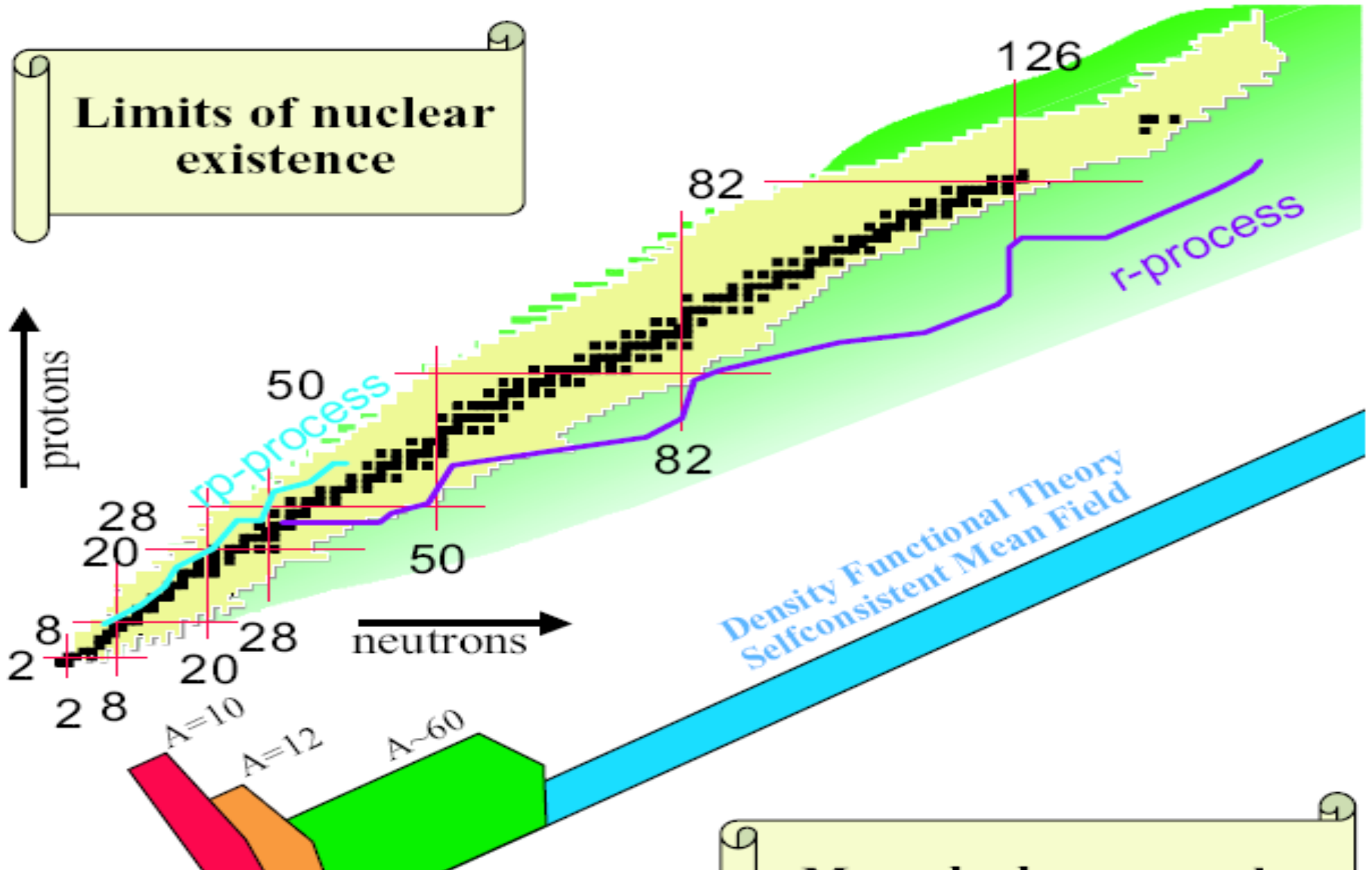


# Density Functional Theory for Processes of Astrophysical Interest

H. Lenske

Institut für Theoretische Physik, U. Giessen





$$E(A)/A = -16\text{MeV} + E_{\text{surf}}/A^{1/3} + E_{\text{pair}} + E_{\text{shell}} + E_{\text{coul}} + [(N-Z)/A]^2(a_4 + C_{\text{sym}}/A^{1/3})$$

## Agenda:

- Sketch of Density Functional Theory
- Pairing in the Continuum
- Dynamical Correlations and Fano-Resonances
- New Modes of Excitation: Pygmy Dipole States



...the Mass Formula:  
a functional of neutron number N and proton number Z

$$E(N,Z)/A = -16\text{MeV} + E_{\text{surf}}/A^{1/3} + E_{\text{pair}} + E_{\text{shell}} + E_{\text{coul}} \\ + [(N-Z)/A]^2(a_4 + C_{\text{sym}}/A^{1/3})$$

# The Nuclear Energy Density Functional

$$\begin{aligned}
 E(\rho, \kappa) \approx E(\rho_0, \kappa_0) &+ \sum_{q=p,n,\Lambda \dots} \left. \frac{\partial E(\rho, \kappa)}{\partial \rho_q} \right|_{\rho_0} \delta \rho_q + \sum_{q,q'=p,n,\Lambda \dots} \left. \frac{\partial^2 E(\rho, \kappa)}{\partial \rho_q \partial \rho_{q'}} \right|_{\rho_0} \delta \rho_q \delta \rho_{q'} + \dots \\
 &+ \sum_{q=p,n,\Lambda \dots} \left. \frac{\partial E(\rho, \kappa)}{\partial \kappa_q} \right|_{\kappa_0} \delta \kappa_q + \sum_{q,q'=p,n,\Lambda \dots} \left. \frac{\partial^2 E(\rho, \kappa)}{\partial \kappa_q \partial \kappa_{q'}} \right|_{\kappa_0} \delta \kappa_q \delta \kappa_{q'} + \dots
 \end{aligned}$$

$$\begin{aligned}
 E(\rho, \kappa) \approx E(\rho_0, \kappa_0) &+ \sum_{q=p,n} \left( (T_q + U_q(\rho_0)) \delta \rho_q + \Delta_q \delta \kappa_q \right) \\
 &+ \sum_{q,q'=p,n} f_{qq'}(\rho_0) \delta \rho_q \delta \rho_{q'} + \dots
 \end{aligned}$$

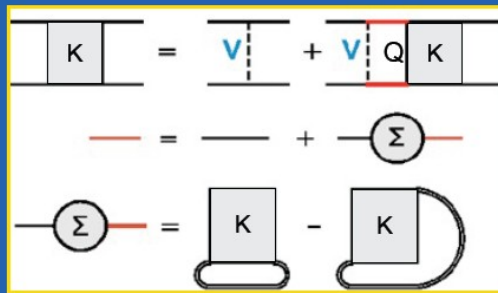
## Fermi-Liquid Theory

(A. Ataie, A. Fedoseew, Nadia Tsoneva H.L.)

# The Giessen Program: *ab initio* Nuclear DFT:

- NN-Interaction in free space
- in-medium interactions by Dirac-Brueckner Theory
- adding 3-body interactions
- density dependent meson-nucleon vertex functionals
- equation of state, nuclear binding energies, single particle spectra, excitations, reactions...

