From Lagrangian to Hadron:

bethe, salpeter, and the constituent quark model

Eric Swanson

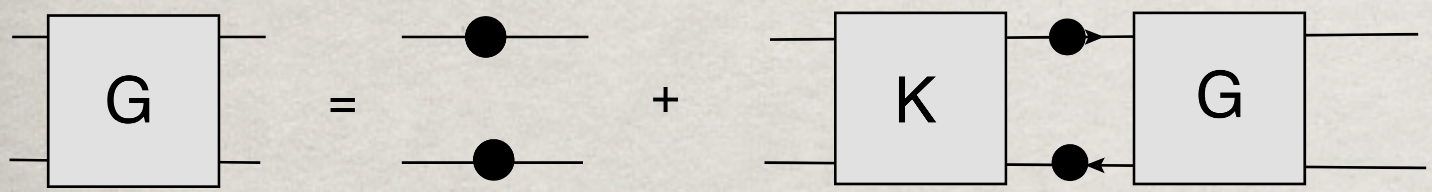
Hadronic Many-body Properties

- relationship of the Bethe-Salpeter formalism to the Random Phase approximation
- relationship of the Random Phase approximation to quantum mechanics

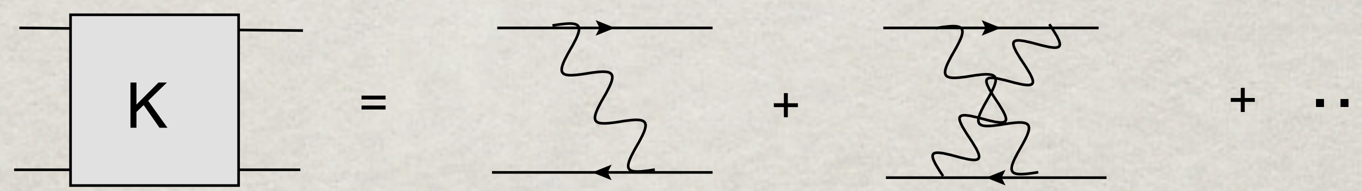
  is quantum mechanics an emergent quantum field theory?
- when is the instantaneous approximation valid?
- multiplicative renormalisability and gauge invariance

Bethe-Salpeter Equation

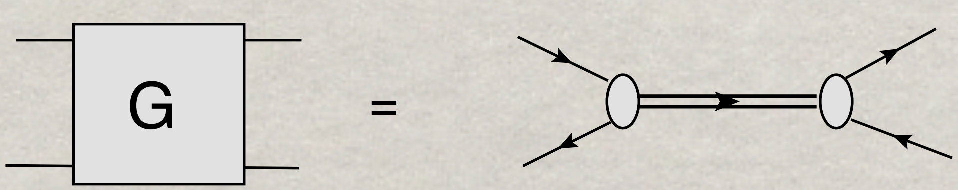
inhomogeneous

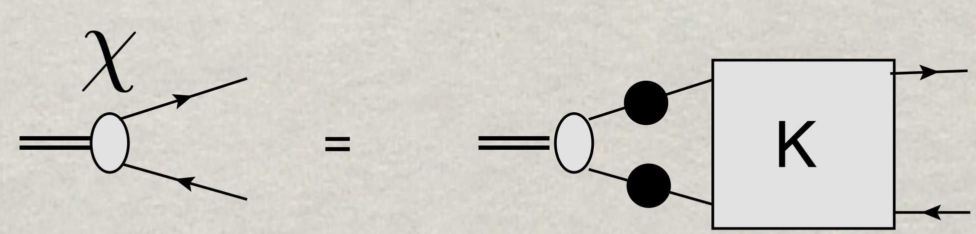


“two valence-particle irreducible”



