How to measure $M_F^{0\nu}$ in charge-exchange reactions?

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Outline

- Introduction
- $M_F^{0\nu} \propto \langle IAS|\hat{T}^+|0_f\rangle$
- Reaction analysis
- Conclusions

Neutrinos

massive particles

(talk of F. Šimkovic, 23.09)

Dirac vs. Majorana

$\bar{\nu} \neq \nu$ $\qquad$ $\bar{\nu} = \nu$

$$0\nu\beta\beta \Longleftrightarrow \bar{\nu} = \nu,\ m_\nu > 0$$

Modes of $\beta^-\beta^-$-decay

$$2\nu\beta^-\beta^- \quad (Z,A) \rightarrow (Z+2,A) + e^- + e^- + \overline{\nu}_e + \overline{\nu}_e$$

$$0\nu\beta^-\beta^- \quad (Z,A) \rightarrow (Z+2,A) + e^- + e^-$$

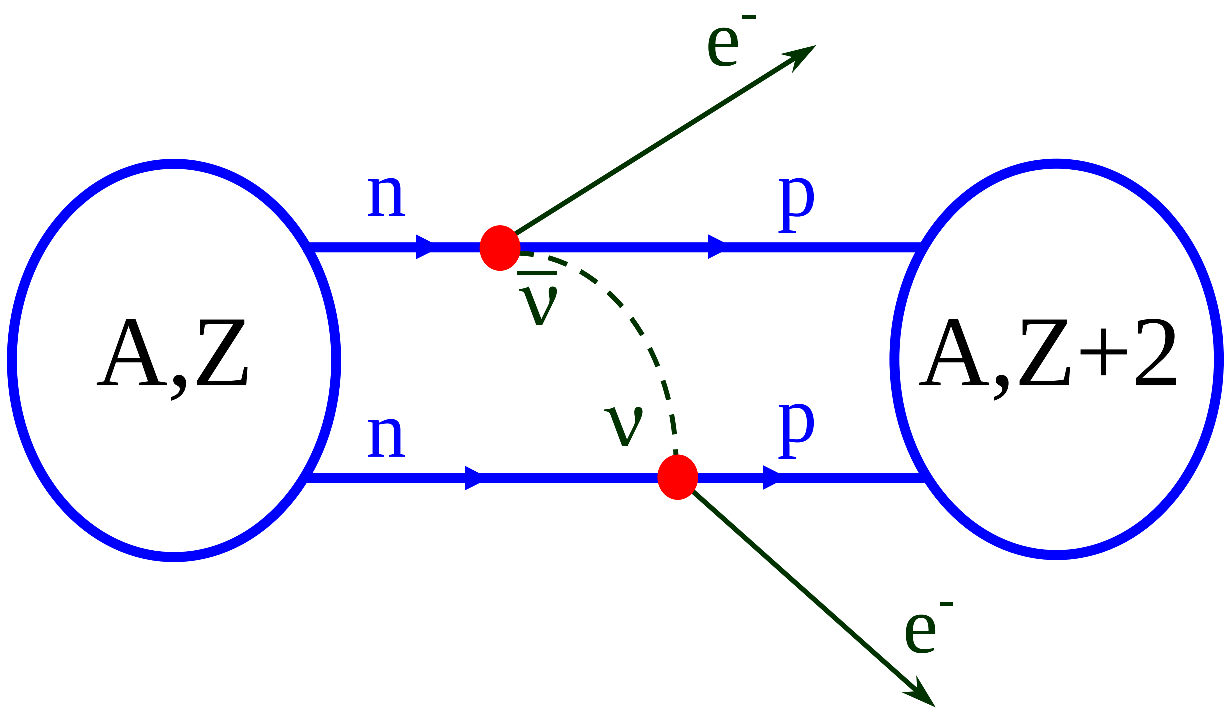
Other modes of $\beta\beta$-decay (smaller Q-values)

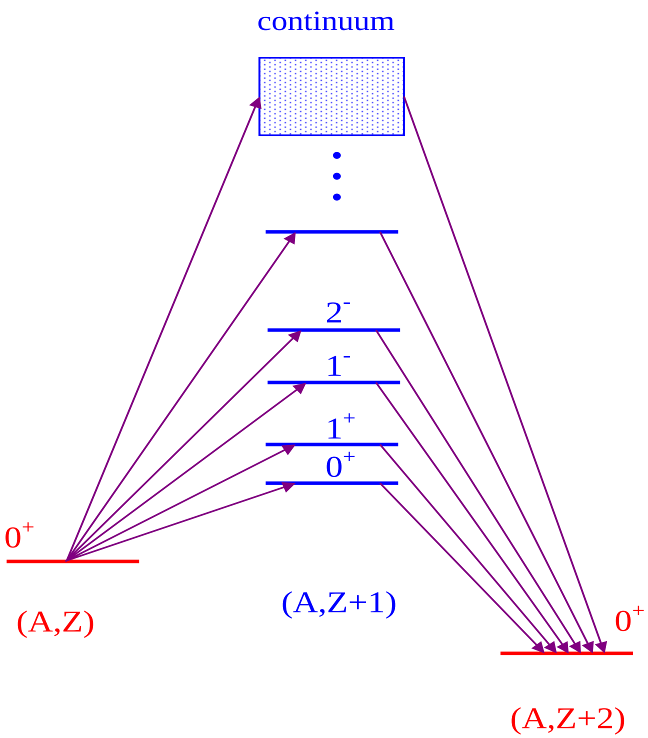
$$2\nu\beta^+\beta^+ \quad (Z,A) \rightarrow (Z-2,A) + e^+ + e^+ + \nu_e + \nu_e$$

$$\mathrm{EC\,EC} \quad (Z,A) + e_b^- + e_b^- \rightarrow (Z+2,A) + \nu_e + \nu_e$$

Nuclear $0\nu\beta\beta$-decay ($\overline{\nu} = \nu$)

