# Neutrino Mass and Unification

### R. N. Mohapatra



**2009 Erice School on Neutrinos** 



# Implications of neutrino mass results for Theory

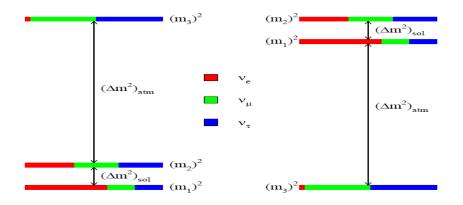
What we may have learnt?

What we need to learn and how?

A subjective overview

# Present information: Masses and mixings

- Masses:  $\Delta m_{sol}^2 \cong 7.67 \times 10^{-5} \, eV^2$ ;  $\Delta m_{Atm}^2 \cong 2.39 \times 10^{-3} \, eV^2$
- Mixings:  $\sin^2 \theta_{12} \cong .312$ ;  $\sin^2 \theta_{23} \cong .466$ ;  $\sin^2 \theta_{13} \leq .04$
- Overall mass scale: < .1- 1 eV (roughly) (WMAP,..)</p>
- Mass ordering not known:



#### Need to know...

(i) Majorana or Dirac  $\beta \beta_{0\nu}$ 

(Nucl matrix element : Fassler et al.; Talk by Simkovic Suhonen ..)



(ii) Absolute mass scale:





(iii) Mass ordering:







(iv) Value of  $\theta_{13}$ 





(v) CP phase

### Dirac vs Majorana

- Very important for determining the nature of new physics:
- Can we tell experimentally?
- Observe  $\beta \beta_{0\nu} \rightarrow Majorana$ .
- If no signal till 20 meV → could be Majorana with normal hierarchy;
- However, no  $\beta\beta_{0\nu}$  signal till 20 meV +  $\Delta m_{23}^2 < 0$  from long Baseline expts  $\rightarrow$  strong hint for

Dirac Rest of this talk assumes Majorana nu

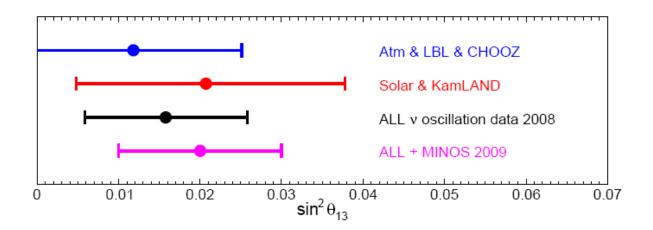


#### What is value of



?

Recent indications ? (Fogli, Lisi, Marrone, Pallazo and Rotunno'08)



#### Too early for definite conclusion--However

Value of  $\theta_{13}$  significant for new physics



#### **Goal of Theory**

- Determining and understanding the Neutrino mass matrix :
- Two parts to the story:

$$M_{\nu} = m_{\nu} \times A_{F}$$

- (i) Scale  $m_{\nu}$
- (ii) Flavor structure  $A_F$  (The neutrino matrix)

#### Challenges

(i) Scale issue: Why

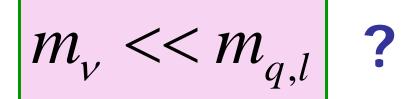
$$m_{\nu} \ll m_{q,l}$$
 ?

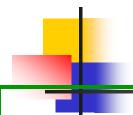
- (ii) Flavor issues:  $A_F$ ?
- A. Milder mass hierarchy compared to quarks and charged leptons:  $m_{\mu}$   $m_{\mu}$

charged leptons: 
$$\frac{m_{sol}}{m_{atm}} \approx \theta_C >> \frac{m_{\mu}}{m_{\tau}}, \frac{m_s}{m_b}$$

- B. Neutrino mixing angles much larger than quark mixings: e.g.  $V_{23}^{l} \approx 0.7 >> V_{23}^{CKM} \approx 0.04$  etc.
- C. Quarks and leptons so different- are they unifiable?

