QUANTUM MECHANICS OF LEPTOGENESIS

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DESY

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Outline

- Introduction
- Non-equilibrium dynamics
  - Boltzmann equations
  - Kadanoff-Baym
- Solution to the Kadanoff-Baym
- Thermal leptogenesis
  - Boltzmann equations
  - Kadanoff-Baym
- Conclusions

Matter-antimater asymmetry

BBN : $\frac{n_B}{n_\gamma} = (5.1 - 6.5) \times 10^{-10}$

WMAP: $\frac{n_B}{n_\gamma} = (6.19 \pm 0.15) \times 10^{-10}$

Sakharov's conditions

- Baryon number violation
- C, CP violation
- Departure from thermal equilibrium

Asymmetry in the SM

The Sakharov's conditions are satisfied in the SM

- Baryon number violation via the triangle anomaly
- CP violation with the Kobayashi-Maskawa mechanism
- Out-of-equilibrium in the electro-weak phase transition

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- Out-of-equilibrium in the electro-weak phase transition

Not enough asymmetry is generated!

Extension to the SM is required

See-saw mechanism and Thermal Leptogenesis

Insertion of right-handed neutrinos ν_R

$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{\nu}_R \partial\!\!\!/ \nu_R - \bar{l}_L \tilde{\Phi} \lambda \nu_R - \frac{1}{2} \bar{\nu}_R^c M \nu_R + h.c.$$

- see-saw mechanism explains small neutrino masses
- mass term of $N \approx \nu_R + \nu_R^c$ violates lepton number
- mass and flavour eigenstates are not identical
- complex phases appear
- out-of-equilibrium decay of N generally violates CP and generates the asymmetry

See-saw mechanism and Thermal Leptogenesis

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⇒the study of the non-equilibrium dynamics of the Majorana particle is fundamental to understand leptogenesis

Non-Equilibrium Dynamics of Quantum Systems

Methods

- Boltzmann equations (BE)
- (full) quantum Boltzmann equations (QBE)
- kinetic equations for *matrices of density*
- Kadanoff-Baym equations (KBE)

Quantum Boltzmann Equations

$$\dot{n}(t) + 3H(t)n(t) + \Gamma(t)(n(t) - n_{eq}) = 0$$

- describe time evolution of classical particle numbers
- cross sections are imported from quantum field theory
- BE are known to work well in some examples, e.g.
  - photon decoupling
  - big bang nucleosynthesis
- but note that
  - BE cannot describe coherent oscillations
  - BE assume particles move freely between scatterings
  - BE are Markovian , they are local in time
  - classical particle number is not well defined in interacting quantum field theory

Full Quantum Boltzmann Equations

First order solution to a gradient expansion of the Kadanoff-Baym equations

- assumes close to equilibrium condition
- slow variation with respect to the center of mass coordinate
- many quantum-interferences are lost
- ill-defined quantities such particle densities are used

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A full quantum treatment needs to be performed

Is a Quantum Treatment possible?

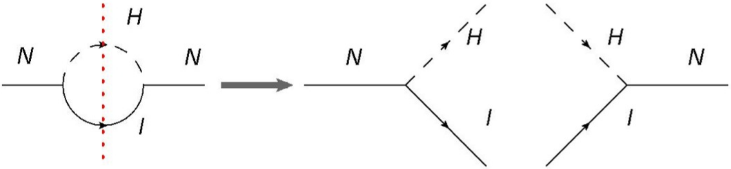
- spacial homogeneity
- weak coupling ⇒ perturbative
- background plasma is in equilibrium
- backreaction can be neglected

'Weak coupling to a thermal bath'

Boltzmann vs Kadanoff-Baym Equations

- initial value problem for density matrix $\rho(t)\ldots$
- $\ldots$or for correlation functions $\langle\ldots\rangle = \mathrm{tr}(\rho\ldots)$
- KBE contain full quantum mechanics

$$\text{particle numbers} \Leftrightarrow \text{correlation functions}$$
$$\text{collision term} \Leftrightarrow \text{self energies}$$