Light neutrino exchange mechanism

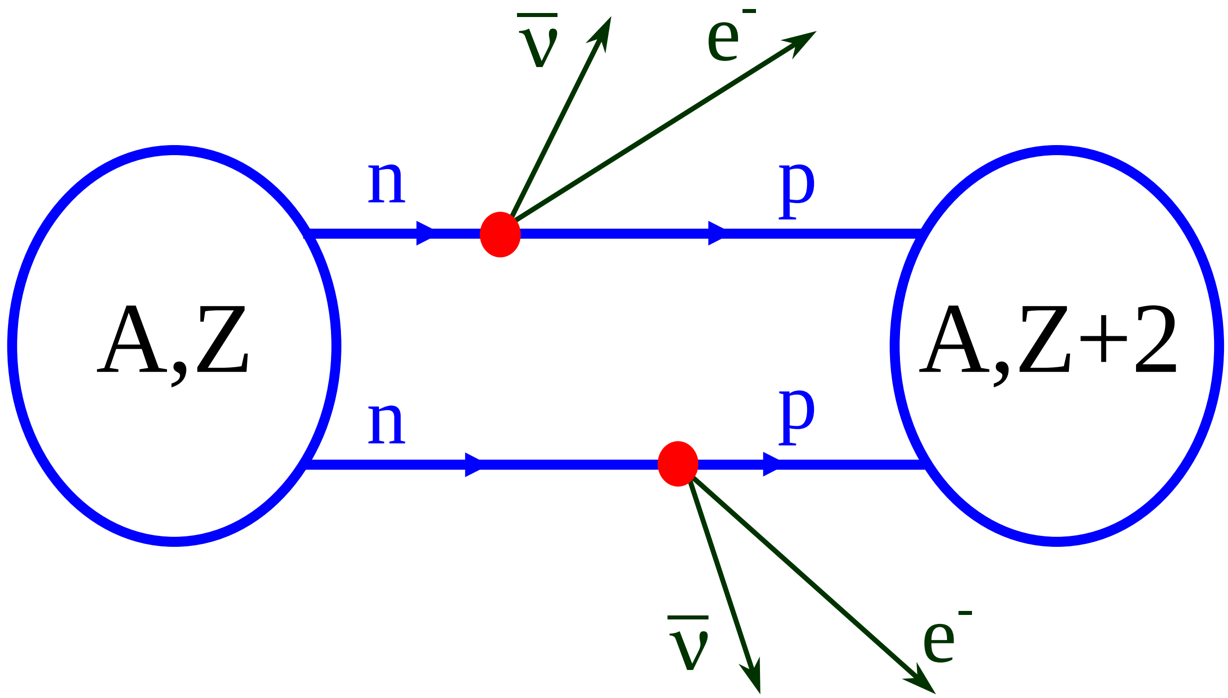




virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

Nuclear $2\nu\beta\beta$-decay

second order weak process
within SM



$2\nu\beta\beta$ $0\nu\beta\beta$

Inverse Half-Lives $[T_{1/2}(0^+ \to 0^+)]^{-1}$

$$G^{2\nu}(Q,Z)\,|M^{2\nu}_{GT}|^2$$

$$m^2_{\beta\beta}\,G^{0\nu}(Q,Z)\,\left|M^{0\nu}_{GT} - \frac{g_V^2}{g_A^2}M^{0\nu}_{F}\right|^2$$

Eff. neutrino mass $m_{\beta\beta} = \sum_j m_j U^2_{ej}$

U_{ej} — first raw of the neutrino mixing matrix

$2\nu\beta\beta$ $0\nu\beta\beta$

Nuclear Matrix Elements

$$M_{GT}^{2\nu} = \sum_s \frac{\langle 0_f || \hat{\beta}^- || s \rangle \langle s || \hat{\beta}^- || 0_i \rangle}{E_s - (M_i + M_f)/2}$$

$$\hat{\beta}^- = \sum_k \boldsymbol{\sigma}_k \tau_k^-$$

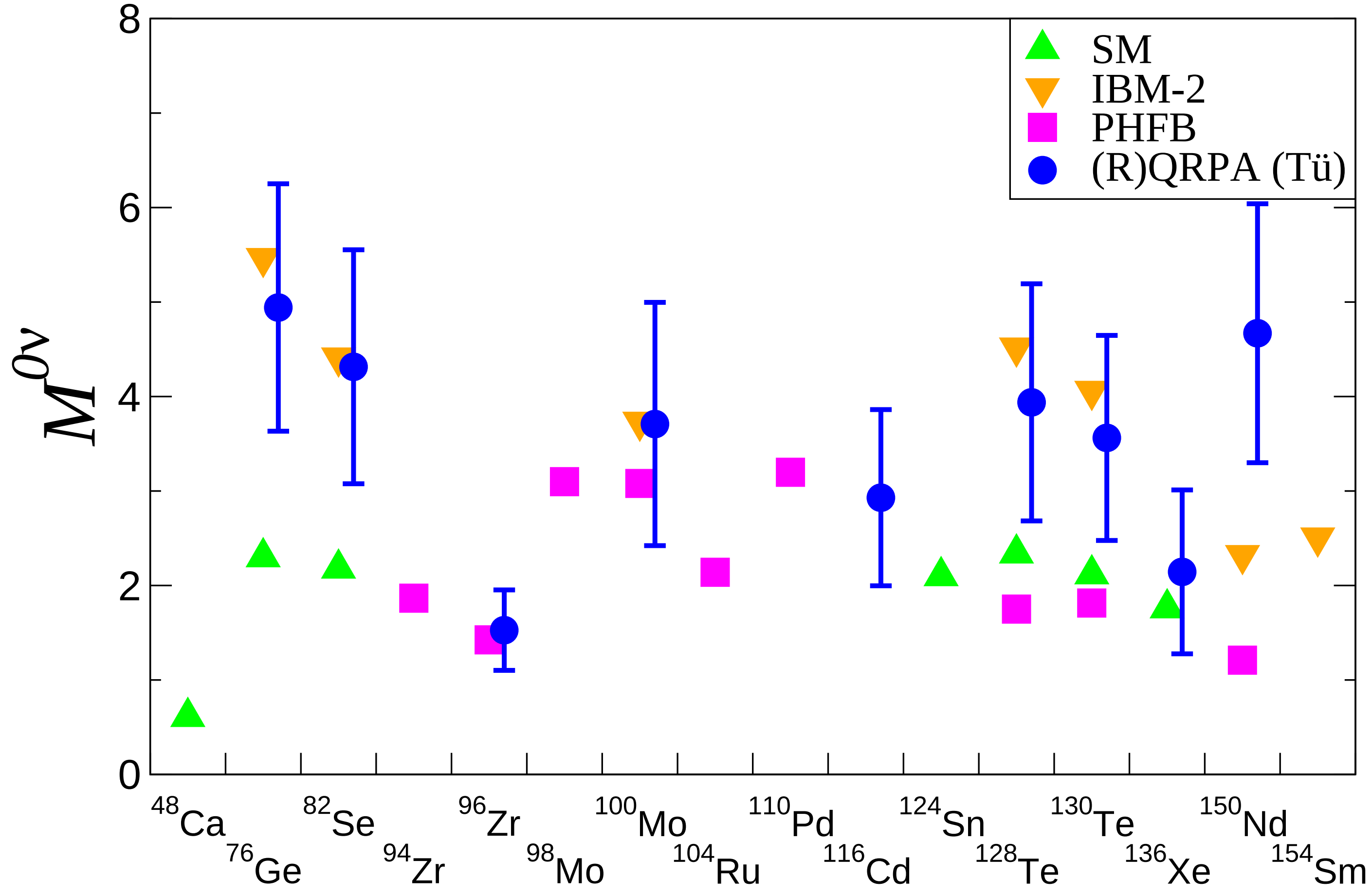
$$M_{GT}^{0\nu} = \langle 0_f | \sum_{ik} P_\nu(r_{ik}, \overline{\omega}) \tau_i^- \tau_k^- \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k | 0_i \rangle$$