homogeneous



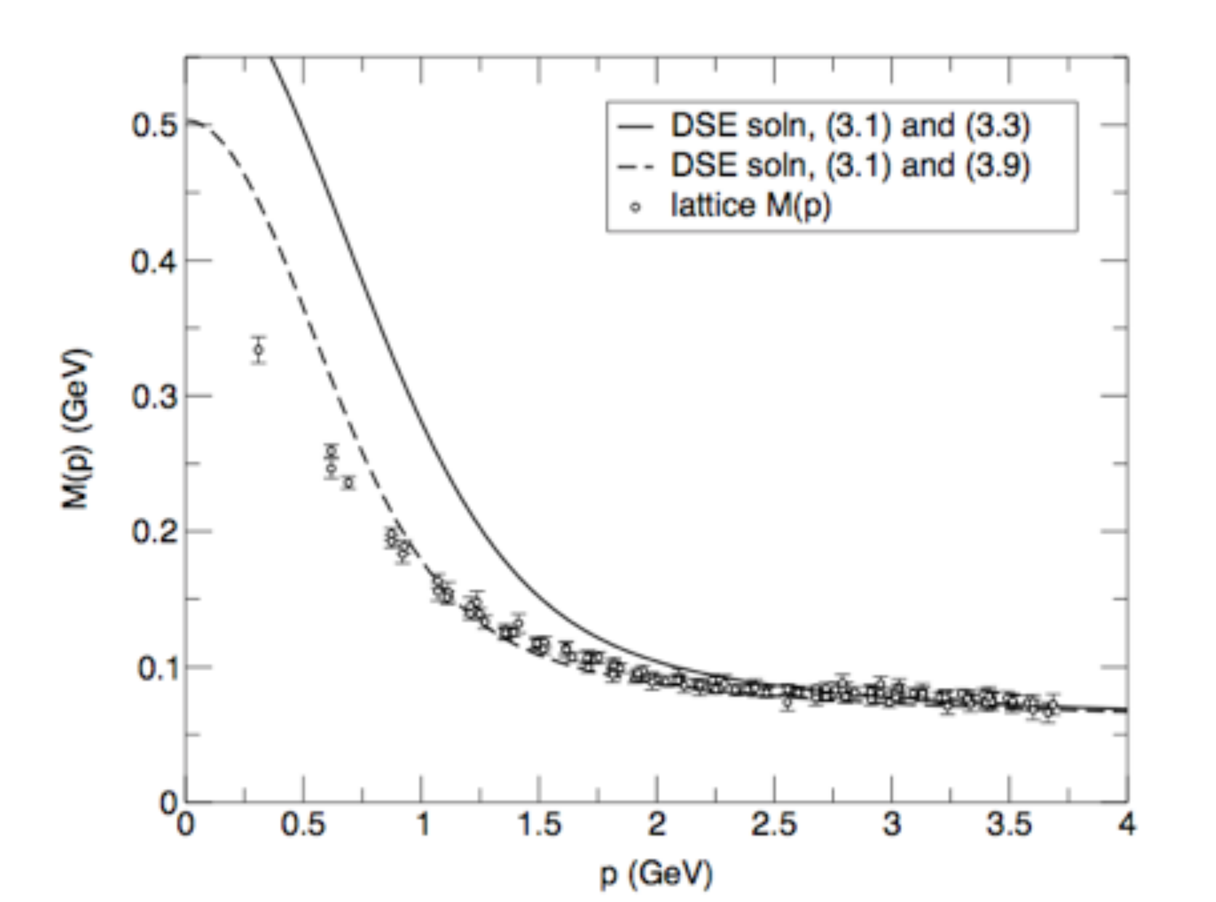


the pion

$$\chi_\pi(P,k) = i\gamma_5 E(P,k) + \gamma_5 \not{P} F(P,k) + \gamma_5 \not{k} G(P,k) + i\gamma_5 \left[\not{P}, \not{k}\right] H(P,k)$$

	m_π	m_K	f_π	f_K
Calc. [51]	138	497	93	109
Expt. [53]	138	496	92	113

	m_ρ	m_{K^*}	m_ϕ	f_ρ	f_{K^*}	f_ϕ
Calc. [51]	742	936	1072	207	241	259
Expt. [53]	771	892	1019	217	227	228
Rel.-Error	0.04	-0.05	-0.05	0.05	-0.06	-0.14



