Beyond mean field approach to the β decay of medium mass nuclei relevant for nuclear astrophysics

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Gamow-Teller β decay of

- **rp-process waiting-point nuclei in the A ~80 region**
- **r-process neutron-rich nuclei in the A~100 region**

Characteristic features of proton-rich A~80 and neutron-rich A~100 nuclei

- ***shape-coexistence and -mixing***
- ***drastic changes in structure with particle number, angular momentum, and excitation energy***

Self-consistent description of structure and dynamics requires

- ***beyond mean field approaches***
- ***realistic effective interactions in large model spaces***

complex EXCITED VAMPIR approach

- **the model space is defined by a finite dimensional set of spherical single particle states**

- **the effective many-body Hamiltonian is represented as a sum of one- and two-body terms**

- **the basic building blocks are Hartree-Fock-Bogoliubov (HFB) vacua**

- **the HFB transformations are essentially *complex* and allow for proton-neutron, parity, and angular momentum mixing being restricted by time-reversal and axial symmetry**

- **the broken symmetries (s=N, Z, I, p) are restored by projection before variation**

Beyond mean field variational procedure

complex VAMPIR

$$E^s[F_1^s] = \frac{\langle F_1^s | \hat{H} \hat{\Theta}_{00}^s | F_1^s \rangle}{\langle F_1^s | \hat{\Theta}_{00}^s | F_1^s \rangle} \qquad |\psi(F_1^s); sM\rangle = \frac{\hat{\Theta}_{M0}^s | F_1^s \rangle}{\sqrt{\langle F_1^s | \hat{\Theta}_{00}^s | F_1^s \rangle}}$$

complex EXCITED VAMPIR

$$|\psi(F_i^s); sM\rangle = \Sigma_{j=1}^{i} |\phi(F_j^s)\rangle \, \alpha_j^i \qquad \text{for} \quad i = 1, ..., n-1$$

$$|\phi(F_i^s); sM\rangle = \hat{\Theta}_{M0}^s | F_i^s \rangle$$

$$|\psi(F_n^s); sM\rangle = \Sigma_{j=1}^{n-1} |\phi(F_j^s)\rangle \, \alpha_j^n + |\phi(F_n^s)\rangle \, \alpha_n^n$$

$$(H - E^{(n)} N) f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi_\alpha^{(n)}; sM > = \sum_{i=1}^{n} |\psi_i; sM > f_{i\alpha}^{(n)}, \qquad \alpha = 1, ..., n$$

A = 60 – 90 mass region

^{40}Ca - core

model space (π, ν):

$1p_{1/2}\ 1p_{3/2}\ 0f_{5/2}\ 0f_{7/2}\ 1d_{5/2}\ 0g_{9/2}$

(charge-symmetric basis + Coulomb contributions to the π-spe from the core)

extended model space $\{\ 1d_{3/2}\ 0g_{7/2}\ \ 2s_{1/2} \}$

renormalized G–matrix (OBEP, Bonn A) (Bonn CD)

- short range Gaussians in the *nn, pp, np channels*
- monopole shifts:

$\langle 0g_{9/2}0f; T=0|\hat{G}|0g_{9/2}0f; T=0\rangle$

$\langle 1p1d_{5/2}; T=0|\hat{G}|1p1d_{5/2}; T=0\rangle$

$A \sim 100$ mass *region*

^{40}Ca *- core*

model space for both: protons and neutrons

$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $2s_{1/2}$ $1d_{3/2}$ $1d_{5/2}$ $0g_{7/2}$ $0g_{9/2}$ $0h_{11/2}$

renormalized G-matrix (OBEP, Bonn A)

- short range Gaussians in the pp, np, nn channels
- monopole shifts:

$\langle 0g_{9/2}\ 0f;\ T=0|G|\ 0g_{9/2}\ 0f; T=0\rangle$

$\langle 0g_{7/2}\ 0g_{9/2};\ T=0|G|\ 0g_{7/2}\ 0g_{9/2}; T=0\rangle$

$\langle 1d_{5/2}\ 0h_{11/2};\ T=0|G|\ 1d_{5/2}\ 0h_{11/2}; T=0\rangle$

Gamow-Teller β decay of the rp-process waiting point ^{72}Kr

CERN/ISOLDE I. Piqueras, Eur. Phys. J. A16(2003)313

$^{72}Kr \rightarrow {}^{72}Br$

$Q_{EC} = 5.040 \pm 0.375$ MeV

$0^+_{gs} \rightarrow 1^+$

open problem: ***possible β-decay contribution from low-lying states***

$0^+_I \rightarrow 1+$

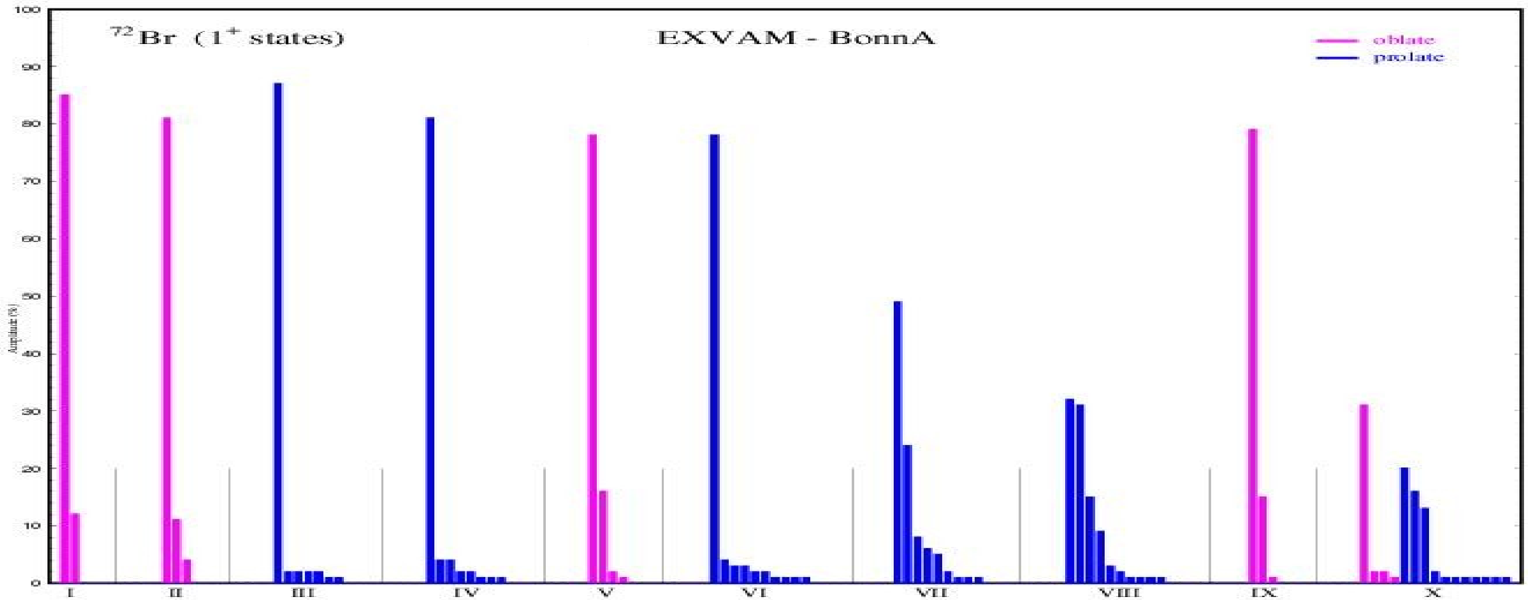
$E_{0^+_I} = 0.671$ MeV

$2^+_{yrast} \rightarrow 1^+ \; / \; 2^+ \; / 3^+$

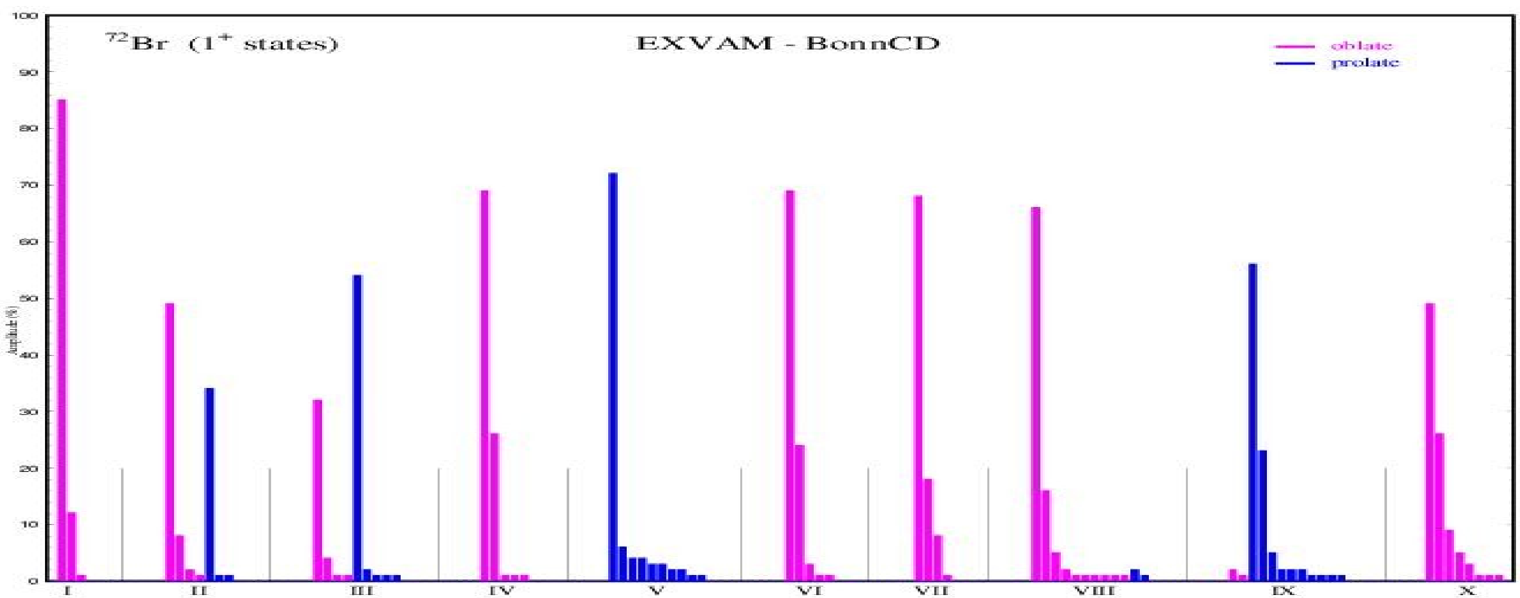
$E_{2^+_{yrast}} = 0.710$ MeV

The amount of mixing for the considered states of the ^{72}Kr nucleus (ms3).

