Problems with proton radii

Ingo Sick

Interest: rms-radii = fundamental quantities to describe size

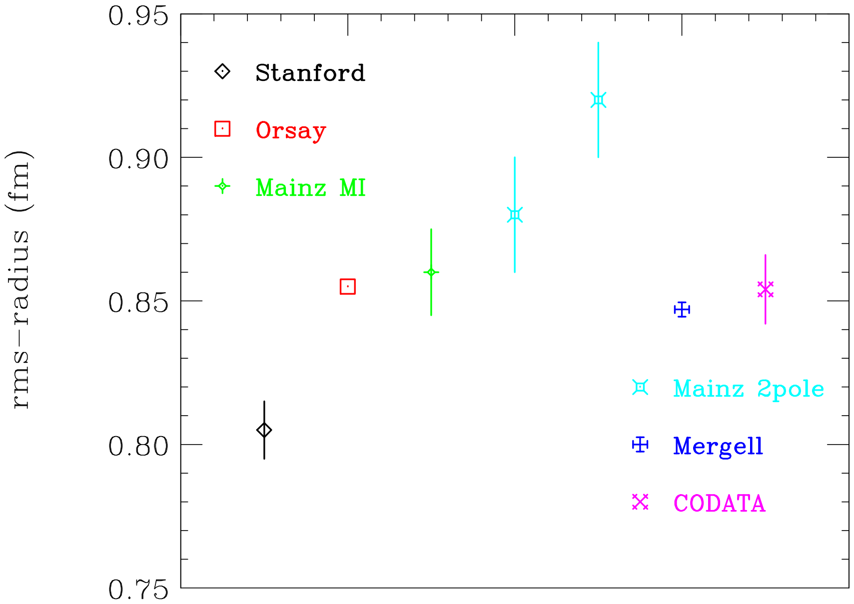
needed for interpretation of atomic hydrogen

hyper-precise transition energies, 13 digits

δR_{rms} = biggest uncertainty

R_{rms} = quantity from (e,e) of most interest to physicists from *outside* (e,e)

History of charge R_{rms} from (e,e): very checkered



Reasons for scatter

$G_e(q)$ and $G_m(q)$ obtained from L/T-separation

$$\frac{d\sigma}{d\Omega} = \sigma_{Mott} f_{recoil} \left[(G_e^2 + \tau G_m^2)/(1+\tau) + 2\tau G_m^2 tg^2(\theta/2) \right]$$

$\tau = q^2/4m^2$, m = proton mass, $q \sim 2Esin(\theta/2)$ = momentum transfer

- fit of G's from *individual* experiments with chosen parameterization
- no Coulomb corrections
- problems with convergence radius of parameterization used

Solution

- use *world* cross sections
  optimal L/T-separation during fit
- use Coulomb corrections
- use Pade approximants to parameterize

