Eberhard Karls Universität Tübingen

NEUTRON STAR DYNAMICS

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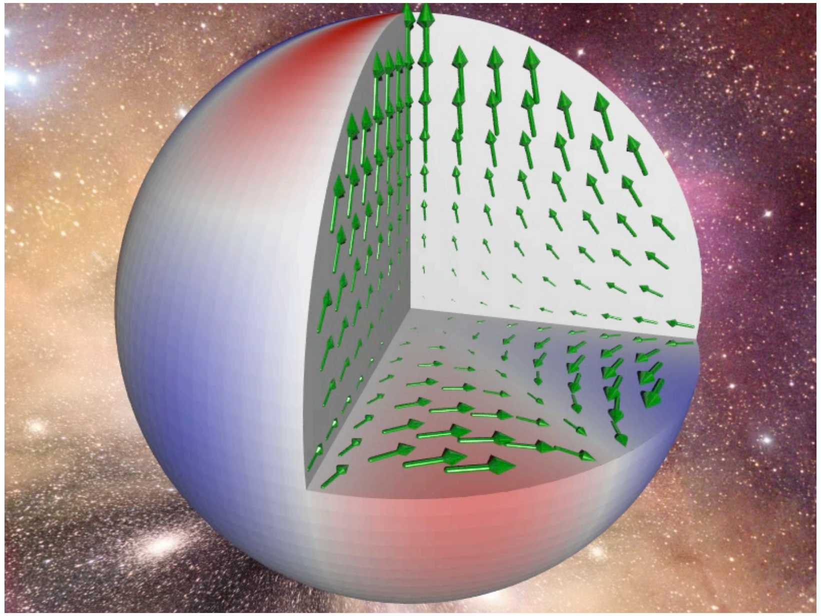
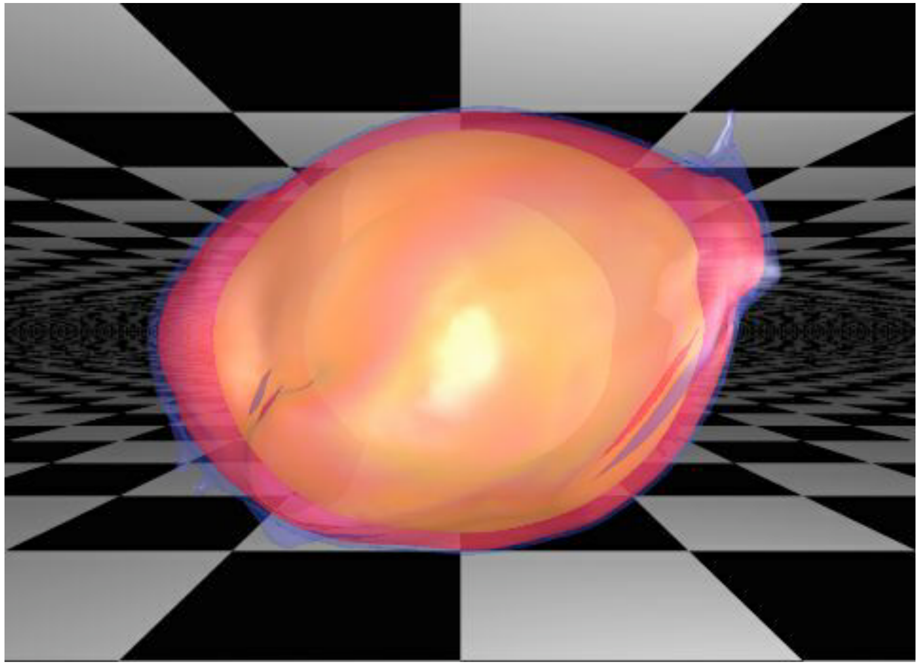
Gravitational Wave Asteroseismology



We can estimate their masses, radii, equations of state by analysing the seismic data via the emitted gravitational waves

Neutron Stars oscillate wildly during the very first seconds of their life

Rotation is responsible for a number of instabilities which emit copious amounts of GWs



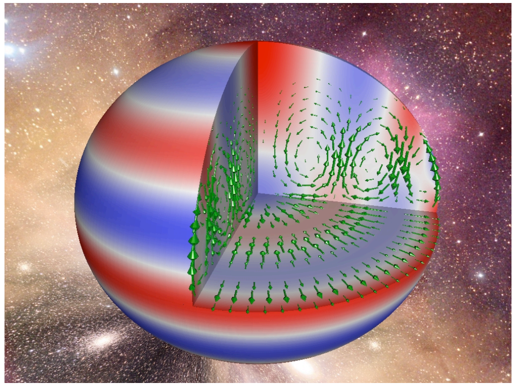
A Laboratory for Theoretical Physics

NS modelling involves the very extremes of physics:

General Relativity	**Alternative theories, Boson, Q-stars,…**
Rotation	**Slow, Fast, Differential** **Instabilities**
Equation of State	**Exotic nuclear physics, strange quarks, hyperons,… Superfluidity, Cold vs Warm**
Magnetic Fields	**Magnetars, Slowdown, Suppression of Instabilities, Uniform Rotation**
Crust – Core interface	**Supression of Instabilities,**

Can GW, x-ray, γ-ray observations constrain the theoretical models?

Neutron Star "ringing"



p-modes: main restoring force is the pressure (**f-mode**) *(>1.5 kHz)*

$$\sigma \approx \sqrt{\frac{M}{R^3}}$$

Inertial modes: (r-modes) main restoring force is the Coriolis force

$$\sigma \approx \Omega$$

w-modes: pure space-time modes (only in GR) *(>5kHz)*

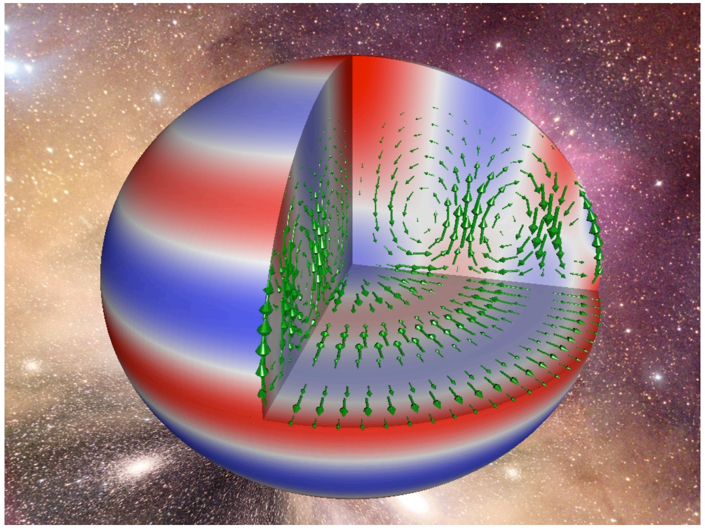
$$\sigma \approx \frac{1}{R}$$

Torsional modes (t-modes) *(>20 Hz)* shear deformations. Restoring force, the weak Coulomb force of the crystal ions.

$$\sigma \approx \sqrt{\frac{v_S}{R}}$$

… and many more

GW Asteroseismology



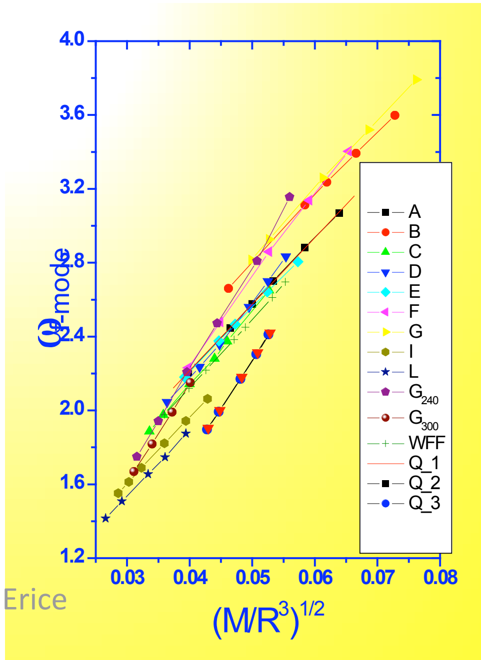
Oscillation patters can reveal the internal structure of neutron stars : mass, radius, EoS, rotation, B-field, crust,…

$$\sigma_f(kHz) \approx 0.78 + 1.64\left(\frac{\bar{M}}{\bar{R}^3}\right)^{1/2}$$ f-mode frequency

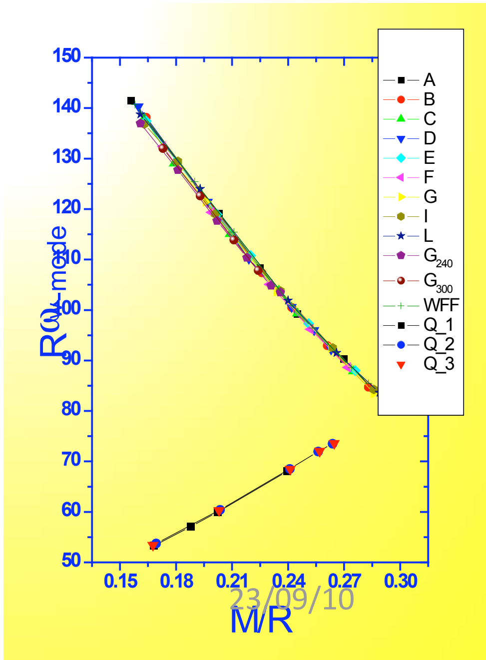
$$\frac{1}{\tau_f(s)} \approx \frac{\bar{M}^3}{\bar{R}^4}\left[22.85 - 14.65\frac{\bar{M}}{\bar{R}}\right]$$ f-mode frequency

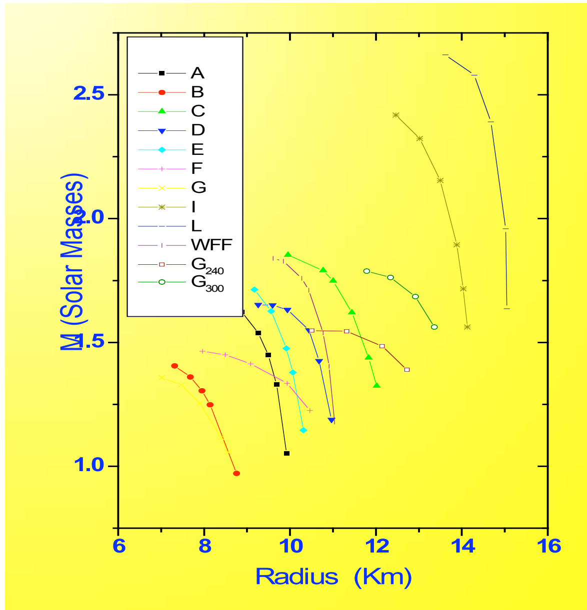
$$\sigma_w(kHz) \approx \frac{1}{\bar{R}}\left(20.9 - 9.1\frac{\bar{M}}{\bar{R}}\right)$$ w-mode frequency

Andersson,KK 1996,1998,2001



+





Effect of Rotation & Magnetic Fields

ROTATION

- Frame dragging
- Quadrupole deformation
- Rotational instabilities
- The degeneracy in m is removed and the nonrotating mode of index ℓ is split into 2ℓ+1 different (ℓ,m) modes
- Shifting of the frequencies and damping times
- Coupling of polar ℓ-term to an axial ℓ±1 term and v-v