Π, Π encode information about all decay and scattering processes

KBE Formalism

Kadanoff-Baym Equations

$$(\partial_1^2 + m^2)\Delta^-(x_1,x_2) = -\int d^3\mathbf{x}' \int_{t_2}^{t_1} dt'\,\Pi^-(x_1,x')\Delta^-(x',x_2)$$

$$(\partial_1^2 + m^2)\Delta^+(x_1,x_2) = -\int d^3\mathbf{x}' \int_{t_i}^{t_1} dt'\,\Pi^-(x_1,x')\Delta^+(x',x_2) + \int d^3\mathbf{x}' \int_{t_i}^{t_2} dt'\,\Pi^+(x_1,x')\Delta^-(x',x_2)$$

$$-(i\partial\!\!\!/_1 - m)S^-(x_1,x_2) = -\int d^3\mathbf{x}' \int_{t_2}^{t_1} dt'\,\Pi^-(x_1,x')S^-(x',x_2)$$

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Weak Coupling to a thermal Bath

- consider fields that are weakly coupled to a large bath in equilibrium
- assume interaction mainly with bath fields $\mathcal{X}$
  - then self energies are computed with equilibrium propagators
  - in practice realised by using couplings that are linear in the field of interest, e.g. $g\phi\mathcal{O}[\mathcal{X}]$, $g\Psi\mathcal{O}[\mathcal{X}]$, at leading order in g

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For such systems

- spectral propagators Δ^-, S^-, G^- are time translation invariant
- KBE are equivalent to a stochastic Langevin equation
- KBE can be solved analytically up to a *memory integral*

Kadanoff-Baym Equations

$$
\begin{aligned}
(\partial_{t_1}^2 + \omega_{\mathbf{q}}^2)\Delta_{\mathbf{q}}^-(t_1 - t_2) &= -\int_{t_2}^{t_1} dt'\,\Pi_{\mathbf{q}}^-(t_1 - t')\Delta_{\mathbf{q}}^-(t' - t_2) \\
(\partial_{t_1}^2 + \omega_{\mathbf{q}}^2)\Delta_{\mathbf{q}}^+(t_1, t_2) &= -\int_{t_i}^{t_1} dt'\,\Pi_{\mathbf{q}}^-(t_1 - t')\Delta_{\mathbf{q}}^+(t', t_2) \\
&\quad + \int_{t_i}^{t_2} dt'\,\Pi_{\mathbf{q}}^+(t_1 - t')\Delta_{\mathbf{q}}^-(t' - t_2)
\end{aligned}
$$

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\begin{aligned}
(i\gamma_0\partial_{t_1} - \mathbf{q}\gamma - m)S_{\mathbf{q}}^-(t_1 - t_2) &= -\int_{t_1}^{t_2} dt'\,\Pi_{\mathbf{q}}^-(t_1 - t')S_{\mathbf{q}}^-(t' - t_2) \\
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Solutions

Properties of the Solutions

- retarded self-energy $\Pi^R = \Pi^R|_{T=0} + \delta\Pi^R(T)$ is the decisive quantity
- $\mathrm{Re}\Pi^R$ gives thermal mass
- $\mathrm{Im}\Pi^R$ decay width Γ to resonance

Three regimes

1. $|\mathrm{Re}\Pi|, |\mathrm{Im}\Pi| \ll \omega_{\mathbf{q}}^2$
2. $|\mathrm{Re}\Pi| \approx \omega_{\mathbf{q}}^2, |\mathrm{Im}\Pi| \ll \omega_{\mathbf{q}}^2$
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   single resonance kinematically behaves like quasiparticle
   but total energy receives vacuum contribution
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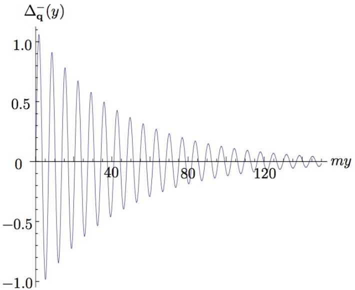
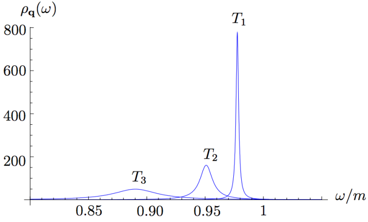
   single resonance kinematically behaves like quasiparticle
   but total energy receives vacuum contribution
3. $|\mathrm{Re}\Pi|, |\mathrm{Im}\Pi| \approx \omega_{\mathbf{q}}^2$
   particle interpretation and Boltzmann equations break down,
   at large T possibly even in a weakly coupled theory