#### Why



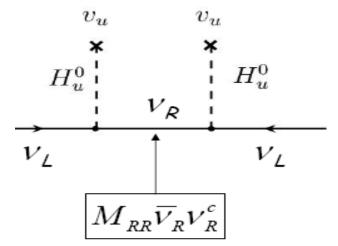


#### **Seesaw Paradigm**

• Add right handed neutrinos  $N_R$  to SM with Majorana mass:

$$L_{Y} = h_{v} \overline{L} H N_{R} + M_{R} N N$$

$$m_{v} \cong -\frac{h_{v}^{2} v_{wk}^{2}}{M_{R}}$$



Standard seesaw (Type I):

■Minkowski,Gell-Mann, Ramond, Slansky,Yanagida, Mohapatra,Senjanovic,Glashow

### Neutrino mass → new symmetry of Nature: B-L

- Why B-L Symmetry?
- $\blacksquare \ \, \mathbf{Seesaw} \,\, \mathbf{scale} \,\, \boldsymbol{M}_{\,R} \,\, \mathbf{breaks} \,\, \mathbf{this} \,\, \mathbf{symmetry} \,\,$
- The question is why  $M_R << M_{Pl}$ ?
- Having a B-L symmetry explains this.
- Is it a global or local symmetry?
- Most likely local since adding RH nu's to SM makes B-L local sym. → Z'

# Need to learn: What is the B-L Scale $M_R$ ?

- Two extreme cases:  $m_D = hv_{wk}$
- $m_D \approx m_t$  (suggested by Q-L unif as in GUTs) leads to
  - $M_R \approx 10^{14} GeV$  -SCALE CLOSE TO GUT SCALE-
- Corresponding theory of seesaw is: SO(10):
- $m_D \approx m_e$  Scale much lower  $M_R \approx \text{TeV}$ ;
- Theory is left-right model based on  $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$  motivated independently Considerations e.g. parity, CP etc.
  - Physics accessible to LHC,  $\beta\beta_{0\nu}$  decay etc.

### Double (Inverse) Seesaw

- Low scale seesaw with no small couplings:
- RH neutrinos + 3 additional gauge singlet fermions → 3x3 neutrino matrix:

$$\begin{pmatrix} 0 & hv_{wk} & 0 \\ hv_{wk} & 0 & M \\ 0 & M & \mu \end{pmatrix} \qquad m_D = hv_{wk} \quad \text{(RNM, Valle, 86)}$$
 
$$m_V \cong -m_D^T M^{-1} \mu M^{-1} m_D$$

- µ determines both scale and flavor structure;
- ullet  $m_D$  decoupled from scale unlike type I; Seesaw scale can be in TeV range without small Yukawas

### Second Challenge for theory: Large lepton mixings:

- Could they be hints of new symmetries for leptons:
- (i) Near maximal  $\theta_{23}$ : very suggestive of  $\mu \tau$  exchange sym. for neutrino matrix: How maximal?
  - (ii) Solar angle  $\theta_{12} \approx 35^{\circ}$ suggests tribimaximal scheme if  $\theta_{13} = 0$ ->

Wolfenstein; Harrison, Perkins, Scott; Xing,

$$V = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0\\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

#### The Neutrino Matrix:

Flavor of the Neutrino flavor research

Find A<sub>F</sub>

Generic mass matrix (NH)  $\varepsilon_i \approx \lambda_{Cabibbo} << 1$ 

$$\varepsilon_i \approx \lambda_{Cabibbo} << 1$$

 $\theta_{23}$  NEAR MAXIMAL

$$heta_{23}$$
maximal

$$\begin{pmatrix}
\varepsilon_{5}^{n\geq 1} & \varepsilon_{4} & \varepsilon_{3} \\
\varepsilon_{4} & 1+\varepsilon_{1} & -1 \\
\varepsilon_{3} & -1 & 1+\varepsilon_{2}
\end{pmatrix}
\longrightarrow
\begin{pmatrix}
\varepsilon_{5}^{n\geq 1} & \varepsilon_{3} & \varepsilon_{3} \\
\varepsilon_{3} & 1+\varepsilon_{1} & -1 \\
\varepsilon_{3} & -1 & 1+\varepsilon_{1}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{1} & \varepsilon_{3} & \varepsilon_{3} \\
\varepsilon_{3} & 1+\varepsilon_{1} & -1+\varepsilon_{3} \\
\varepsilon_{3} & -1+\varepsilon_{3} & 1+\varepsilon_{1}
\end{pmatrix}$$