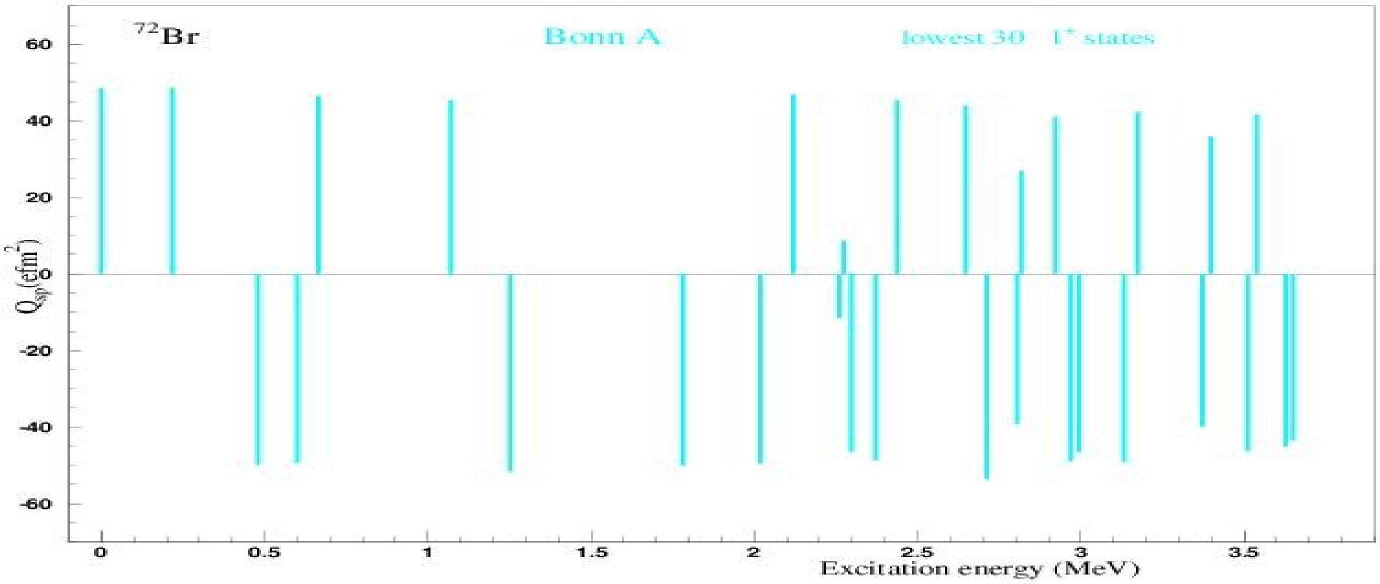
$I[\hbar]$	Bonn A o-mixing	p-mixing	Bonn CD o-mixing	p-mixing
0_1^+	64(2)%	29(2)(1)(1)%	50(3)%	38(5)(3)%
0_2^+	35(2)%	57(3)(1)(1)%	49(2)%	46(3)%
2_1^+	92(1)%	6%	76(1)%	20(3)%



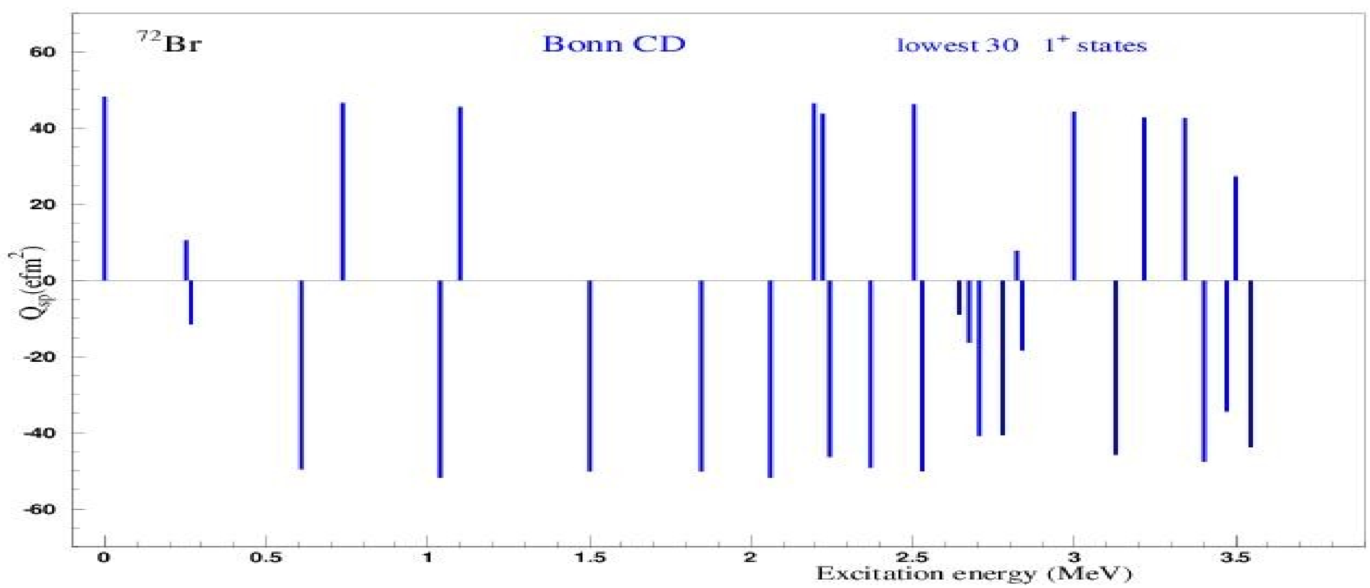
72Br (1+ states)
EXVAM - BonnA
oblate
prolate
Amplitude (%)
0 10 20 30 40 50 60 70 80 90 100
I II III IV V VI VII VIII IX X



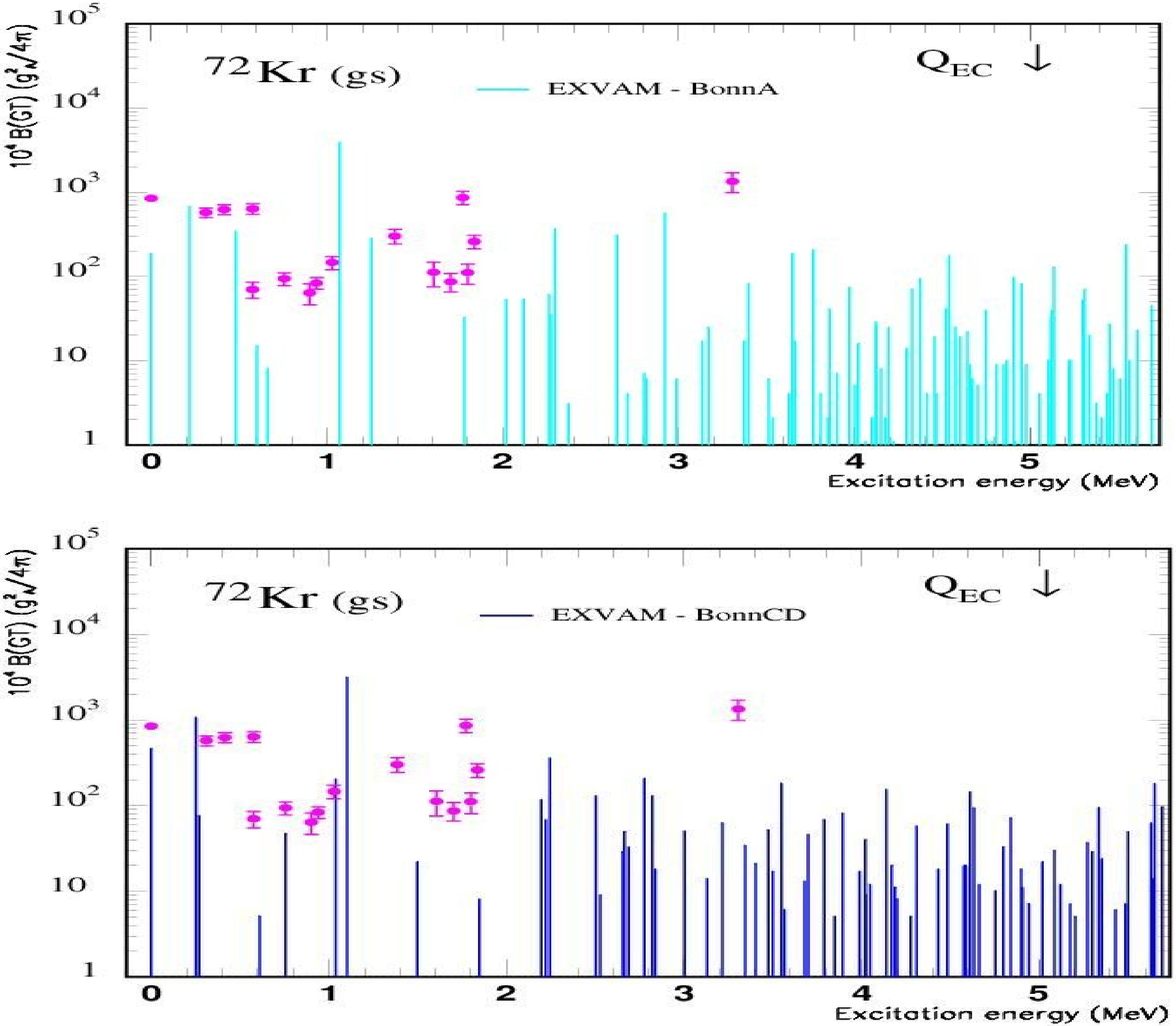
72Br (1+ states)
EXVAM - BonnCD
oblate
prolate
Amplitude (%)
0 10 20 30 40 50 60 70 80 90 100
I II III IV V VI VII VIII IX X