Result

I.S. Phys.Lett. B576 (03) 62

$R_{rms}^{ch} = 0.895 \pm 0.018 fm$

$R_{rms}^{m} = 0.855 \pm 0.035 fm$

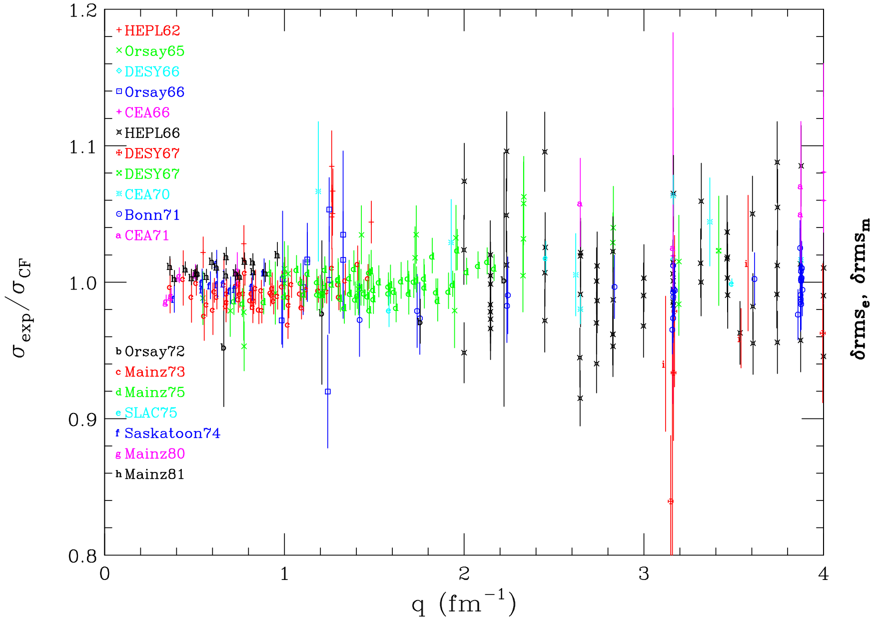
Very conservative error bars

statistical errors from error matrix

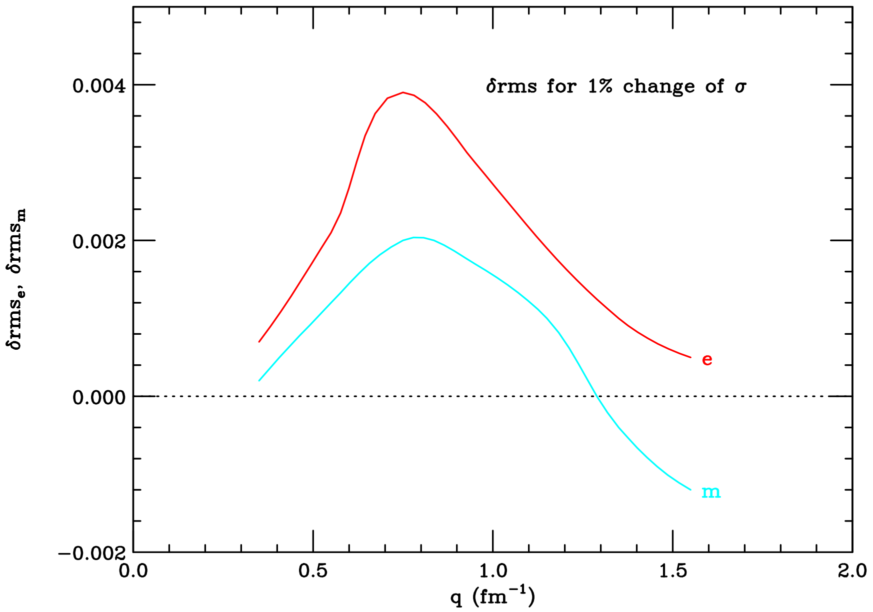
systematic errors of data included

change σ by syst.error, refit, add changes quadratically

Good fit



sensitivity to R_{rms}: 0.5÷1.2fm^{-1}



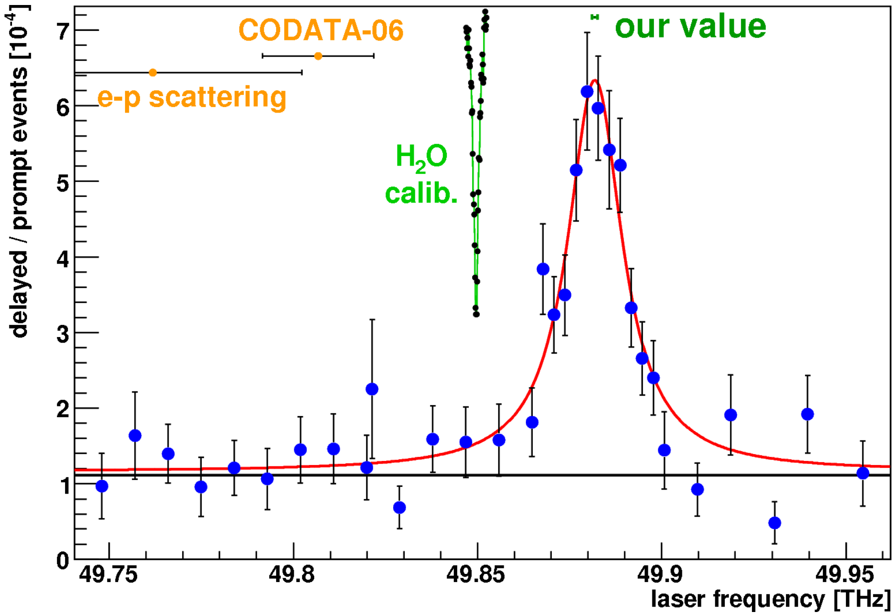
Identify and understand deviations of previous determinations