$$\longrightarrow \mu \leftrightarrow \tau \text{ sym.}$$

5 parameters

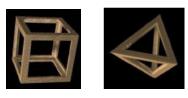
3 param.

2 param.

## Understanding the pattern

- **GUTs**: **SO(10)**
- Family symmetries motivated by TBM:

$$S_{2(\mu-\tau)} \subset S_3, S_4, A_4, Z_2, \Delta(3n^2),...$$



• Non-zero  $\theta_{13}$  will provide important clue about new physics- is it symmetry + corrections or perhaps TBM an accident ?

(For extensive references, see G. Altarelli, Fermilab Neutrino summer school lectures; Talks at this school by M. Chen, R. Volkas, F. Feruglio)



-0.01

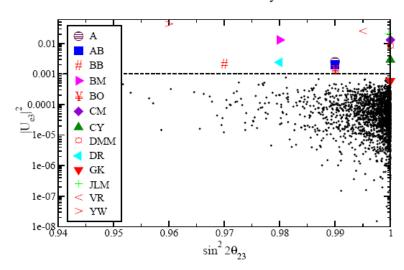
-0.6

# 9<sub>13</sub>:Discriminator between Symmetry vs GUTs

mu-tau sym

0.015 0.01 0.005 0.005 TBM (Albright, Rodejohann)

Normal hierarchy



correlation with atm mixing –

cos2⊕A

■ GUT predictions generally larger than 0.03.

0.2

# NEUTRINO MASS: A SIGNAL OF GRAND UNIFICATION?

Grand unification hypothesis: all forces and all matter become one at high energies no matter how different

they are at low energies. Leptons ->

quarks→

become same.

- ---- Explains charge quantization;
- ----High scale goes well with ideas in cosmology;
- ----Goes well with high scale version of seesaw.



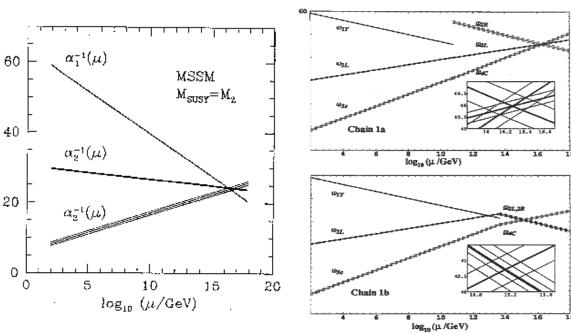
#### Some examples:

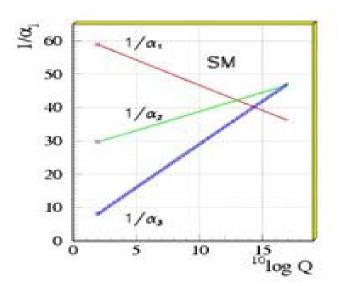
SUSY

Non-SUSY SO(10)

SM

#### with seesaw





Scale of grand unification ~  $10^{16} GeV$ 

### Which GUT group?

- Two simplest are: SU(5) and SO(10):
- (i) SU(5):

(I) SU(5):

minimal: > Fermions: 
$$5 = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e^- \end{pmatrix}$$
 and  $10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ 0 & u_1^c & u_2 & u_3 \\ 0 & u_3 & d_3 \\ e^+ \\ 0 \end{pmatrix}$ 