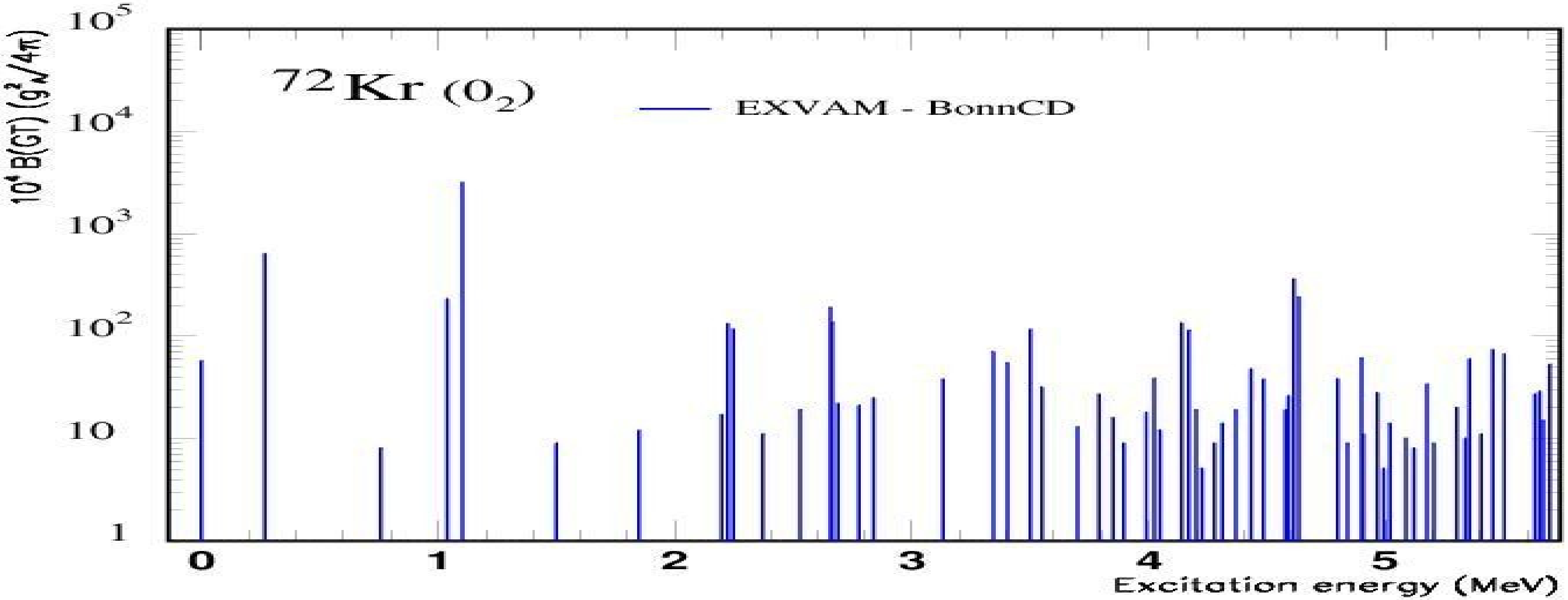
^{72}Br
Bonn A
lowest 30 1^{+} states
Q_{sp}(efm^2)
Excitation energy (MeV)



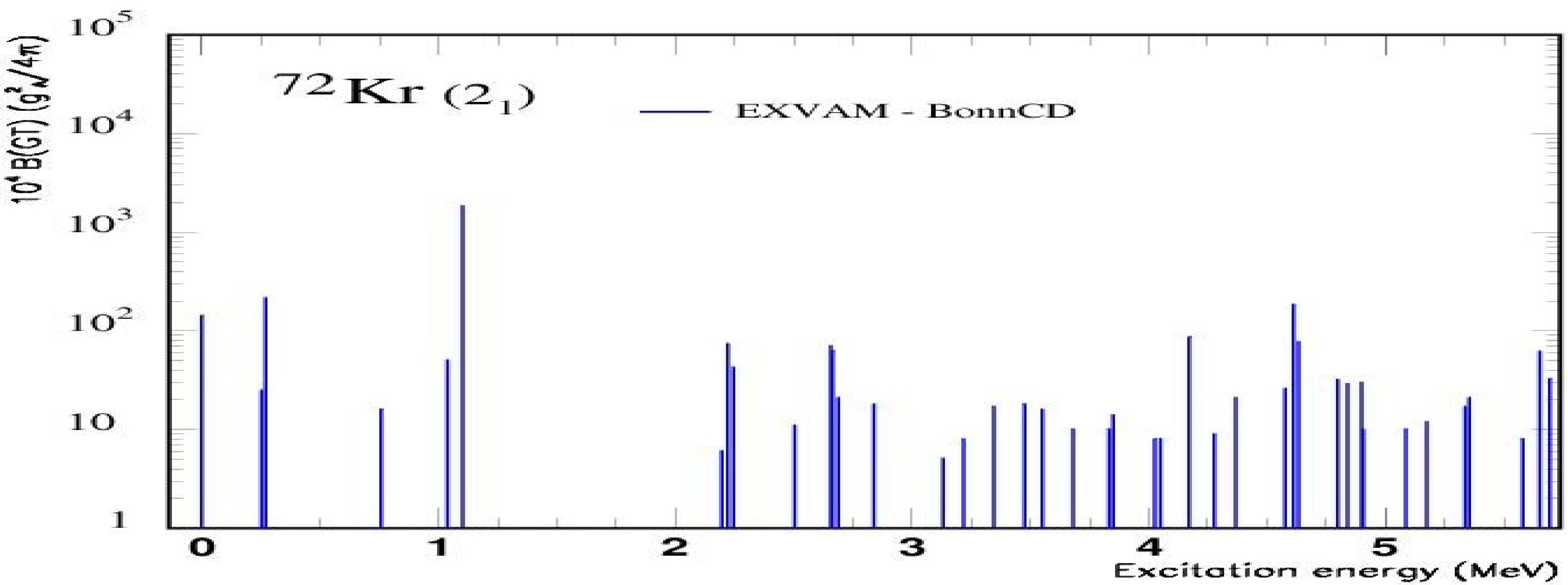
^{72}Br
Bonn CD
lowest 30 1^{+} states
Q_{sp}(efm^2)
Excitation energy (MeV)



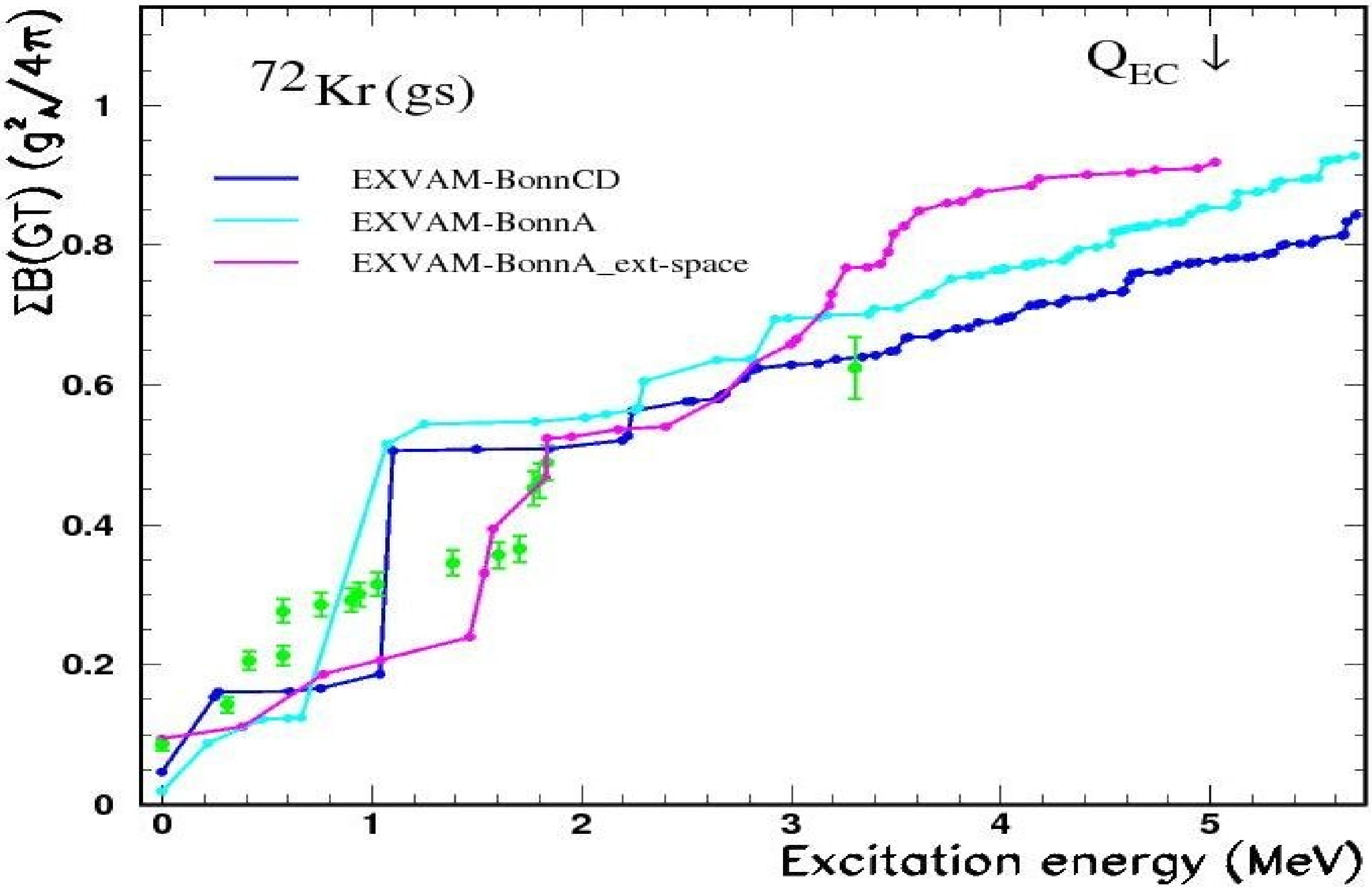
^{72}Kr (gs)
Q_{EC}
EXVAM - BonnA
10^4 B(GT) ($g_A^2/4\pi$)
Excitation energy (MeV)
^{72}Kr (gs)
Q_{EC}
EXVAM - BonnCD
10^4 B(GT) ($g_A^2/4\pi$)
Excitation energy (MeV)



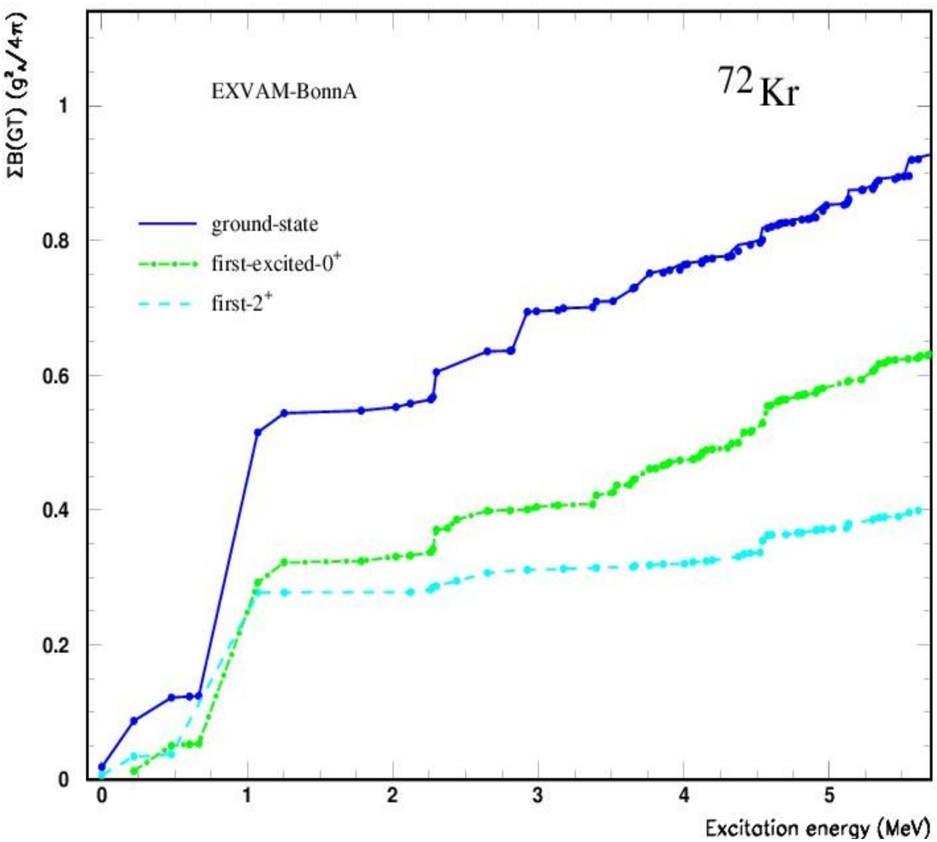
^{72}Kr (0_2)
EXVAM - BonnCD
10^4 B(GT) ($g_A^2/4\pi$)
Excitation energy (MeV)