P. Maris and C.D. Roberts,nucl-th/0301049

the pion

$$\chi(x,y) \equiv \langle \Psi_0 | T \psi(x) \bar{\psi}(y) | M \rangle$$

expand in particle basis

$$\chi(x,y) = \sum_{k_1,k_2,s_1,s_2} \Big(U_1(x)\bar{U}_2(y) B + V_1(x)\bar{V}_2(y) D + V_1(x)\bar{U}_2(y) X + U_1(x)\bar{V}_2(y) Y \Big)$$

$$B = \langle \Psi_0 | b(\vec{k}_1, s_1) b^\dagger(\vec{k}_2, s_2) | M \rangle$$

$$D = \langle \Psi_0 | d^\dagger(\vec{k}_1, s_1) d(\vec{k}_2, s_2) | M \rangle$$

$$Y = \langle \Psi_0 | d^\dagger(\vec{k}_1, s_1) b^\dagger(\vec{k}_2, s_2) | M \rangle$$

$$X = \langle \Psi_0 | b(\vec{k}_1, s_1) d(\vec{k}_2, s_2) | M \rangle$$

the pion

relate E, F, G, H to B, D, X, Y

ultrarelativistic

$$
\begin{aligned}
4iE(k_1,k_2) \;\; = \;\; & \frac{1}{2}\chi_s^\dagger(1-\sigma\cdot\hat{k}_2\sigma\cdot\hat{k}_1)\tilde{\chi}_s\,X_{s's}(k_1,k_2) - \frac{1}{2}\,\tilde{\chi}_s^\dagger(1-\sigma\cdot\hat{k}_2\sigma\cdot\hat{k}_1)\chi_s\,Y_{s's}(k_1,k_2) \\
& + \frac{1}{2}\chi_s^\dagger(\sigma\cdot\hat{k}_1-\sigma\cdot\hat{k}_2)\chi_s\,B_{s's}(k_1,k_2) - \frac{1}{2}\tilde{\chi}_s^\dagger(\sigma\cdot\hat{k}_1-\sigma\cdot\hat{k}_2)\tilde{\chi}_s\,D_{s's}(k_1,k_2)
\end{aligned}
$$

nonrelativistic

$$4iE \rightarrow \chi^\dagger\cdot\tilde{\chi}\,(X-Y)$$

$$4P_0^2\vec{k}^2\,F \rightarrow P_0\vec{k}^2\,\chi^\dagger\cdot\tilde{\chi}\,(X+Y) - P_0k_0\,\sigma\cdot\vec{k}\,(B+D)$$

$$4P_0^2\vec{k}^2\,G \rightarrow P_0^2\,\sigma\cdot\vec{k}\,(B+D)$$

$$H \rightarrow 0$$

the pion

instantaneous

$$4iE \to \chi^\dagger \cdot \tilde{\chi}\,(X - Y)$$
$$4P_0\vec{k}^2\,F \to P_0\vec{k}^2\,\chi^\dagger \cdot \tilde{\chi}\,(X + Y)$$
$$G \to 0$$
$$H \to 0$$

instantaneous, chiral

$$2iE \to \chi^\dagger \cdot \chi\,X$$
$$F \to 0$$
$$G \to 0$$
$$H \to 0$$

the pion

only tensors available: $\mathbb{1}$ $\vec{\sigma}$ $\underbrace{\sigma \cdot \hat{k}}_{\text{suppressed by 1/M}}$ $\underbrace{\sigma \cdot \hat{P}}_{\text{zero in rest frame}}$ $\underbrace{\sigma \cdot (\hat{k} \times \hat{P})}_{\text{zero in rest frame}}$

All that the extra terms do is implement boosts. Thus they are not important if one is interested in masses or static properties.