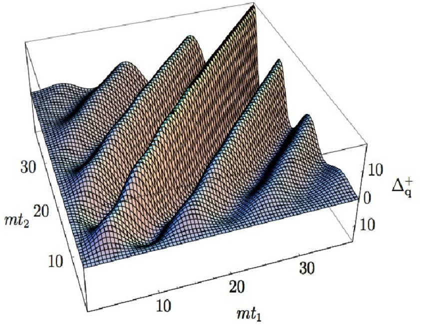
The Spectral Function

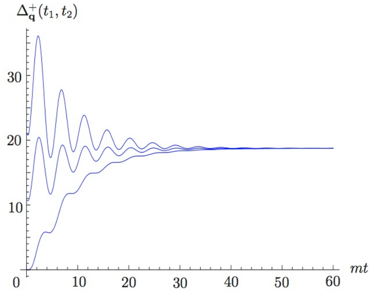




- Damped oscillatory behaviour
- Breit-Wigner breaks down at high temperatures

The Statistical Propagator





- depends on two time arguments
- equilibrates independent of initial conditions after characteristic time $\tau \sim {}^{1}/_{\Gamma}$
- oscillates with plasma frequency

Application to Leptogenesis

Boltzmann approach for asymmetry

The coupled differential equations in a static universe are given by

$$\frac{\partial f_N}{\partial t} = \mathcal{C}[f_N] \qquad \text{for Majorana}$$

$$\frac{\partial f_{l-\bar{l}}}{\partial t} = \mathcal{C}[f_{l-\bar{l}}, f_N] \qquad \text{for lepton asymmetry}$$

$\mathcal{C}[f]$: collision term

Inserting the solution for the Majoranas to the asymmetry equation, and without wash-out terms

$$f_{l-\bar{l}} = -\epsilon_{CP}\frac{1}{k}\int_{\mathbf{q},\mathbf{p}}(2\pi)^4\delta^4(k+q-p)p\cdot k \times f_{l\phi}f_N^{eq}\frac{1}{\Gamma}(1-e^{-\Gamma t})$$

KBE approach for the Asymmetry

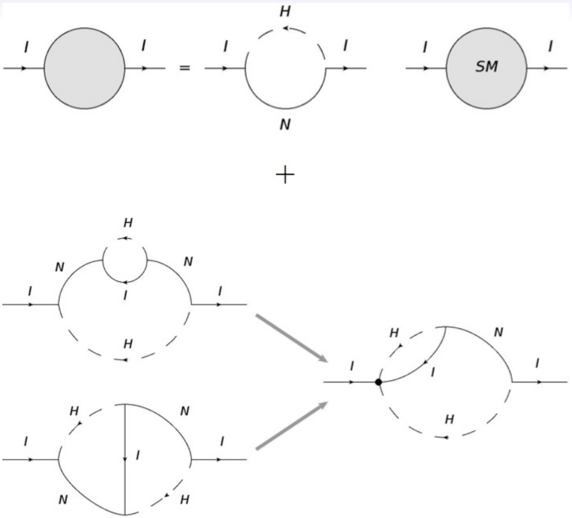
How to calculate the asymmetry without reference to particle number or distribution function?

- define *lepton number matrix*

$$L_{\mathbf{k}ij}(t_1, t_2) = -\mathrm{tr}[\gamma^0 S^+_{\mathbf{k}ij}(t_1, t_2)].$$

- $L_{\mathbf{k}ii}(t, t)$ gives leptonic charge in flavour i at time t
- CP-violation comes from interference between LO and NLO terms (tree level and 1-loop level)
- Considering an stationary universe and neglecting wash-out terms