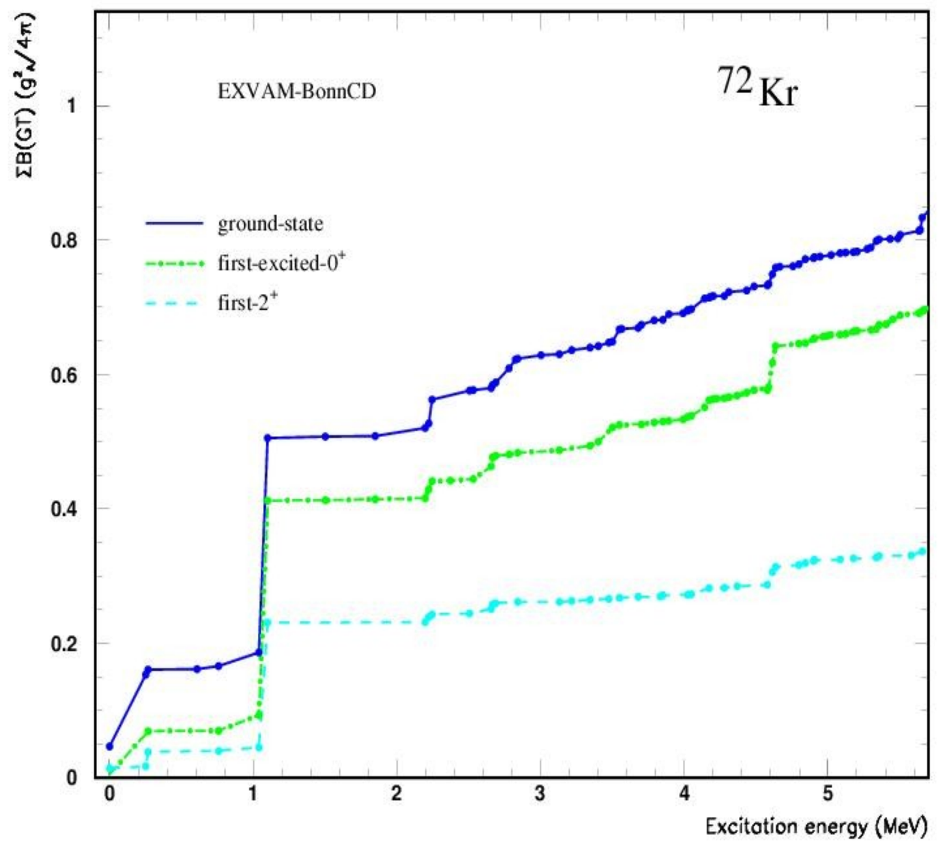
^{72}Kr (2_1)
EXVAM - BonnCD
10^4 B(GT) ($g_A^2/4\pi$)
Excitation energy (MeV)



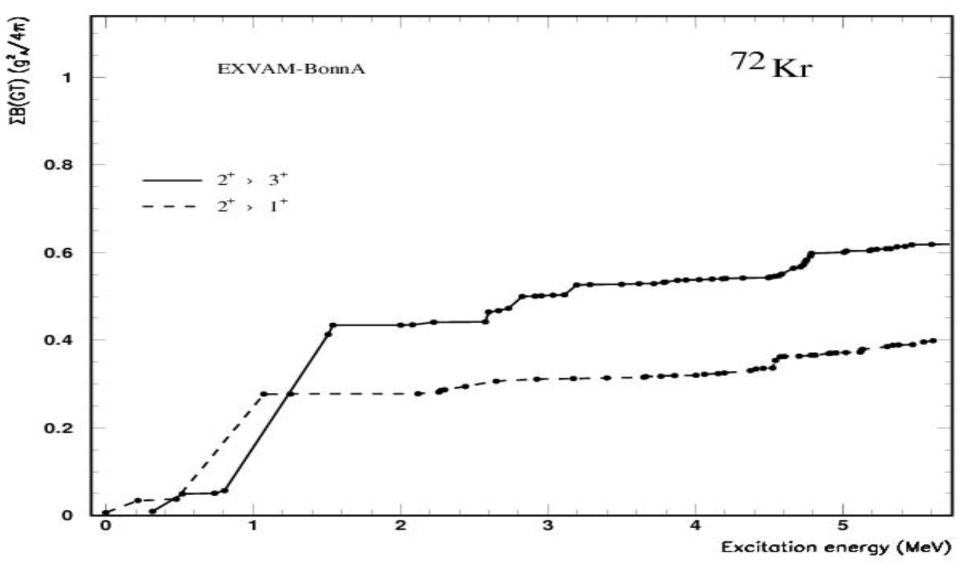
A.Petrovici, J. Phys. G: Nucl. Part. Phys. 37(2010)064036



EXVAM-BonnA
72Kr
ground-state
first-excited-0+
first-2+
ΣB(GT) (g²A/4π)
Excitation energy (MeV)



EXVAM-BonnCD
72Kr
ground-state
first-excited-0+
first-2+
ΣB(GT) (g²A/4π)
Excitation energy (MeV)



EXVAM-BonnA
72Kr
2+ > 3+
2+ > 1+
ΣB(GT) (g²A/4π)
Excitation energy (MeV)

$$\frac{1}{T_{1/2}} = \frac{g_A^2}{D} \sum_i f(Z, E_i) | \langle 1_i^+ || \beta^+ || 0^+ \rangle |^2$$

$D = 6146$ s $\quad g_A = 1.26$

$T_{1/2}^{exp} = 17.1(2)$ s

$T_{1/2}$ (gs) = 20.8 s (Bonn A) 18.9 s (Bonn CD)

$T_{1/2}$ (first-excited 0^+) = 17.3 s (Bonn A) 12.9 s (Bonn CD)

$T_{1/2}$ (yrast $2^+ \to 1^+$) = 18.7 s (Bonn A) 21.6 s (Bonn CD)

$T_{1/2}$ (yrast $2^+ \to 3^+$) = 19.5 s (Bonn A)

$$\lambda = \ln 2/K \sum_i [(2J_i+1)\, e^{-E_i/(kT)}] / G(Z,A,T) \sum_j B_{ij} \Phi_{ij}$$

i – parent states *j – daughter states*

$$G(Z,A,T) = \sum_i e^{-E_i/(\kappa T)}$$ *(partition function of the parent nucleus)*

$Bij = Bij\ (GT)$

Φ_{ij} *– phase space integral*

$T < 2$ GK *X-ray bursts*

In the astrophysical environment of the X-ray bursts the effect of the decay of the first excited 0^+ state of ^{72}Kr is within the uncertainty of the ground-state half-life