the equations

ultrarelativistic

$$\chi(k_1, k_2) = \int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\, S_F(q_1)\chi(q_1, q_2)S_F(q_2)K(q_1, q_2; k_1, k_2)\delta(\ldots)$$

instantaneous

$$[E - 2E(k)]X(\vec{k}) = \int \frac{d^3q}{(2\pi)^3}\,[V^{++}(k, q)X(q) + V^{+-}(k, q)Y(q)]$$

$$[-E - 2E(k)]Y(\vec{k}) = \int \frac{d^3q}{(2\pi)^3}\,[V^{+-}(k, q)X(q) + V^{--}(k, q)Y(q)]$$

$$\begin{pmatrix} H_0 + V^{++} & V^{+-} \\ V^{-+} & H_0 + V^{--} \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = E\begin{pmatrix} X \\ -Y \end{pmatrix}$$

the equations

nonrelativistic

$$(H_0 + V^{++})X = EX$$

normalisation

ultrarelativistic

$$\lim_{P^2 \to M^2} \frac{\bar{\chi}(V - [S^{(1)} S^{(2)}]^{-1})\chi}{P^2 - M^2} = i$$

instantaneous

$$\int d^3x \left(|X|^2 - |Y|^2\right) = 1$$

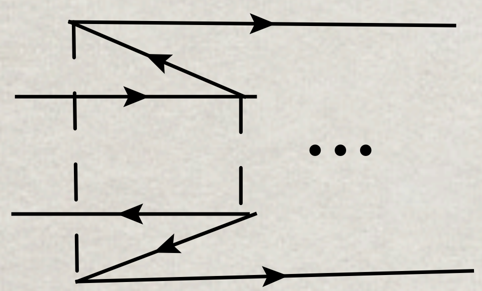
nonrelativistic

$$\int d^3x |\psi|^2 = 1$$

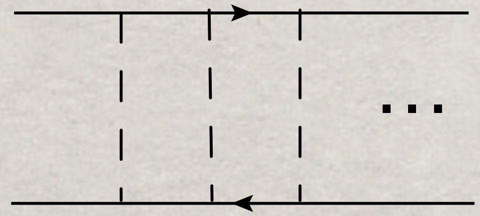
this permits the extension of QFT to QM

particle content

instantaneous



nonrelativistic

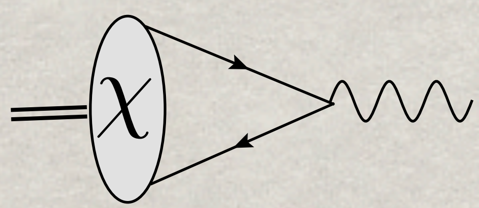


chiral symmetry breaking

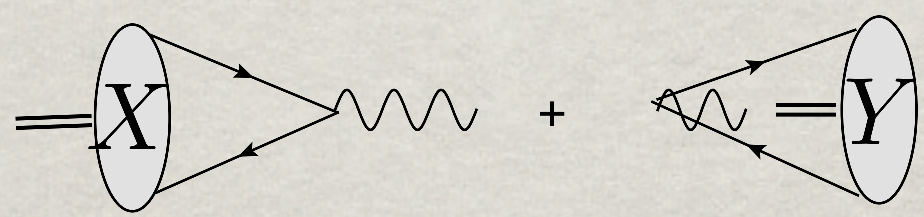
ultrarelativistic	instantaneous	nonrelativistic
yes	yes	no
		modelling must occur in the broken phase