Zemach moments also determined to ±1%

needed for atomic HFS (second Zemach moment)
needed for muonic Hydrogen (third Zemach moment)

New: Precise data on atomic Hydrogen

- energies in electronic H measured to 13 digits
- Lamb shift in muonic H measured, see spectrum below



Problem with R_{rms}^{ch}

(e,e) *world*	$0.895 \pm 0.018\ fm$	
e-H	$0.877 \pm 0.007\ fm$	Udem, PRL79(97)2646
		Melnikov, PRL 84(00)1673
μ-H	$0.8418 \pm 0.0007\ fm$	Pohl, Nature 466(10)213

Severe discrepancy with radius from muonic Hydrogen

→ much excitement, many (wild) ideas to reconcile, no convincing way out!

Since: new (e,e) data from Bernauer *et al.* PRL 105(10)242001

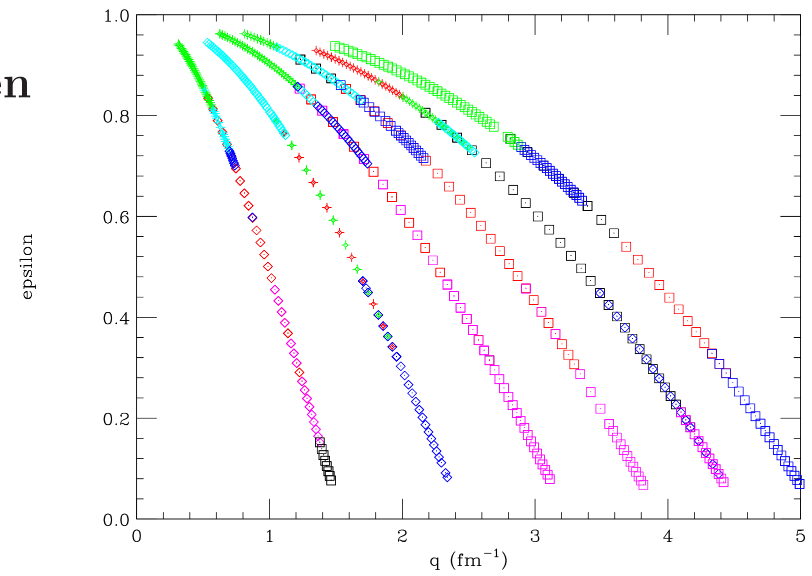
completely new set of data 0.4 ... 5 fm^{-1}
different data taking philosophy

Better

many data points, ∼1400
use second spectrometer to check luminosity
explore several parameterizations

Worse

no absolute cross sections
incorrect Coulomb corrections (minor)
34 data sets with 31 free normalizations, most occurring in 2 data sets
nightmare to fit
systematic errors not given



Bernauer result

$$R^{ch}_{rms} = 0.879 \pm 0.007 \ fm$$
$$R^{m}_{rms} = 0.777 \pm 0.02 \ fm$$

At first sight nice confirmation of previous R^{ch}_{rms}
(although I find larger model dependence)