Neutrino potential : $P_\nu(r, \overline{\omega}) =$

$$\frac{2R}{\pi r} \int_0^\infty dq \frac{q \sin(qr)}{\omega(\omega + \overline{\omega})} \approx \frac{R}{r} \phi(\overline{\omega} r)$$

World status of $M^{0\nu}$, light neutrino mass mechanism

A. Escuderos, A. Faessler, V. R., F. Šimkovic, arXiv:1001.3519 [nucl-th]



(R)QRPA (Tü) = F. Šimkovic, A. Faessler, V.R., P. Vogel and J. Engel, PRC **77** (2008)

SM = E. Caurier, J. Menendez, F. Nowacki, A. Poves, PRL **100** (2008)

IBM-2 = J. Barea and F. Iachello, PRC **79** (2009)

PHFB = K. Chaturvedi *et al.*, PRC **78** (2008)

Measuring $M_F^{0\nu}$

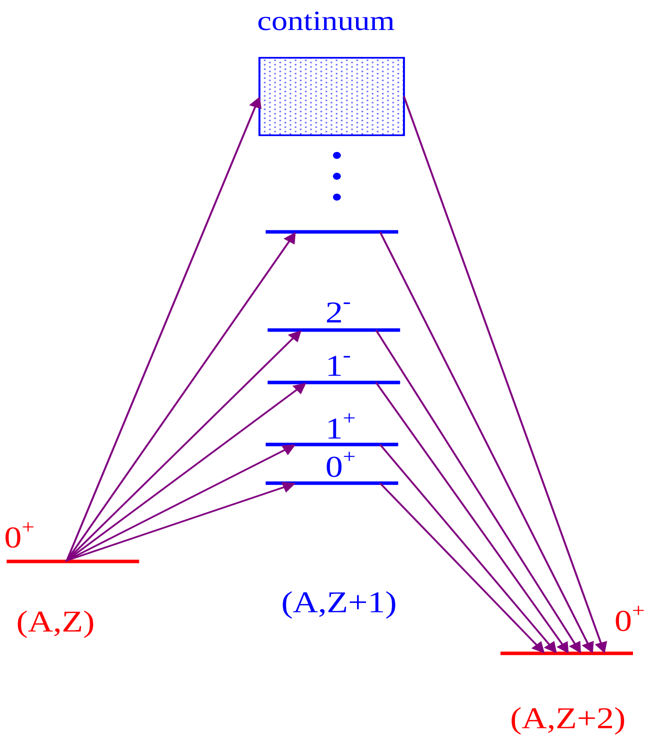
Can one measure nuclear matrix elements of neutrinoless double beta decay?

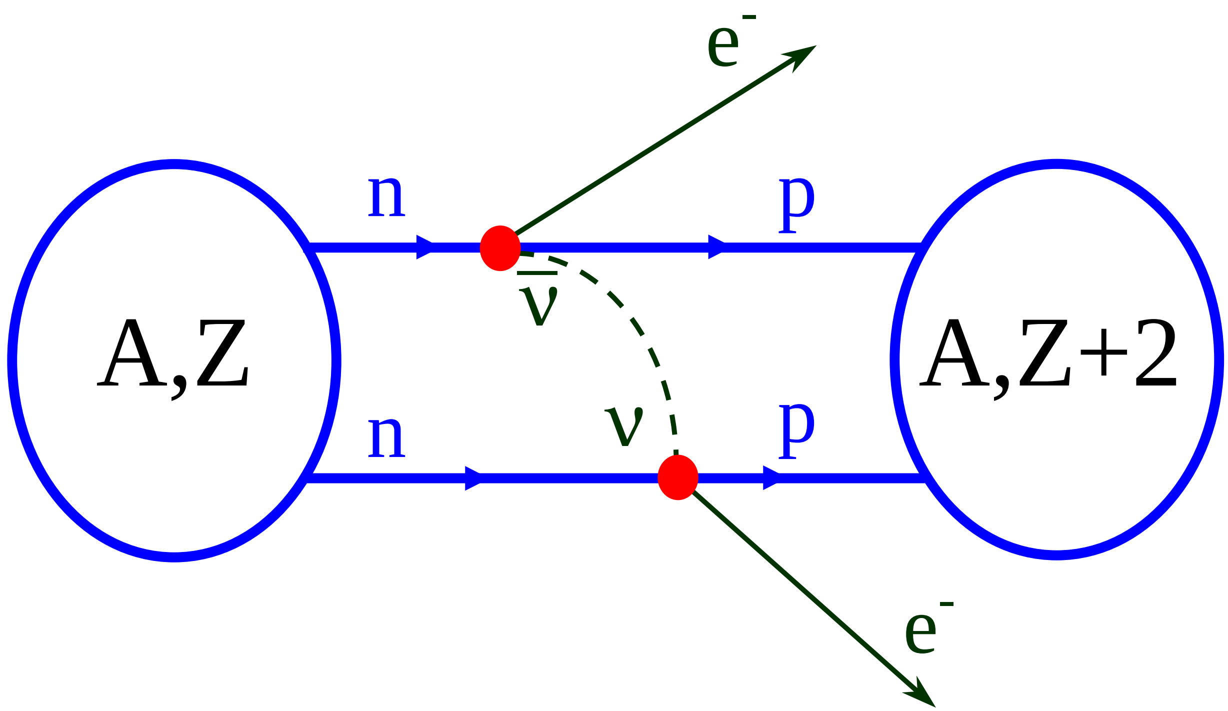
V.R., A. Faessler, PRC **80** , 041302(R) (2009) [arXiv:0906.1759 [nucl-th]]

Measuring $M_F^{0\nu}$

Nuclear $0\nu\beta\beta$-decay ($\overline{\nu} = \nu$)