- > : Higgs 5⊕5 ⊕ 24.
- ightharpoonup Predicts: at  $M_U$ ,  $m_b = m_{\tau}$ ; very good prediction Also predicts  $m_s = m_\mu$ ;  $m_d = m_e$ ; VERY BAD PREDICTION!!
- No explanation of neutrino mass:

### Why SU(5) not satisfactory

- Minimal model ruled out by proton decay!
- Not predictive for neutrinos- so no advantage of GUTs except scale!
- However one nice feature:  $m_b = m_ au$

# SO(10)-Just right for neutrinos

 Minimal GUT group with complete fermion unification (per family) is SO(10)-its spinor rep contains all 16 needed fermions (including RH nu) in single rep.

 $\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{r,p}$ 

- Georgi; Fritzsch, Minkowski (74)
- Contains B-L needed to understand why MR<<</li>
   M\_Planck .
- B-L if properly broken also allows a naturally stable dark matter in MSSM.
- Also helps proton decay problem.

# Appraising SO(10) as a theory of neutrinos

Quark lepton unif. means:

$$M_d = M_l + \delta M$$
 and  $M_u = M_{v^D} + \delta M'$ 

With  $\delta M$  small.

- This means quark and lepton mixings are similar Disaster
- Most models keep breaking symmetry till they get  $\delta M_{v^D} >> M_{u,d,l}$  and one gets large nu-mixings and a model. What trace is left of SO(10) ?

One exception!!

### Minimal Predictive SO(10)

- **Minimal model:** 10+126+.. (Babu, Mohapatra, 93)
- Gives naturally stable dark matter without additional assumption.
- Relates RH neutrino spectrum to charged fermion spectrum reducing seesaw parameters;
   i.e. RH mass M<sub>N</sub> has similar hierarchy as m<sub>D</sub>
- Consequently type I inadequate: a new possibility emerges within the model.

#### How does it work?

T126}-Higgs relates nu matrix AF to quark-lepton flavor

$$M_{\nu} \cong c(M_d - M_l)$$

(Bajc, Senjanovic, Vissani'02)

Even though quark and lepton masses are strongly <u>hierarchical</u>, due to  $m_b \approx m_{\tau}$ ,  $M_{\nu}$ becomes less so and gives

$$\mathcal{M}_{\nu} = m_b c \lambda^2 \begin{pmatrix} \lambda^2 & \lambda^2 & \lambda \\ \lambda^2 & 1 + \lambda & 1 \\ \lambda & 1 & 1 \end{pmatrix} \qquad \begin{array}{c} \lambda = \text{Cabibbo angle.} \\ \lambda^{\text{(Goh, RNM, Ng'03)}} \end{pmatrix}$$

#### **Predictions**

- Large solar and near maximal atmosph. mixing; diluted mass hierarchy, large  $\theta_{13}$
- Predictions: (qualitatively work very well.)
- $\theta_{12}, \theta_{23}$  large

$$\theta_{13} \approx \lambda$$

$$\frac{m_{solar}}{m_{atmos}} \sim \lambda$$
 (Diluted hierarchy)

### A quantitative model that works: Improved SO(10)

- 10+126 model-hard to include CKM CP violation!
- Requires cancellation for proton decay!
- 10+120+126 model with spontaneous CP solves CP problem, proton decay problems while keeping neutrino Sector predictive; :(Dutta, Mimura, RNM,2005,06,07)
- Solution to proton decay dictates flavor texture:

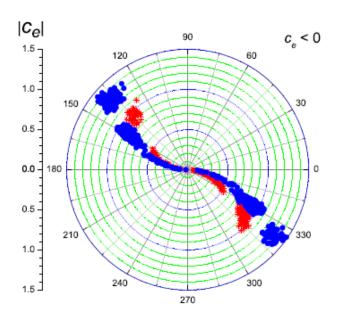
$$h_{10} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; h_{126} = \begin{pmatrix} 0 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}; h_{120} = \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix};$$