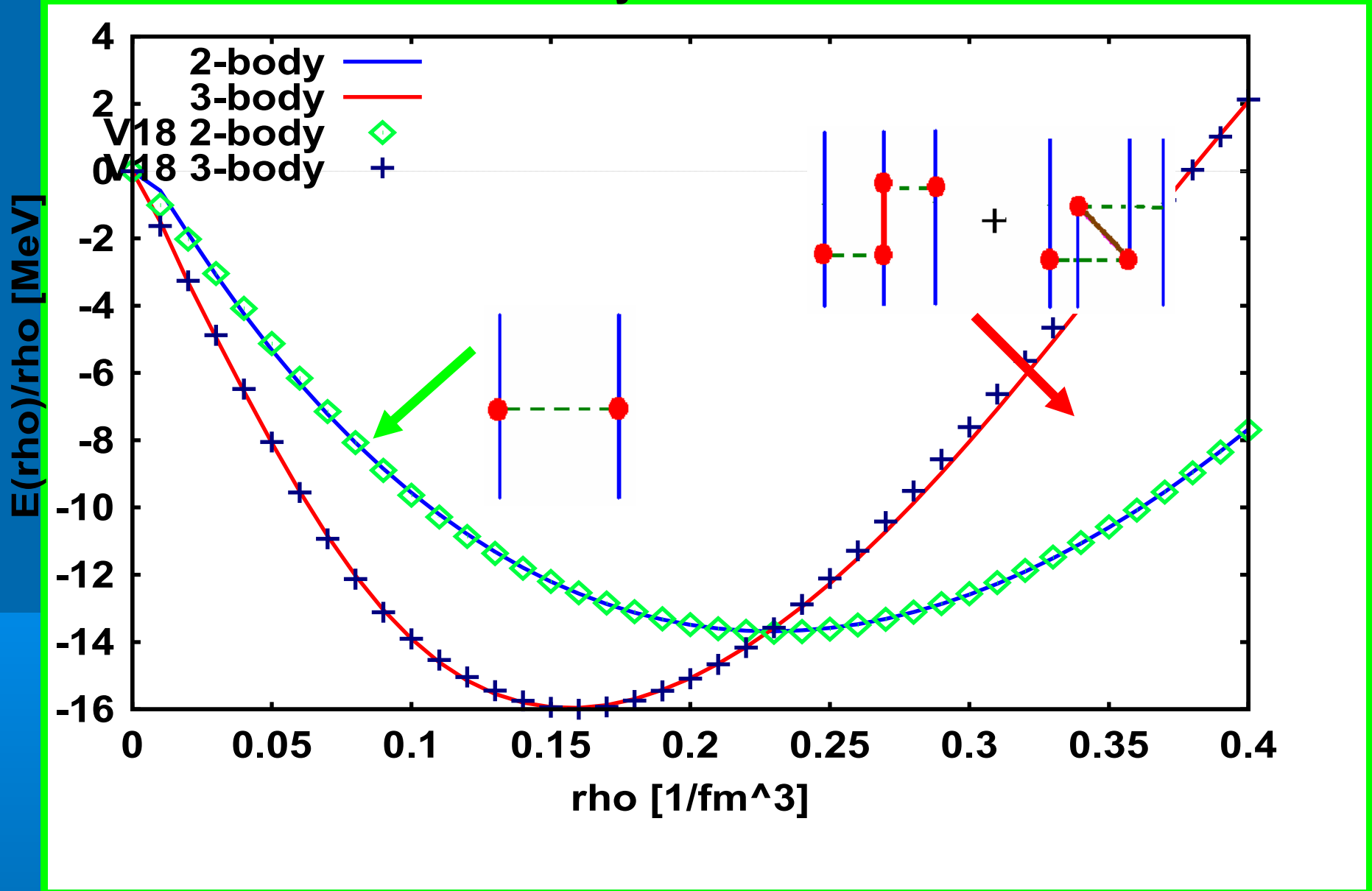
$$K = V + \int V Q_F K$$



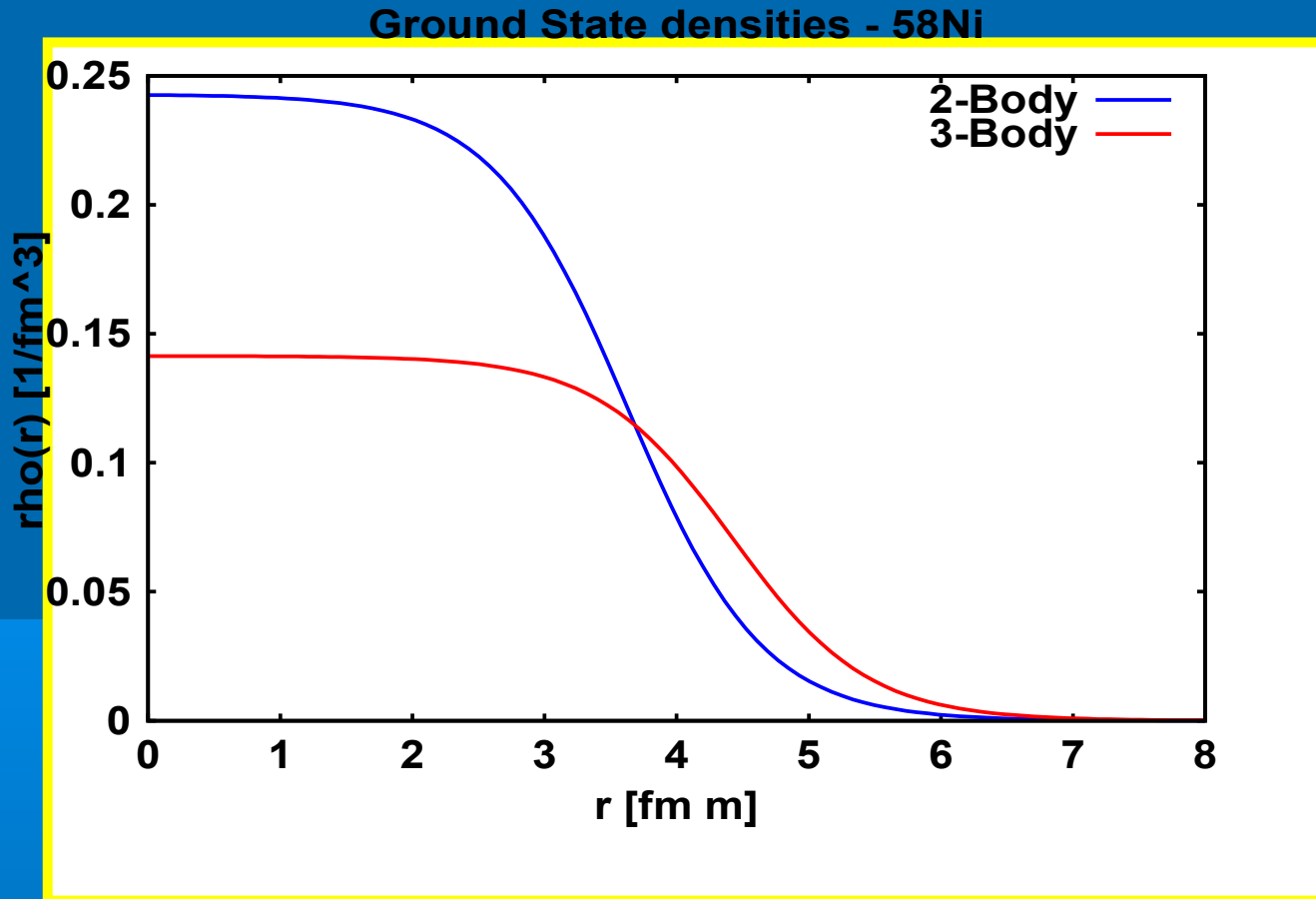
- Ladder Kernel
- Map the ab-initio calculations on an effective Lagrangian
- Medium dependent renormalization

$$V_{OBE} = \sum_{\alpha} g_{\alpha}^2 D_{\alpha}(t) \langle \bar{u}_1 \hat{O}_{\alpha} u_3 \rangle \langle \bar{u}_2 \hat{O}_{\alpha} u_4 \rangle$$

# EoS of Symmetric Matter



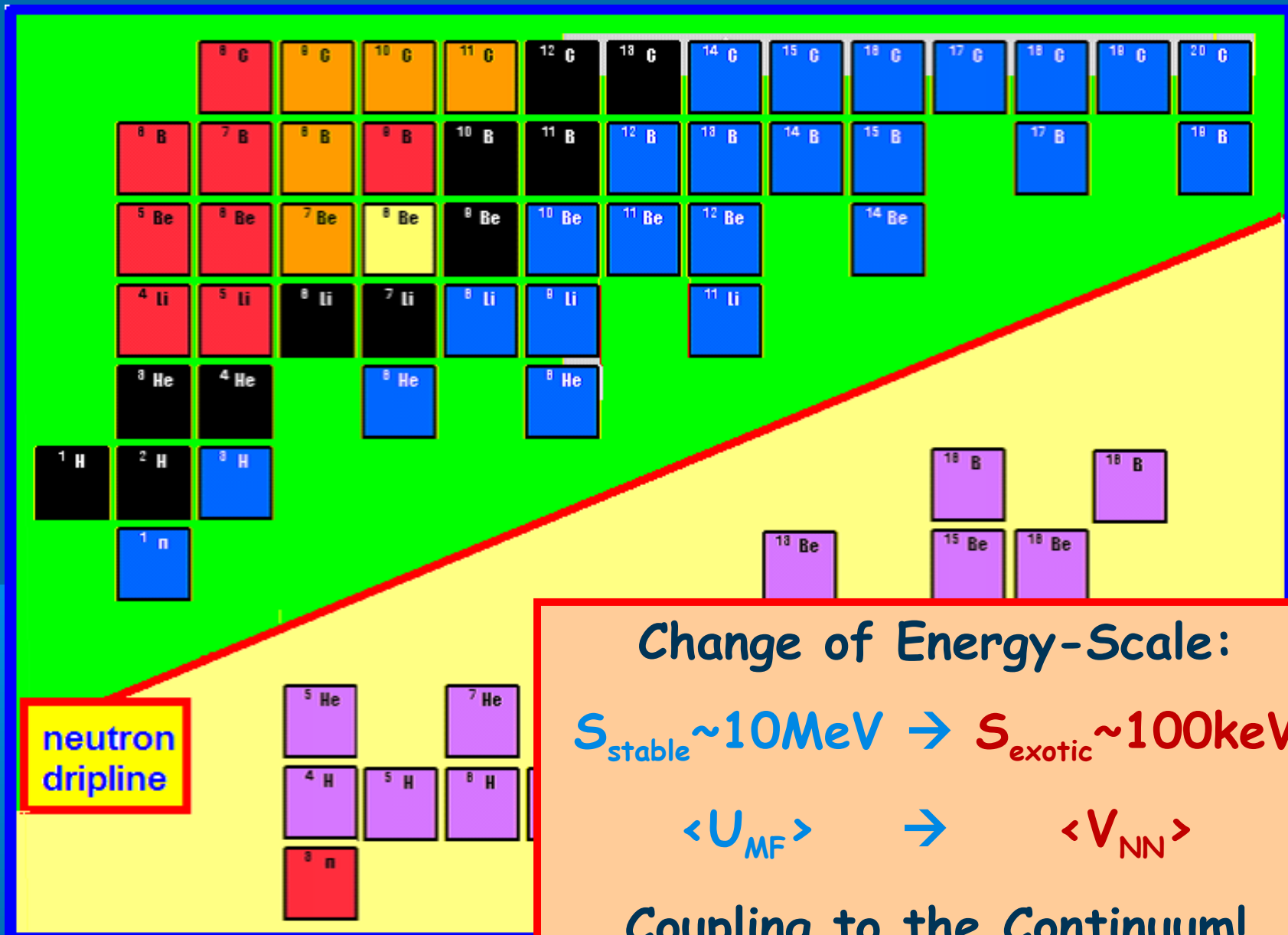
# 2-body and 3-body Interactions in Nuclei



...two different worlds!



# Fluctuating (Fractal?) Structure of the Neutron Dripline



Change of Energy-Scale:

$$S_{\text{stable}} \sim 10 \text{ MeV} \rightarrow S_{\text{exotic}} \sim 100 \text{ keV}$$

$$\langle U_{\text{MF}} \rangle \rightarrow \langle V_{\text{NN}} \rangle$$

Coupling to the Continuum!

# Pairing Theory as Coupled Channels Problem: The Gorkov-Equations

$$\begin{pmatrix} H - \lambda & -\Delta \\ -\Delta^\dagger & -(H - \lambda) \end{pmatrix} \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix} = E \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix}$$

$$\phi_+ \sim u_{lj}^{(q)}(r) |(\ell s) jm\rangle; \quad \phi_- \sim v_{lj}^{(q)}(r) |(\ell s) jm\rangle$$

Mean-Field Hamiltonian (q = p,n):

$$H = -\frac{\hbar^2}{2m} \vec{\nabla}^2 + U(\rho)$$

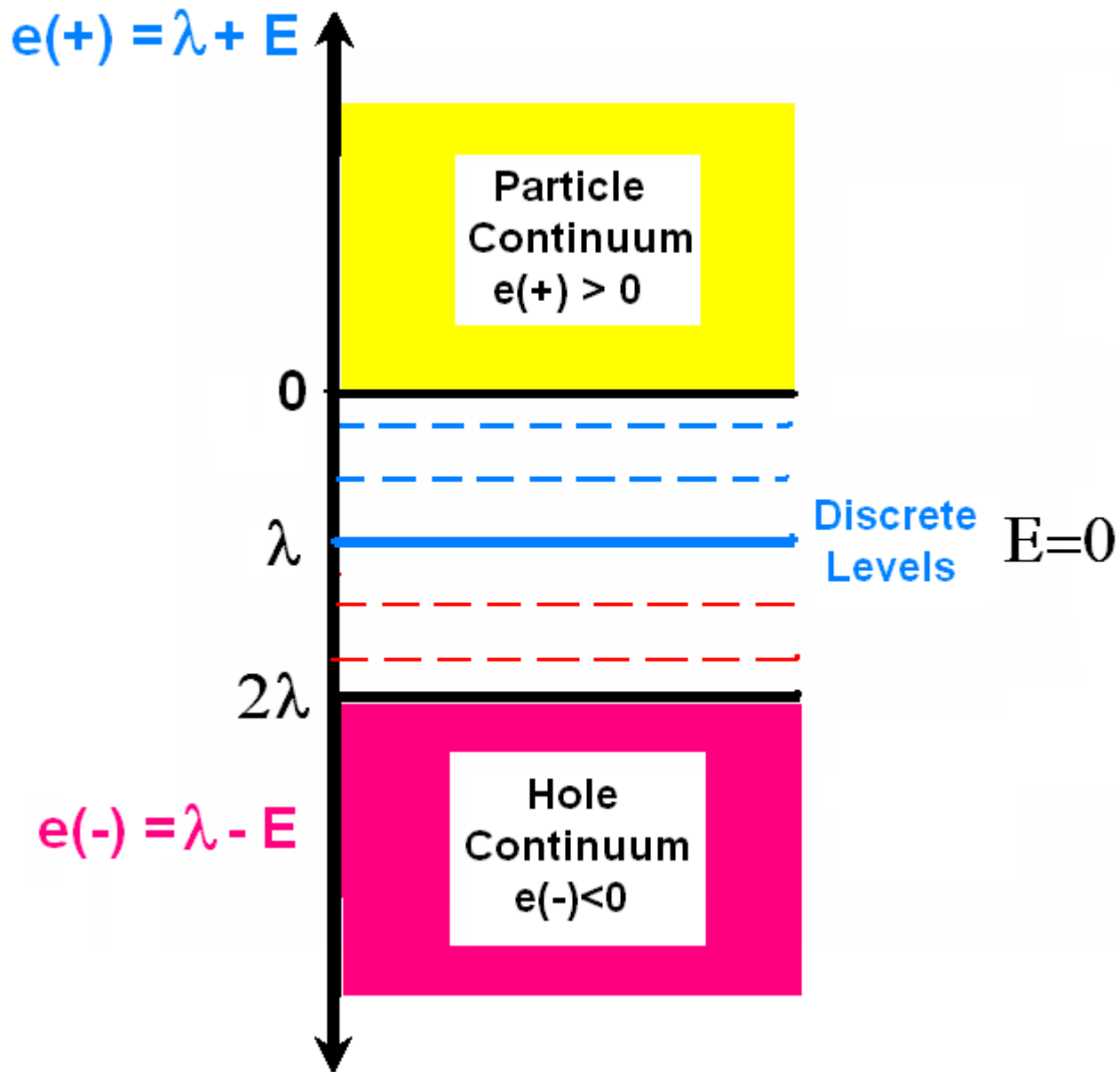
$$\rho_q(r) = \sum_{nlj} \frac{2j+1}{4\pi} |v_{nlj}^{(q)}(r)|^2$$