Light neutrino exchange mechanism

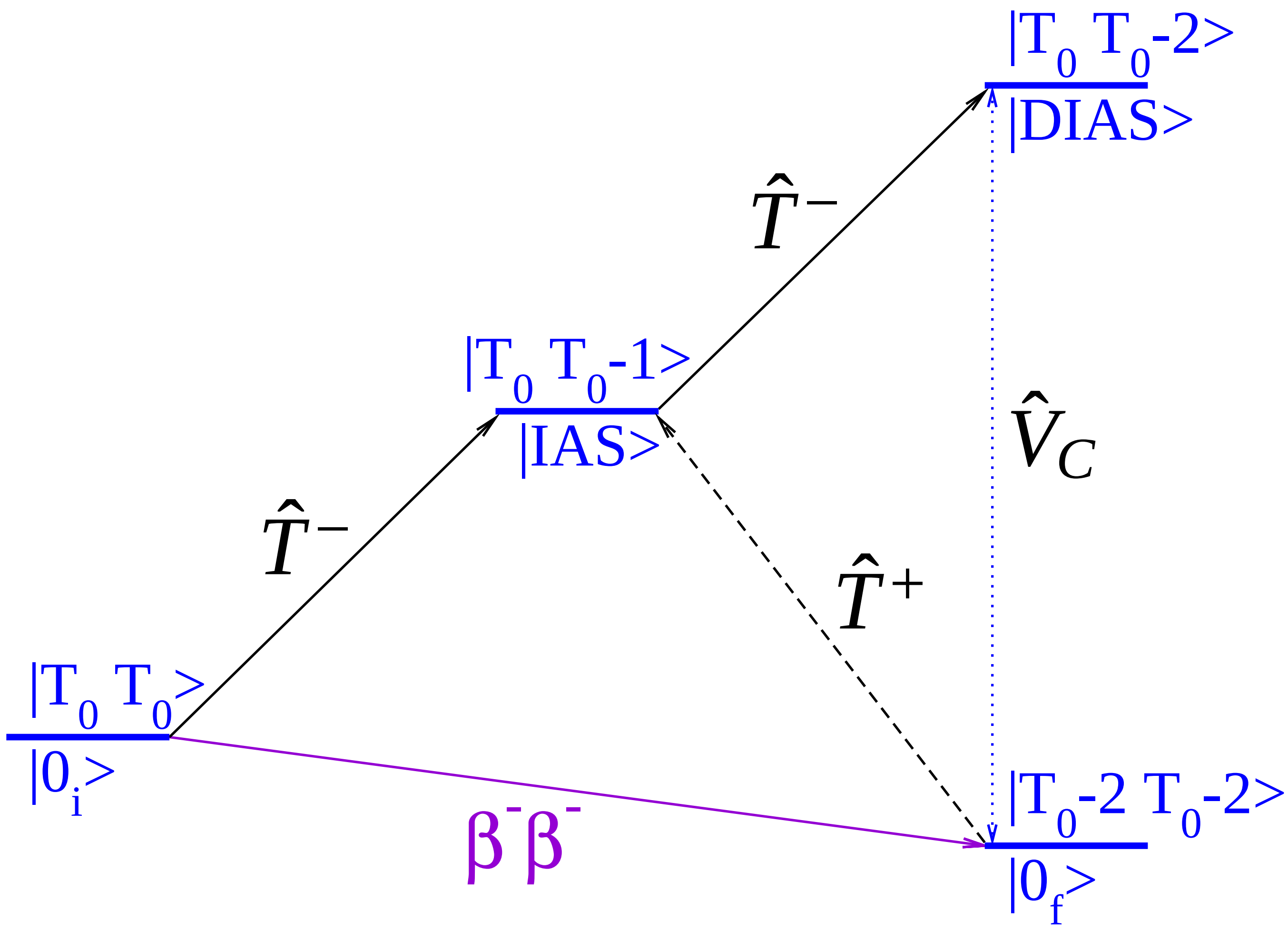




B(GT) of 1^+ — from charge-exchange reactions
(D. Frekers, H. Sakai, R. Zegers, et al.)

Measuring $M_F^{0\nu}$

Double Fermi transition ($J_s^\pi = 0^+$)



$M_F^{2\nu} = 0$ if isospin SU(2) symmetry is exact — Violated by Coulomb

Measuring $M_F^{0\nu}$

$$\hat{W}_F^{0\nu} = \sum_{ab} P_\nu(r_{ab})\tau_a^- \tau_b^- = \frac{1}{e^2}\left[\hat{T}^-, [\hat{T}^-, \hat{V}_C]\right]$$

Isospin lowering operator $\hat{T}^- = \sum_a \tau_a^-$; Coulomb interaction $\hat{V}_C = \frac{e^2}{8}\sum_{a\neq b}\frac{(1-\tau_a^{(3)})(1-\tau_b^{(3)})}{r_{ab}}$

Measuring $M_F^{0\nu}$

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$$\hat{V}_C = \hat{V}_C^{(0)} + \hat{V}_C^{(1)} + \hat{V}_C^{(2)}$$

$$\hat{V}_C^{(0)} = \frac{e^2}{8}\sum_{a \neq b} \frac{1 + \frac{\boldsymbol{\tau}_a \boldsymbol{\tau}_b}{3}}{r_{ab}} \qquad \hat{V}_C^{(1)} = -\frac{e^2}{8}\sum_{a \neq b} \frac{\tau_a^{(3)} + \tau_b^{(3)}}{r_{ab}} \qquad \hat{V}_C^{(2)} = \frac{e^2}{8}\sum_{ab} \frac{T_{ab}^{(2)}}{r_{ab}} \quad \left(T_{ab}^{(2)} \equiv \tau_a^{(3)}\tau_b^{(3)} - \frac{\boldsymbol{\tau}_a \boldsymbol{\tau}_b}{3}\right)$$

Only isotensor $\hat{V}_C^{(2)}$ contributes to $\left[\hat{T}^-, [\hat{T}^-, \hat{V}_C]\right]$

Measuring $M_F^{0\nu}$

$$\hat{W}_F^{0\nu} = \sum_{ab} P_\nu(r_{ab})\tau_a^- \tau_b^- = \frac{1}{e^2}\left[\hat{T}^-, [\hat{T}^-, \hat{V}_C]\right]$$

$$\left[\hat{T}^-, [\hat{T}^-, \hat{V}_C^{(2)}]\right] = \hat{V}_C^{(2)}\left(\hat{T}^-\right)^2 + \left(\hat{T}^-\right)^2 \hat{V}_C^{(2)} - 2\hat{T}^-\hat{V}_C^{(2)}\hat{T}^-$$

$$e^2 M_F^{0\nu} \approx \langle 0_f^+ | V_C^{(2)} \left(\hat{T}^-\right)^2 | 0_i^+ \rangle =$$
$$\langle 0_f^+ | \hat{V}_C^{(2)} | DIAS \rangle \langle DIAS | \left(\hat{T}^-\right)^2 | 0_i^+ \rangle$$