• Predicts 
$$\theta_{13} \ge 0.06$$



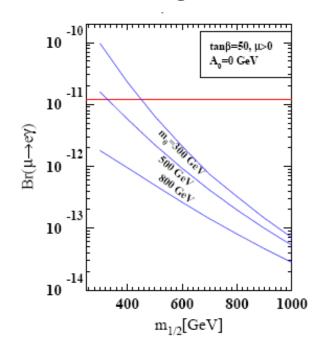
#### **Other Predictions:**

#### Dirac Phase:



$$B(\mu \rightarrow e + \gamma) > 10^{-14}$$
 within range of

#### mu→ e+gamma





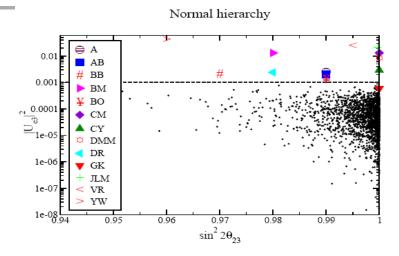
# Further work on SO(10) + 126 models

- Fukuyama, Okada
- Goh, RNM, Ng
- Babu, Macesanu
- Bertolini, Malinsky, Frigerio
- Bertolini, Malinsky, Schwetz
- Aulakh, Bajc, Melfo, Senjanovic, Vissani
- Fukuyama, Okada, Kikuchi, Melajnac, Iljakovic
- Aulakh, Giridhar
- Dutta, Mimura, RNM
- Grimus, Kuhboch
- Aulakh, Garg
- Joshipura, Kodrani, Patel

# Neutrino SUSY GUT summary:

#### **PREDICTIONS**

Large theta\_13-;



■ Mu→e+gamma within MEG range

#### **ISSUES**

- (i) True test of GUTs proton decay  $\tau \approx 10^{35} 10^{36} yrs$
- (ii) How to suppress dim 5 planck induced operator for proton decay:  $\{\psi_m(16)\}^4/M_{Pl}$ ; strength has to be less than  $10^{-7}$



Very hard to test high scale seesaw models!!

Understanding the origin of matter within seesaw—

# Seesaw and Origin of Matter:

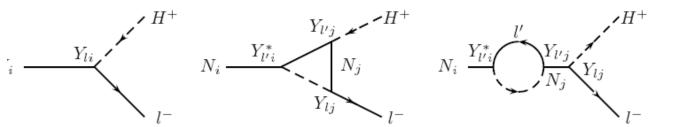
- One advantage of seesaw is the possibility to understand origin of matter, using RH neutrino seesaw couplings. <u>Leptogenesis</u>
- **Proposal:** Heavy  $\nu_R$  decays:

$$\nu_R \to L + H$$
 $R = (1 + \varepsilon)$ 
 $\nu_R \to \overline{L} + \overline{H}$ 
 $R = (1 - \varepsilon)$ 

 Generates lepton asymmetry which gets converted to baryons via sphaleron interactions; (Fukugita, Yanagida'86);



Diagrams:



- Two classes of models depending on RH masses
- High Scale leptogenesis: Adequate asymmetry; lightest RH nu  $M \ge 10^9 GeV$  for hierarchical RH nu's. (Buchmuller, Plumacher, di Bari; Davidson, Ibarra)
- Resonant leptogenesis: degenerate N's, self energy diagram dominates:  $\sim \frac{1}{M_i^2 M_j^2 + M\Gamma}$ ; Resonance  $M_i \cong M_j$ ; works for all B-L scales.

(Liu, Segre'94; Covi et al. Flanz et al.'95; Pilaftsis'97)

## ISSUES WITH HIGH SCALE SUSY LEPTOGENESIS

- Adequate baryogenesis requires the lightest RH neutrino mass  $M_N \ge 3 \times 10^9$  GeV (Davidson, Ibarra)
- Problem for supersymmetric models:

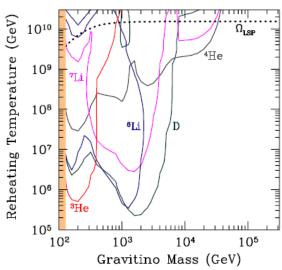
they have gravitinos with TeV mass that are produced during

inflation reheat along with all SM particles

If stable Will overclose the universe for T<sub>R</sub>>10^9 GeV.