Lepton Self-Energy



Solution of the Asymmetry

$$L_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\,\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{q}'}\frac{k\cdot k'}{|\mathbf{k}'|\omega}f_{l\phi}(k,q)f_N^{eq}(\omega)f_{l\phi}(k',q')$$

$$\times\,\frac{\frac{1}{4}\Gamma}{((\omega-|\mathbf{k}|-|\mathbf{q}|)^2+\frac{\Gamma^2}{4})((\omega-|\mathbf{k}'|-|\mathbf{q}'|)^2+\frac{\Gamma^2}{4})}$$

$$\times\,\Big(\cos[(|\mathbf{k}|+|\mathbf{q}|-|\mathbf{k}'|-|\mathbf{q}'|)t]+e^{-\Gamma t}$$

$$-(\cos[(\omega-|\mathbf{k}|-|\mathbf{q}|)t]+\cos[(\omega-|\mathbf{k}'|-|\mathbf{q}'|)t])e^{-\frac{\Gamma t}{2}}\Big)\,,$$

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$$\times \Big(\cos[(|\mathbf{k}|+|\mathbf{q}|-|\mathbf{k}'|-|\mathbf{q}'|)t]+e^{-\Gamma t}$$

$$-(\cos[(\omega-|\mathbf{k}|-|\mathbf{q}|)t]+\cos[(\omega-|\mathbf{k}'|-|\mathbf{q}'|)t])e^{-\frac{\Gamma t}{2}}\Big)\,,$$

$$f_{Li}(t,k) = -\epsilon_{ii}\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{k}'} k\cdot k' f_{l\phi}(k,q) f_N^{eq}(\omega)$$

$$\times \frac{1}{\Gamma}(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p)$$

$$\times \left(1-e^{-\Gamma t}\right)$$

On-Shell Approximation (unjustified!)

$$L^{os}_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\frac{16\pi}{k}\int_{\mathbf{q},\mathbf{q}',\mathbf{p},\mathbf{k}'} k\cdot k' f_{l\phi}(k,q) f_N^{eq}(\omega) f_{l\phi}(k',q')$$
$$\times \frac{1}{\Gamma}(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p)$$
$$\times \left(1-e^{-\frac{\Gamma t}{2}}\right)^2$$

$$f_{Li}(t,k) = -\epsilon_{ii}\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{k}'} k\cdot k' f_{l\phi}(k,q) f_N^{eq}(\omega)$$
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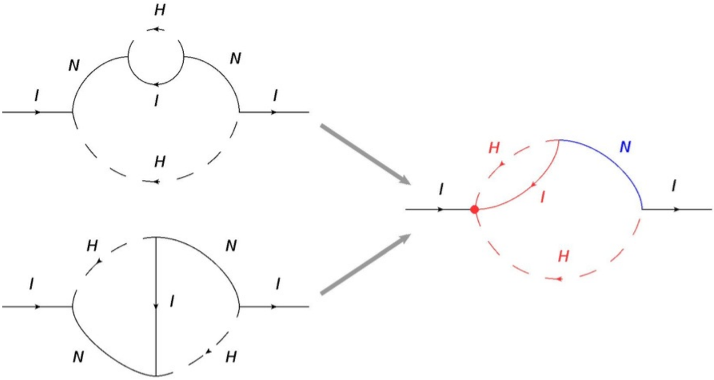
Inclusion of SM widths



Inclusion of SM widths

$$\tilde{L}_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\,\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{q}'}\frac{k\cdot k'}{|\mathbf{k}'|\omega} f_{l\phi}(k,q) f_N^{eq}(\omega) f_{l\phi}(k',q')$$

$$\times\frac{1}{\Gamma}\frac{\frac{1}{4}\Gamma_{l\phi}\Gamma'_{l\phi}}{((\omega-k-q)^2+\frac{1}{4}\Gamma_{l\phi}^2)((\omega-k'-q')^2+\frac{1}{4}\Gamma'^2_{l\phi})}$$

$$\left(1-e^{-\Gamma t}\right)$$

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$$\times\left(1-e^{-\Gamma t}\right)$$

BUT: This is not yet a consistent treatment of gauge interactions!!!

Conclusions

- We computed the generated lepton asymmetry for hierarchical heavy neutrino masses and a constant (or very slowly changing) temperature without semi-classical approximations.
- Quantum and non-Markovian effects can be crucial for leptogenesis.
- We find significant deviations from Boltzmann equations due to off-shell effects, memory effects and temperature dependent corrections.
- The consistent inclusion of all SM corrections remains an issue.