MAGNETIC FIELD

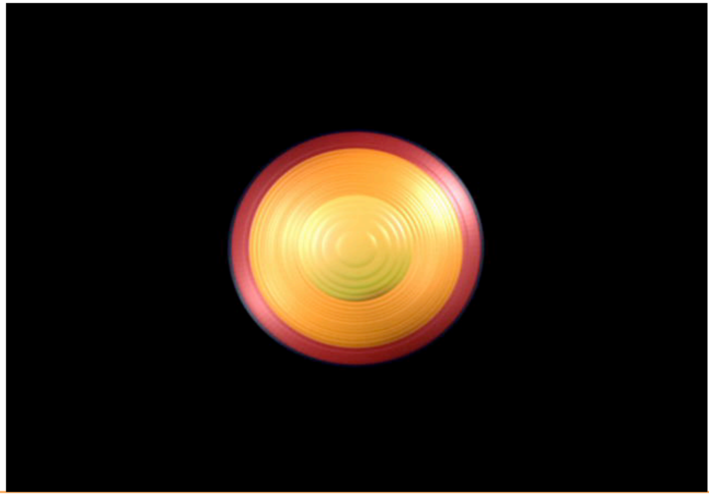
- No significant effect in the fluid frequencies and damping/growth times

$$\frac{\text{magnetic energy}}{\text{gravitational energy}} \sim \frac{B^2 R^3}{GM^2/R} \sim 10^{-4}\left(\frac{B}{10^{16} G}\right)^2$$

- For magnetars we may observe Alfvén oscillations

Stability of Rotating Stars

Non-Axisymmetric Perturbations



A general criterion is:

T : rotational kinetic energy

W : gravitational binding energy

$$\beta = \frac{T}{|W|} \approx \frac{2}{15} e^2 + \ldots$$

Dynamical Instabilities

- Driven by hydrodynamical forces (bar-mode instability)
- Develop at a time scale of about one rotation period

$$e > 0.953 \quad \text{or} \quad \beta > 0.274$$

Secular Instabilities

- Driven by dissipative forces (*viscosity, gravitational radiation*)
- Develop at a time scale of several rotation periods.
  Chandrasekhar-Friedman-Schutz (CFS)

$$e > 0.813 \quad \text{or} \quad \beta > 0.138$$

GR predicts considerably lower β

β*~0.24* for the onset of the dynamical instabilities

β*~0.07* for the onset of the secular instabilities

Bar-mode dynamical instability

- ✓ For rapidly (differentially!) rotating stars with:

$$\beta = \frac{T}{|W|} \sim \frac{1}{R} > \beta_{\rm dyn} \approx 0.27$$

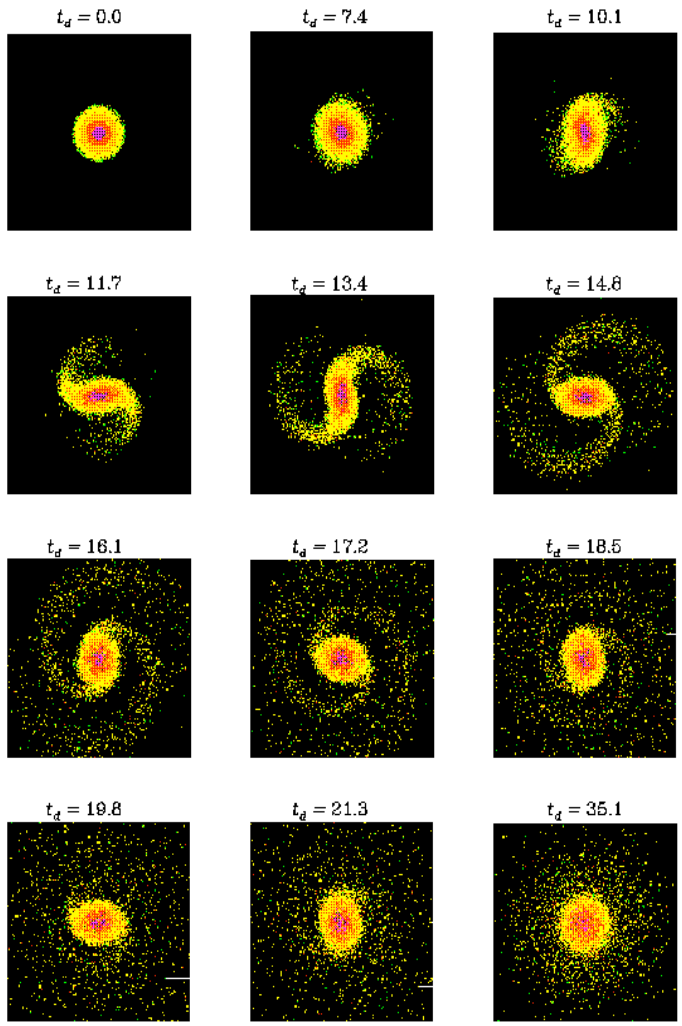
- ✓ **GR enhances** the onset of the instability ($\beta_{dyn} \gtrsim 0.24$) and β decreases with increasing ***M/R***.

- ✓ The "bar-mode" grows on a dynamical timescale.

$$h \approx 10^{-22} \left(\frac{\varepsilon}{0.2}\right)\left(\frac{f}{3\ \rm kHz}\right)^2 \left(\frac{15\ \rm Mpc}{\rm d}\right) M_{1.4} R_{10}^2$$

- ✓ If the bar persists for many (~10-100) rotation periods, the signal will be easily detectable from at least Virgo cluster.

- ✓ **Typical Frequencies ~1.5-3.5kHz**



Bar Mode Dynamical Instability

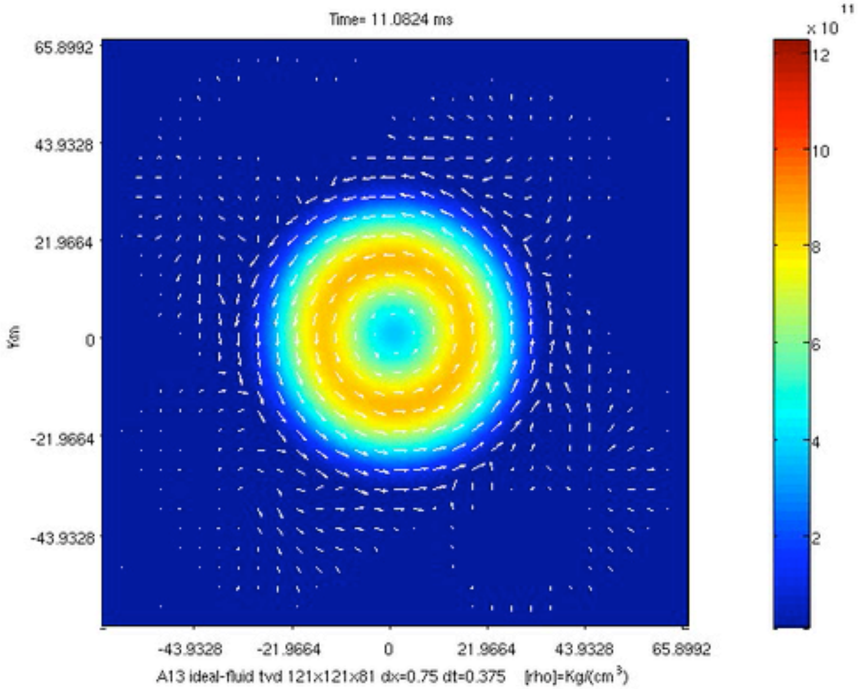
- Bars can be also created during the merging of **NS-NS**, **BH-NS**, **BH-WD** and **Collapsars (type II)**.

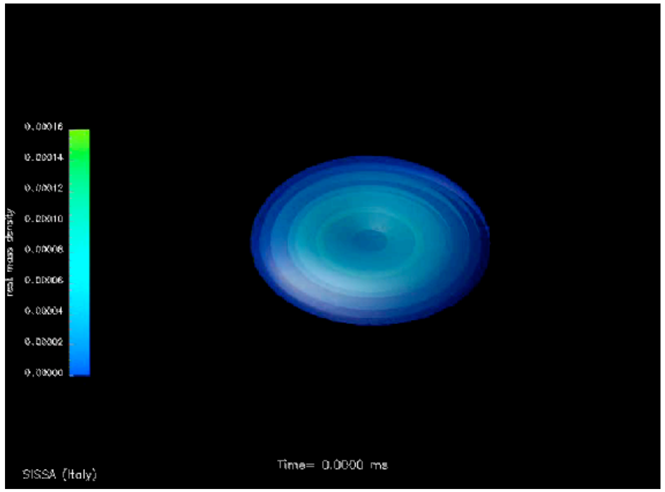
- Bar-mode **instability might happen for much smaller** β if centrifugal forces produce a peak in the density off the source's rotational center.

$$h_{eff} \simeq 3\times10^{-22}\left(\frac{f}{800\,Hz}\right)^{1/2}\left(\frac{R_{eq}}{30\,km}\right)\left(\frac{M}{1.4\,M_{\odot}}\right)^{1/2}\left(\frac{100\,Mpc}{d}\right)$$

- **LOW T/|W| Instability**
  - **Highly differentially rotating stars** are shown to be dynamically unstable for significantly lower β (even when $\beta \gtrsim 0.01$).

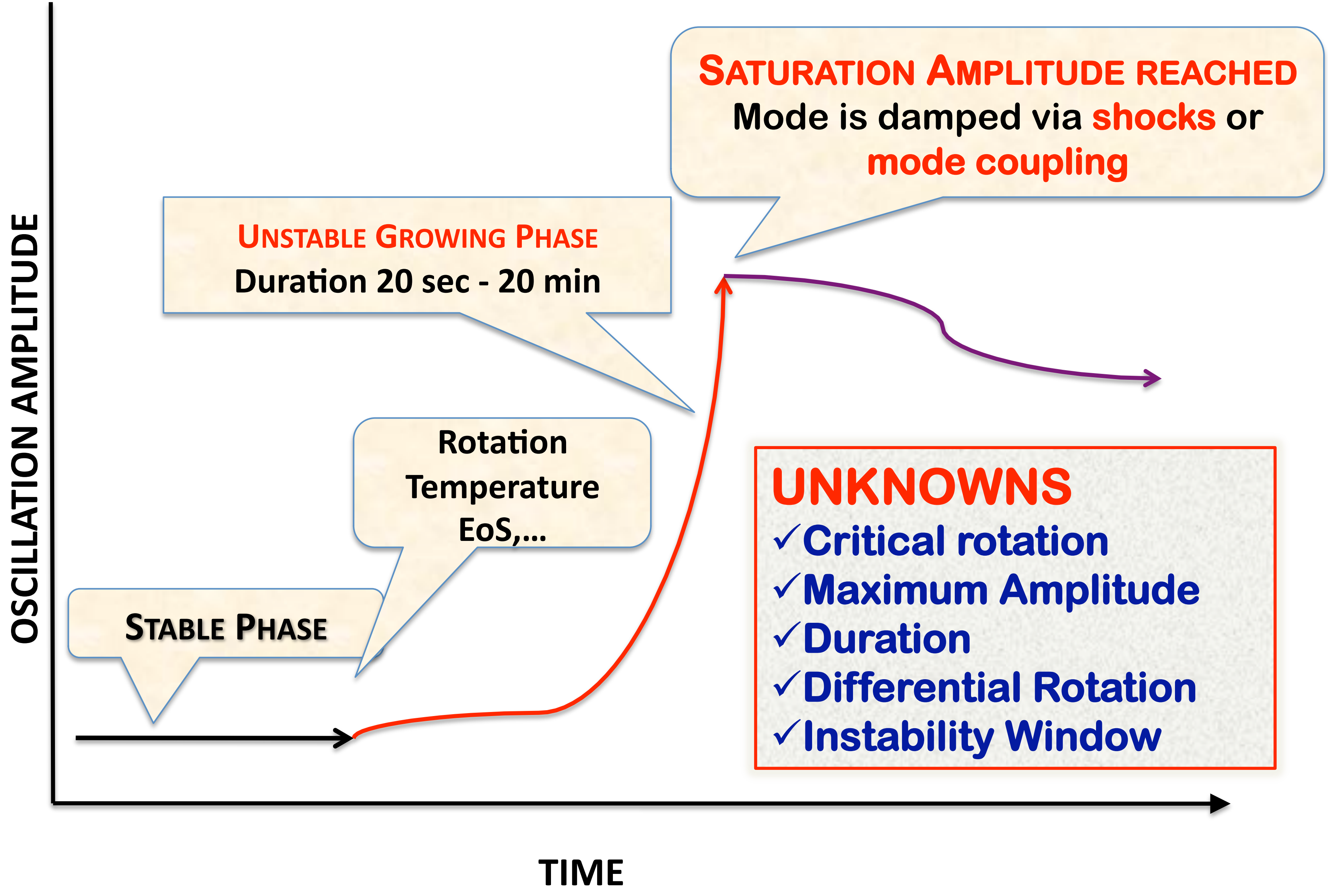
- Bars can be also create during the collapse of a SMS before the creation of a SMBH. **Ideal sources for LISA.**