A. Petrovici et al., Phys. Rev. C78 (2008) 044315

Gamow-Teller β decay of the rp-process waiting point ^{68}Se

CERN/ISOLDE *P. Baumann et al., Phys. Rev. C50 (1994) 1180*

$^{68}Se \rightarrow {}^{68}As$

$Q_{EC} = 4.730 \pm 0.310$ MeV

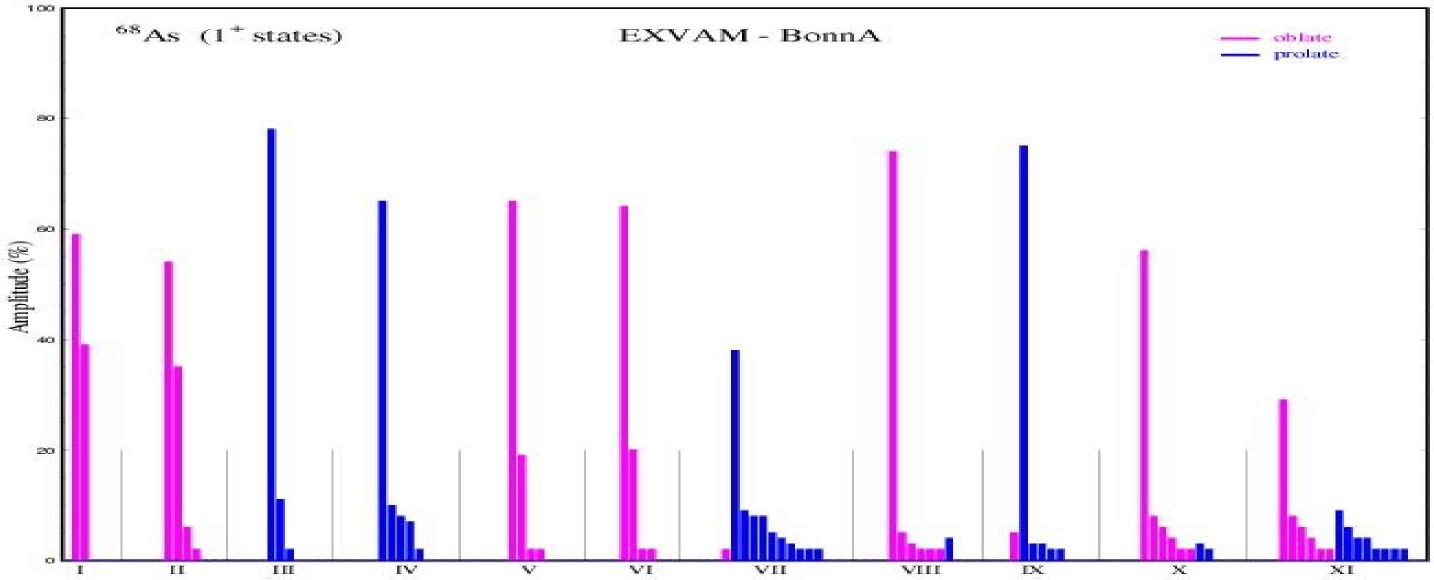
$0^+_{gs} \rightarrow 1^+$

open problem: ***β-decay contribution from possible isomeric excited states***

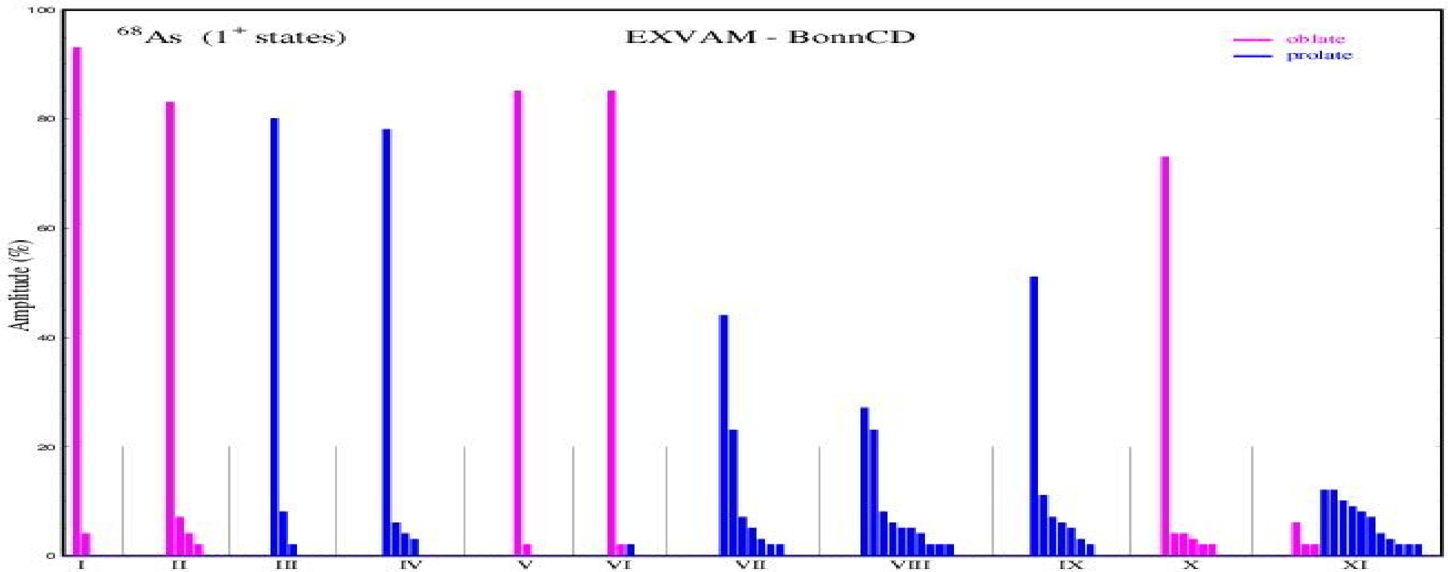
$0^+_{I} \rightarrow 1^+$ $0^+_{II} \rightarrow 1^+$

The amount of mixing for the lowest 0^+ states of the ^{68}Se nucleus (ms3).

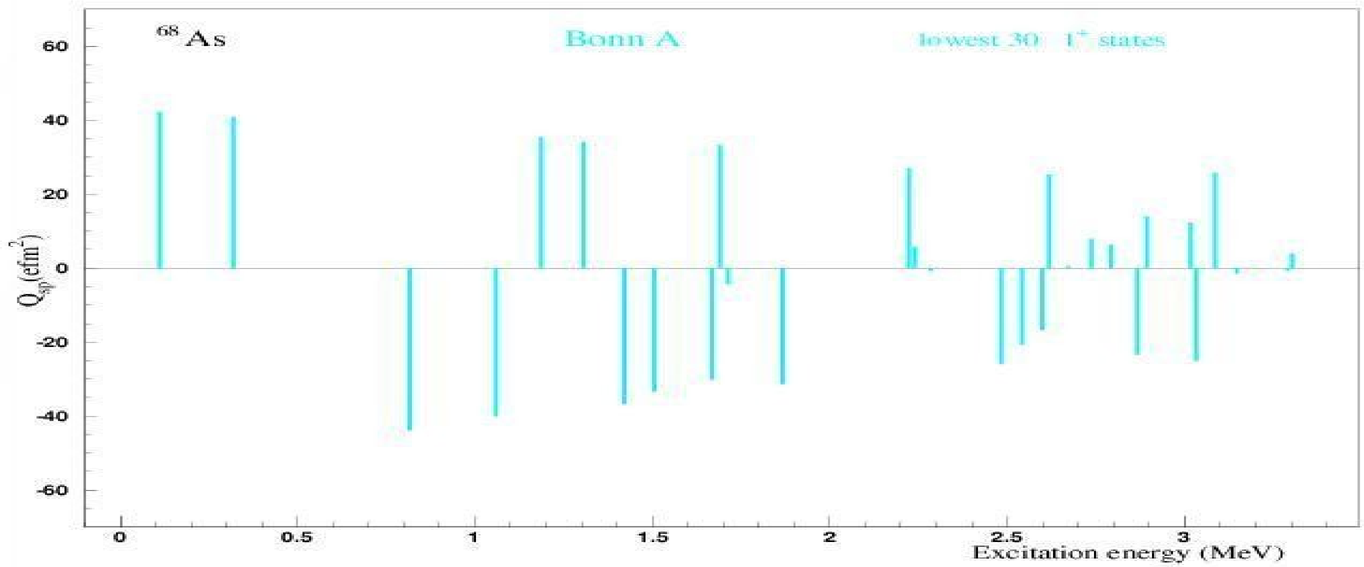
$I[\hbar]$	Bonn A o-mixing	p-mixing	Bonn CD o-mixing	p-mixing
0^+_1	58(2)%	22(10)(4)%	53(2)%	24(11)(4)%
0^+_2	10(6)%	73(5)(3)%	5(5)%	84(3)%
0^+_3	16(7)(3)%	38(20)(10)(2)%	26%	32(16)(11)(10)(2)%



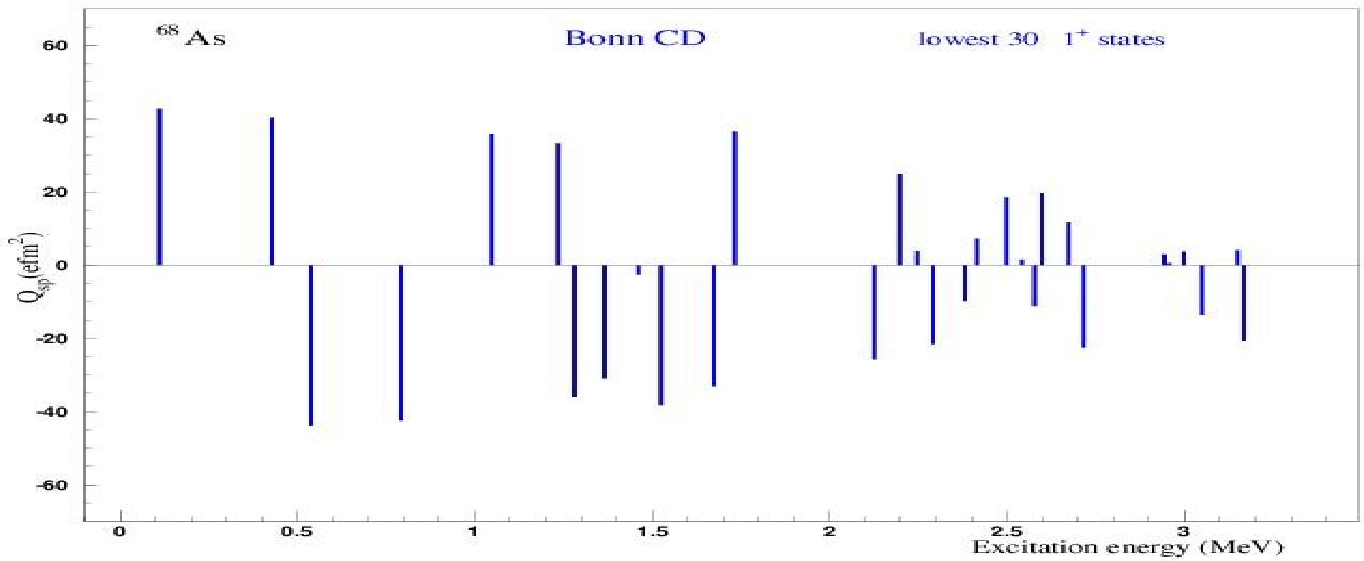
^{68}As (1$^+$ states)
EXVAM - BonnA
oblate
prolate
Amplitude (%)
I
II
III
IV
V
VI
VII
VIII
IX
X
XI