Problematic: disagreement with *world* value $R^{m}_{rms} = 0.855 \pm 0.035 fm$

Understanding

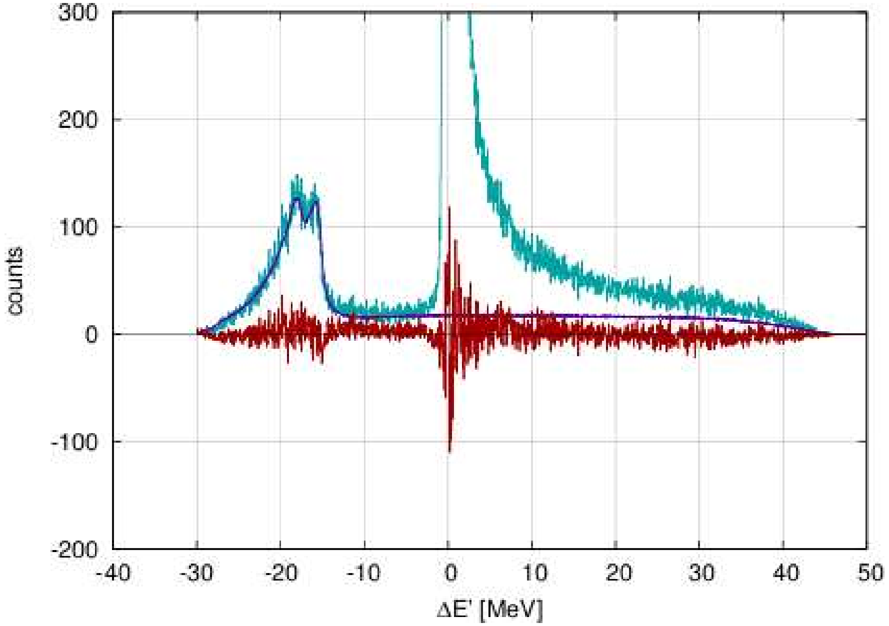
effect of R^{m}_{rms}-discrepancy only 0.3% at q of maximal sensitivity to rms-radius
(data oriented towards determination of R^{ch}_{rms}!)

At this level background subtraction no good

background from Havar target-window 4 ... 10%
not measured!
primitive model: rad. tail Havar + quasielastic contr. in Fermigas model
no inelastic scattering on Havar
Fermi-gas model in threshold region *very* poor

Spectrum shown in thesis

shows misfit amounting to 1.2% in cross section!



1.2% *very* significant as compared to 0.3%!

My conclusion

R^{m}_{rms} **is not significant, → ignore**

On the positive side

tests with various assumptions on background show:

effect upon R^{ch}_{rms} **small**

Unsatisfactory in general: size error bar of R_{rms}

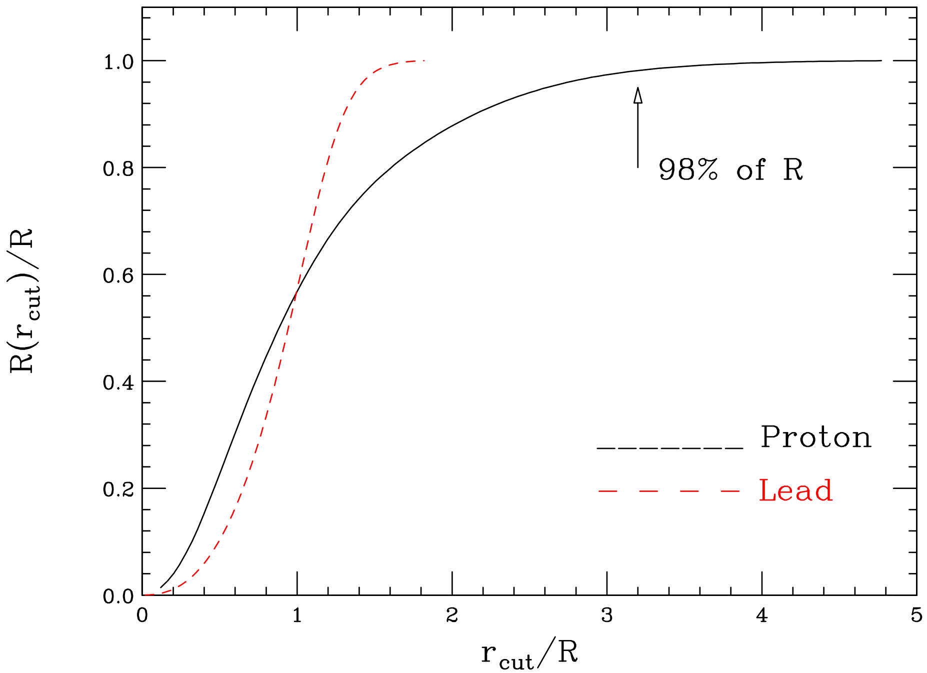
for A>1 δR_{rms} smaller, despite poorer data base
fits to $\pm$ same data gave radii differing by 4% (Arrington, Borisyuk,...)