Pairing-Field & Density (q = p,n):

$$\Delta_q = \frac{1}{2} V_{SE} K_q$$

$$K_q(r) = \sum_{nlj} \frac{2j+1}{4\pi} u_{nlj}^{(q)}(r) v_{nlj}^{(q)*}(r)$$

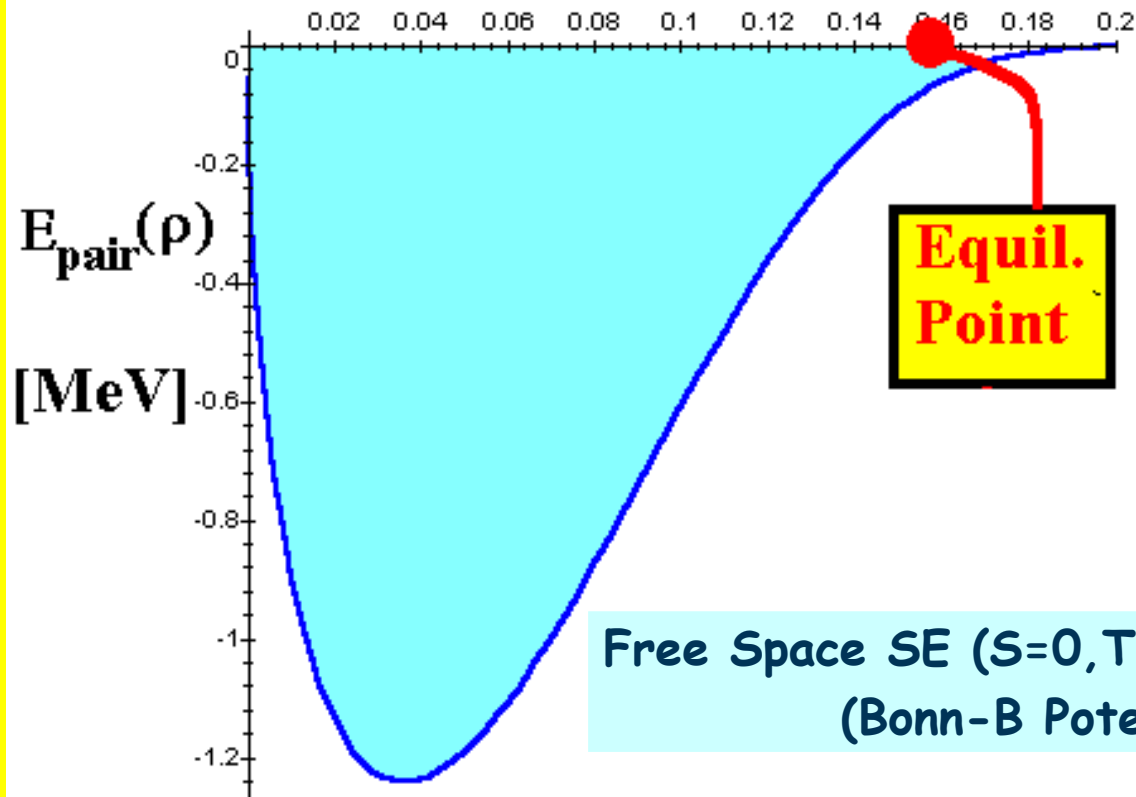
# Spectrum of the Gorkov Equation:



# Pairing in Infinite Nuclear Matter

## Pairing Energy per Particle

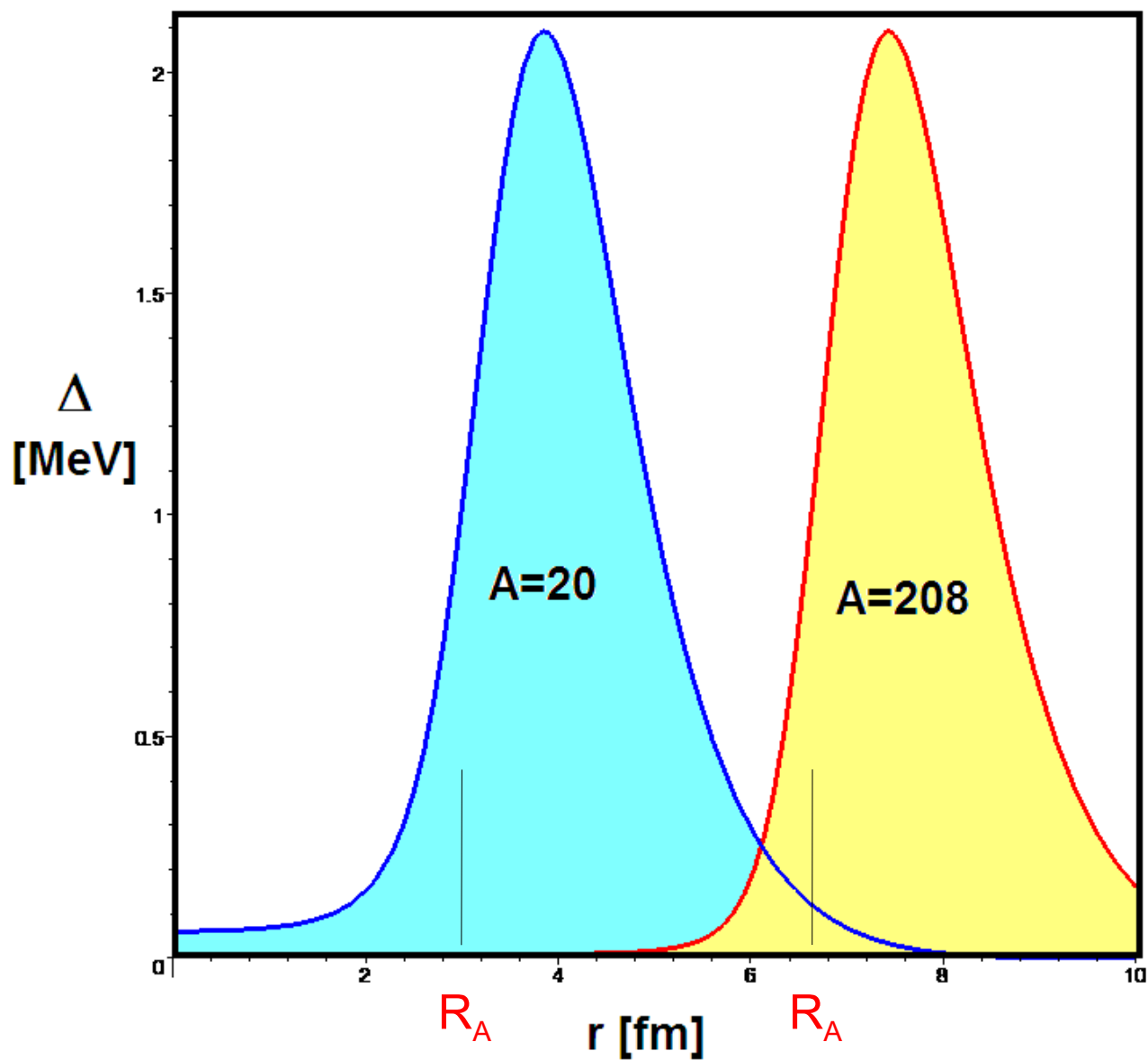
$\rho$  [ $1/\text{fm}^3$ ]



Free Space SE ( $S=0, T=1$ ) Interaction:  
(Bonn-B Potential)

Pairing is a **LOW DENSITY** Phenomenon

# The Pairing-Field in Finite Nuclei



# Pairing in the Continuum

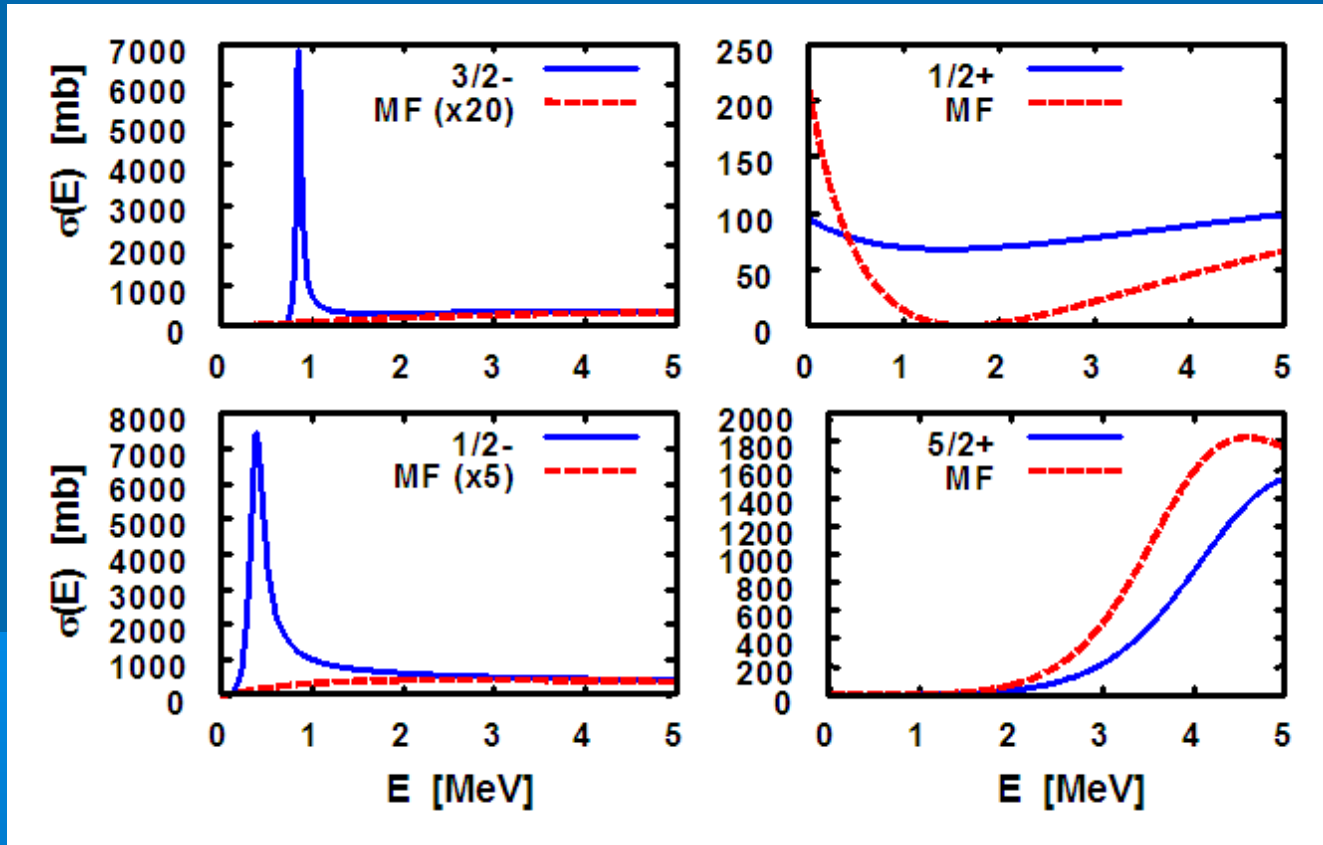
$$\Sigma_q^{(c)} = \Delta_q^\dagger \frac{1}{2\lambda_q - e - T_q - U_q} \Delta_q$$

$$u_\alpha(r) \rightarrow \cos(\delta_\alpha^{(c)}) f_\alpha(r) + \sin(\delta_\alpha^{(c)}) g_\alpha(r),$$

$$\tan(\delta_\alpha^{(c)}) = -\frac{2\tilde{m}k_\alpha}{4\pi\hbar^2} \langle f_\alpha | \Sigma_q^{(c)} | u_\alpha \rangle \sim -\frac{2\tilde{m}k_\alpha}{4\pi\hbar^2} \langle f_\alpha | \Sigma_q^{(c)} | f_\alpha \rangle$$