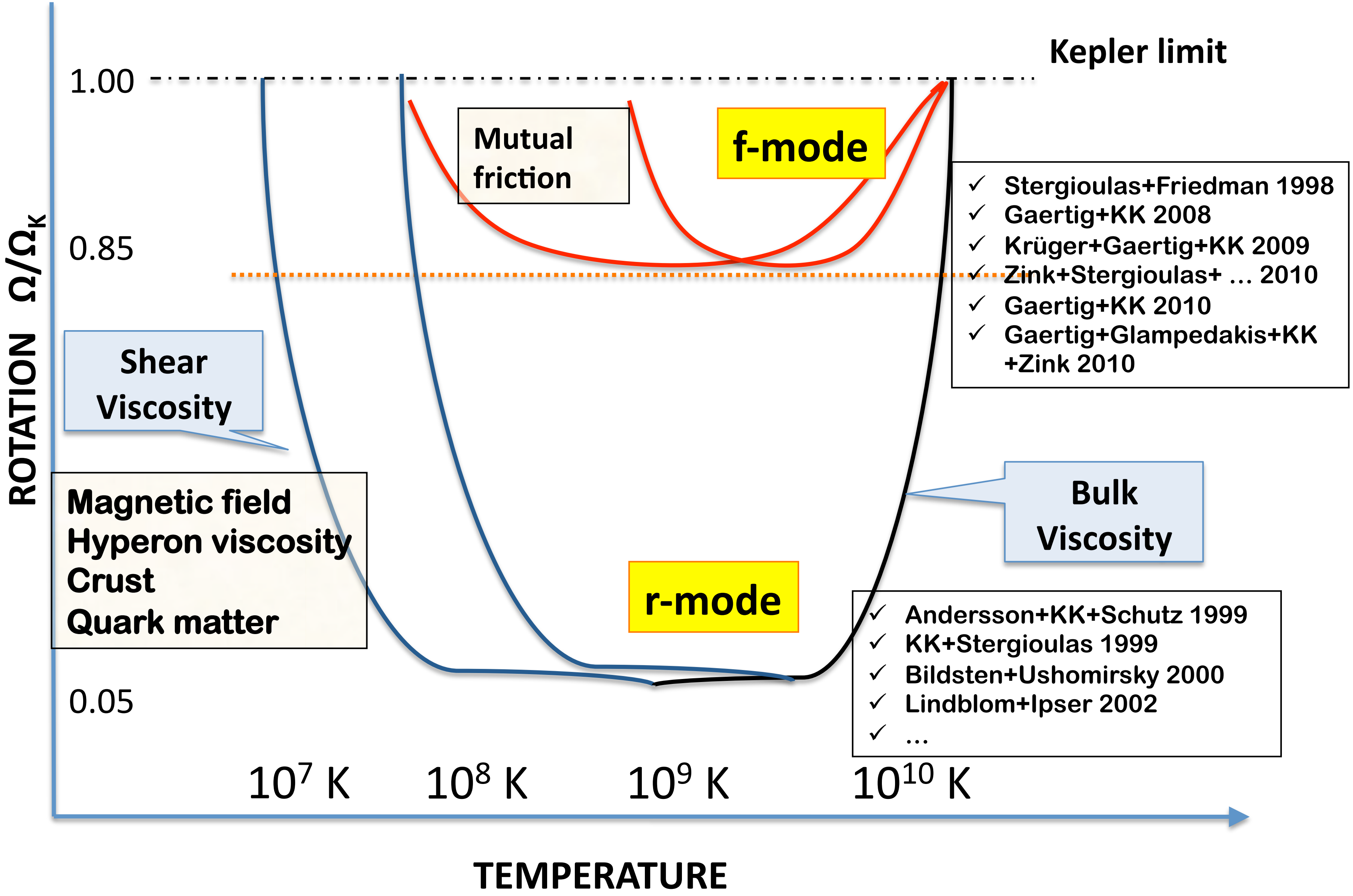


Rezzola et al

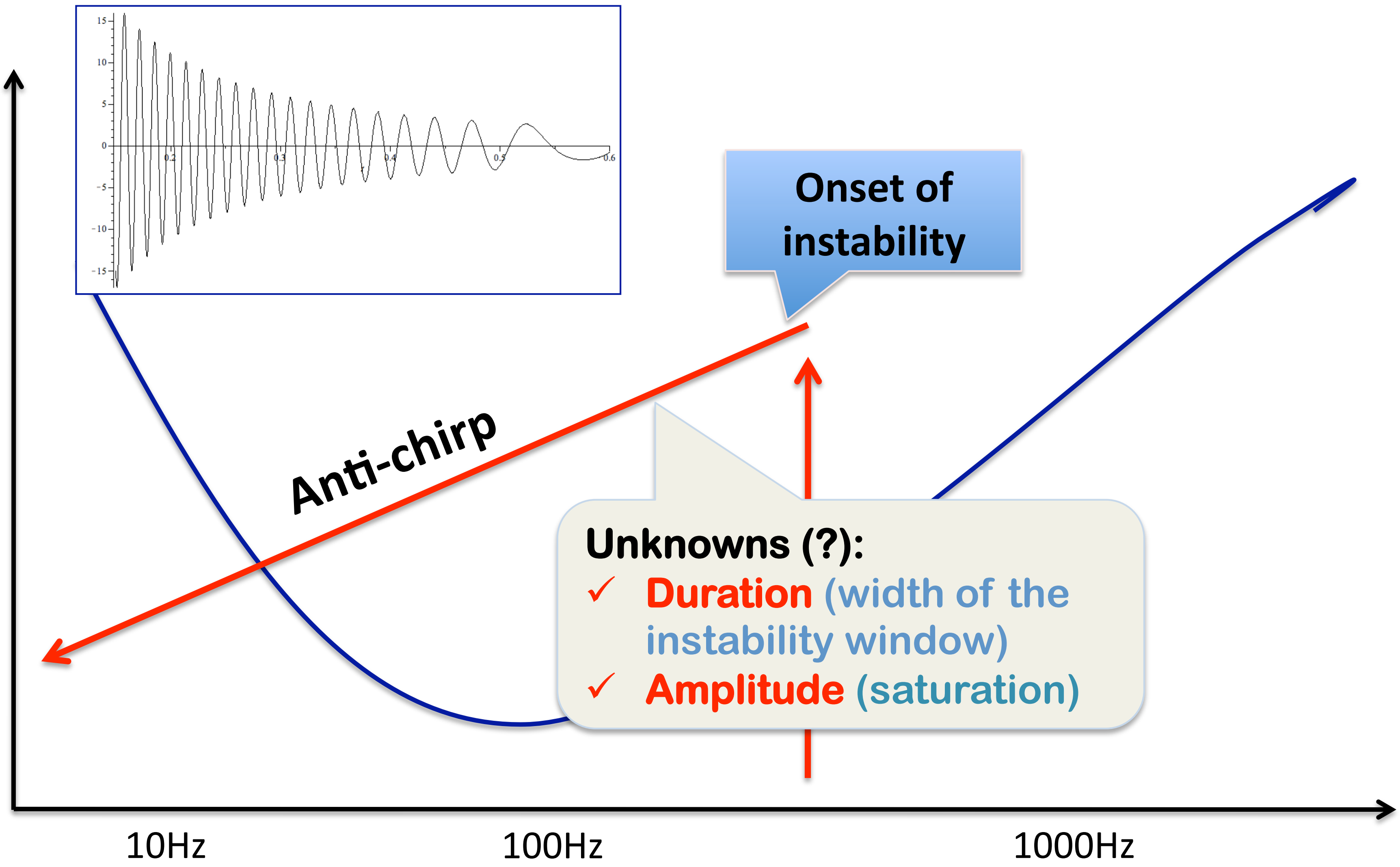
The Excitation of Secular Instabilities



INSTABILITY WINDOW

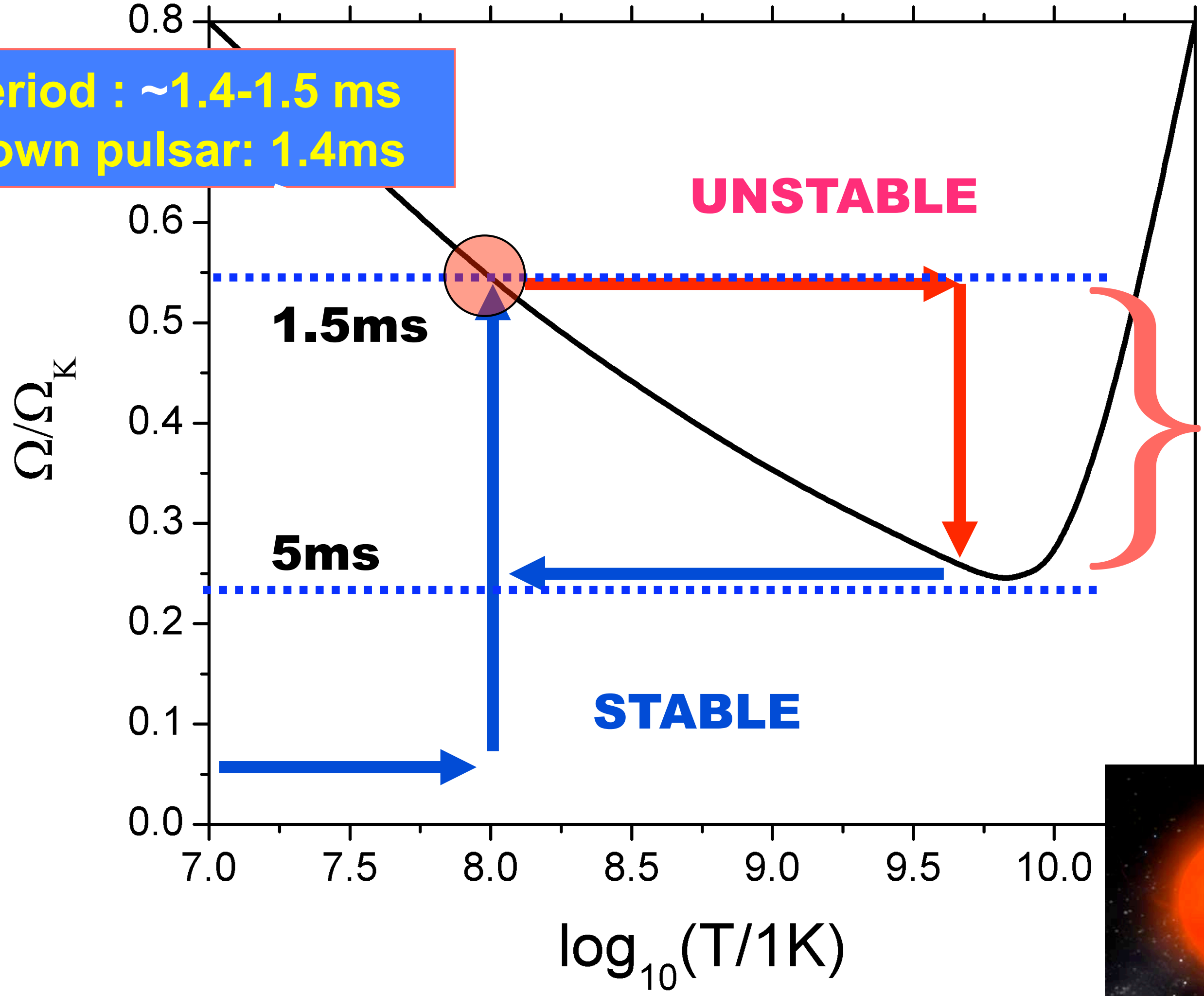


f-mode Instability

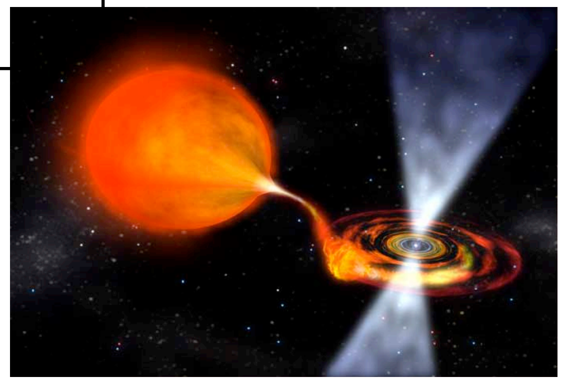


LMXBs & r-modes

Limiting Period : ~1.4-1.5 ms
Fastest known pulsar: 1.4ms

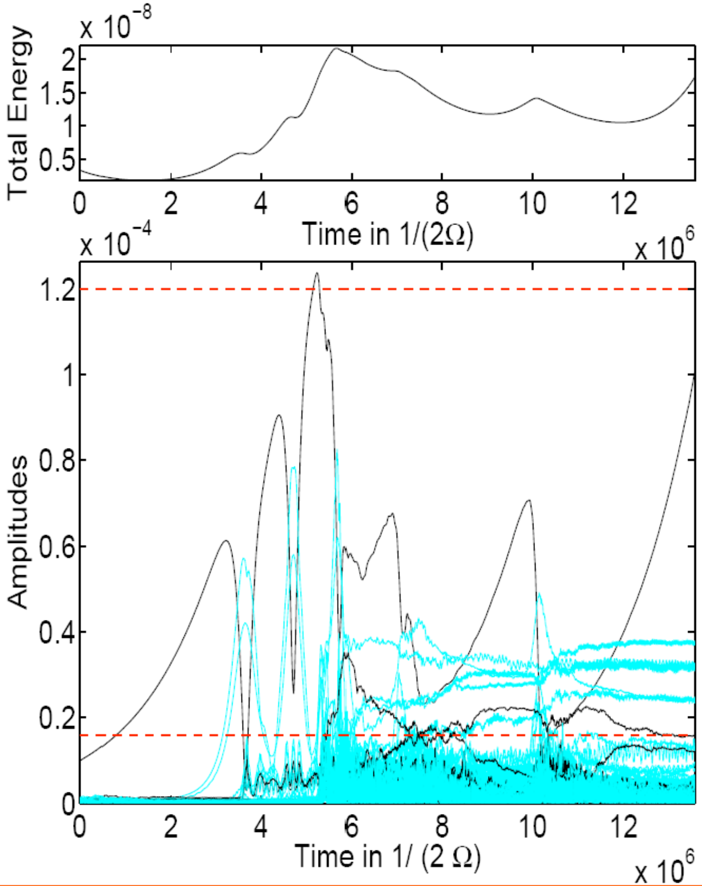


Period clustering of ms pulsars



Andersson, KK, Stergioulas 1999, Levin 2000
Andersson, Jones, KK, Stergioulas 2000, Heyl 2002
Andersson, Jones, KK 2002

R-modes



$$h(t) \approx 10^{-20} \alpha \left(\frac{\Omega}{1 \text{ kHz}} \right) \left(\frac{10\text{Kpc}}{d} \right)$$

$$\alpha \simeq 10^{-3} - 10^{-4}$$

GW amplitude depends on the saturation amplitude

- ✓ **Mode coupling** might not allow the growth of instability to high amplitudes (Cornell group `04-`07)
- ✓ The existence of ***crust***, **hyperons** in the core, **magnetic fields**, affects the efficiency of the instability.