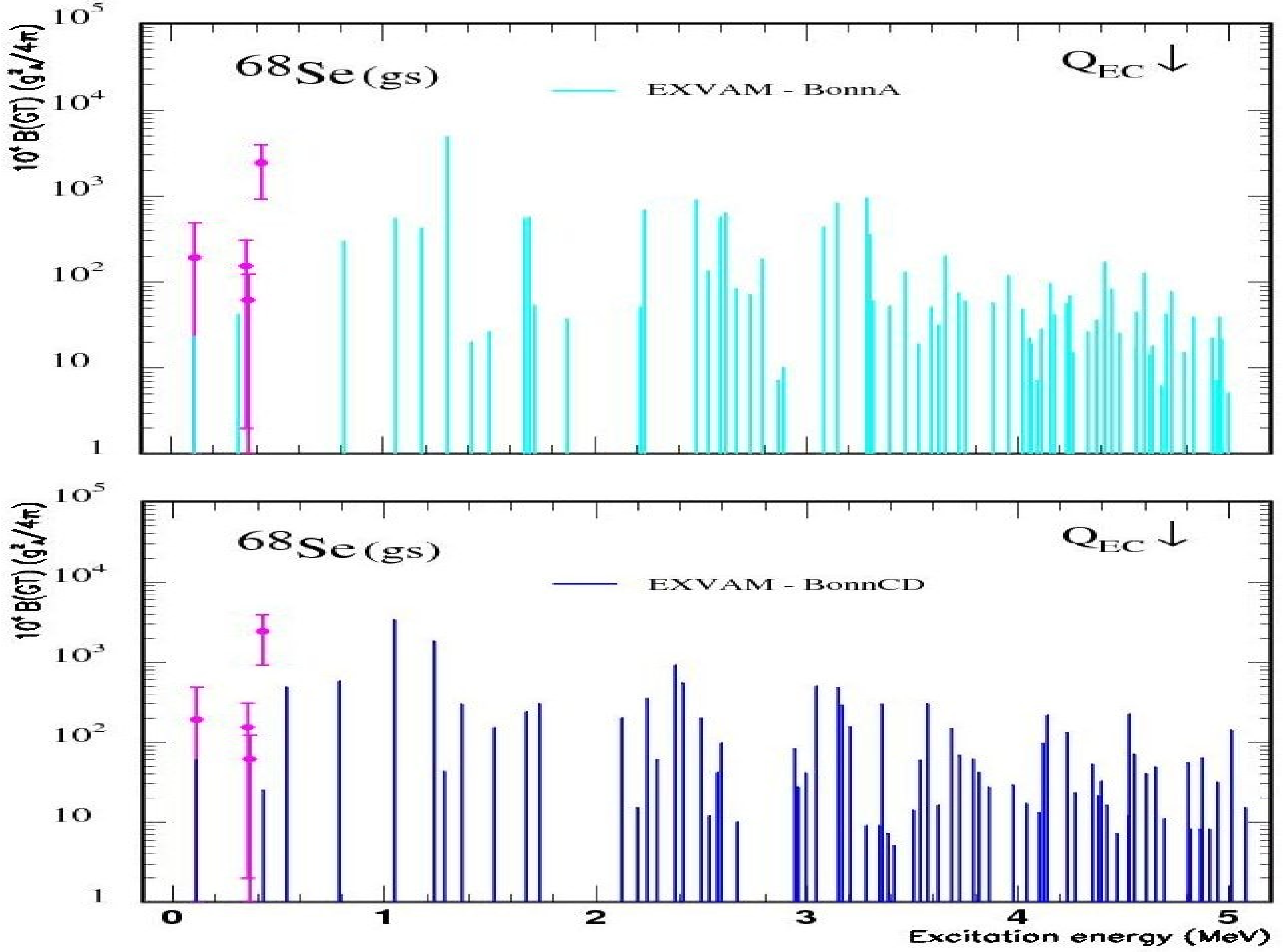
^{68}As (1$^+$ states)
EXVAM - BonnCD
oblate
prolate
Amplitude (%)
I
II
III
IV
V
VI
VII
VIII
IX
X
XI



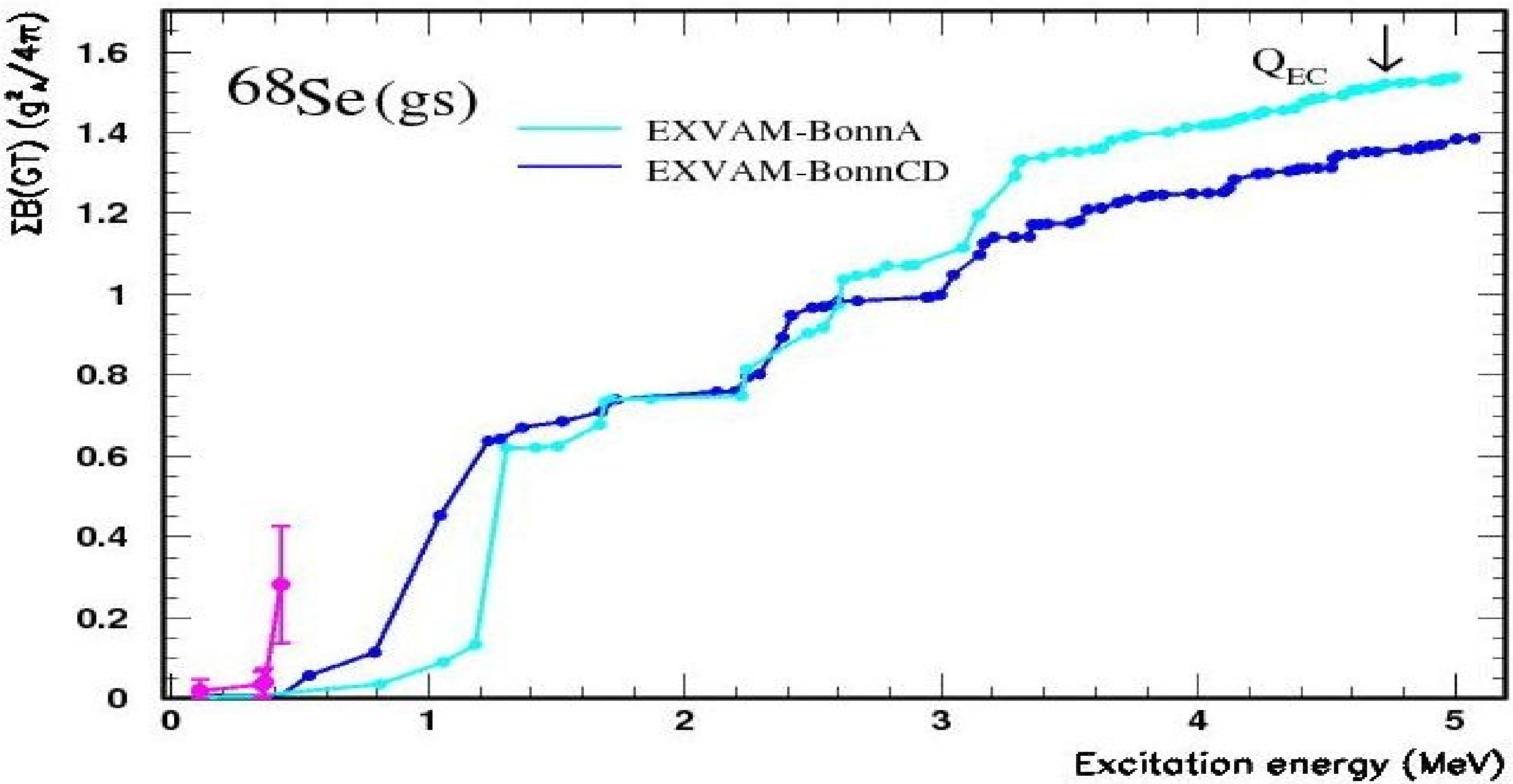
^{68}As
Bonn A
lowest 30 1^+ states
$Q_{sp}(efm^2)$
Excitation energy (MeV)



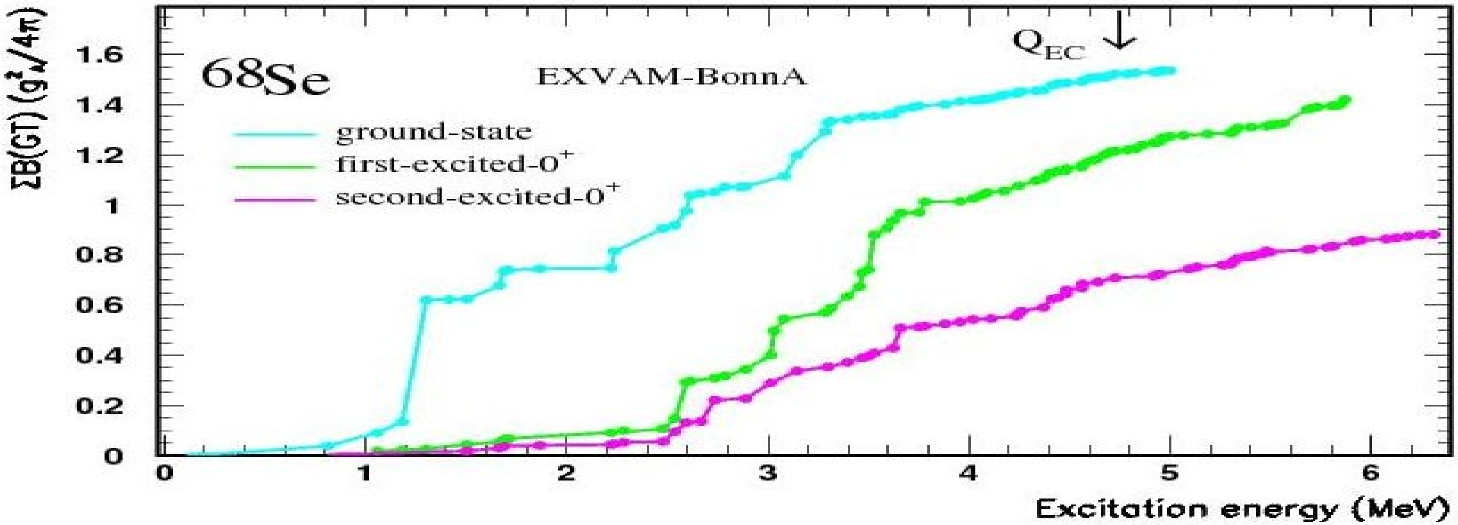
^{68}As
Bonn CD
lowest 30 1^+ states
$Q_{sp}(efm^2)$
Excitation energy (MeV)



68Se(gs)
EXVAM - BonnA
Q_{EC} ↓
68Se(gs)
EXVAM - BonnCD
Q_{EC} ↓
10^4 B(GT) $(g_A^2/4\pi)$
Excitation energy (MeV)



$T_{1/2}^{exp} = 35.5(7)$ s $T_{1/2}^{BonnCD} = 33.9$ s $T_{1/2}^{BonnA} = 48.5$ s

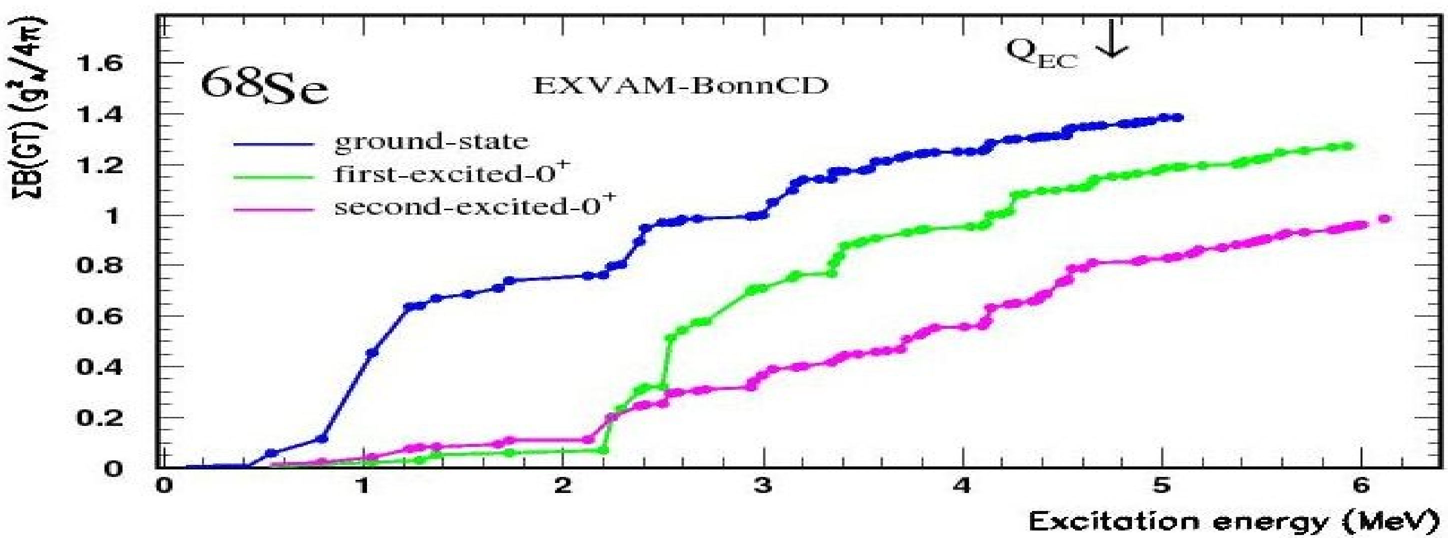


Bonn A

$E^{I}_{0+} = 1.117 MeV$

$T^{I}_{1/2} = 60.7\ s$

$E^{II}_{0+} = 2.155\ MeV$



Bonn CD

$E^{I}_{0+} = 0.843\ MeV$

$T^{I}_{1/2} = 62.8\ s$

$E^{II}_{0+} = 2.004\ MeV$

In the astrophysical environment of the X-ray bursts the decay of the lowest excited 0^+ states of ^{68}Se will not influence the effective half-life.