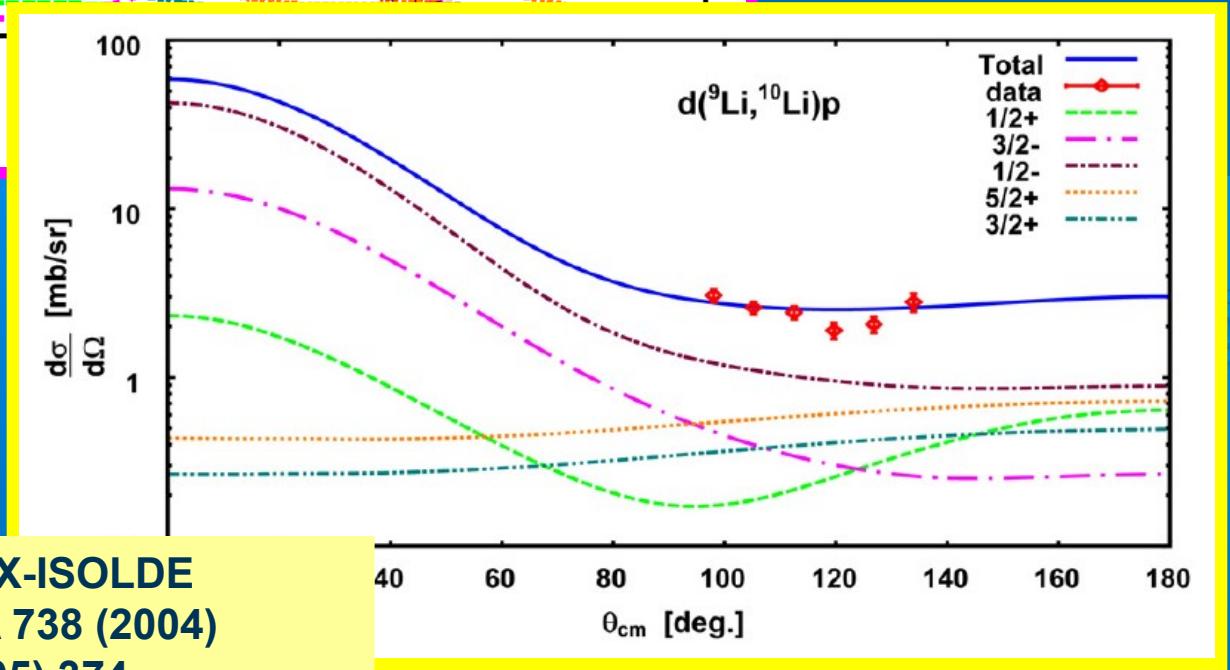
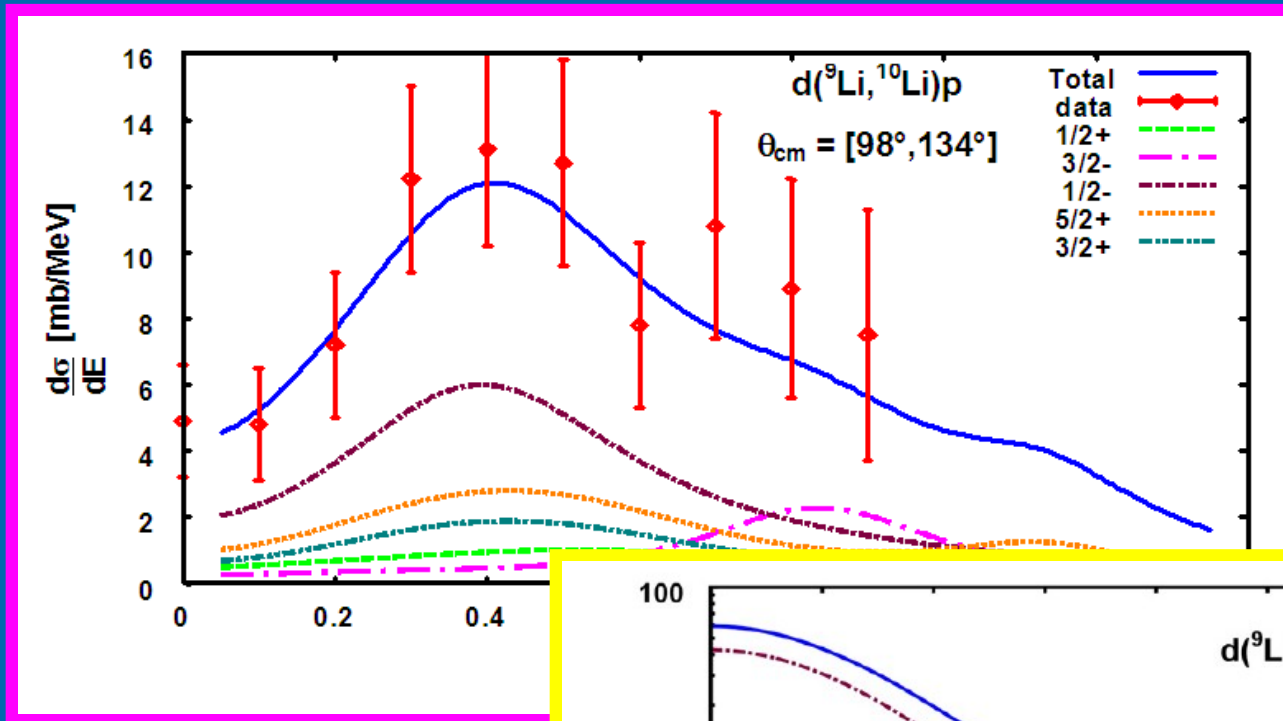
$$\Gamma_\alpha^{(c)} \sim N(k_\alpha) |\langle f_\alpha | \Delta_\alpha | v_b \rangle|^2$$

# Pairing Resonances in Dripline Nuclei



$$\begin{pmatrix} T_q + U_q - 2\lambda_{q\alpha} + e\Delta & r_{q\alpha} \begin{pmatrix} - \\ \end{pmatrix} \\ -\Delta_q^\dagger(\vec{r}) & -\left(T_q + U_{q\alpha} - e\right) \end{pmatrix} \begin{pmatrix} u_{\alpha q}(\vec{r}) \\ v_{\alpha q}(\vec{r}) \end{pmatrix} = 0$$

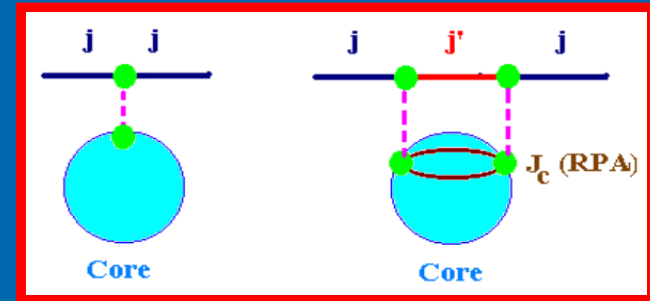
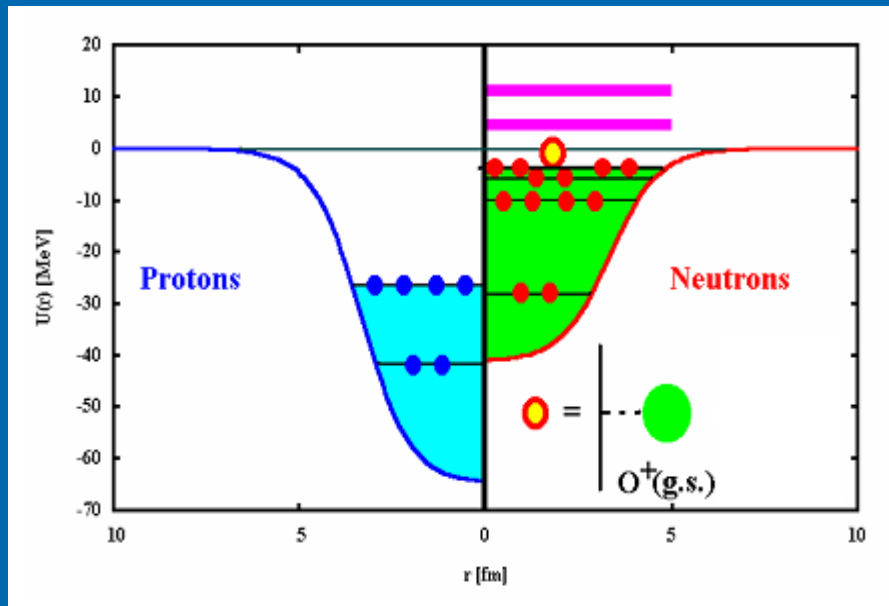
# Continuum Spectroscopy: $^{10}\text{Li} = ^9\text{Li} + n$ @ 2.36 A MeV



Data: H. Jeppesen et al., REX-ISOLDE Collaboration, Nucl. Phys. A 738 (2004) 511 & Nucl. Phys. A 748 (2005) 374.