- ✓ For **newly born** neutron stars might be **quite weak** ; unless we have the creation of a strange star
- ✓ **Old accreting neutron** (or strange) stars, probably the best source!

Fast Rotating NS in GR: f-mode

Frequency

$$\omega^2 = \frac{2l(l-1)}{2l+1}\left(\frac{GM}{R^3}\right)$$

Gaertig+KK 2008,09,10
Krueger,,Gaertig, KK 2009
Zink etal 2010

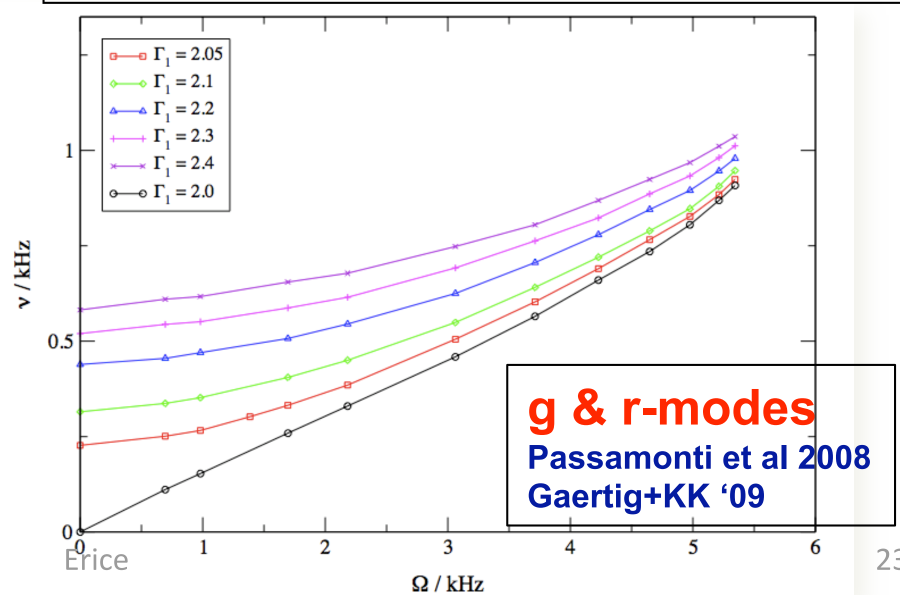
Damping/Growth time

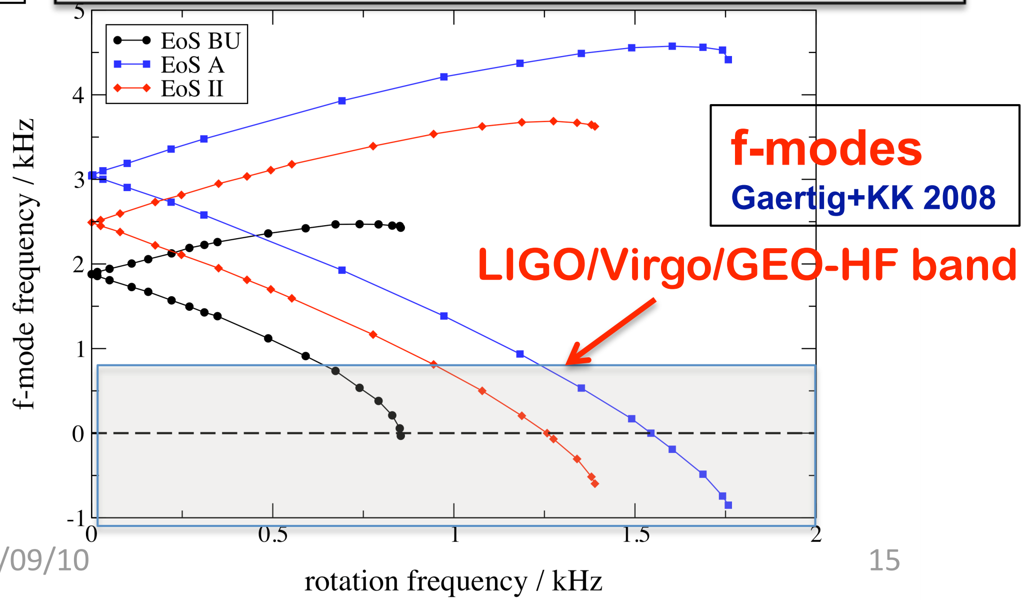
$$t_{GW} \approx f(l)R\left(\frac{R}{M}\right)^{l+1} \sim 0.07\left(\frac{1.4M_\odot}{M}\right)^3\left(\frac{R}{10km}\right)^4 \text{ sec}$$

- ✓ In GR the ***m=2*** mode becomes unstable for ***Ω>0.85Ω_{Kepler}***
- ✓ **Differential rotation** affects the onset of the instability
- ✓ Up to **10%** of energy and angular momentum will be dissipated by GWs.

Major uncertainties:

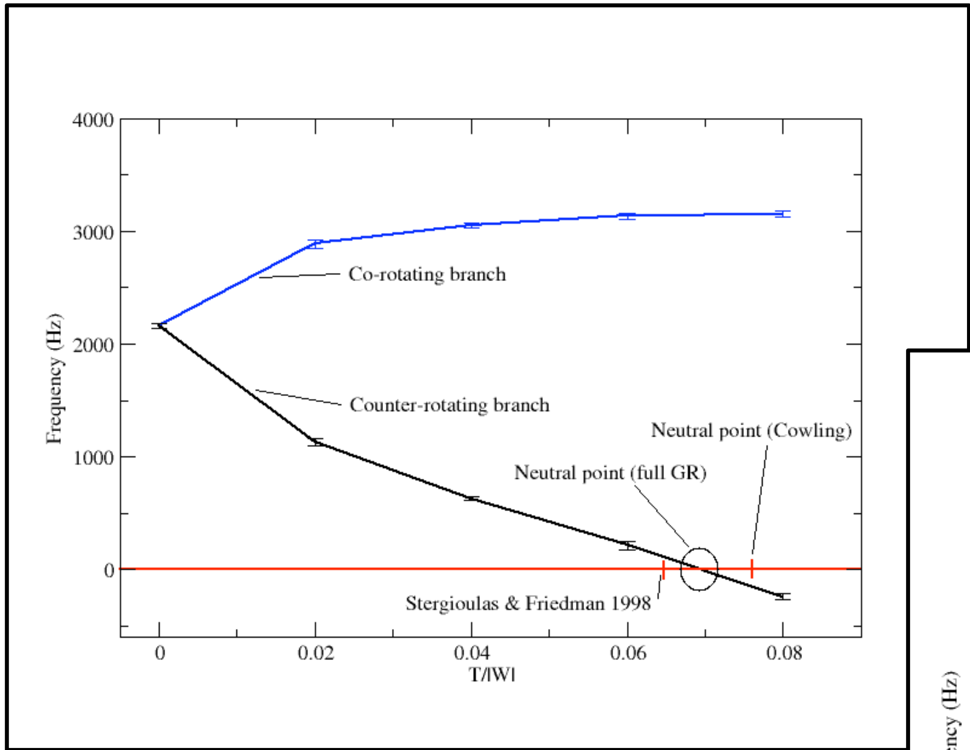
1. Relativistic growth times ✔
2. Nonlinear saturation ✔
3. Initial rotation rates of proto-neutron stars ✖