decay constant f_π

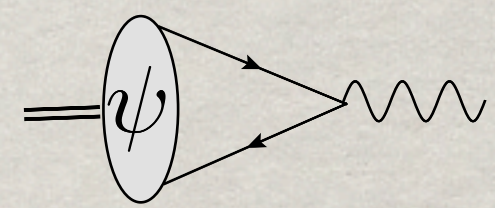
ultrarelativistic



instantaneous

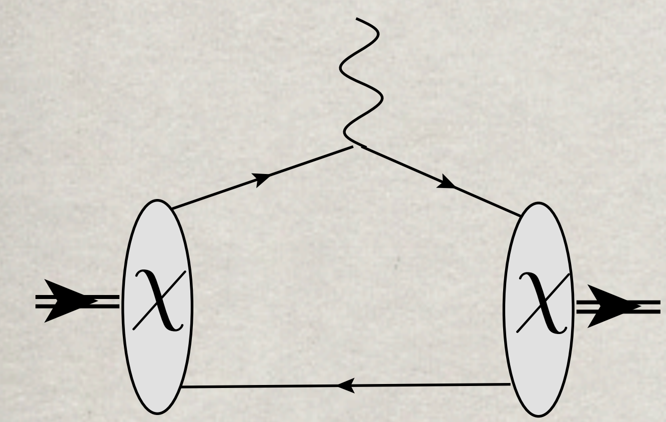


nonrelativistic

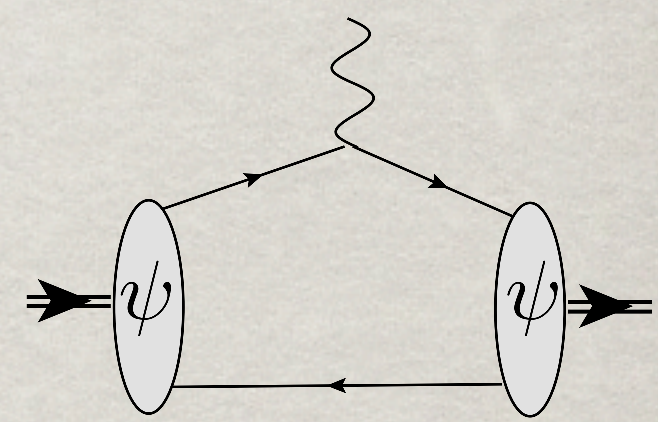


electromagnetic form factor

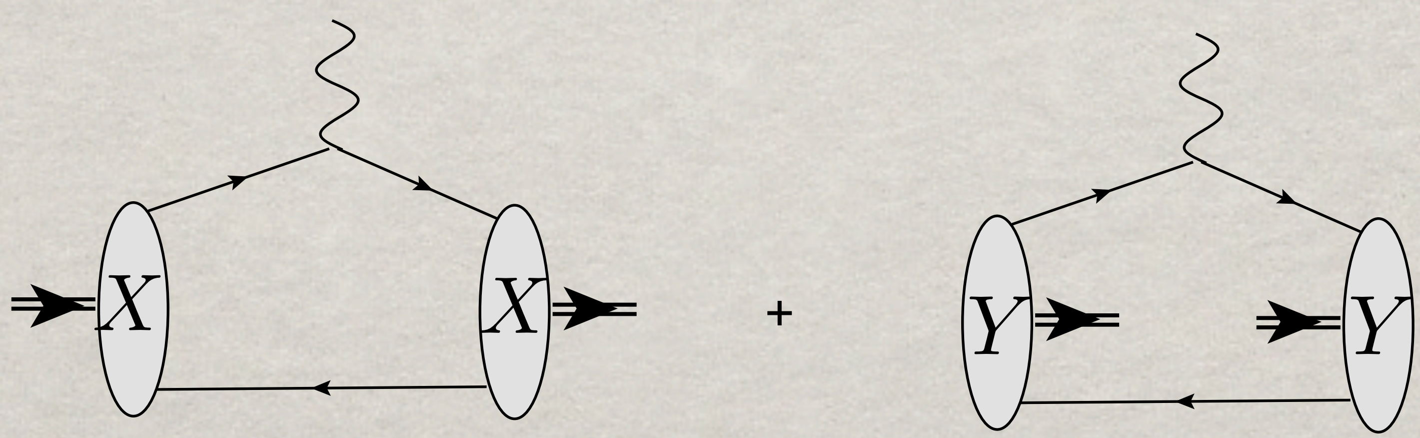
ultrarelativistic



nonrelativistic



instantaneous