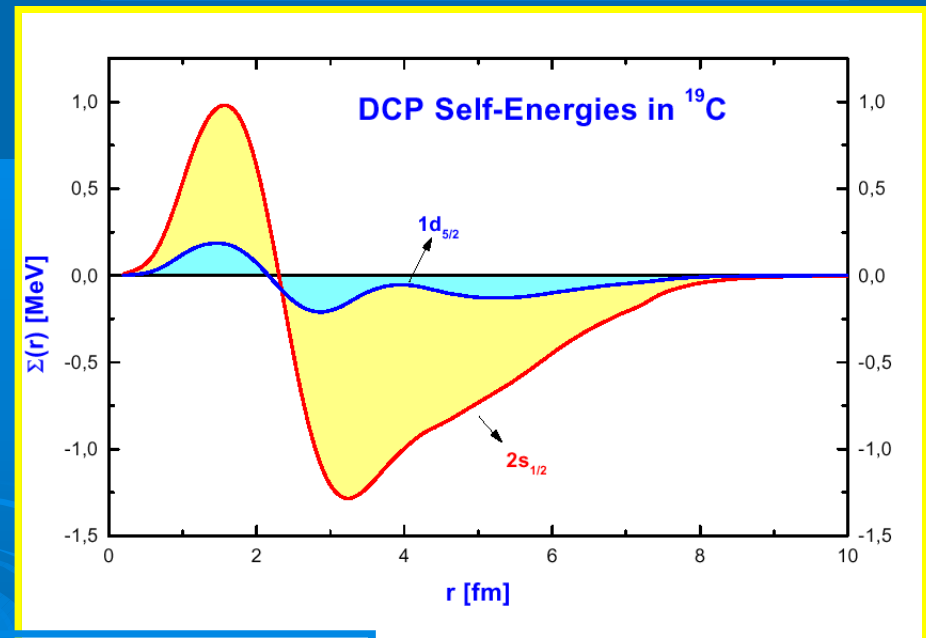
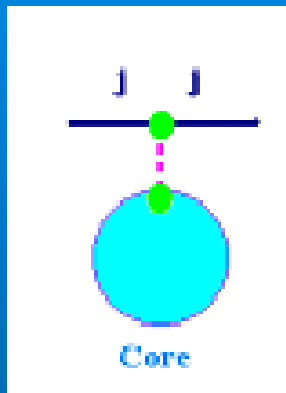


# 1n-Neutron Halo States: Transition from Mean-Field to Correlation Dynamics

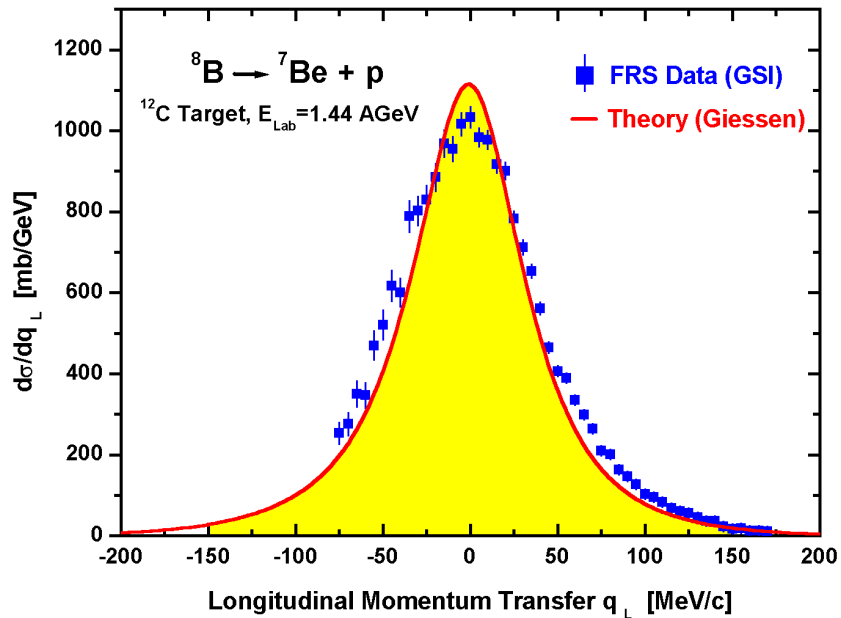


The DCP picture: Binding by  
Virtual Continuum Coupling

The s.p. shell model picture



# Investigating the Structure of ${}^8\text{B}$ by breakup reactions

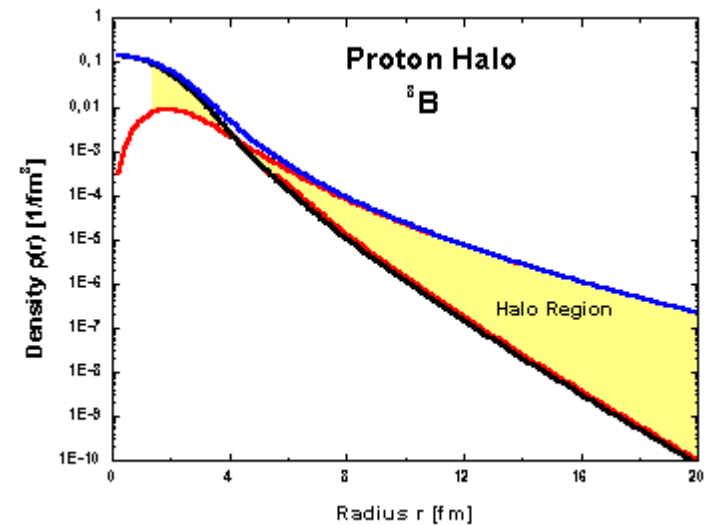


- Relativistic eikonal theory
- NN T-Matrix
- 3-body kinematics
- dynamical Correlations

$\Gamma(\text{the.}): 75$  MeV/c  
 $\Gamma(\text{exp.}): 91 \pm 5$  MeV/c

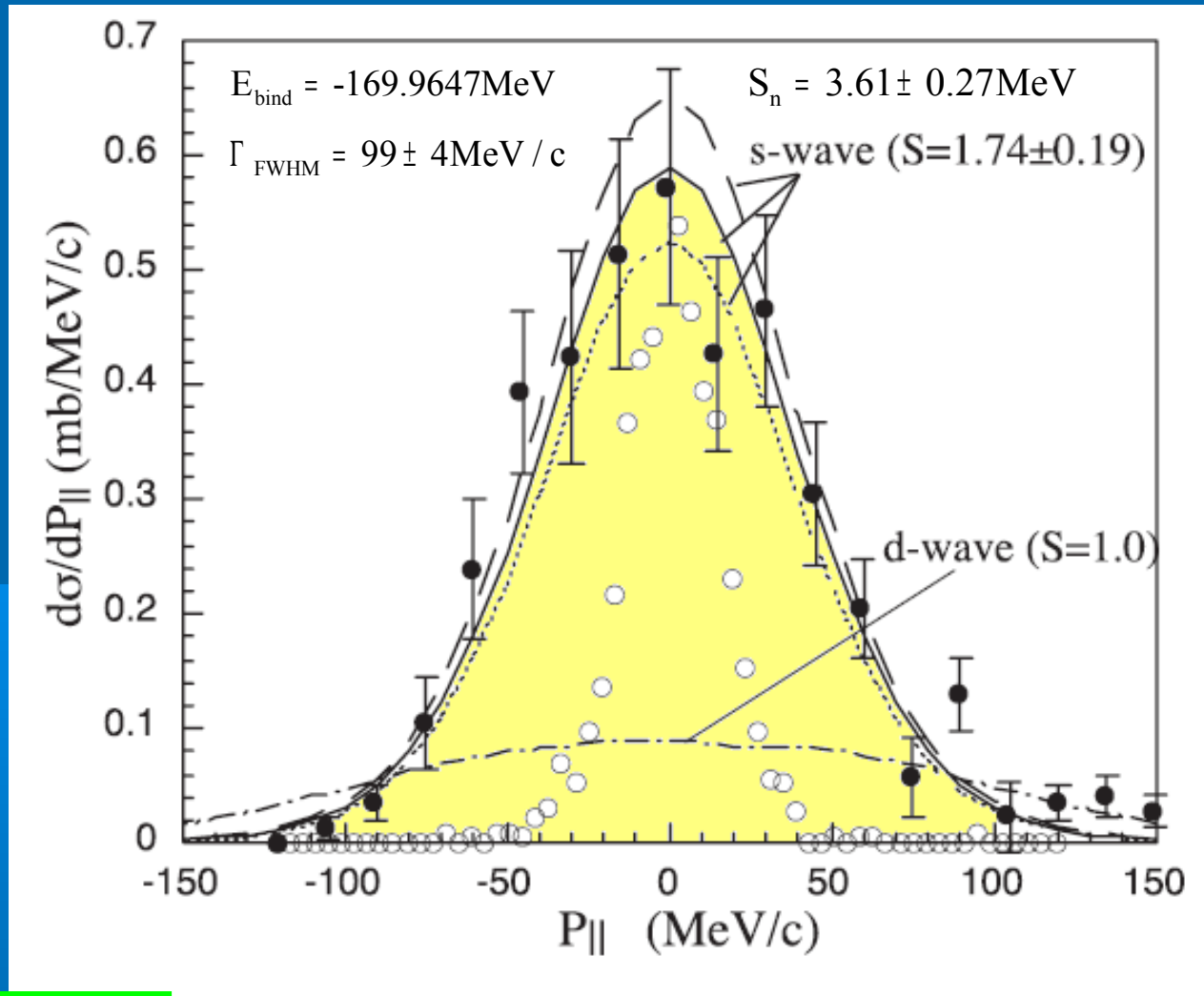
$\sigma(-\text{p}, \text{the.}): 104$  mb  
 $\sigma(-\text{p}, \text{exp.}): 98 \pm 6$  mb

${}^7\text{Be}(3/2^-, 0.0)$  ) p $3/2$ : 71%  
 ${}^7\text{Be}(3/2^-, 0.0)$  ) p $1/2$ : 13%  
 ${}^7\text{Be}(3/2^-, 0.0)$  ) f  $7/2$ : 11%  
 ${}^7\text{Be}(3/2^-, 0.0)$  ) f  $5/2$ : 5%  
 ${}^7\text{Be}(1/2^-, 0.420)$  ) : 15%

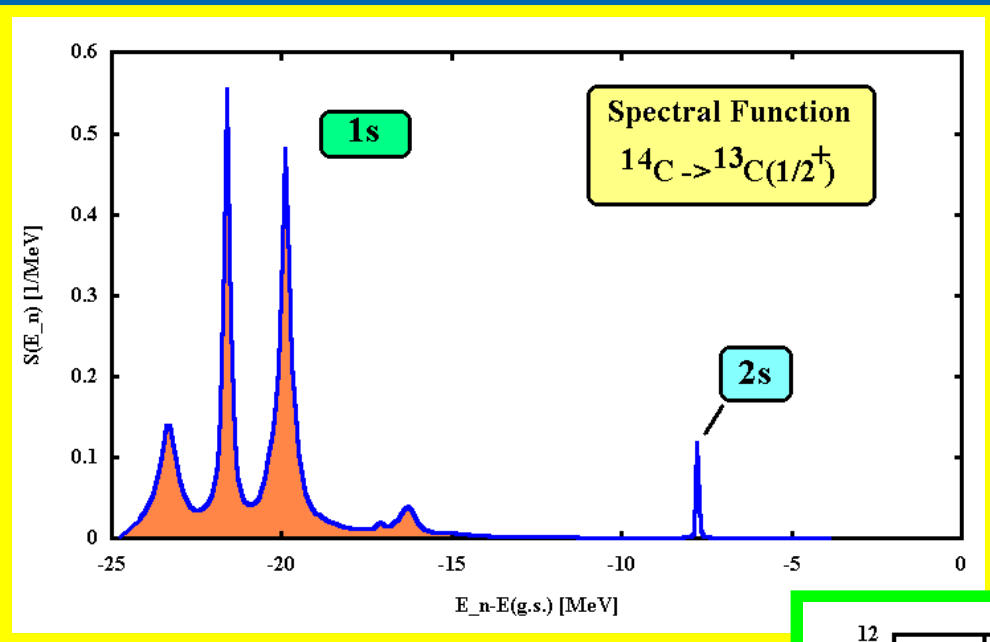


# FRS@GSI: $^{24}\text{O}$ Breakup at 920A MeV:

$^{24}\text{O}$  a new doubly magic Nucleus ( $Z=8, N=16$ )