Reason

for proton shape $\rho(r) \sim$ exponential since $G_e \sim$ dipole

$\rightarrow$ important role of large-radius tail, see $R(r_{cut})/R = [\int_0^{r_{cut}} \rho(r) r^4 dr / \int_0^{\infty} \rho(r) r^4 dr]^{1/2}$

in tail ρ small, poorly determined



for 98% of $\int$ need to integrate to $r_{cut} = 3 \cdot R_{rms}$

remaining 2%:

effect upon $\sigma(q > 0.5 fm^{-1})$ is <0.2%
not measurable

R_{rms} with 1%-type accuracy is an illusion!

To do better: constrain *shape* of large-r tail

add physics → get more accurate rms-radius

Physical model for large r

least-bound Fock state: p = n + π^+, n = p + π^-

dominates $\rho(r)$ completely at large-enough r ($> 0.8 fm$ in cloudy bag model)

will use as constraint

To exploit need relation $G_e(q) \leftrightarrow \rho(r)$

for accurate shape need data up to largest q's

must account for relativistic corrections

not discussed here

have been accounted for

for large r minor uncertainty due to ambiguities involved

Calculation of density at very large r

a priori: asymptotic form = Whittaker function $W_{-\eta,3/2}(2\kappa r)/r$

with physical masses m_N, m_π, l=1

with separation energy = m_π, include CM-correction

makes sense only at *large* n-π relative distance: $R^p_{rms} = 0.89 fm$, R^π_{rms}=$0.66 fm$

only at large r overlap n, π small

potential difficulty

need to fold W^2/r^2 with charge distribution of n, π

could get into trouble with $r = 0$ divergence of W/r

In practice

calculate w.f. in square well potential, $V(r > R) = 0$ (courtesy D.Trautmann)

radius $R = 0.8 fm$ (not important) , depth adjusted to separation energy

for $r > R$ shape of $\psi^2 \equiv$ shape of Whittaker function

can easily fold

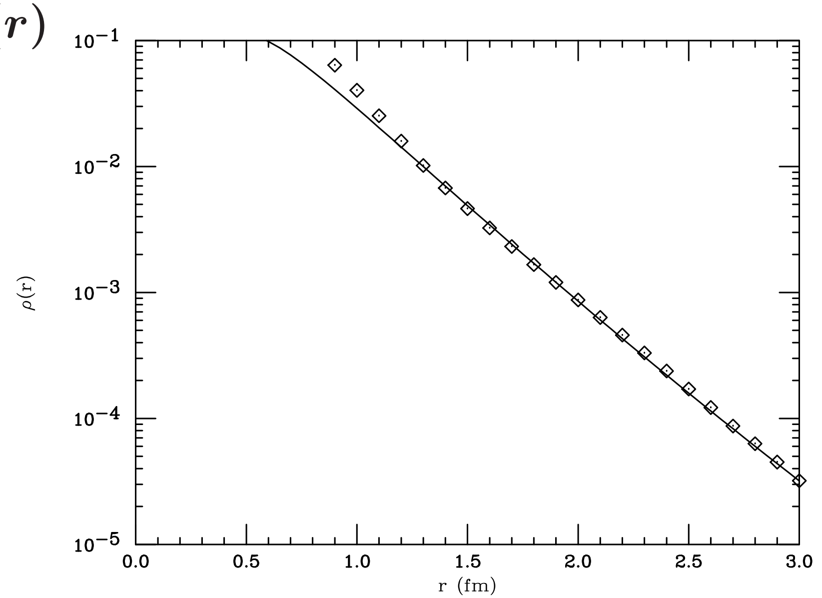
expect small difference Schrödinger-Klein-Gordon (DT)