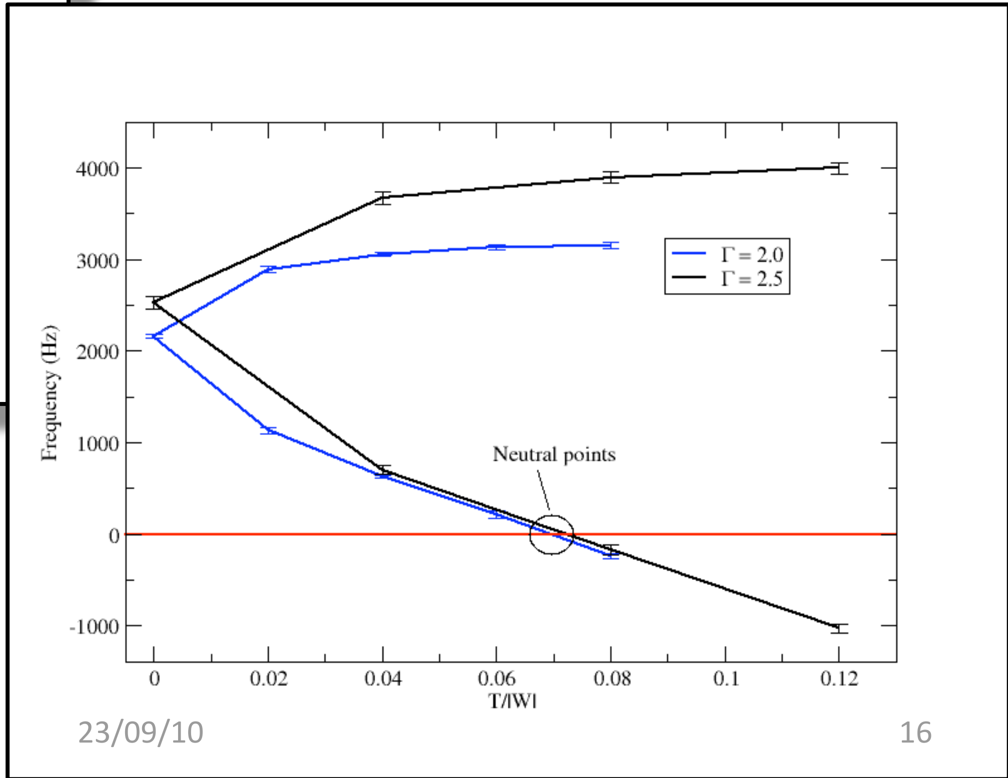


f-mode : non-linear results

B. Zink, N. Stergioulas, O. Korobkin, P. Diener, E. Schnetter ***(2010)***



Top: *Frequencies of l = |m| = 2 f-modes in rapidly rotating Γ= 2 polytropes*



Bottom: *Comparison between Γ= 2 and Γ= 2.5*

f-modes: Asteroseismology

- ✓ **We can trace the effect of rotation of the f-mode (and any p, i or g-mode) frequency and the onset of the CFS instability**
- ✓ **We can produce empirical relation relating the parameters of the neutron stars to the observed frequencies.**

$$\frac{\sigma}{\sigma_0} \approx 1 + 0.63\left(\frac{\Omega}{\Omega_K}\right) - 0.32\left(\frac{\Omega}{\Omega_K}\right)^2 + \ldots \quad (\mathrm{m}=2)$$

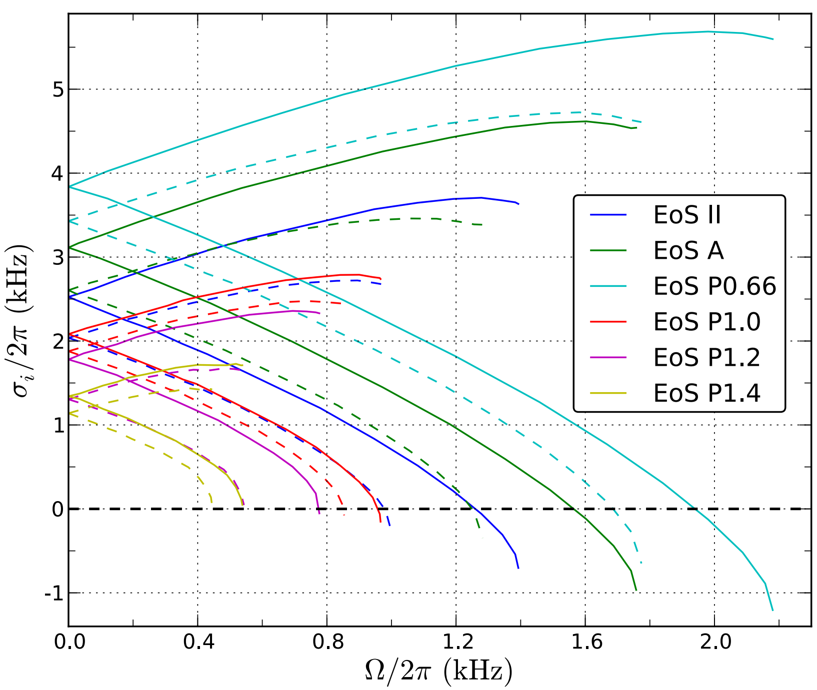
$$\frac{\sigma}{\sigma_0} \approx 1 - 0.41\left(\frac{\Omega}{\Omega_K}\right) - 0.53\left(\frac{\Omega}{\Omega_K}\right)^2 + \ldots \quad (\mathrm{m}=-2)$$

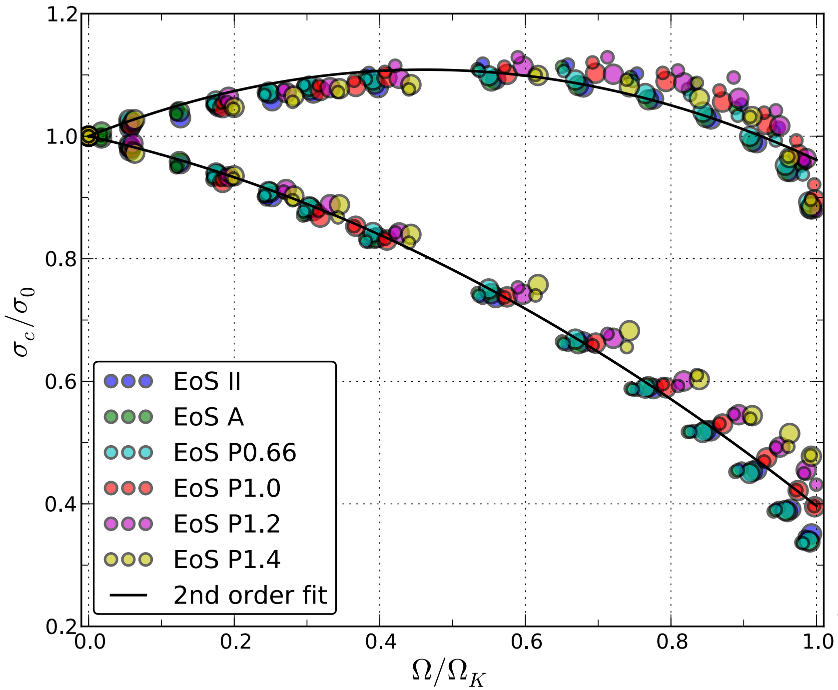
$$\Omega_K \approx 0.67\left(\frac{M}{R^3}\right)^{1/2} \quad \text{or} \quad \Omega_K \approx C(\chi)\left(\frac{M}{R^3}\right)^{1/2}$$

$$C(\chi) = 0.47 + 0.76\frac{M}{R}$$

Gaertig-KK 2008, 2010

(i)





f-mode: Damping/Growth time

Gaertig-KK 2010

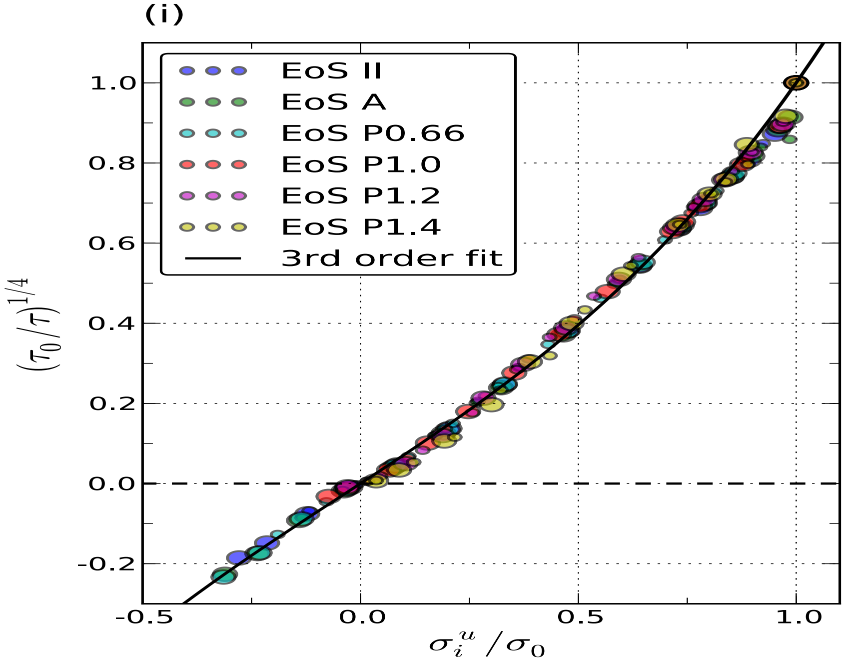
$$E = \frac{1}{2}\int\left[\rho\delta u^a \delta u_a^* + \frac{\delta p}{\rho}\delta\rho^*\right]d^3x \quad \Rightarrow \quad E \approx \sigma^2$$

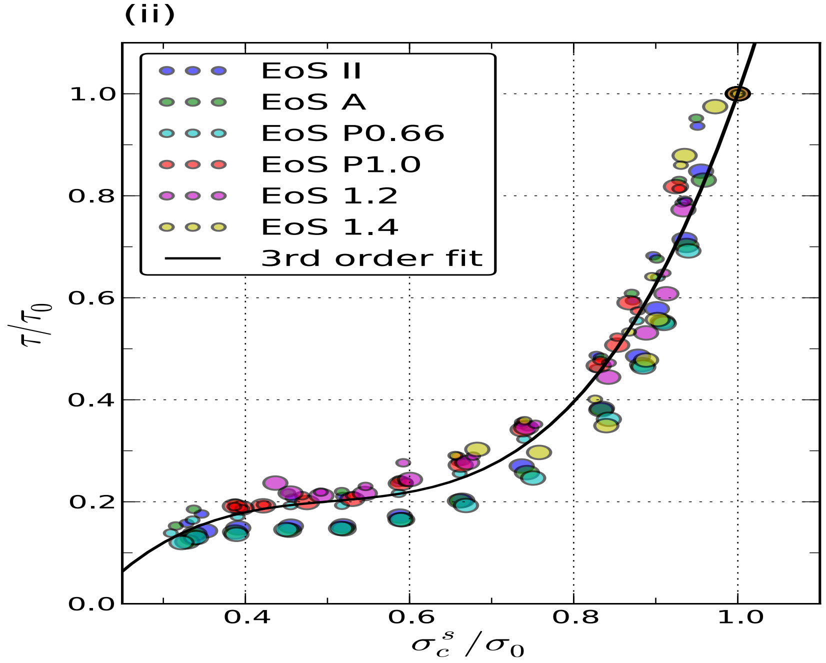
$$\frac{1}{\tau_{GR}} = -\frac{1}{2E}\left(\frac{dE}{dt}\right) \quad \approx \quad \sigma^4$$

$$\frac{dE}{dt} = -\sigma_i\left(\sigma_i + m\Omega\right)N_\ell\left|\delta D_{\ell m}\right|\sigma_i^4 \quad \Rightarrow \quad \frac{dE}{dt} \approx \sigma_i^6$$

$$\left[\frac{\tau_0}{\tau}\right]^{1/4} \approx \mathrm{sgn}(\sigma_i)0.71\left(\frac{\sigma_i}{\sigma_0}\right)\left[1 + 0.048\left(\frac{\sigma_i}{\sigma_0}\right) + 0.35\left(\frac{\sigma_i}{\sigma_0}\right)^2\right]$$

$$\left[\frac{\tau_0}{\tau}\right] \approx -0.66\left[1 - 7.33\left(\frac{\sigma_c}{\sigma_0}\right) + 15.06\left(\frac{\sigma_c}{\sigma_0}\right)^2 - 9.26\left(\frac{\sigma_c}{\sigma_0}\right)^3\right]$$