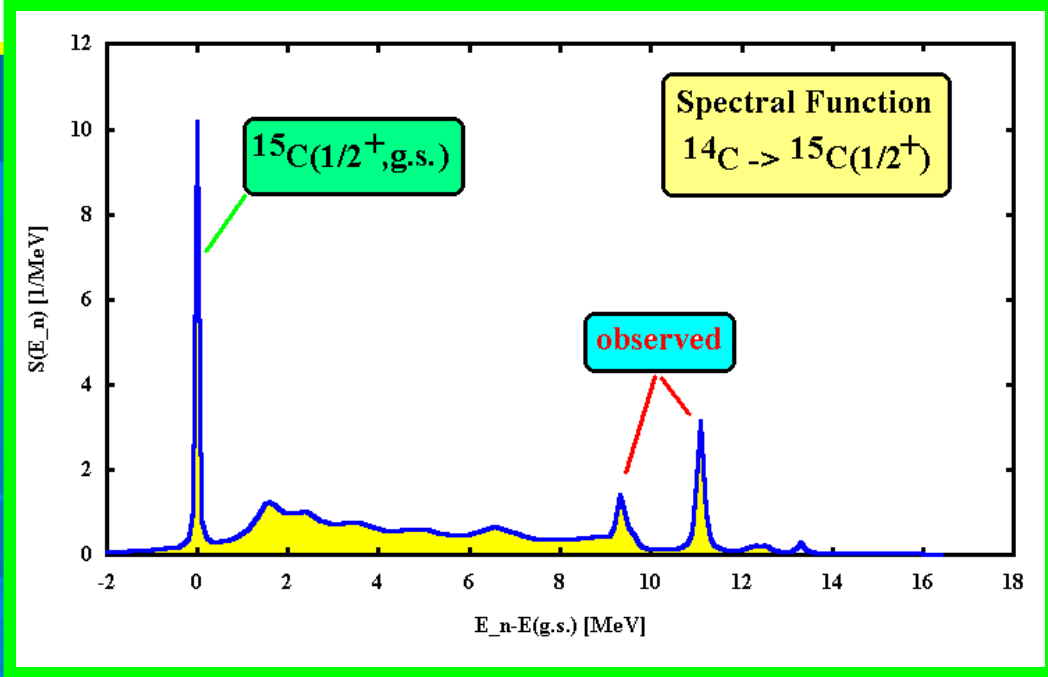


# 1/2<sup>+</sup> Particle and Hole Strength Functions in <sup>14</sup>C

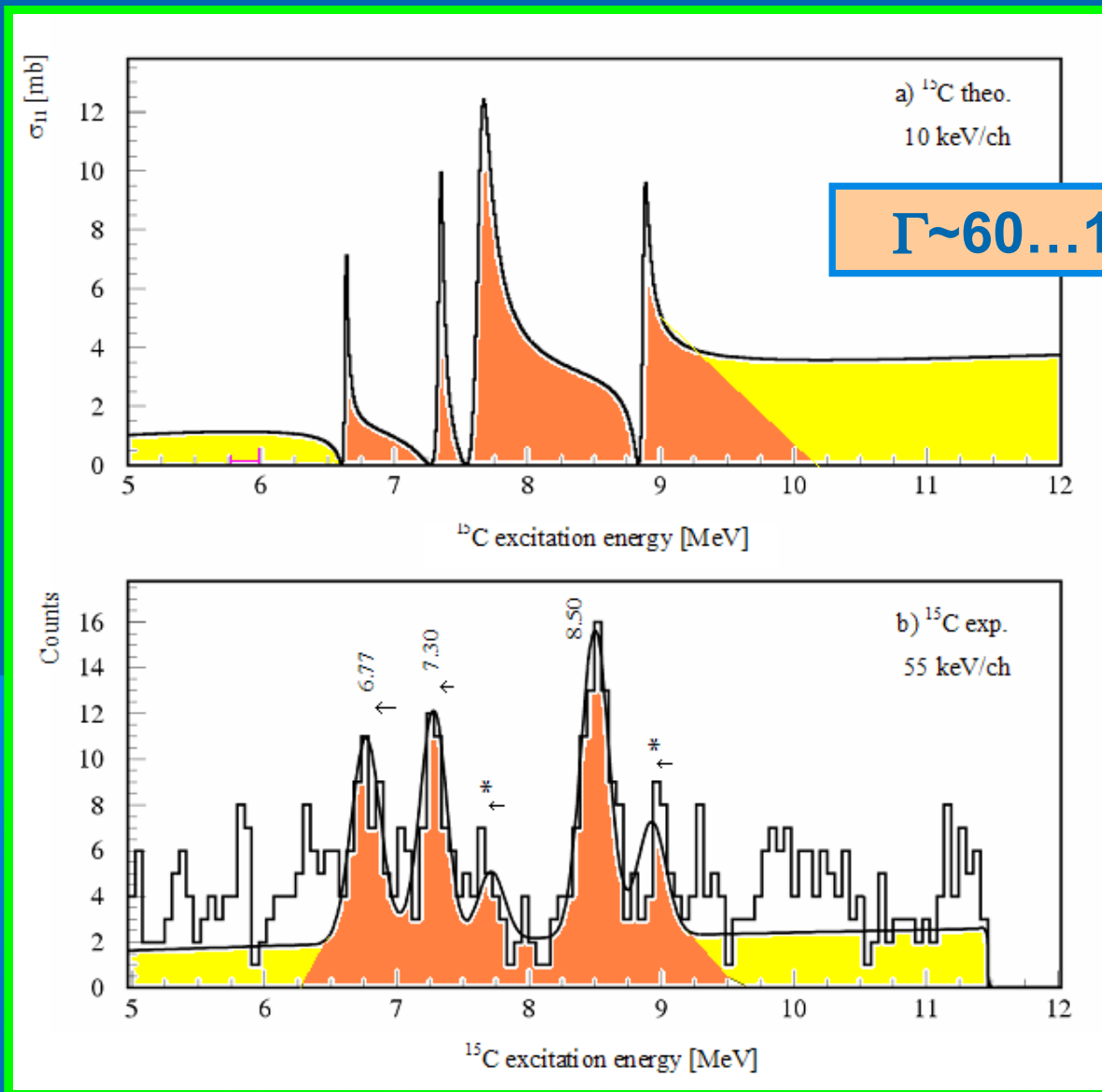


Hole strength function

Particle strength function

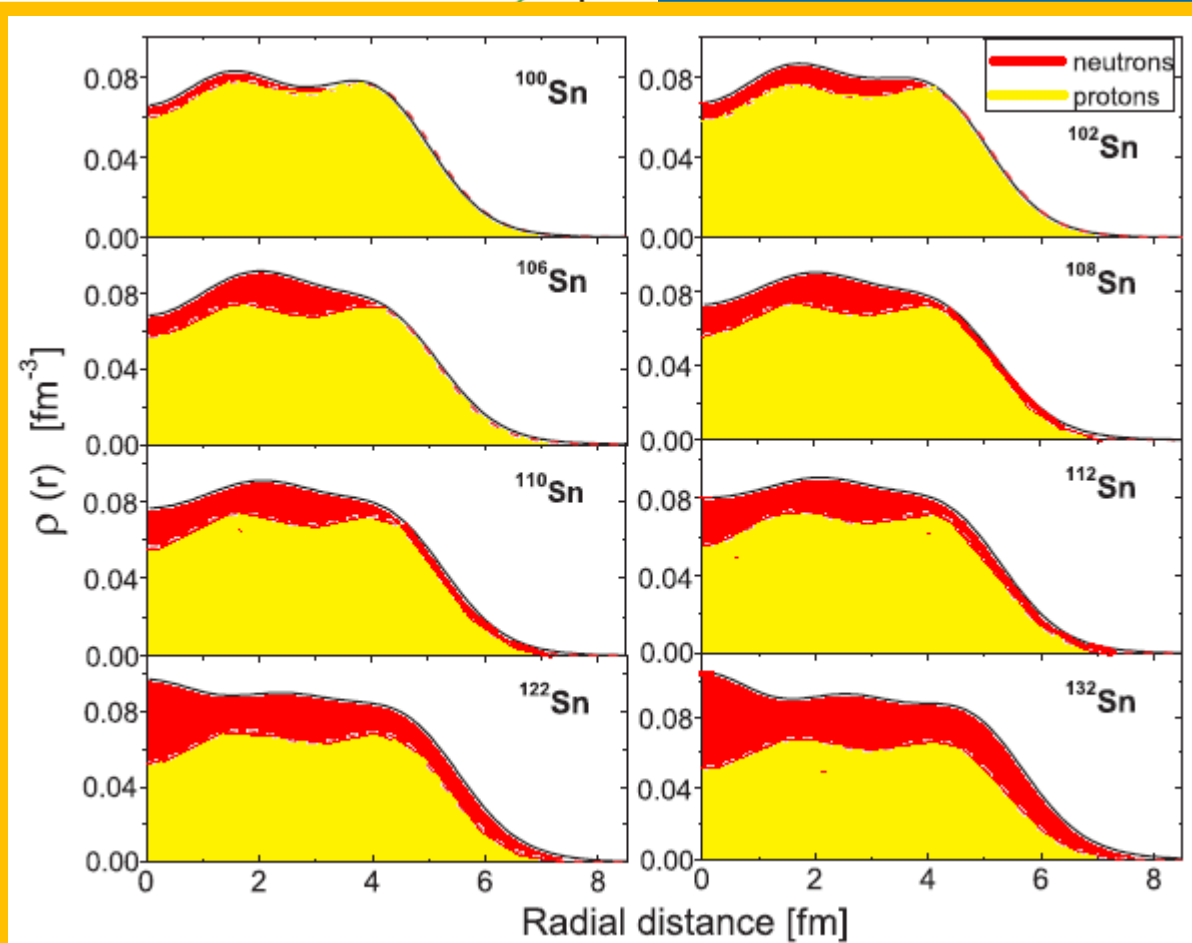
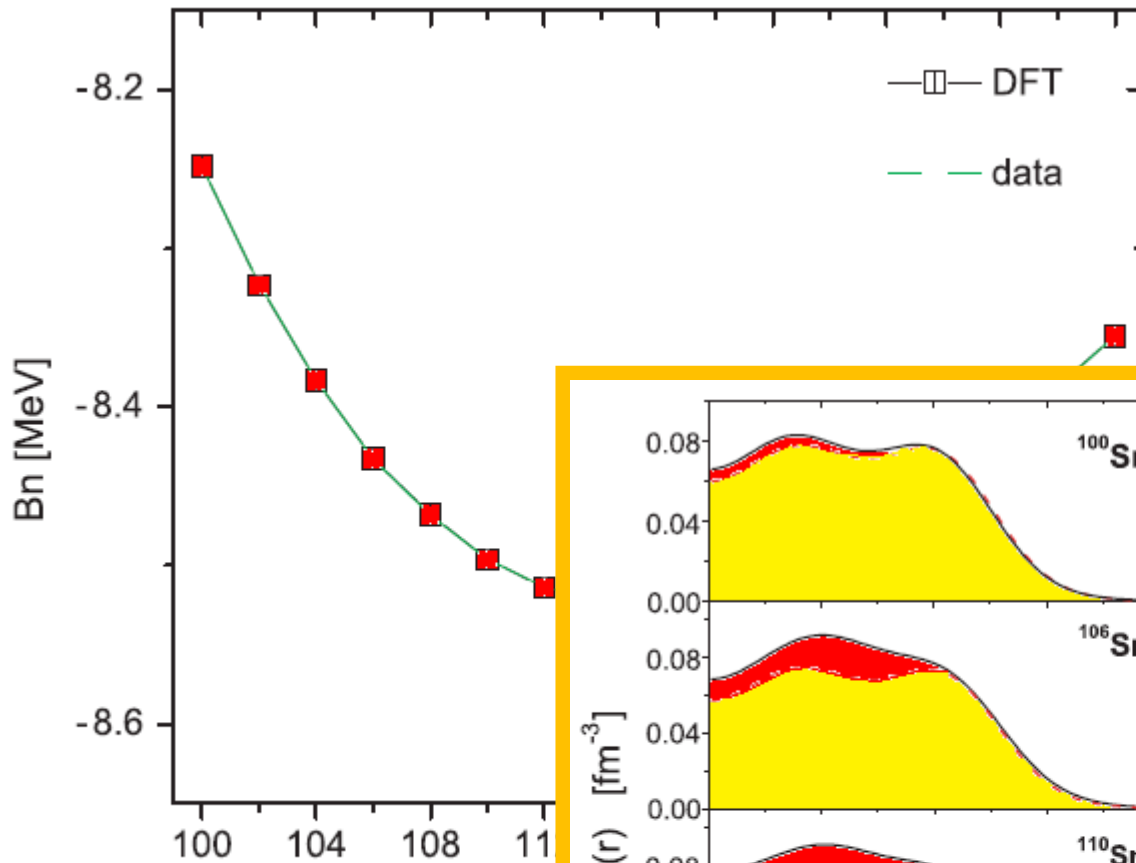