Measuring $M_F^{0\nu}$

$$\hat{H}_{tot} = \hat{T} + \hat{H}_{str} + \hat{V}_C$$

If $\hat{H}_{str}$ exactly isospin-symmetric: $\left[\hat{T}^-, \hat{H}_{str}\right] = 0$

$$\Downarrow$$

$$\hat{W}_F^{0\nu} = \frac{1}{e^2}\left[\hat{T}^-, [\hat{T}^-, \hat{H}_{tot}]\right]$$

Measuring $M_F^{0\nu}$

$$M_F^{0\nu} =$$

$$-\frac{2}{e^2}\sum_s \bar{\omega}_s \langle 0_f^+|\hat{T}^-|0_s^+\rangle\langle 0_s^+|\hat{T}^-|0_i^+\rangle$$

$$\bar{\omega}_s = E_s - (E_{0_i^+} + E_{0_f^+})/2$$

Just equivalent representation of

$$M_F^{0\nu} = \frac{1}{e^2}\langle 0_f^+|\left[\hat{T}^-,[\hat{T}^-,\hat{V}_C^{(2)}]\right]|0_i^+\rangle$$

Measuring $M_F^{0\nu}$

$$M_F^{0\nu} \approx -\frac{2}{e^2}\,\bar{\omega}_{IAS}\langle 0_f^+|\hat{T}^-|IAS\rangle\langle IAS|\hat{T}^-|0_i^+\rangle$$

$$\approx \frac{1}{e^2}\langle 0_f^+|\hat{V}_C^{(2)}|DIAS\rangle\langle DIAS|\left(\hat{T}^-\right)^2|0_i^+\rangle$$

Measuring $M_F^{0\nu}$

Measure the $\Delta T = 2$ isospin-forbidden matrix element $\langle 0_f^+ | \hat{T}^- | IAS \rangle$

charge-exchange (n, p)-type reaction

Challenge: $\langle 0_f^+ | \hat{T}^- | IAS \rangle \sim 0.005$

$\langle IAS | \hat{T}^- | 0_i^+ \rangle \approx \sqrt{N - Z} \sim 5$

$$M_F^{0\nu}(QRPA)/M_F^{0\nu}(SM) \approx 3 \div 5$$

Measuring $M_F^{0\nu}$

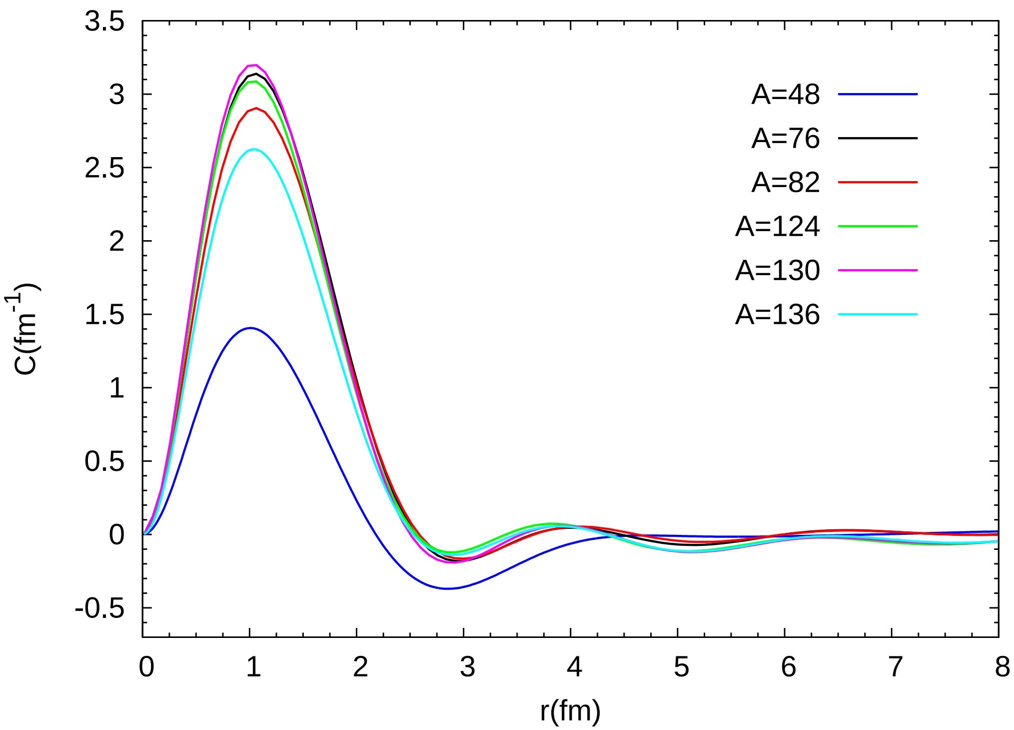
$$\text{But } M_F^{0\nu}/M_{GT}^{0\nu} \approx 0.3$$

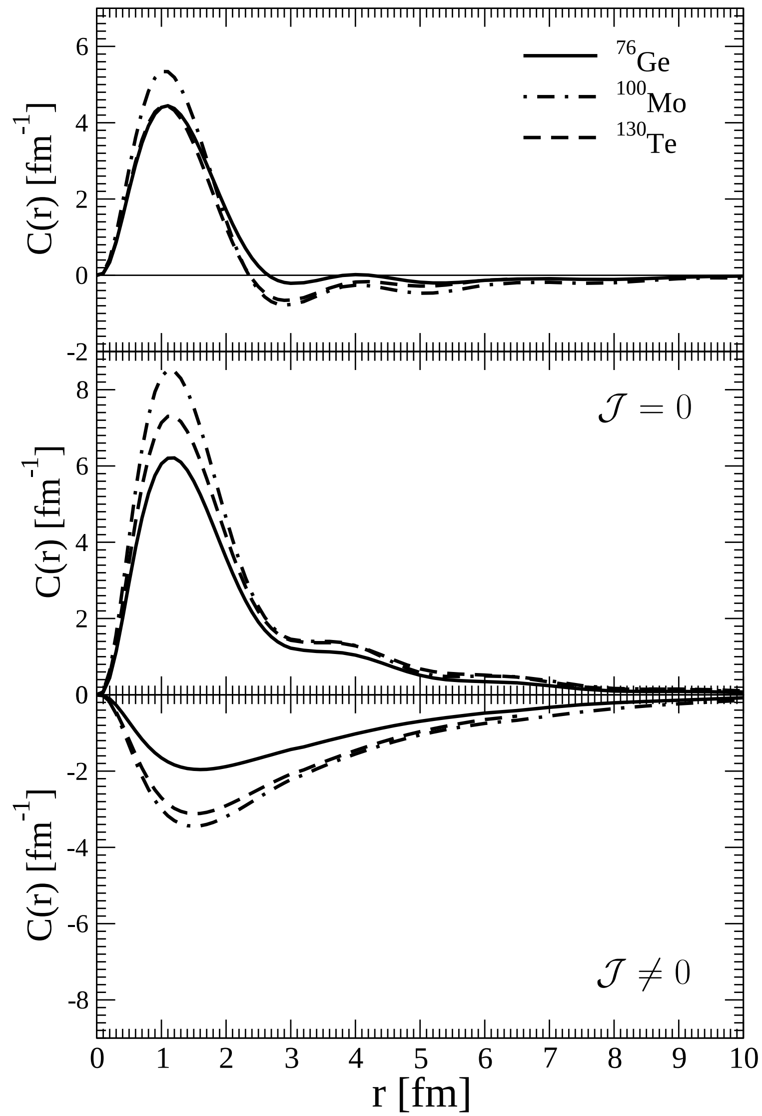
$$\text{Ratio } M_F^{0\nu}/M_{GT}^{0\nu}$$