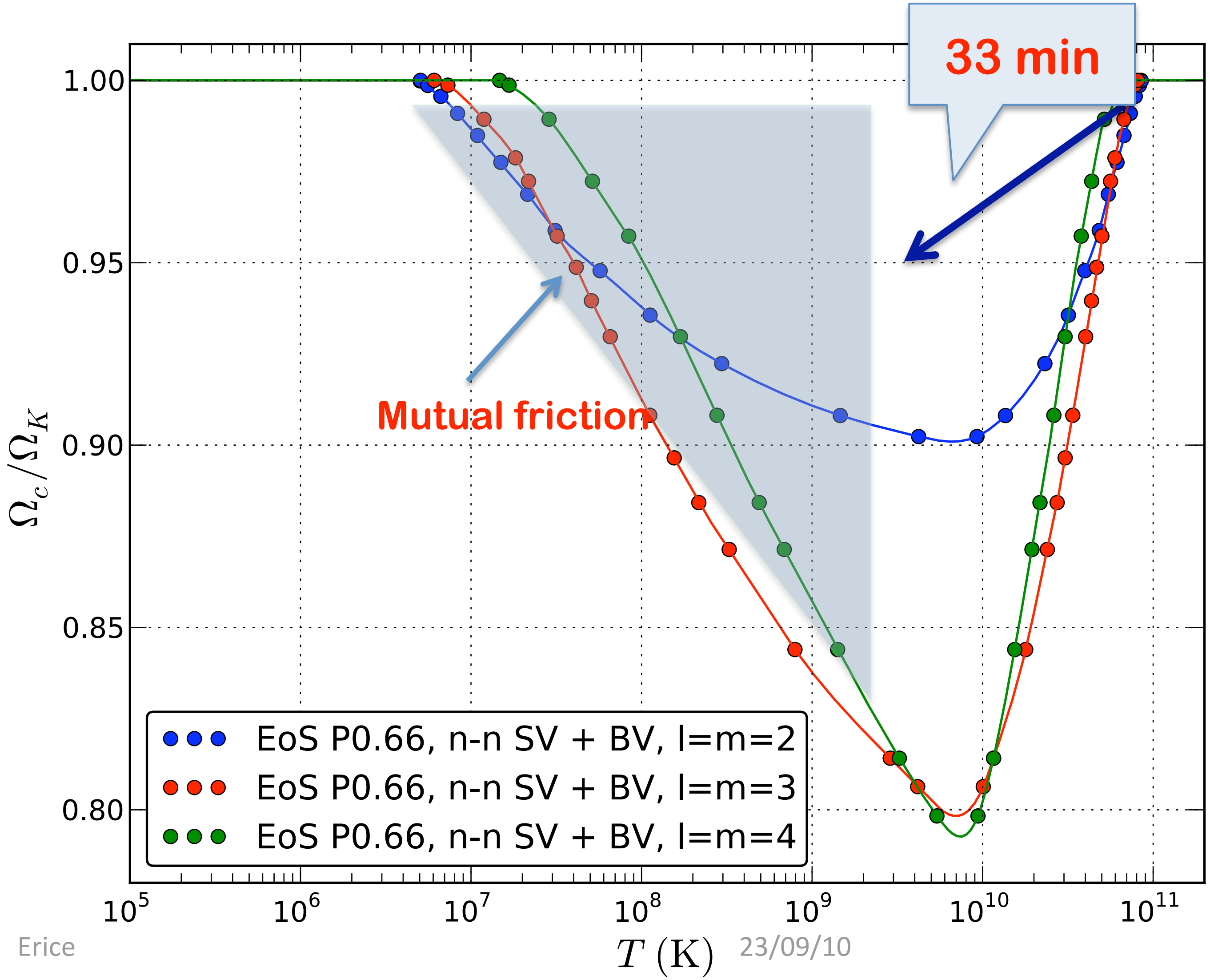




Instability Window

For the first time we have the window of f-mode instability in GR

Gaertig, Glampedakis, KK, Zink (2010)

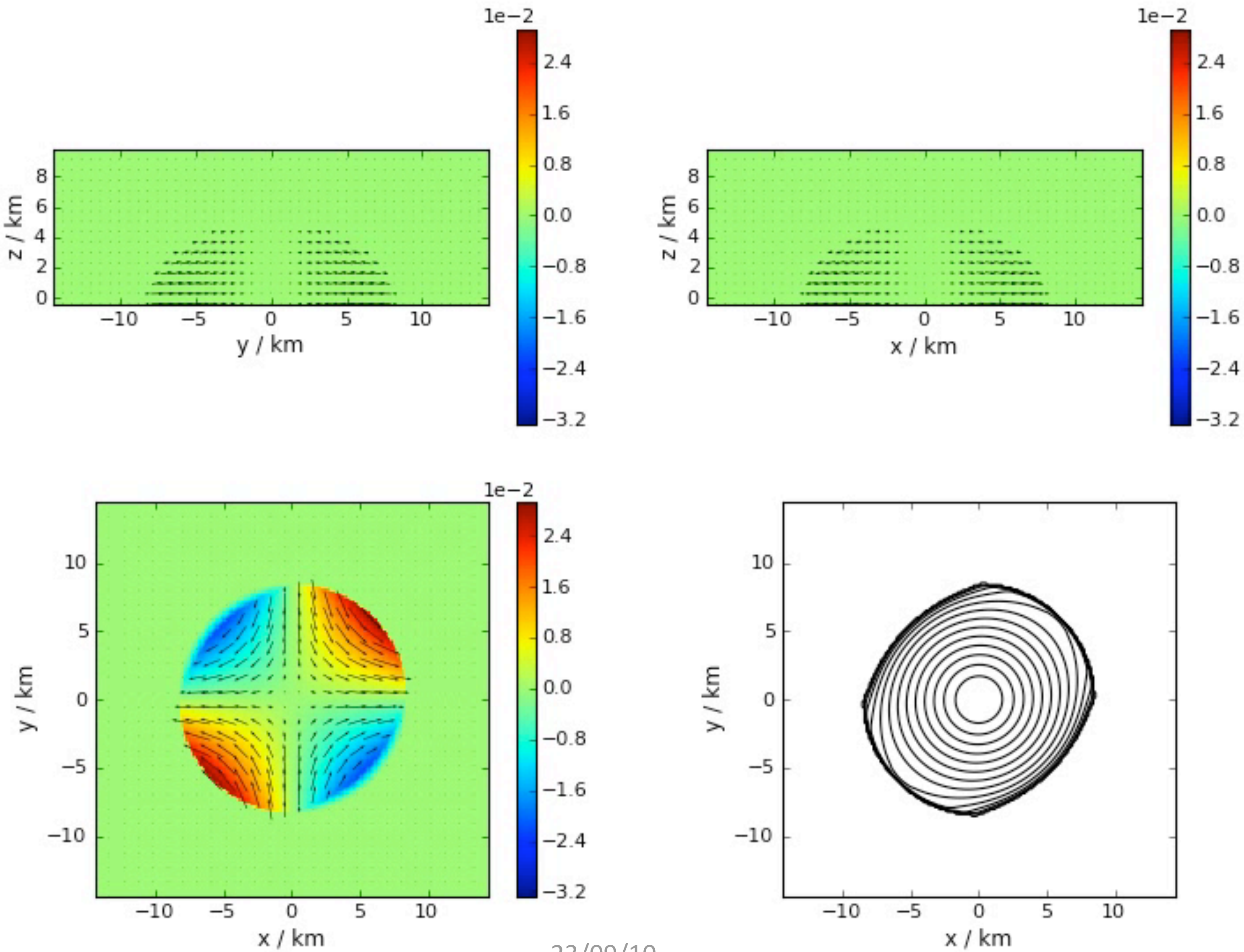


…typical
Growth Time

~28 sec l=2
~48 sec l=3
~90 sec l=4

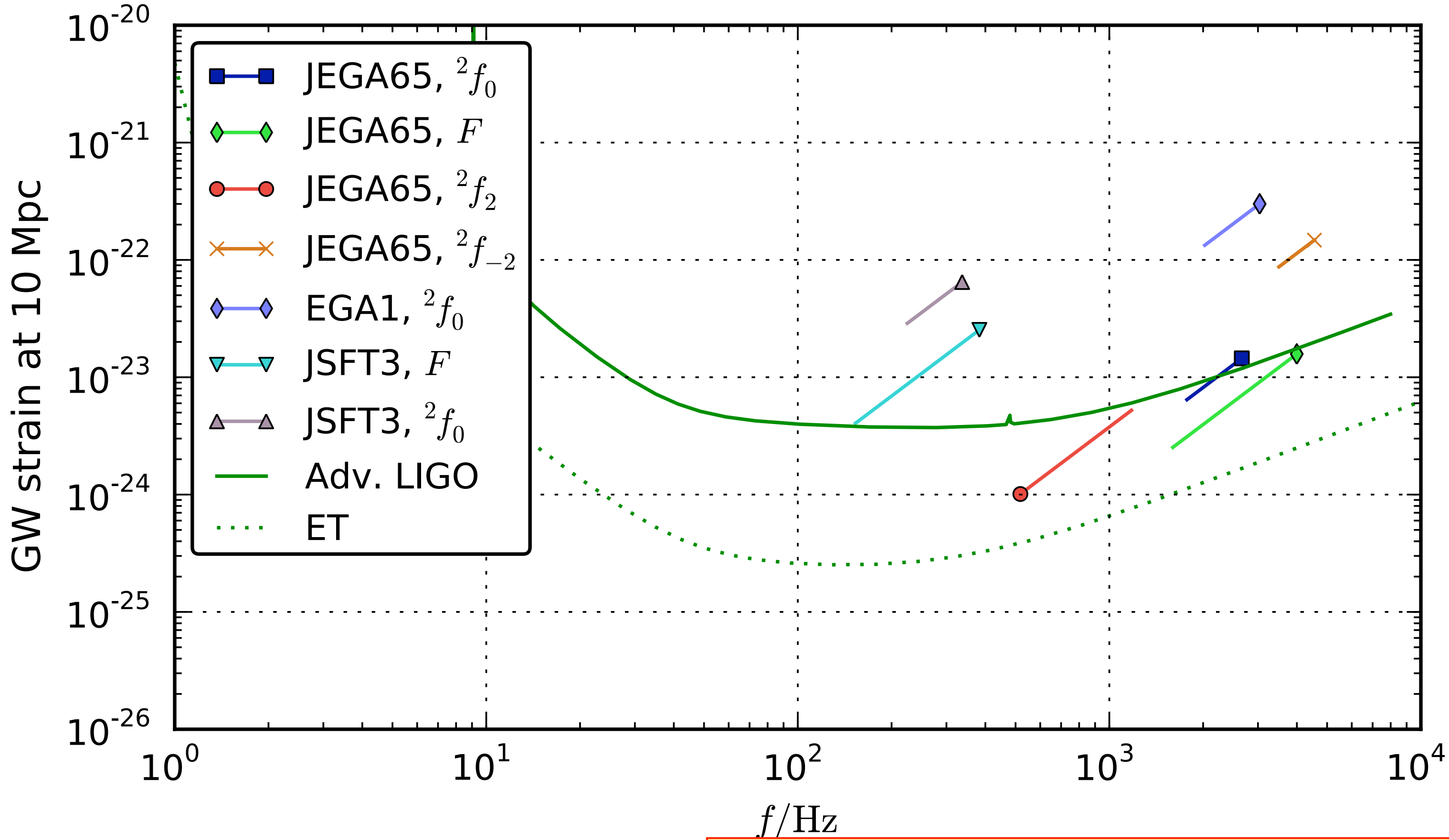
Animation of the l=m=2 f-mode

Kastaun, Willburger, KK (2010)



Detectability (10Mpc)

Kastaun, Willburger, KK (2010)