# Correlation Dynamics in an Open Quantum System: Fano-Resonances in $^{15}\text{C}$

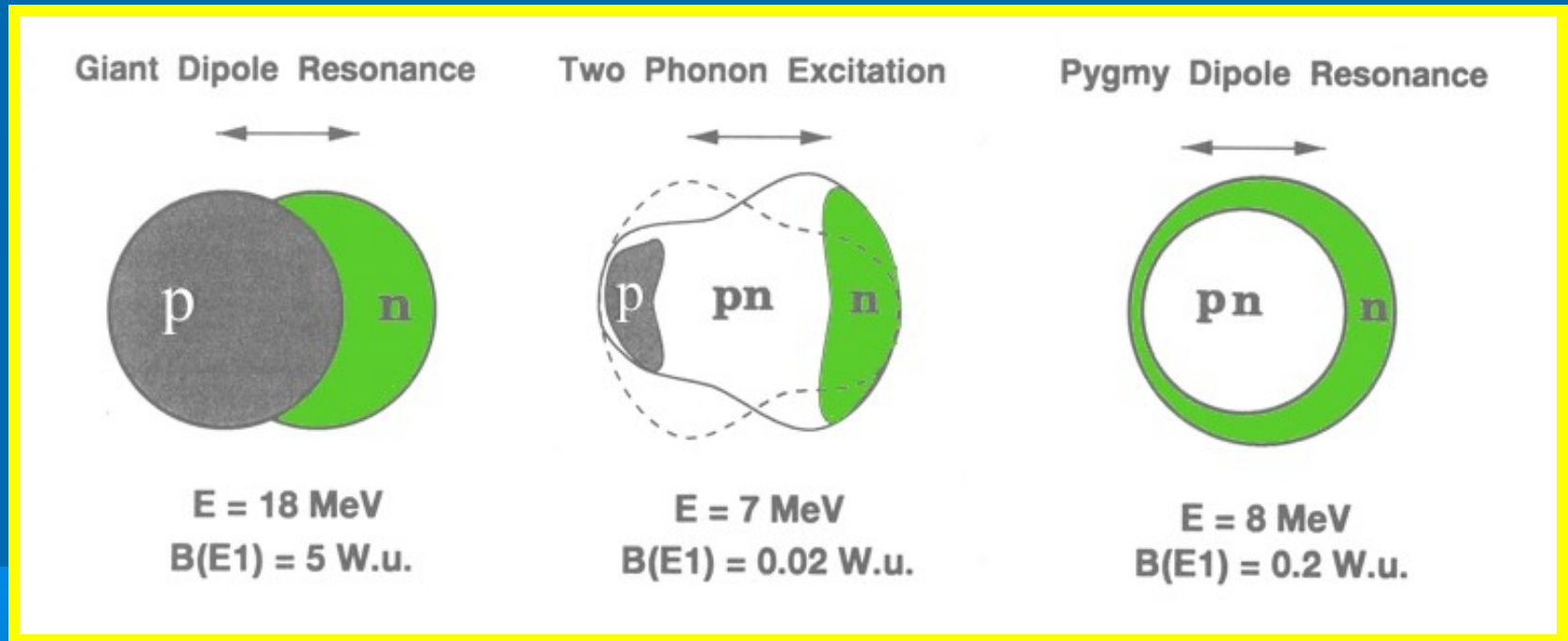


# Sn Isotopes: DFT-HFB Results

(N. Tsoneva, HL,  
PRC77 (2008))



# Electric Dipole Response of Exotic Nuclei



$$\vec{D} = \frac{1}{2} \sum_i \vec{\xi}_i (1 - \tau_{3i}) = -\frac{1}{2} \sum_i \vec{\xi}_i \tau_{3i}.$$

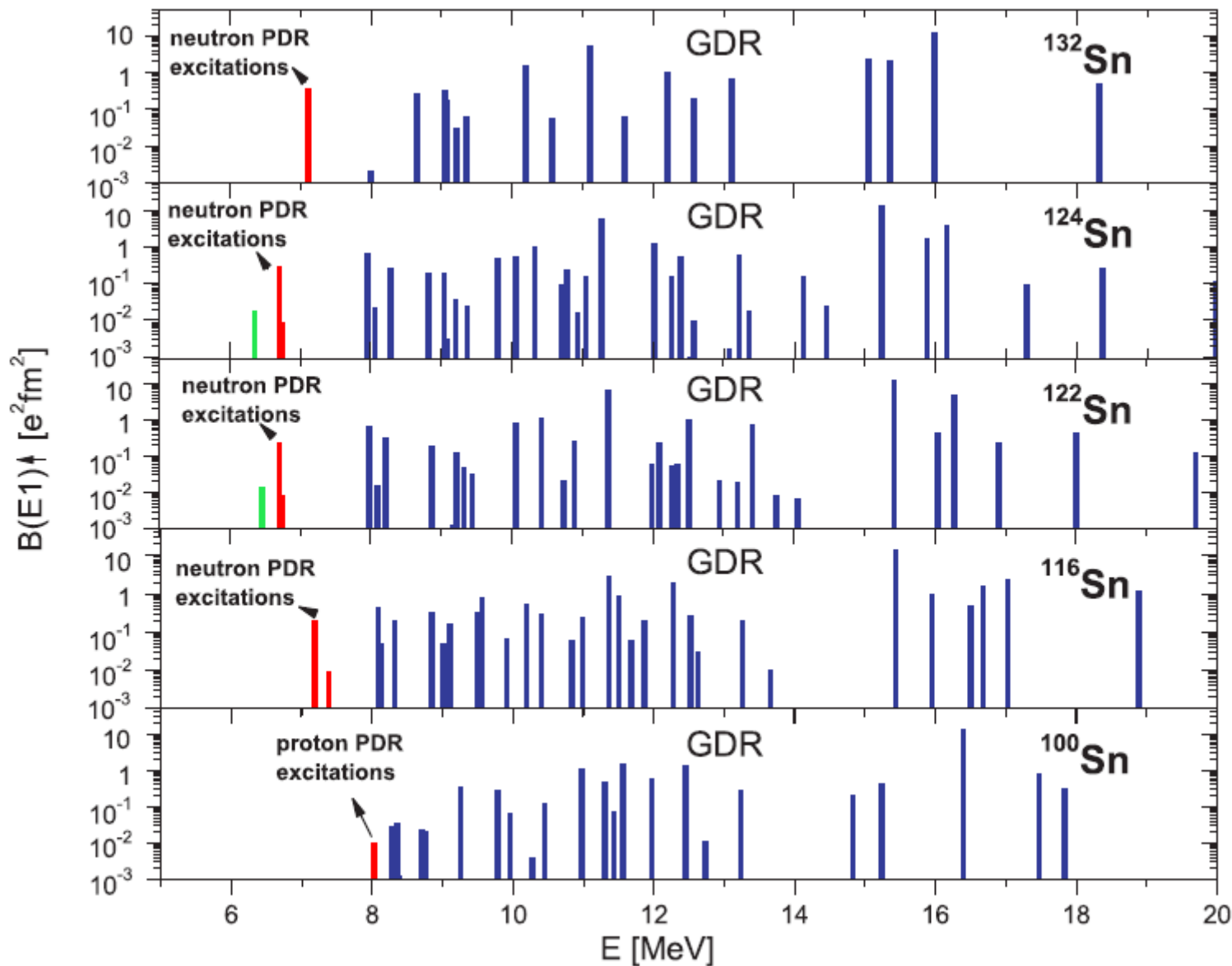
# HFB g.s. and Multi-Phonon QRPA Theory:

## The Multi-Configuration (multi-phonon) Wave Function

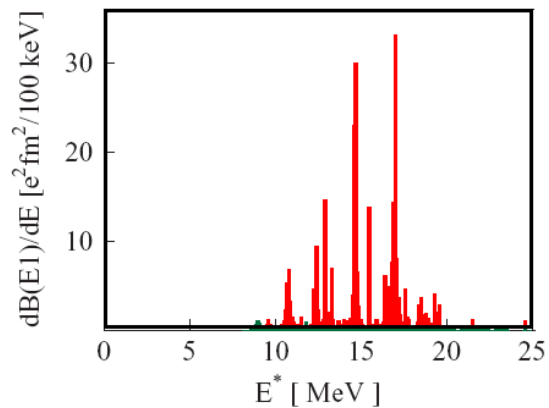
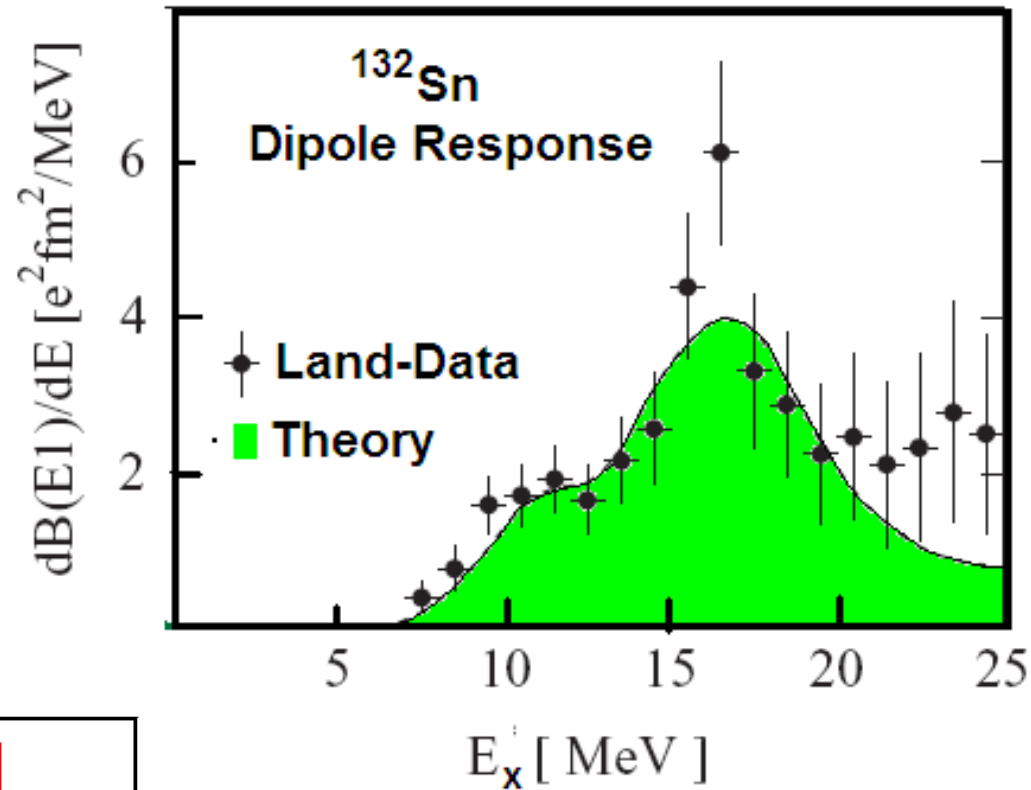
$$\Psi_\nu(JM) = \left\{ \begin{aligned} & \sum_i R_i(J\nu) Q_{JM_i}^+ + \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) \\ & \times [Q_{\lambda_1 \mu_1 i_1}^+ \times Q_{\lambda_2 \mu_2 i_2}^+]_{JM} + \sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ \lambda_3 i_3 I}} T_{\lambda_3 i_3}^{\lambda_1 i_1 \lambda_2 i_2 I}(J\nu) \\ & \times [[Q_{\lambda_1 \mu_1 i_1}^+ \otimes Q_{\lambda_2 \mu_2 i_2}^+]_{IK} \otimes Q_{\lambda_3 \mu_3 i_3}^+]_{JM} \end{aligned} \right\} \Psi_0$$