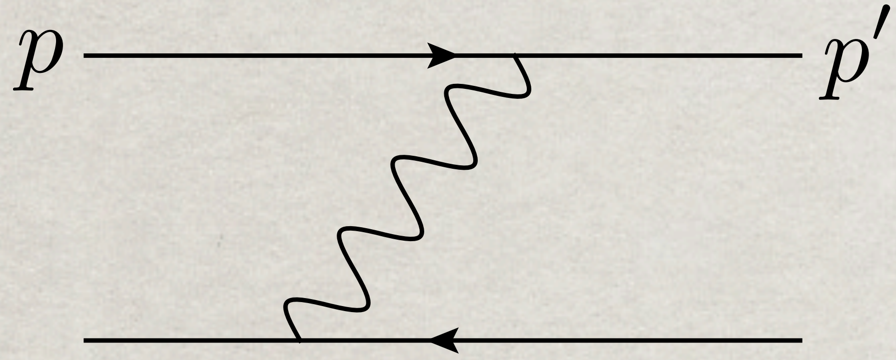


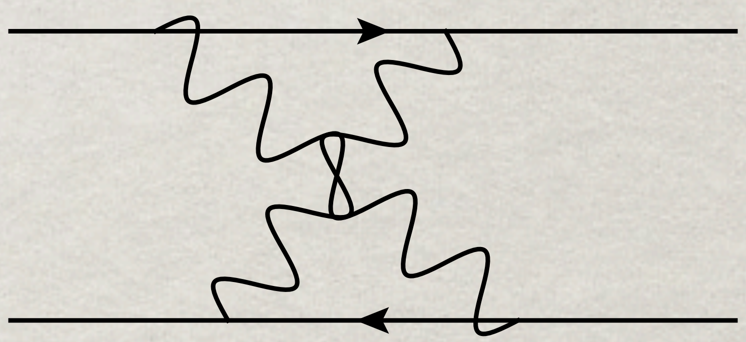
electromagnetic form factor

$$Q^2 F_\pi(Q^2) \to 16\alpha_s(Q^2) f_\pi^2$$

ultrarelativistic	instantaneous	nonrelativistic
yes	no	no

the instantaneous approximation





$$\frac{k_0}{\vec{k}} = \mathcal{O}(\frac{|\vec{p} - \vec{p}'|}{M})$$

NN fails (?)