If unstable, live too long -effect the success of BBN.

Kawasaki, Kohri, Moroi, Yatsuyanagi, 2008)



# Could Seesaw be a TeV scale phenomenon?

- If so it is very likely that there is a new gauge symmety of Nature beyond SM that couples to RH neutrinos:
- The symmetry could involve either an extra Z' as in  $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$  or both WR and Z' as in left-right models.
- If masses are in few TeV range, production and decays at LHC could provide evidence of their existence via Z' decays  $Z' \to NN; N \to lH$  and WR decays:  $W_R^{\phantom{R}+} \to l^+N \; ; N \to lH$

### Current bounds on WR, Z'

- Collider limits on W<sub>R</sub>: around 780
- Low energy limits: K-K-bar, CPV, edm etc:
   WR mass > 2.5 TeV(Zhang,An,Ji,RNM,2008; adopted by PDG)
- Limits from Neutrinoless double beta decay+ vacuum stability:

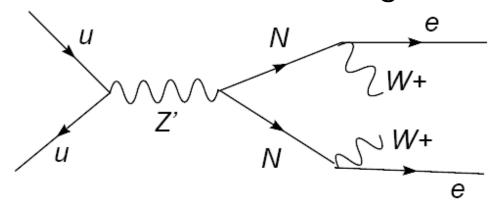
WR mass > 1.5 TeV. (Paes, Simkovic talk)

- Limits are lower for SUSYLR due to sparticle FCNC effects. (Zhang,An,Ji 2008)
- Z' mass bound: > 995 GeV (Langacker, Erler, Munir, Pena)

## 4

### Collider Signatures

- Seesaw effect observable at LHC even with tiny v-N mixings as in generic neutrino models.
- pp $\rightarrow$  Z'+X; Z' $\rightarrow$ NN followed by N-decay;
- Like sign dileptons is the tell-tale seesaw signal.

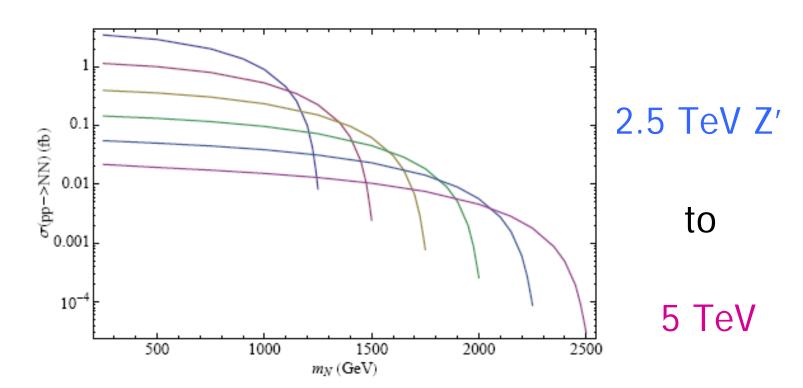


• Keung-Senjanovic  $pp \to W_R^+ \to l^+ N$ 

### 4

#### TeV Z' cross section at LHC

- LHC Z' reach 4 TeV
- Cross section for pp $\rightarrow$ Z' $\rightarrow$ NN (Z' $\rightarrow$ NN branching ratio ~20%)



# Testing seesaw with Z' decay

- PP→Z'+X; xsection for a 3 TeV Z' ~fb
- Seesaw signal: N=Majorana
- N→  $\ell^{\pm}$ W  $\bar{\nu}$  ,  $\bar{\nu}$  + Z W→jj ,  $l\nu$
- Di and Multi-lepton events: (X=jjjj) $pp \rightarrow l^{\pm}l^{\pm}X, l^{\pm}l^{\pm}l^{\mp} + E, l^{\pm}l^{\pm}l^{\mp}l^{\mp} + E$
- Important for signal to bg: very high pT leptons coming from N-decay; inv mass reconstruction:

(Del Aguila, Aguilar-Saavedra; P. Perez, Han, T. Li)

### Does leptogenesis work with TeV Z' and WR?

#### Conditions:

- (i) RH neutrinos must be degenerate in mass to the level of  $M_1 M_2 \sim 10^{-10} M$  since h~10^-5;
- (ii) Since there are fast processes at that temperature, the net lepton asymmetry and primordial lepton asym are related by  $\eta_B \simeq 10^{-2} \sum \varepsilon_{i\alpha} \, \kappa_{i\alpha}$

$$\eta_B \simeq 10^{-2} \sum_{i} \varepsilon_{i\alpha} \, \kappa_{i\alpha}$$

where  $\kappa$  <1- depends on Z' mediated  $e^+e^- \rightarrow NN$ and inverse decay  $lH \rightarrow N$ 

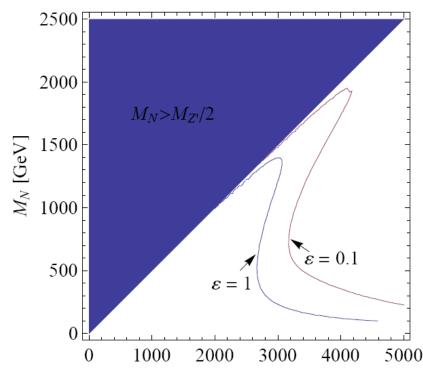
Not clear that a TeV scale Z' is even allowed by baryogenesis due to rapid rates?

# Lower bound on Z' mass from leptogenesis

Lower the Z' mass, faster the scattering and

less the efficiency implying a lower limit on Z' mass !!

•(BLANCHET, CHACKO, GRANOR, RNM: ARXIV:0904.2974)



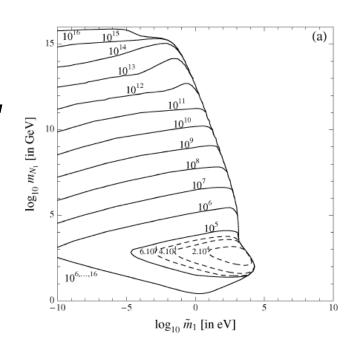


### Limits on WR

- Left-right Model:  $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$
- New fast processes that erase the lepton asymmetry:  $e_R + u_R \rightarrow N + d_R$
- Except when Mw<sub>R</sub> > 18 TeV,

(Frere, Hambye and Vertongen)

Sym br. to U(1)<sub>13R</sub>XU(1)<sub>B-L</sub> then to SM at TeV-to do resonant lepto.

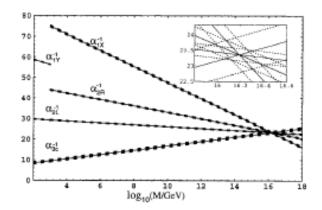


# Unification Prospects for TeV seesaw: An SO(10) possibility

- TeV scale Triplets with B-L=2 hard to unify to SUSY SO(10).
- Both for TeV Z' and WR, unification possible with B-L

=1 doublets breaking  $U(1)_{B-L}$ ; (Deshpande, Keith and Rizzo; 93;

Malinsky, Romao, Valle'05);



This has implications for neutrino mixing:

### Double seesaw for Neutrino masses

B-L=1 breaking -> inverse seesaw for neutrino masses

$$\begin{pmatrix}
0 & hv_{wk} & 0 \\
hv_{wk} & 0 & M \\
0 & M & \mu
\end{pmatrix}
\quad m_{v} \cong -m_{D}^{T}M^{-1}\mu M^{-1}m_{D}$$

- Unlike type I, nu-N mixing  $m_D/M$  decoupled from neutrino mass- so can be large enhancing Nproduction at LHC.
- Unlike type I, Majorana character of RH N (amount) of like sign dileptons) depends on how large  $\frac{\mu}{M}$  is.