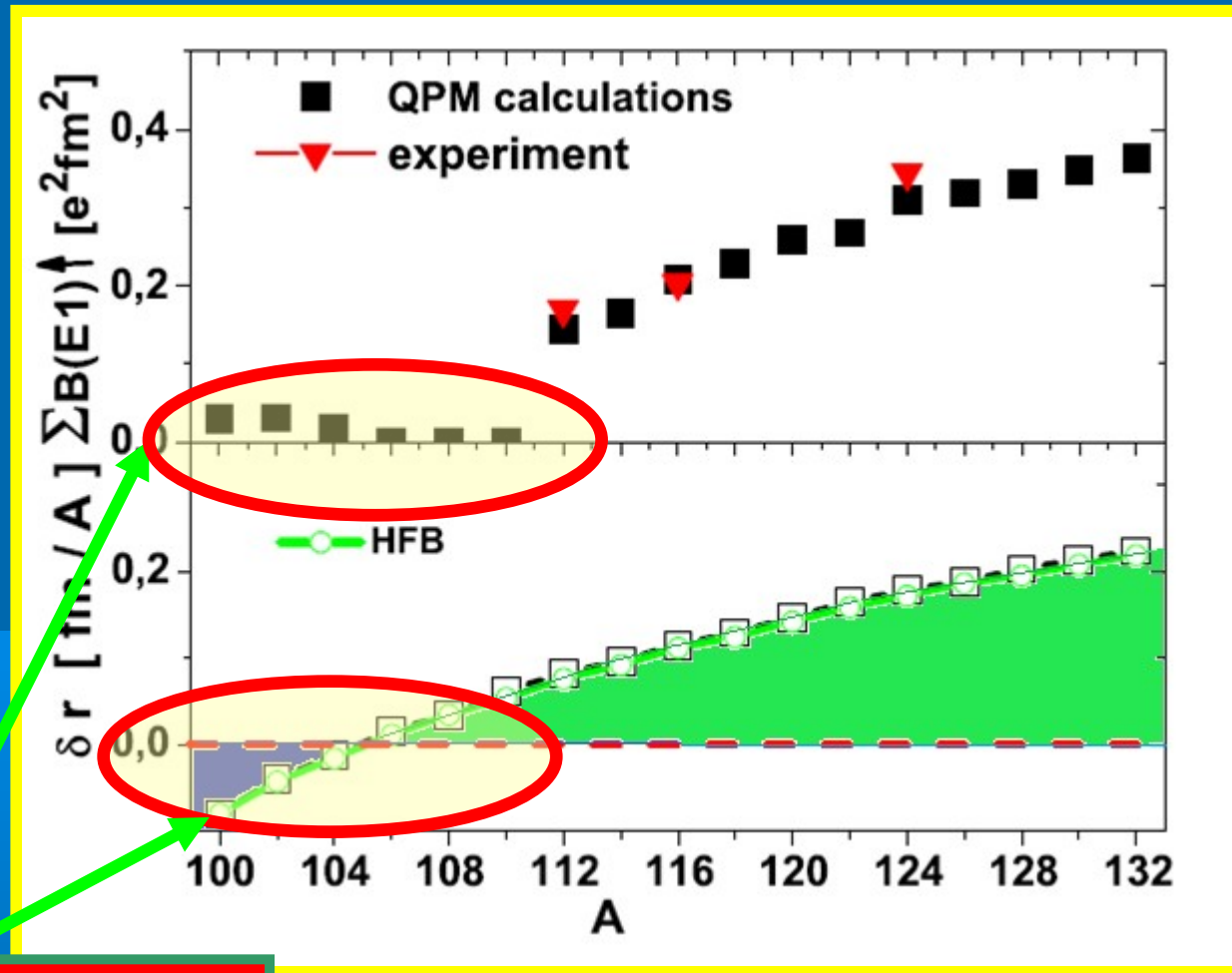
# Electric Dipole Response in Sn-Isotopes



# Relativistic Coulex@LAND



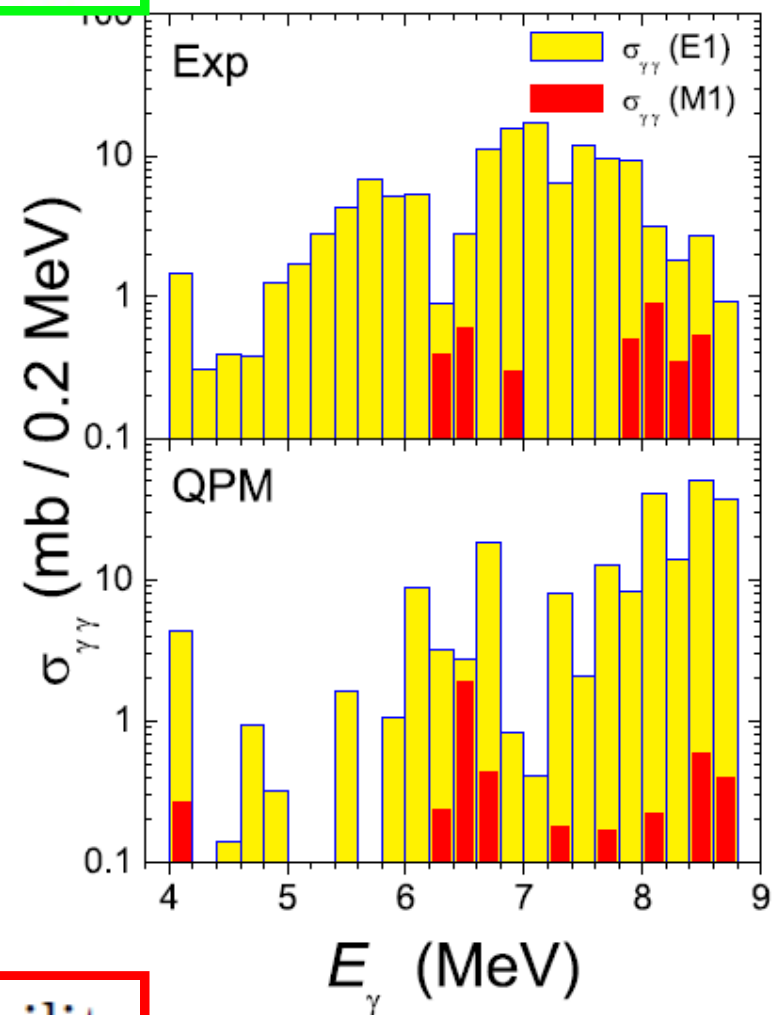
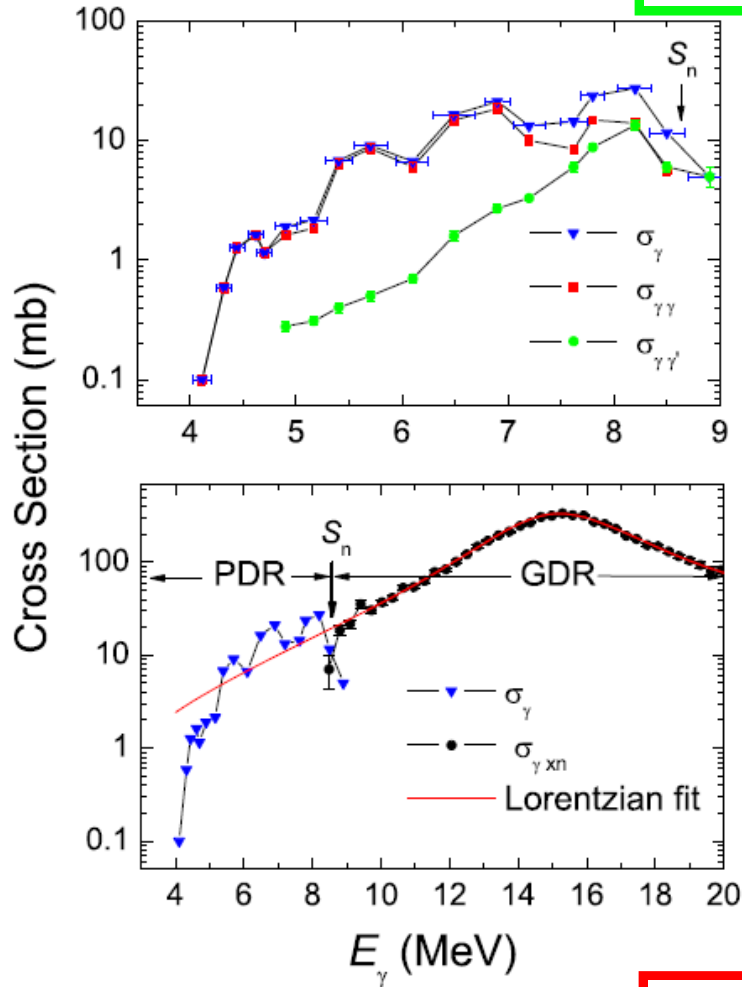
# PDR Response and Nuclear Skins in the Sn Isotopes: HFB g.s. and Multi-Phonon QRPA Theory



**Coulomb-driven  
proton skins & PDR!**

# Low Energy Dipole Response: Parity Assignment

$^{138}\text{Ba}(\vec{\gamma}, \gamma')$



HI $\vec{\gamma}$ S facility

# Summary and Outlook

- Exotic Nuclei as Open Quantum Systems
- Continuum Spectroscopy and Dynamical Correlations
- New Generic Modes of Exotic Nuclei - PDR, PQR...
- Challenges:
  - Many-body Dynamics at extreme Isospin
  - Nuclear Structure at weak Binding
  - Reaction Theory for weakly Bound Systems

Credits to: Nadia Tsoneva, Urnaa Badarch, A. Ataie, A. Fedoseew,  
P. Konrad, Anika Obermann, M. Strecker