may be more reliably calculable than $M_F^{0\nu}$ and $M_{GT}^{0\nu}$ separately

Measuring $M_F^{0\nu}$

$$\int_0^\infty C(r)dr = M^{0\nu}$$





Measuring $M_F^{0\nu}$

Only small r_{ab} ~1–2 fm determine $M^{0\nu}$

$\Rightarrow$ nucleon pairs in the relative s-wave contribute $\Rightarrow$ $T = 1, S = 0$ pairs

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 |S = 0, T = 1\rangle = -3|S = 0, T = 1\rangle$$

$$\Downarrow$$

$$M_{GT}^{0\nu} = -3 M_F^{0\nu}$$

provided the neutrino potential is the same in both F and GT cases

High-order terms of nucleon weak current $\Rightarrow$ $M_{GT}^{0\nu}/M_F^{0\nu} \approx -2.5$

Basic requirements for a charge-exchange probe

Measure cross section $\equiv$ Know $\langle IAS|\hat{T}^{+}|0^{+}_{f}\rangle$
???

Reaction analysis

Any hadronic probe adds isospin to nuclear system
(weak interaction probe would be ideal)

to probe small admixture of $|DIAS\rangle$ to $|0_f^+\rangle$
$\Rightarrow$ must be forbidden to connect in reaction
main components of $|IAS\rangle$ and $|0_f^+\rangle$ ($\Delta T = 2$)

Only $T = \frac{1}{2}$ probes ((n, p), $(t, {}^3\mathrm{He})$, ...)

Reaction analysis

$$\sigma_{np}(0^+_f \to IAS) \propto \langle IAS|\hat{T}^+|0^+_f\rangle$$

???

$$|0^+_i\rangle = |T_0\, T_0\rangle; \qquad |IAS\rangle = \frac{\hat{T}^-}{\sqrt{2T_0}}|0^+_i\rangle + \alpha\,|T_0-1\; T_0-1\rangle$$

$$|0^+_f\rangle = |T_0-2\; T_0-2\rangle + \beta\,|T_0-1\; T_0-2\rangle + \gamma\,\frac{(\hat{T}^-)^2}{\sqrt{4T_0(2T_0-1)}}|0^+_i\rangle$$

$$= |DIAS\rangle$$

Reaction analysis

$$|0_i^+\rangle = |T_0\,T_0\rangle; \qquad |IAS\rangle = \frac{\hat{T}^-}{\sqrt{2T_0}}|0_i^+\rangle + \alpha\,|T_0-1\;T_0-1\rangle$$

$$|0_f^+\rangle = |T_0-2\;T_0-2\rangle + \beta\,|T_0-1\;T_0-2\rangle + \gamma\,\frac{(\hat{T}^-)^2}{\sqrt{4T_0(2T_0-1)}}|0_i^+\rangle$$
$$= |DIAS\rangle$$

$$|n\rangle \otimes |0_f^+\rangle \Rightarrow |p\rangle \otimes |IAS\rangle$$

Need: $\gamma\,|n\rangle \otimes |T_0\,T_0-2\rangle \Rightarrow |p\rangle \otimes |T_0\,T_0-1\rangle$

$$T = T_0 \pm \tfrac{1}{2},\; T_z = T_0 - \tfrac{3}{2}$$

Competitive channels:

$|n\rangle \otimes \beta|T_0-1\;T_0-2\rangle \Rightarrow |p\rangle \otimes |T_0\;T_0-1\rangle$ $\quad(T = T_0 - \tfrac{1}{2})$ $\quad$ (1)

$|n\rangle \otimes |T_0-2\;T_0-2\rangle \Rightarrow |p\rangle \otimes \alpha|T_0-1\;T_0-1\rangle$ $\quad(T = T_0 - \tfrac{3}{2})$ $\quad$ (2)

Reaction analysis

$$T^{(1)}_{np}(0^+_f \to IAS) \propto$$

$$\langle p| \otimes \langle T_0\, T_0 - 1| \;\; \hat{V}_{str} \;\; |n\rangle \otimes \sum_s \beta_s |T_0 - 1\; T_0 - 2\rangle_s$$

$$= \frac{\hat{T}^-}{\sqrt{2T_0 - 2}} |T_0 - 1\; T_0 - 1\rangle_s$$

$$\beta_s = \frac{{}_s\langle T_0 - 1\; T_0 - 2|\hat{V}_C|T_0 - 2\; T_0 - 2\rangle}{E_{s_>} - E_{0^+_f}}$$

$$= \frac{{}_s\langle T_0 - 1\; T_0 - 1|\hat{V}^+_C|T_0 - 2\; T_0 - 2\rangle}{\sqrt{2(T_0 - 1)}(E_s + \Delta_C - E_{0^+_f})}$$

$$\hat{V}^+_C = [\hat{T}^+, \hat{V}_C] = \frac{Ze^2}{2R} \sum_a (3 - \frac{r_a^2}{R^2})\tau^+_a \;\; (r_a < R)$$

Reaction analysis

$$T^{(1)}_{np} \propto \frac{\langle T_0-2\; T_0-2|\hat{V}^{-}_{C}|T_0-1\; T_0-1\rangle_s}{\sqrt{2T_0}(E_s+\Delta_C-E_{0^+_f})}\times$$

$$\times\; {}_s\langle T_0-1\; T_0-1|a_n(k')\,\hat{V}_{str}\,a^\dagger_p(k)|T_0\; T_0\rangle$$