the instantaneous approximation

adopt an effective field theory approach

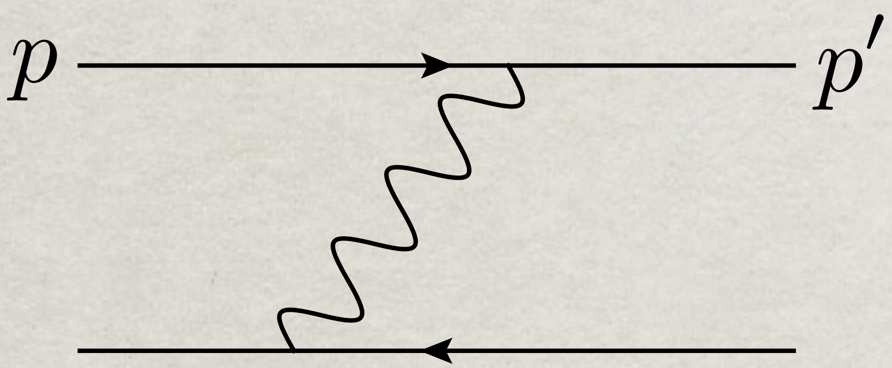
positronium

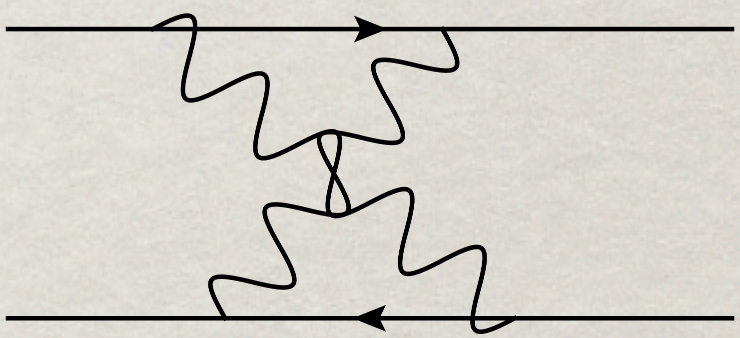
$$K_e < \Lambda < K_\gamma$$

$$\alpha^2 m_e < \Lambda < \alpha m_e$$

the instantaneous approximation

adopt an effective field theory approach





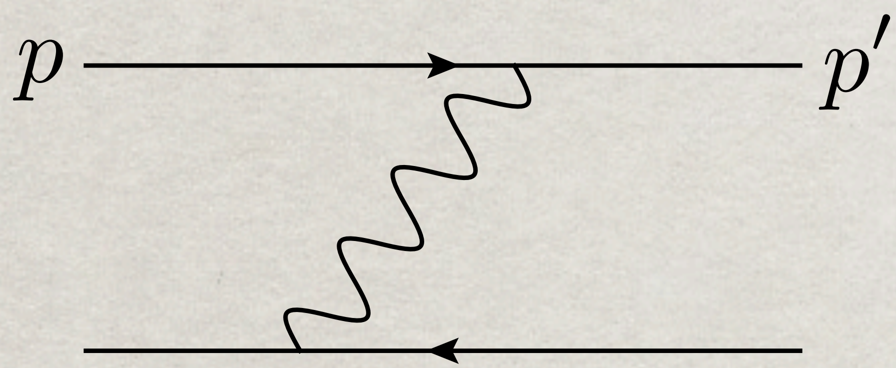
nucleon-nucleon

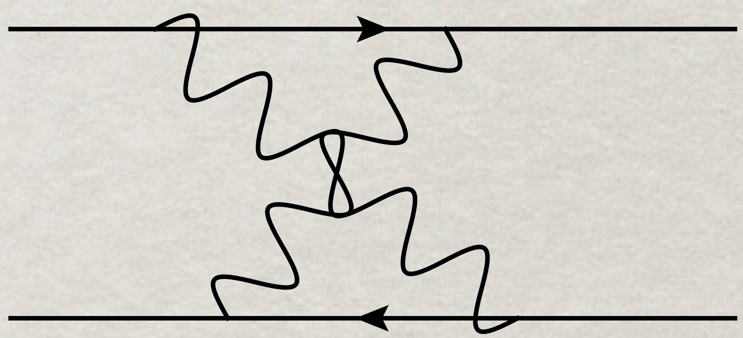
$$E_\pi > m_\pi > K_N \sim 40 \text{ MeV}$$