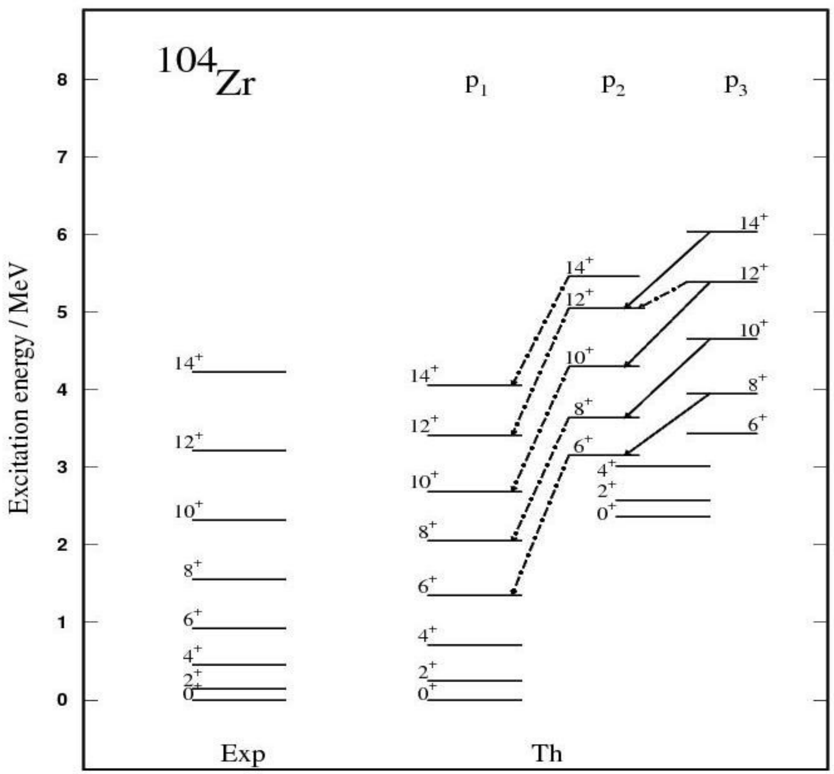
A. Petrovici et al, Phys. Rev. C80 (2009) 044319

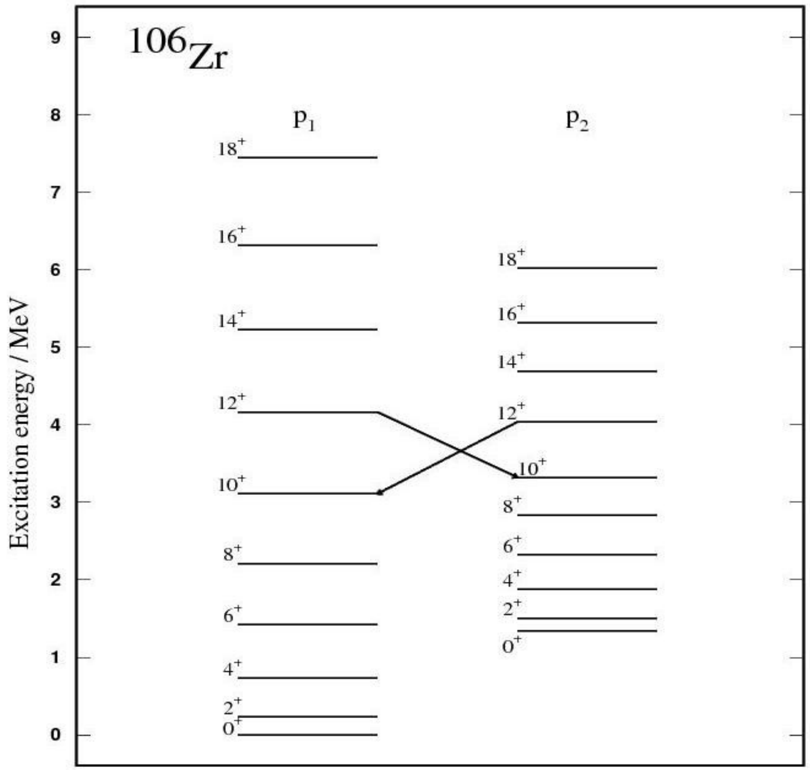
Neutron-rich Zr nuclei relevant for r process

A = 104, 106

A ~ 98-110: rapid transition from spherical to deformed shape, shape coexistence,

competing prolate, oblate, and spherical shapes





Variable mixing of largely- and moderately-deformed prolate configurations

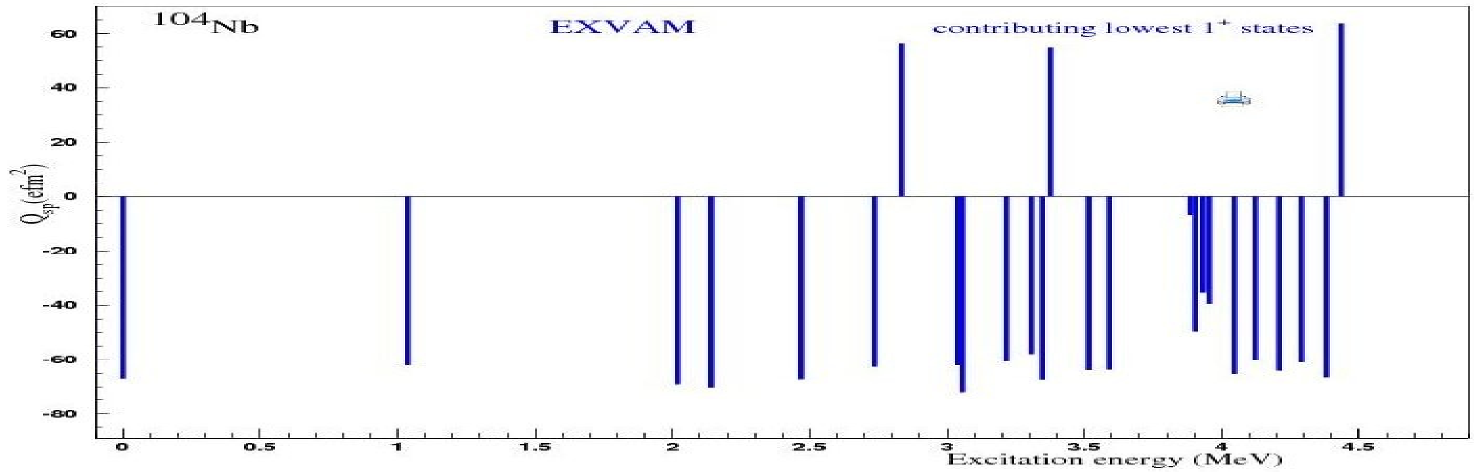
A.Petrovici, INPC 2010, Vancouver, Canada

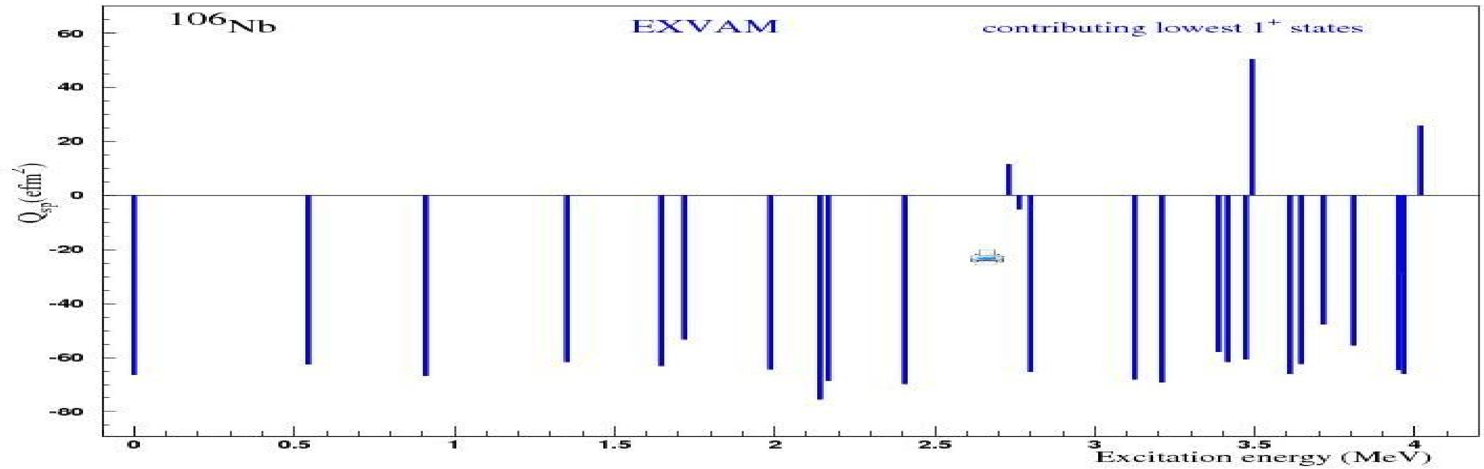
β-decay half-lives and β-delayed neutron emission probabilities