Assume $\sigma_{pn}(0^+_i \to IVMR_s) = \sigma_0 \left|\langle IVMR_s|\hat{R}^-|0^+_i\rangle\right|^2, \quad \hat{R}^- = \sum_a \frac{r_a^2}{R^2}\tau^-_a$

$$r^{(1)} = \frac{\sigma^{(1)}_{np}(0^+_f \to IAS)}{\sigma_{pn}(0^+_i \to IVMR)} = \frac{1}{2T_0}\left(\frac{Ze^2}{2R}\right)^2 \frac{\left|\sum_s \frac{\langle 0^+_f|\hat{R}^-|s\rangle\langle s|\hat{R}^-|0^+_i\rangle}{E_s+\Delta_C-E_{0^+_f}}\right|^2}{\sum_s\left|\langle s|\hat{R}^-|0^+_i\rangle\right|^2}$$

Reaction analysis

$$T_{np}^{(2)} \propto \langle T_0 - 2\, T_0 - 2 | a_n(k')\, \hat{V}_{str}\, a_p^{\dagger}(k) | T_0 - 1\, T_0 - 1 \rangle_s \times$$

$$\times \frac{{}_s\langle T_0 - 1\, T_0 - 1 | \hat{V}_C^- | T_0\, T_0 \rangle}{\sqrt{2T_0}(E_s - \Delta_C - E_{0_i^+})}$$

$$r^{(2)} = \frac{\sigma_{np}^{(2)}(0_f^+ \to IAS)}{\sigma_{np}(0_f^+ \to IVMR)} = \frac{1}{2T_0}\left(\frac{Ze^2}{2R}\right)^2 \frac{\left|\sum_s \frac{\langle 0_f^+ | \hat{R}^- | s \rangle \langle s | \hat{R}^- | 0_i^+ \rangle}{E_s - \Delta_C - E_{0_i^+}}\right|^2}{\sum_s \left|\langle s | \hat{R}^+ | 0_f^+ \rangle\right|^2}$$

Reaction analysis

^{82}Se, $7\hbar\omega$ s.p. space, $\Delta N = 2$ p-h excitations, independent Bogolyubov quasiparticles

$$r^{(1)} = 6.6 \cdot 10^{-7}\,; \quad r^{(2)} = 1.3 \cdot 10^{-5}$$

$$\frac{\sigma^{(1)}_{np}(0^+_f \to IAS)}{\sigma_{np}(\gamma DIAS \to IAS)} \approx 10^6 r^{(1)} \cdot \frac{\sigma_{pn}(0^+_i \to IVMR)}{\sigma_{pn}(0^+_i \to IAS)}$$

$$\frac{\sigma^{(2)}_{np}(0^+_f \to IAS)}{\sigma_{np}(\gamma DIAS \to IAS)} \approx 10^6 r^{(2)} \cdot \frac{\sigma_{np}(0^+_f \to IVMR)}{\sigma_{pn}(0^+_i \to IAS)}$$

with $\sigma_{np}(\gamma DIAS \to IAS) \approx 10^{-6} \sigma_{pn}(0^+_i \to IAS)$

Reaction analysis

^{82}Se, $\Delta N = 0$ p-h excitations

subtler, one needs good isospin in calculations

continuum-QRPA, zero-range forces, $\hat{R}^- \rightarrow \hat{R}^- - a\hat{T}^-$ ($a = \frac{\langle IAS|\hat{R}^-|0\rangle}{\langle IAS|\hat{T}^-|0\rangle}$)

$$r^{(1)} = 6.4 \cdot 10^{-8}\,; \quad r^{(2)} = 2.2 \cdot 10^{-6}$$

IAS of ^{48}Ca ($T = 4, T_z = 3$) in ^{48}Sc

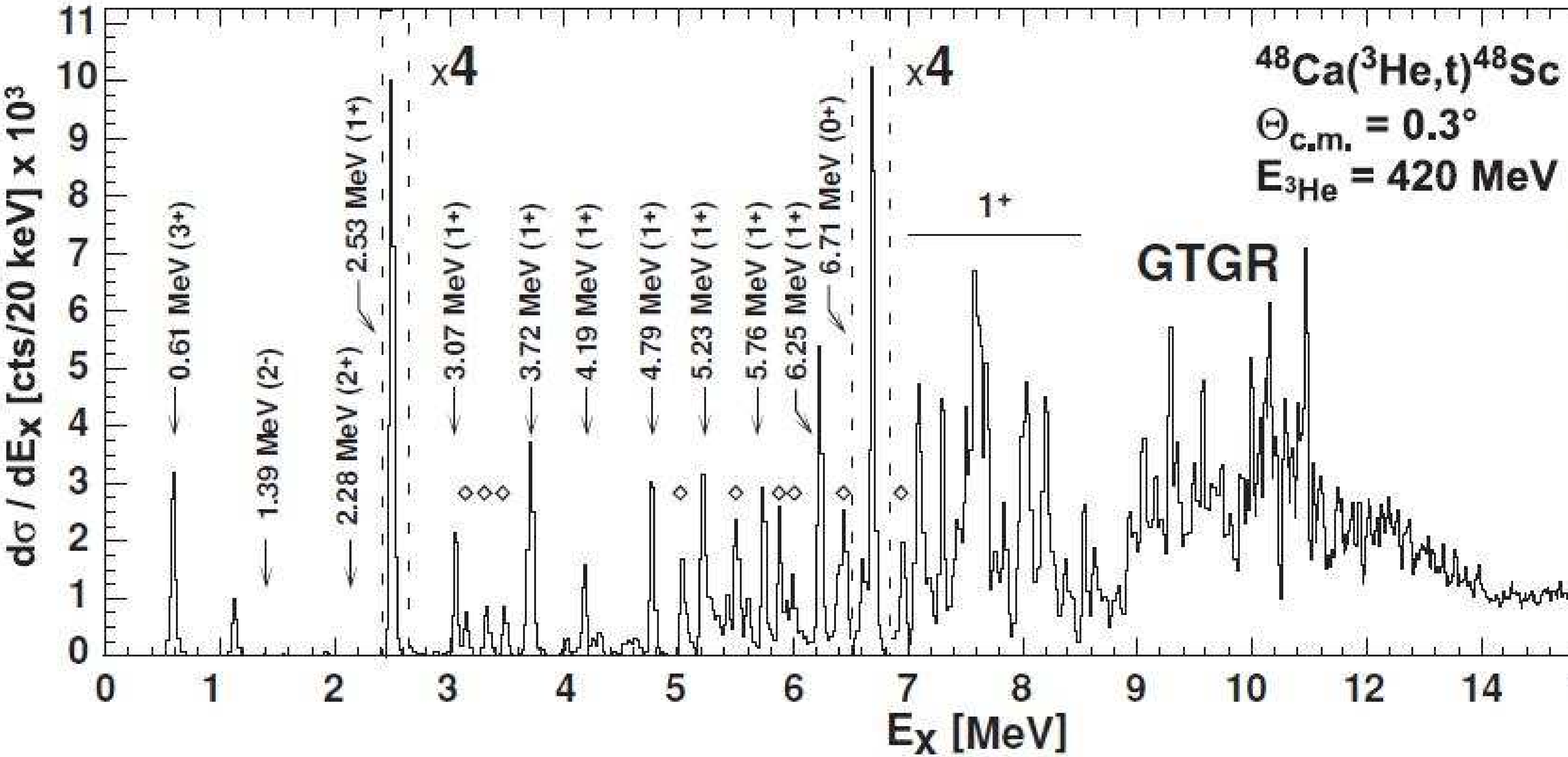
1. locates at E_x =6.678 MeV
2. 100% γ-decay to 1^+ state at E_x =2.517 MeV (E_γ=4.160 MeV)

