin{pmatrix} h_{\text{eff}}^{C\text{-odd}} & 0 \\ 0 & h_{\text{eff}}^{C\text{-even}} \end{pmatrix} \begin{pmatrix} \nu^- \\ \nu^+ \end{pmatrix}$$

Resonant C-odd oscillations

- C-odd effective Hamiltonian

$$h_{\text{eff}}^{C\text{-odd}} = \text{diag}\left(E + \frac{\Sigma m^2}{4E}\right) + \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta - \frac{(b_e + c_{ee})E}{2} & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta - \frac{(b_\mu + c_{\mu\mu})E}{2} \end{pmatrix}$$

- Effective mixing angle

$$\tan 2\theta_{C\text{-odd}} = \frac{\Delta m^2 \sin 2\theta}{(b_e - b_\mu + c_{ee} - c_{\mu\mu})E^2 + \Delta m^2 \cos 2\theta}$$

- Resonant mixing

$$E_{\text{res}}^{C\text{-odd}} = \sqrt{\frac{\Delta m^2 \cos 2\theta}{b_\mu - b_e + c_{\mu\mu} - c_{ee}}}$$

- Effective mass eigenvalues

$$m_1^2 = \frac{\Sigma m^2}{2} - \frac{1}{2}[(b_e + b_\mu + c_{ee} + c_{\mu\mu})E^2 + \kappa_{C\text{-odd}}]$$

$$m_2^2 = \frac{\Sigma m^2}{2} - \frac{1}{2}[(b_e + b_\mu + c_{ee} + c_{\mu\mu})E^2 - \kappa_{C\text{-odd}}]$$

where

$$\kappa_{C\text{-odd}}^2 = (b_e - b_\mu + c_{ee} - c_{\mu\mu})^2 E^4 + 2\Delta m^2 (b_e - b_\mu + c_{ee} - c_{\mu\mu}) \cos 2\theta E^2 + (\Delta m^2)^2$$

Resonances

- Effective mixing angle

$$\tan 2\theta_{C\text{-even}} = \frac{\Delta m^2 \sin 2\theta}{(b_\mu - b_e + c_{ee} - c_{\mu\mu})E^2 + \Delta m^2 \cos 2\theta}$$

Resonant mixing for states with an energy

$$E_{\text{res}}^{C\text{-even}} = \sqrt{\frac{\Delta m^2 \cos 2\theta}{b_e - b_\mu + c_{\mu\mu} - c_{ee}}}$$

- Diagonalization connects C-eigenstates with mass eigenstates

$$\begin{pmatrix} \nu_e^+ \\ \nu_\mu^+ \end{pmatrix} = \begin{pmatrix} \cos\theta_{C\text{-even}} & \sin\theta_{C\text{-even}} \\ -\sin\theta_{C\text{-even}} & \cos\theta_{C\text{-even}} \end{pmatrix} \begin{pmatrix} \nu_3 \\ \nu_4 \end{pmatrix}$$

- Translation between flavor and mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_e^c \\ \nu_\mu^c \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\theta_{C\text{-odd}} & \sin\theta_{C\text{-odd}} & \cos\theta_{C\text{-even}} & \sin\theta_{C\text{-even}} \\ -\sin\theta_{C\text{-odd}} & \cos\theta_{C\text{-odd}} & -\sin\theta_{C\text{-even}} & \cos\theta_{C\text{-even}} \\ -\cos\theta_{C\text{-odd}} & -\sin\theta_{C\text{-odd}} & \cos\theta_{C\text{-even}} & \sin\theta_{C\text{-even}} \\ \sin\theta_{C\text{-odd}} & -\cos\theta_{C\text{-odd}} & -\sin\theta_{C\text{-even}} & \cos\theta_{C\text{-even}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

Resonances

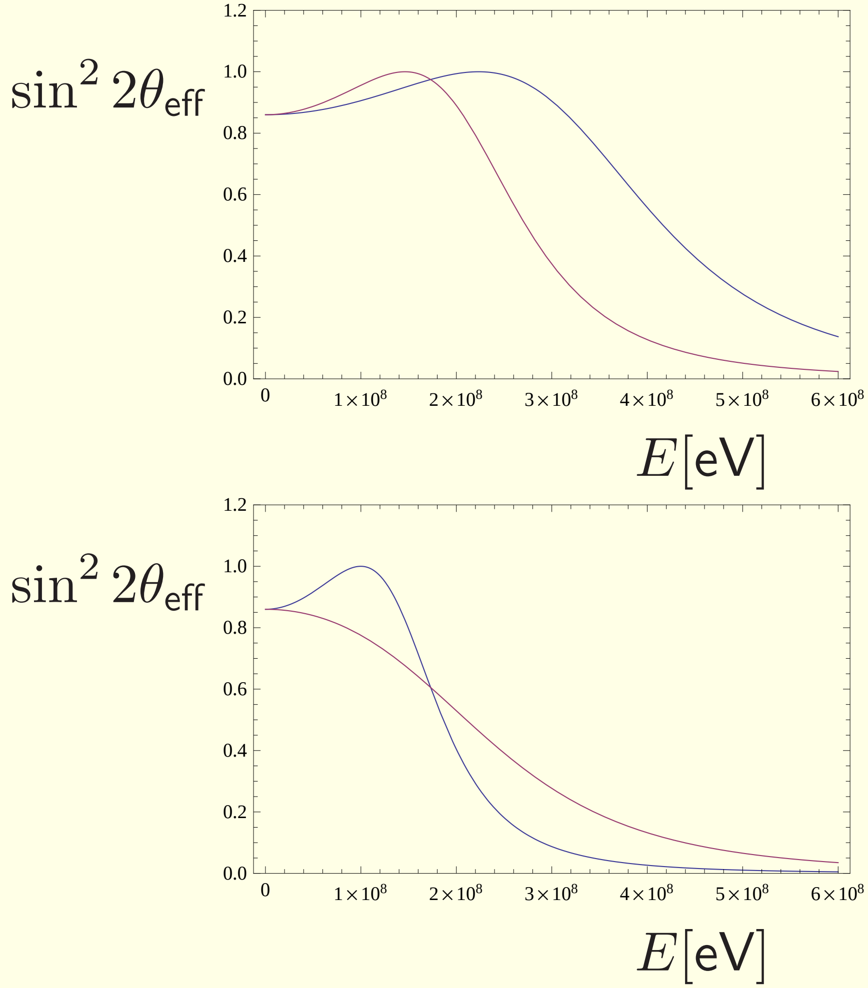


Figure 4: Resonance structures between charge conjugation eigenstates. Shown is the sine-squared of the effective mixing angles $\theta_{C\text{-odd}}$ (blue curve) and $\theta_{C\text{-even}}$ (red curve).

Summary

$\star$ **CPT- and Lorentz-violating terms**

- Neutrino-antineutrino oscillations become possible!
- Can explain low energy MiniBooNE data.
- Generate new resonance peaks which can be at different energies.
- Resonance peaks can be narrower.
- No disappearance due to oscillations into sterile ν's

CPT- and Lorentz-violating effects generate new resonances and can help us understand the LSND & MiniBooNE data!