Result

excellent agreement with shape of $\rho_{exp}(r)$

(fit *world* data with Pade)

norm fit to ρ_{exp}



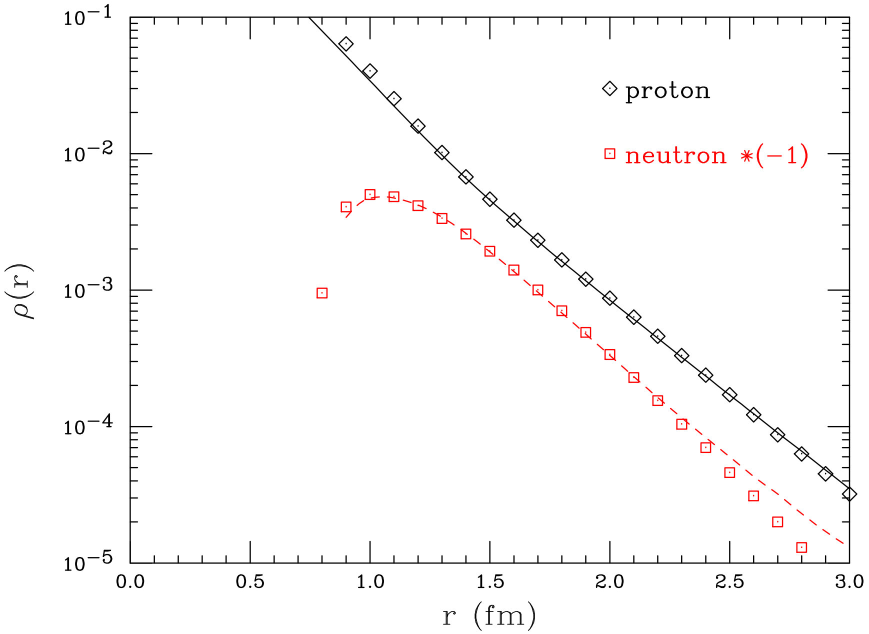
"Refinements" of model

allow also for $\Delta + \pi$ contribution
coefficients of various terms from Dziembowski,...,Speth
'Pionic contribution to nucleon EM properties in light-front approach'
include all states: $\pi^+ n$, $\pi^- p$, $\pi^- \Delta^{++}$, $\pi^+ \Delta^0$, $\pi^- \Delta^+$, $\pi^+ \Delta^-$
calculate similarly

effect on p-tail: small, a bit closer to ρ_{exp} towards smaller r

effect on n-tail: larger, gets close to ρ_{exp} with exactly *same* parameters
will ignore n since components $\neq \pi^- p$ too important

◇ $\rho_p(r)$, ◇ $-\rho_n(r)$, ═══ shape tail



Data used in fit

- *world* (e,e) data up to 12 fm^{-1}
  both cross sections and polarization data, 605 data points

- for some fits add Bernauer σ with 0.2% quadr. added
- two-photon exchange corrections
  needed to make G_{ep} from σ and P agree
  includes both soft+hard photons
  uses phenomenological modification for very large q
  Melnitchouk+Tjon

- (relative) tail density for $r > 1.3 fm$

Parameterization for G_e and G_m

use r-space parameterization to implement constraint
Sum-Of-Gaussians (SOG) parameterization: flexible + convenient

Detail

placed every $\sim 0.3 fm$, for $r < 3.3 fm$
amplitudes fit to σ, P, constraint
30 parameters

Results

average over various data sets and treatments of normalization

$R^{ch}_{rms} = .886 \pm 0.008\ fm$ $\qquad$ $R^{m}_{rms} = .858 \pm .024\ fm$

Great feature

result much less sensitive to use of absolute vs. floated data

Conclusion: disagreement with μ-H confirmed.