Reaction analysis

E. W. Grewe, D. Frekers *et al.*, PRC **76**, 054307 (2007)

Resolution = 40 keV

IAS↓



Conclusions

- $M_F^{0\nu}$ can be related to Coulomb m.e. determining $\Delta T = 2$ isospin admixture of the DIAS in the final g.s.

- $M_F^{0\nu}$ can be reconstructed if one is able to measure Fermi m.e. $\langle IAS|\hat{T}^+|0_f\rangle$ (e.g. charge-exchange (n, p)-type reactions)

- can help to discriminate between nuclear structure models (difference in $M_F^{0\nu}$ as much as the factor of 5)

- Estimate $M_{GT}^{0\nu}/M_F^{0\nu} \approx -2.5$ must hold

Conclusions

- Estimates show that $\sigma_{np}(0_f \to IAS) \propto \langle IAS|\hat{T}^+|0_f\rangle$

- Choice of a target: better well-isolated IAS. ^{48}Ca ?
(in principle, any pair of even-even nuclei with $\Delta Z = 2$)

- Role of spread of IAS in heavy nuclei to be investigated

Supported by: DFG SFB TR27 "Neutrinos and beyond"

Backup

$$M_F^{0\nu} \approx -\frac{2}{e^2}\,\bar{\omega}_{IAS}\langle 0_f|\hat{T}^-|IAS\rangle\langle IAS|\hat{T}^-|0_i\rangle$$

$$\langle 0_f|\hat{T}^-|IAS\rangle\langle IAS|\hat{T}^-|0_i\rangle = \langle 0_f|DIAS\rangle\langle DIAS|\hat{T}^-|IAS\rangle\langle IAS|\hat{T}^-|0_i\rangle$$

$$\langle 0_f|DIAS\rangle = -\frac{\langle 0_f|\hat{V}_C^{(2)}|DIAS\rangle}{E_{DIAS} - E_{0_f}}, \quad \text{with } E_{DIAS} - E_{0_f} \approx 2\bar{\omega}_{IAS}\,.$$

$$M_F^{0\nu} \approx \frac{1}{e^2}\langle 0_f|\hat{V}_C^{(2)}|DIAS\rangle\langle DIAS|\left(\hat{T}^-\right)^2|0_i\rangle$$

Backup

Switching off Coulomb

$$\hat{H}_{tot}(\lambda) = \hat{T} + \hat{H}_{str} + \lambda \hat{V}_C$$

$$\hat{W}_F^{0\nu} = \frac{1}{e^2\lambda}\left[\hat{T}^-, [\hat{T}^-, \hat{H}_{tot}]\right]$$

$$M_F^{0\nu} = -\frac{2}{e^2\lambda}\sum_s \bar{\omega}_s \langle 0_f|\hat{T}^-|0_s^+\rangle\langle 0_s^+|\hat{T}^-|0_i\rangle$$

Backup

$$\lambda \rightarrow 0$$

$$e^2 M_F^{0\nu} = \langle 0_f | \left[\hat{T}^-, [\hat{T}^-, \hat{V}_C] \right] | 0_i \rangle$$

$$= \langle 0_f | \hat{V}_C \hat{T}^- \hat{T}^- | 0_i \rangle$$

$$= \langle 0_f | \hat{V}_C | DIAS \rangle \langle DIAS | \hat{T}^- \hat{T}^- | 0_i \rangle$$

Backup

$$\hat{V}_C = \overline{V}_C + \Delta\hat{V}_C$$

$$\overline{V}_C = \hat{V}_C^{(0)} + \overline{V}_C^{(1)} + \overline{V}_C^{(2)}$$

$$\overline{V}_C^{(1)} = -\frac{e^2}{8}\sum_{a\neq b}\frac{\tau_a^{(3)} + \tau_b^{(3)}}{R_1} = -\frac{e^2 A}{2R_1}\hat{T}^{(3)}$$

$$\Delta\hat{V}_C^{(1)} = -\frac{e^2}{8}\sum_{a\neq b}(\tau_a^{(3)} + \tau_b^{(3)})(\frac{1}{r_{ab}} - \frac{1}{R_1})$$

$$\overline{V}_C^{(2)} = \frac{e^2}{8}\sum_{ab}\frac{T_{ab}^{(2)}}{R_2} = \frac{e^2}{2R_2}(\hat{T}^{(3)}\hat{T}^{(3)} - \frac{\mathbf{T}^2}{3})$$

$$\Delta\hat{V}_C^{(2)} = \frac{e^2}{8}\sum_{ab} T_{ab}^{(2)}(\frac{1}{r_{ab}} - \frac{1}{R_2})$$

$\overline{V}_C$ does not mix $|T\ T_z\rangle$

Backup

$$\langle 0_f | \overline{V}_C^{(2)} \left(\hat{T}^-\right)^2 |0_i\rangle =$$

$$\frac{e^2}{2R_2} \langle 0_f | \left(\hat{T}^-\right)^2 |0_i\rangle = \frac{e^2}{2R_2} \sum_s \langle 0_f | \hat{T}^- | 0_s^+\rangle \langle 0_s^+ | \hat{T}^- | 0_i\rangle$$

Suppression by $\frac{e^2}{4R_2\overline{\omega}_{IAS}} \ll 1$

$$\langle DIAS | \hat{V}_C^{(2)} | 0_f^+\rangle = \langle DIAS | \Delta\hat{V}_C^{(2)} | 0_f^+\rangle.$$