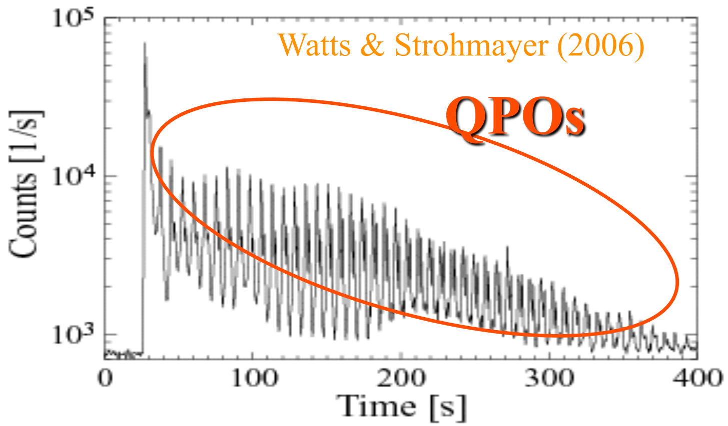


$$h \approx 10^{-23} - 10^{-24}\left(\frac{10\mathrm{Mpc}}{r}\right)$$

$$h_{\mathrm{eff}} = h\sqrt{N} \approx 10^{-23} - 10^{-24}\left(\frac{10\mathrm{Mpc}}{r}\right) \times \sqrt{N}$$

$$N \Rightarrow 10 - 10^8$$

Magnetized Stars
Observations



✓ **Giant flares in SGRs**

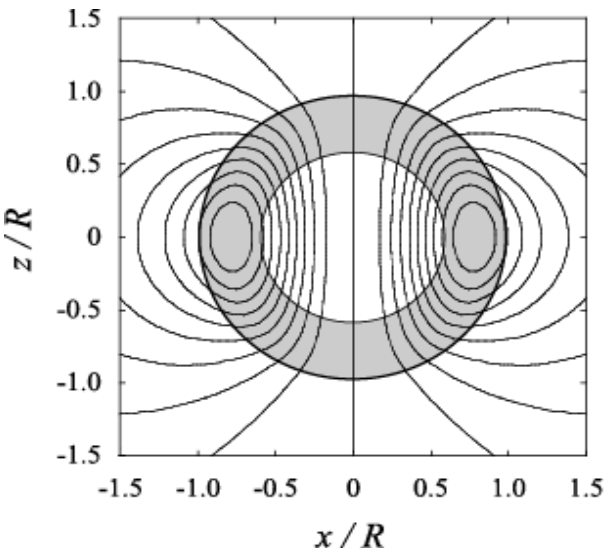
- **Up to now, three giant flares have been detected.**
  - *SGR 0526-66 in 1979,*
  - *SGR 1900+14 in 1998,*
  - *SGR 1806-20 in 2004*
- **Peak luminosities : 10^{44} – 10^{46} erg/s**
- **A decaying tail for several hundred seconds follows the flare.**

✓ **QPOs in decaying tail** (Israel *et al.* 2005; Watts & Strohmayer 2005, 2006)

- **SGR 1900+14 : 28, 54, 84, and 155 Hz**
- **SGR 1806-20 : 18, 26, 29, 92.5, 150, 626.5, & 1837 Hz**
  (possible additional frequencies : 720 & 2384 Hz)

B-field configuration

- ✓ Axisymmetric dipole magnetic fields with poloidal and toroidal components.
- ✓ **Two types of B-field geometries;**
  I. B-fields permeating star.
  II. Magnetic fields are confined in the crust region.



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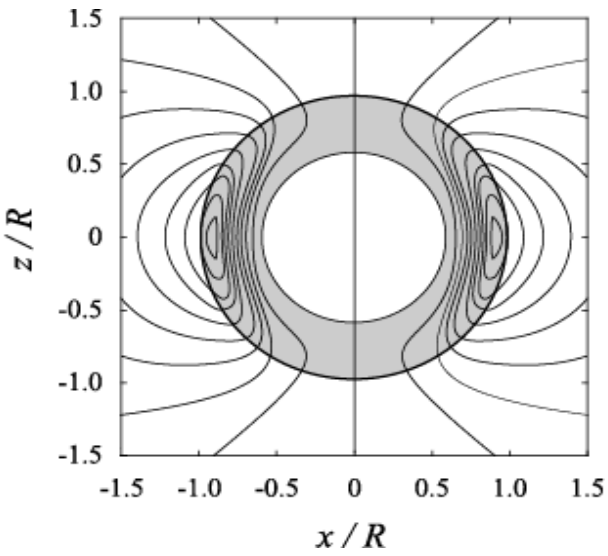
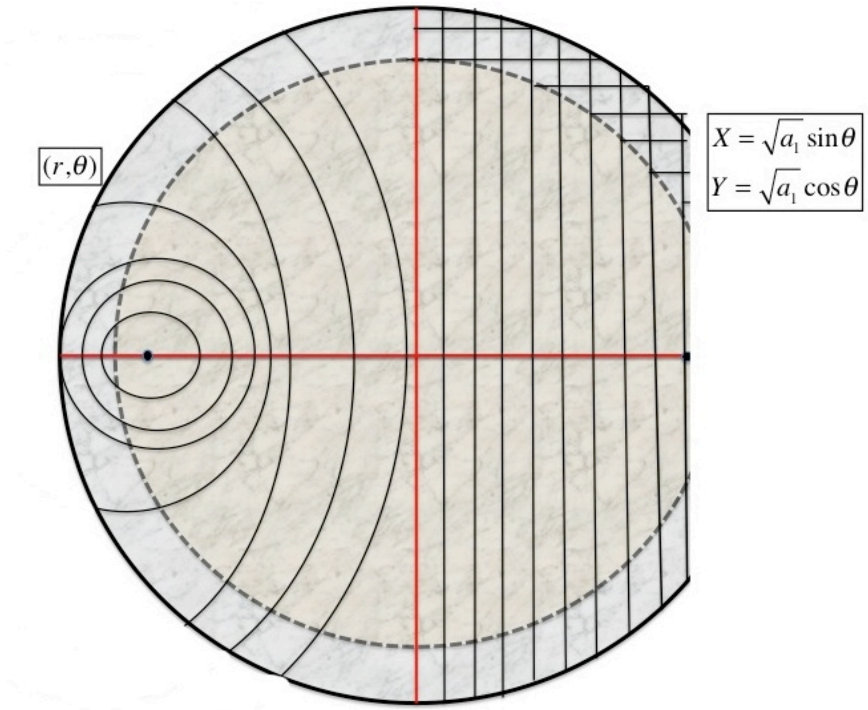
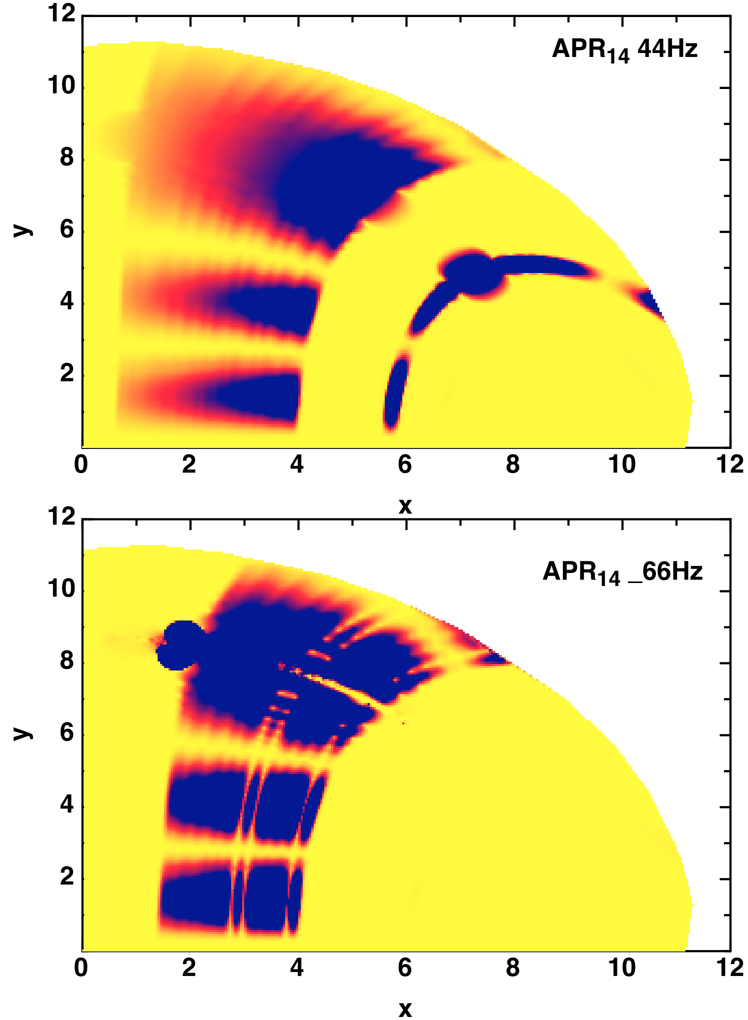


(ii)

- Piro 2005
- Glampedakis, Samuelsson, Andersson 2006
- Sotani, KK, Stergioulas 2007
- Samuelsson, Andersson 2007
- Levin 2007
- Sotani, KK, Stergioulas 2008
- Vavoulidis, Stavridis, KK 2008
- Sotani, Colaiuda, KK 2008
- Colaiuda,Beyer,KK 2009
- Cerda-Duran, Stergioulas, Font 2009
- Sotani, KK 2009
- Steiner, Watts 2009
- Lander, Jones, Passamonti 2009
- Van Hoven, Levin 2010
- Cerda-Duran etal 2010
- Colaiuda+ KK 2010

Alfven Continuum + Discrete oscillations





Without Crust

- **Levin 2007**
- **Sotani,KK, Stergioulas 2008**
- **Colaiuda, Beyer, KK 2009**
- **Cerda-Duran, Stergioulas, Font 2009**

With Crust

- **Van Hoven, Levin 2010**
- **Cerda-Duran, Stergioulas, Font 2010**
- **Colaiuda, KK 2010**

Attempt to Fit the Observational data

- ✓ **Axial type of perturbations** described by **discrete modes** (crust) and a **continuum** (core) and ***they can explain the lower frequencies*** observed
- ✓ **Polar type of perturbations** only **discrete** modes with higher frequencies

Alfvén modes

$$f_n^{\text{even}} \approx (2n+1) f_0$$
$$f_n^{\text{odd}} \approx (n+1) f_0$$
$$f_n^{C} \approx (n+1) f_0^{C}$$

$${}_\ell t_n \approx \left[1 + {}_\ell a_n \left(\frac{B}{B_\mu}\right)^2\right]^{1/2} \cdot {}_\ell t_0$$