Question: to which degree could fit (e,e) with R^{μ}_{rms} as constraint?

redo analysis with various combinations of data sets

floated or fixed normalization

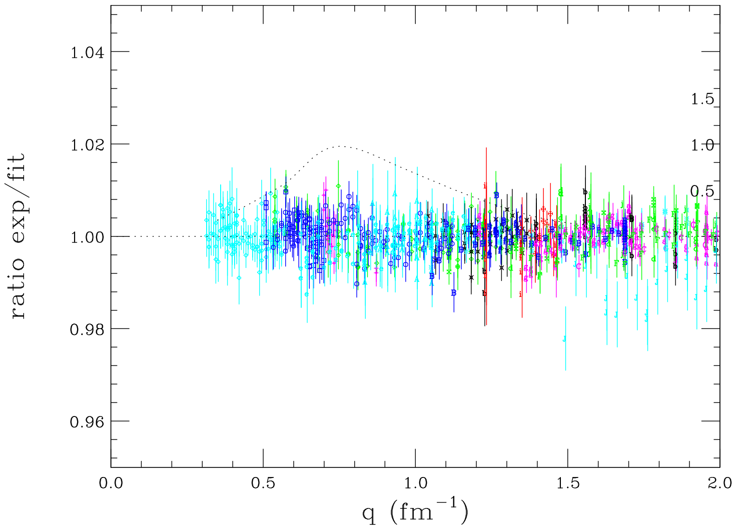
constraint $R^{ch}_{rms} = 0.84\ fm$

Increase in χ^2 due to constraint

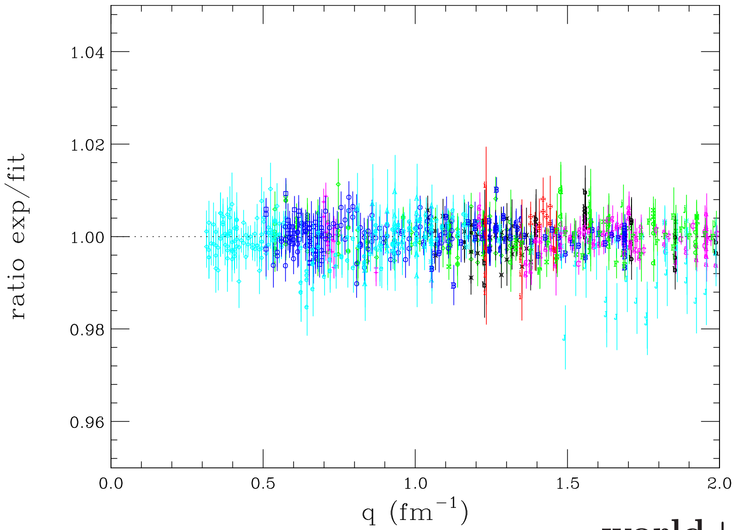
Bernauer	5%
world floated + Bernauer	8%
world floated + tail	10%
world + tail	24%
world + Bernauer + tail	24%