  • Unlike type I,  $\mu \to e + \gamma$  can be large without susy.
  • Leptogenesis possible for  $\frac{\mu}{M} > 10^{-6}$

### Testing double seesaw

Can lead to deviations from Unitarity for neutrino mixings <a>s</a>.

Antusch, C. Biggio, E. Fernandez-Martinez, M.B. Gavela, J. Lopez-Pavon Goswami, Ota; Altarelli, Meloni; Malinsky, Ohlsson, Zhang, Xing;)

- Mixing matrix  $N=(1+\eta)U$
- -Current limits on  $\eta_{\alpha\beta}$ :  $\eta_{\mu\tau} < 5 \times 10^{-3}$ ;  $\eta_{e\mu} < 0.0001$  on 12-element from Lepton flavor violation.
- Observable oscillation effect at near detector in neutrino factories as well as far detector.  $\eta_{\mu\tau} < 10^{-4} \, {\rm attainable} \, \, {\rm from \, SBL \, \, with \, 50 \, \, GeV \, \, E}.$

#### Conclusion:

- Need to know-Dirac vs Majorana:
- What may we have learnt?
- Majorana nu > seesaw good paradigm!!
  - may explain the origin of matter
- What we need to know?
  - Scale of new physics (e.g. B-L sym.)- GUT vs
  - TeV scale? May give a hint as to whether
  - large mixing is from dynamics or symmetry.
  - mass ordering, theta\_13 to understand AF

#### Large mixing from Dynamics: Simple hierarchy to Double hierarchicy

$$m_v = -m_D \frac{1}{M_v} m_D^T$$
 mD hierarchical in most models:

• A possibility within type I seesaw: 
$$m_{v} = -m_{D} \frac{1}{M_{N}} m_{D}^{T} \quad \text{mD hierarchical in most models:}$$
•  $m_{v} \sim \begin{pmatrix} \varepsilon_{5}^{n \geq 1} & \varepsilon_{4} & \varepsilon_{3} \\ \varepsilon_{4} & 1 + \varepsilon_{1} & -1 \\ \varepsilon_{3} & -1 & 1 + \varepsilon_{2} \end{pmatrix}$ 
Not very hierarchical

That means M<sub>N</sub> could be "doubly" hierarchical:

(Altarelli, Feruglio, Masina; '03; He, Law, Volkas'08)

A recent example:  $m_D = \operatorname{diag}(m_e, m_\mu, m_\tau) \rightarrow$ 

$$(M_N)_{\rm NH} = \frac{\lambda'}{6p^2m_0} \begin{pmatrix} 4m_e^2 & -2m_e m_\mu & -2m_e m_\tau \\ -2m_e m_\mu & m_\mu^2 & m_\mu m_\tau \\ -2m_e m_\tau & m_\mu m_\tau & m_\tau^2 \end{pmatrix}$$

### **Hierarchy Dilution by** cancellation

Suppose at very high scale, there is a sum-rule:

$$M_{\nu} \cong c(M_d - M_l)$$

• Since at high scale,  $m_h \approx m_{\tau}$  most hierarchical term cancels out:

$$M_{d,l} = m_{b,\tau} \begin{pmatrix} \sim \lambda^4 & \sim \lambda^3 & \sim \lambda^3 \\ \sim \lambda^3 & \sim \lambda^2 & \sim \lambda^2 \\ \sim \lambda^4 & \sim \lambda^2 & 1 \end{pmatrix}$$

$$\begin{array}{ccc} \lambda = \text{Cabibbo angle.} \\ \bullet & \text{Sum rule} \end{array} \xrightarrow{} & \mathcal{M}_{\nu} & = m_b c \lambda^2 \begin{pmatrix} \lambda^2 & \lambda^2 & \lambda \\ \lambda^2 & 1 + \lambda & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

(Bajc, Senjanovic, Vissani; Goh, RNM, Ng)