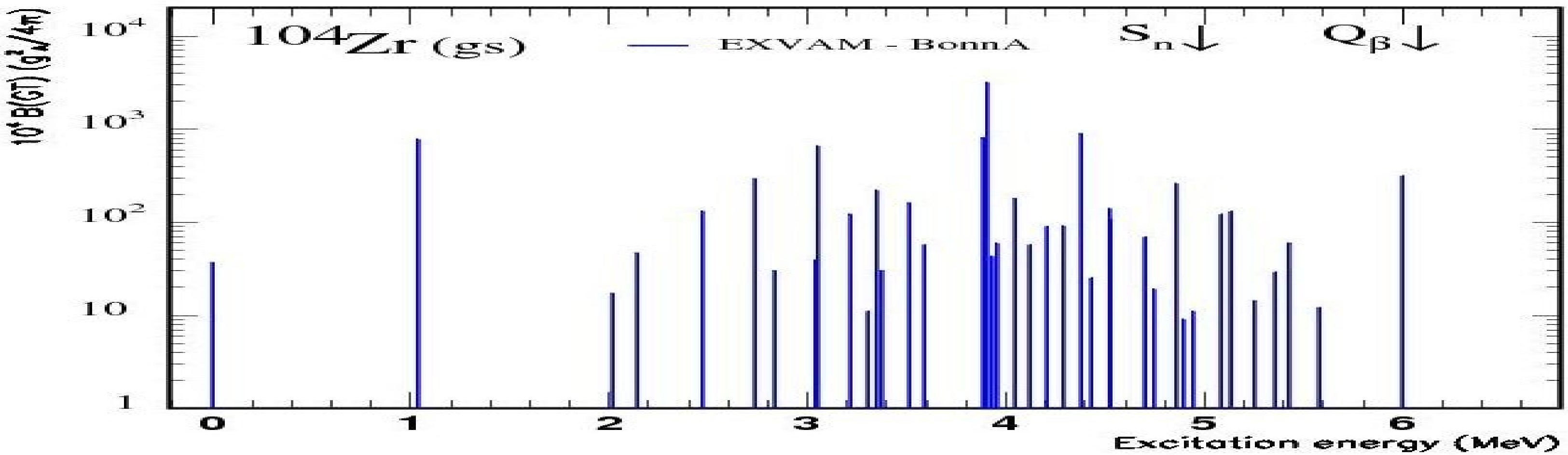
NCSL-MSU / J. Pereira et al., Phys. Rev. C79 (2009) 035801

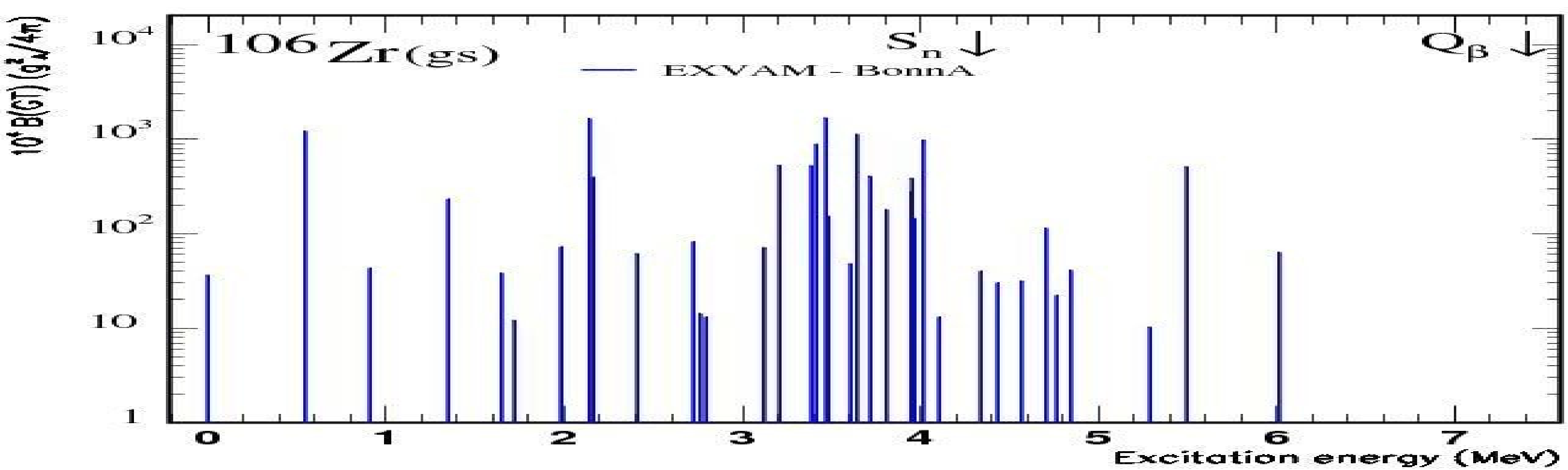
$^{104}Zr \rightarrow {}^{104}Nb$ $^{106}Zr \rightarrow {}^{106}Nb$

complex* Excited Vampir *50 1^+ states in ^{104}Nb and ^{106}Nb

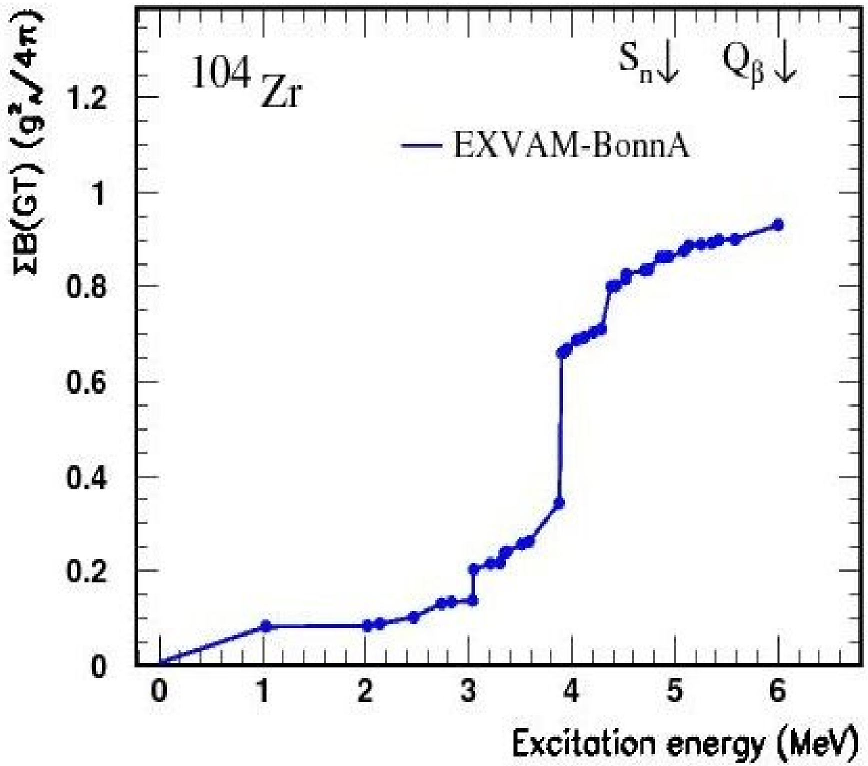


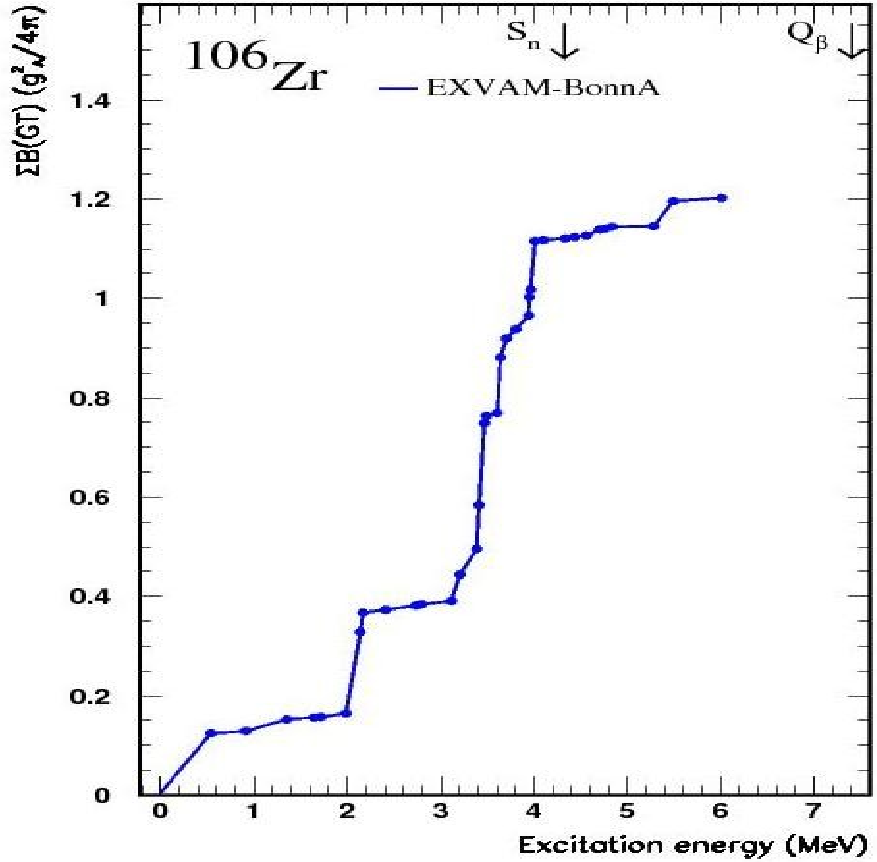






Essential contribution from $g^{\pi}_{9/2} g^{\nu}_{7/2}$, $d^{\pi}_{5/2} d^{\nu}_{3/2}$, and $d^{\pi}_{5/2} d^{\nu}_{5/2}$ matrix elements





$T^{exp}_{1/2}$ 870(50)(30) ms (MSU 2009)

1200(300) ms (Mainz 1980)

P^{exp}_{n} < 1 % (MSU 2009)

$T^{exp}_{1/2}$ 260(20)(30) ms (MSU 2009)

P^{exp}_{n} < 7 % (MSU 2009)

$T^{EXVAM}_{1/2}$ = 2040 ms (50 1^+ states in ^{104}Nb)

P^{EXVAM}_{n} ~ 1 ‰

$T^{EXVAM}_{1/2}$ = 230 ms (50 1+ states in ^{106}Nb)

P^{EXVAM}_{n} ~ 2 %

Summary and outlook

- the *complex* Excited Vampir approach to the Gamow-Teller β decay of the rp-process waiting point nuclei ^{68}Se and ^{72}Kr gives good agreement with the available data

- in the X-ray bursts environment the decay of the lowest isomeric states of ^{68}Se and ^{72}Kr will not influence the effective half-life of these *rp-process waiting point* nuclei

- the *complex* Excited Vampir results on the β-decay half-lives and β-delayed neutron emission probabilities for ^{104}Zr and ^{106}Zr are in agreement with the available data

- in progress: systematic investigations of the structure and dynamics of A ~ 100 neutron-rich nuclei relevant for r process

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