Results show that

R_{rms}=0.84



R_{rms}=0.88

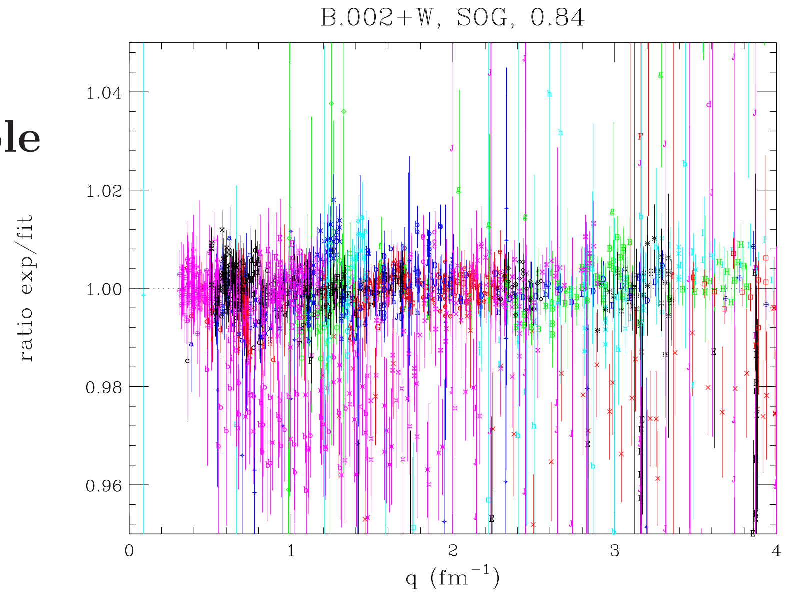


1. **With floating data and no tail constraint:**
   can change R^{ch}_{rms} with modest effect upon χ^2
   for Bernauer data effect on $\sigma_{exp}/\sigma_{fit}$ not visible

2. **With tail constraint: get larger increase**

3. **Absolute σ + tail: fixes *rms*-radius best**
   gives also *visible* disagreement in data/fit
   world data 2-3% below fit

world+Bernauer, 0.84fm



Overall conclusion: problem with R^{ch}_{rms} persists

Many speculations on origin

Missing QED terms?
Two-photon effects in (e,e)?
Polarization of proton in μH?
Problems with radiative corrections?
Wrong Zemach moment?
Recoil terms in μH?
(e,e) and μH not measuring same thing?

.... but there are plausible arguments against all

Upshot: no clear idea available, and there is a real problem

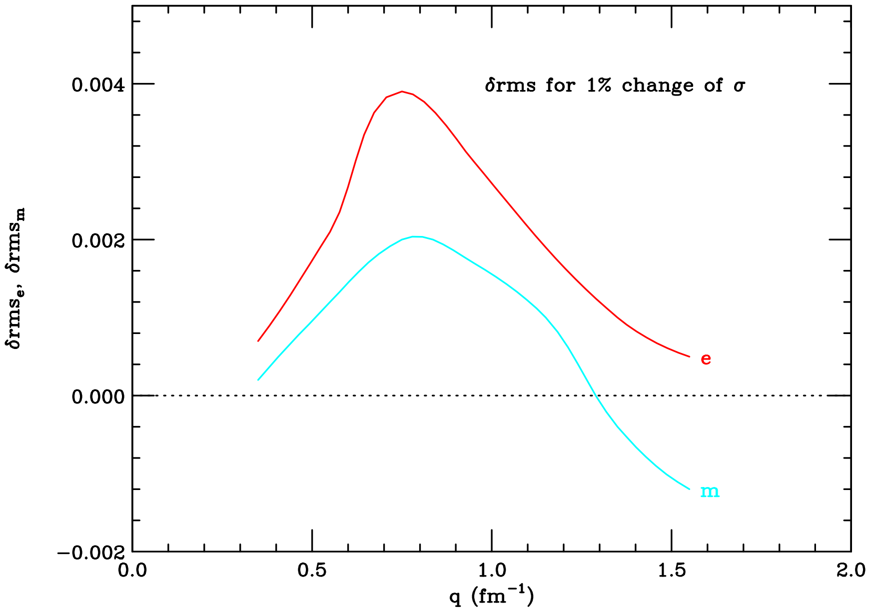
Backup

New value of radius from polarization transfer data?

measure G_e/G_m at JLab using polarization transfer at "low" q

claim to improve knowledge on *rms*-radii, in particular R^m_{rms}

Sensitivity of data to *rms*-radii: $0.5 < q < 1.2 fm^{-1}$



$\uparrow q{=}2.7 fm^{-1}$

radius corresponds to slope of $G(q)$ at $q = 0$!

must go to *real low q* to measure

lowest q-point of polarization transfer is $q{=}2.7 fm^{-1}$

Consequence:

polarization transfer contribute *nothing* to knowledge of *rms*-radii

radii entirely determined by previous cross section data.