Crust modes

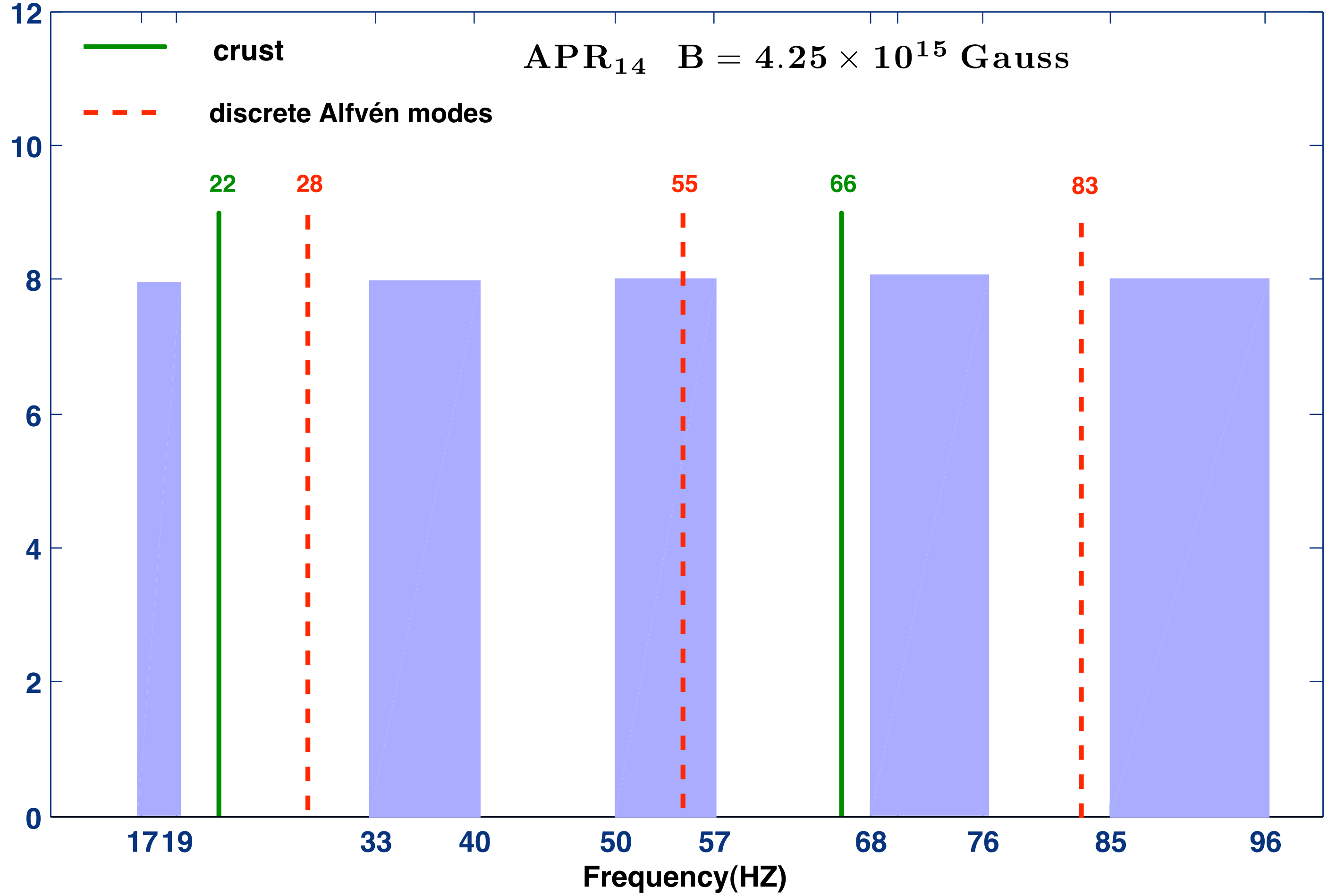
Sotani,KK,Stergioulas 2007

- On the other hand the observed frequencies of QPOs in SGRs
  - SGR 1900+14 : 28, 54, 84, 155 Hz
    x2 x3 ← crust torsional oscillation ? or polar oscillation ?
  - SGR 1806-20 : 18, 26, 30, 92.5, 150 Hz
    0.6 (18 / 30)
    x3 x5
    crust torsional oscillation?

EoS =>APR+DH,
M=1.4 $M_\odot$, **R**=12.1Km
ΔR/R=0.93km,
B=4x10^{15}Gauss

SGR 1806-20

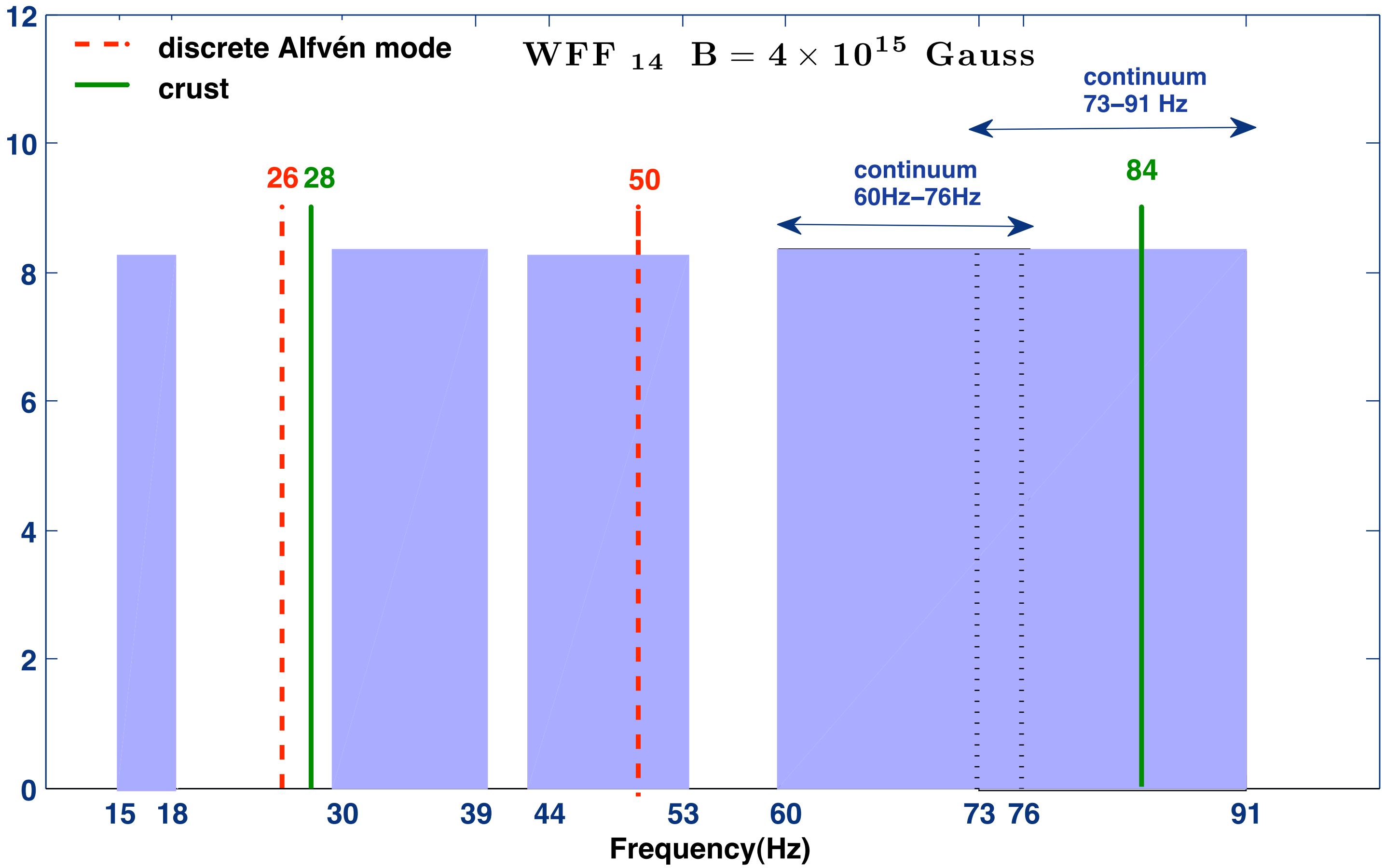
Colaiuda, KK 2010



Identification of the frequencies of SGR 1806-20. We show that a stellar model APR with mass $M = 1.4M_\odot$ can explain all the frequencies observed.

SGR 1900+14

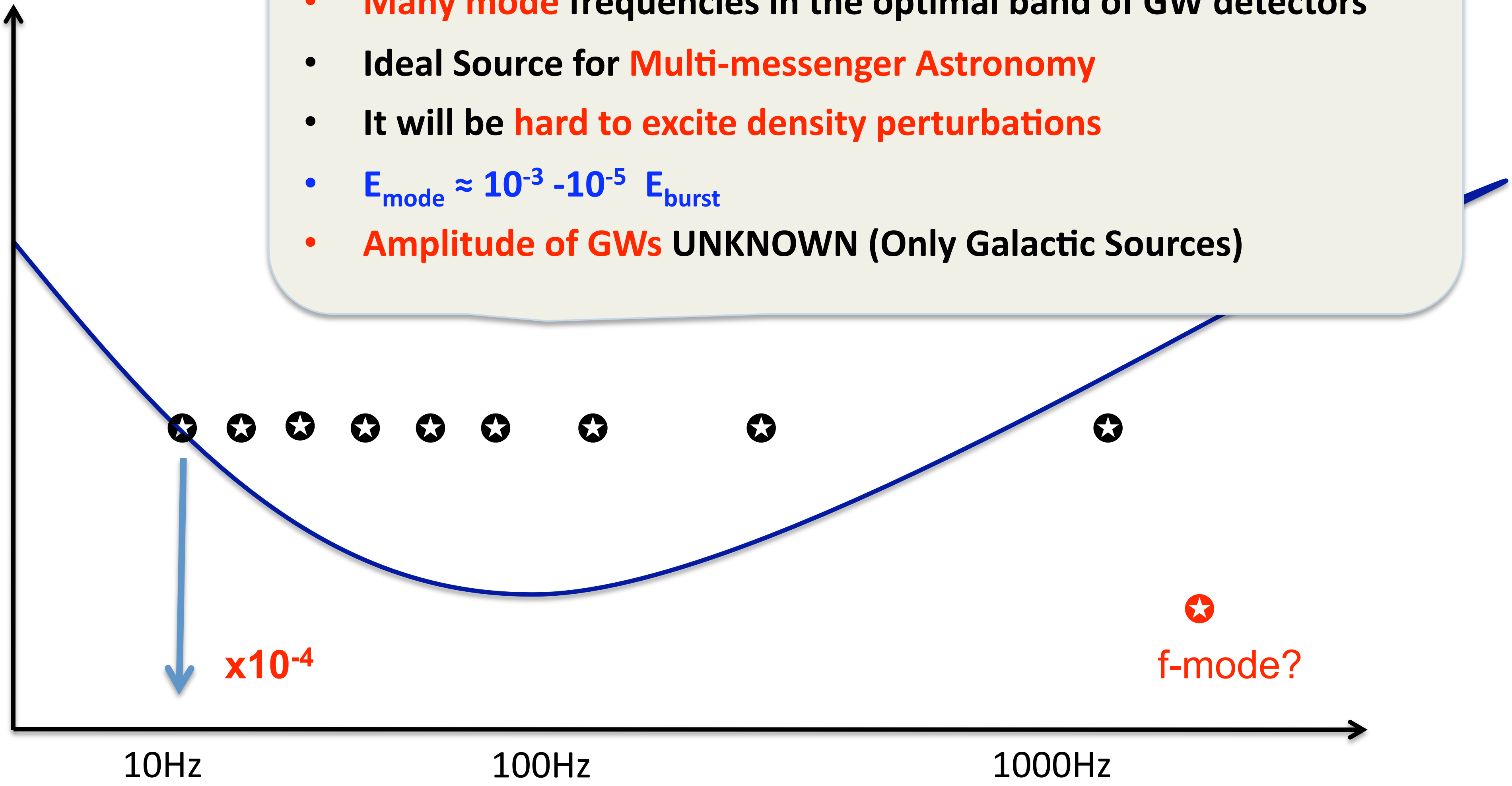
Colaiuda, KK 2010



Identification of the frequencies of SGR 1900+14. We show that a stellar model **WFF** with mass M = 1.4M$_\odot$ can explain all the frequencies observed.

Magnetars & GWs

- Many mode frequencies in the optimal band of GW detectors
- Ideal Source for Multi-messenger Astronomy
- It will be hard to excite density perturbations
- $E_{mode} \approx 10^{-3} - 10^{-5}\ E_{burst}$
- Amplitude of GWs UNKNOWN (Only Galactic Sources)



Conclusions

- **Rotational Instabilities of Neutron Stars**
  - ✓ Are potential sources for GW beyond our galaxy
  - ✓ **Many open issues** (growth time, non-linear coupling,…) are resolved one after the other.

- **Dynamics of magnetars**
  - ✓ Offers the possibility to understand their structure
  - ✓ Most probably a **weak source** for GW with the present generation detectors