Λ_{NN}

the instantaneous approximation

adopt an effective field theory approach





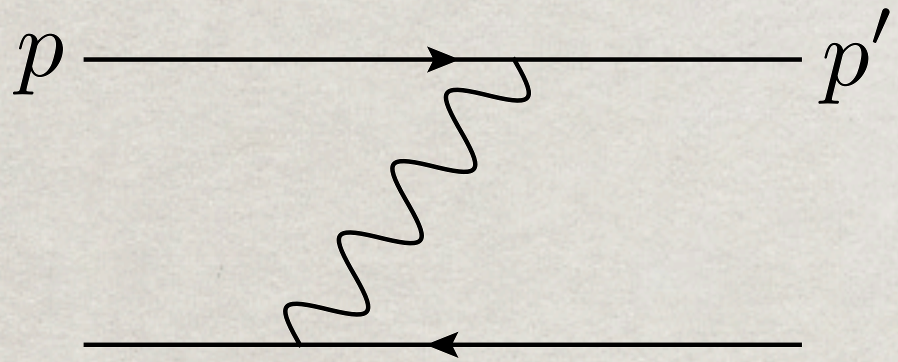
heavy hadron

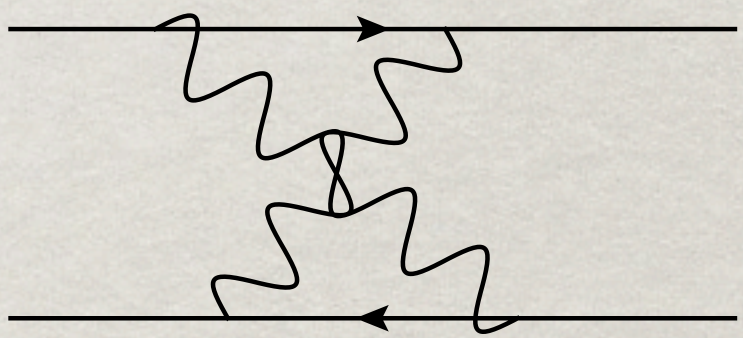
$$K_q < \Lambda < K_g$$

$$\alpha_s(m_q)m_q < \Lambda < \alpha_s(m_q)m_q$$

the instantaneous approximation

adopt an effective field theory approach



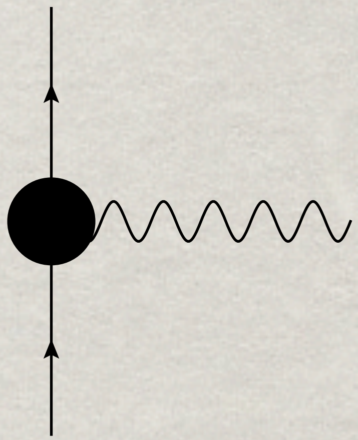


light hadron

???

multiplicative renormalisability

$$(k-k')^\mu \Gamma(k,k')_\mu = S^{-1}(k) - S^{-1}(k')$$



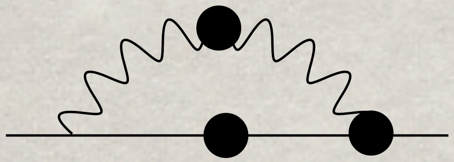
$$\begin{aligned}\Gamma_\mu(k,k') &= \frac{1}{2}(A(k)+A(k'))\gamma_\mu + \frac{1}{2}\frac{A(k)-A(k')}{k^2-k'^2}(k+k')^\mu(\not{k}+\not{k}') \\ &- \frac{B(k)-B(k')}{k^2-k'^2}(k+k')^\mu + \Gamma_\mu^{(T)}\end{aligned}$$

J.S. Ball and T.W. Chiu, Phys. Rev. D22, 1542 (1980).

multiplicative renormalisability

$$A(k;\mu) = Z_2(\Lambda,\mu) A_0(k,\Lambda)$$

$$A_0(k,\Lambda) = \left(\frac{k^2}{\Lambda^2}\right)^a \qquad Z_2(\Lambda,\mu) = \left(\frac{\Lambda^2}{\mu^2}\right)^a$$

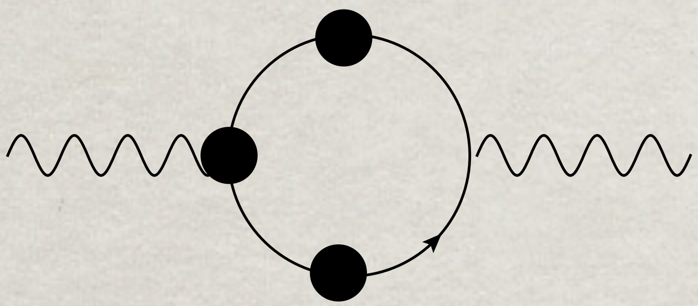


D.C. Curtis and M.R. Pennington, Phys. Rev. D42, 4165 (1990)

$$\Gamma^\mu_{CP} = \frac{1}{2}\frac{A(k)-A(k')}{d(k,k')}\left[\gamma^\mu(k^2-k'^2)-(k+k')^\mu(\not{k}-\not{k}')\right]$$

this makes renormalisation of the vertex multiplicative at LL and NLL orders.

multiplicative renormalisability



A. Kızılersu and M. Pennington, Phys, Rev. D79, 125050 (2009)

conclusions

- the instantaneous/RPA approximation can capture χSB and PCAC properties of the pion
- the instantaneous approximation can be obtained as an effective field theory and is broadly applicable
- the nonrelativistic approximation can capture static properties of the pion with appropriate tuning
- multiplicative renormalisation is a severe constraint and implies all current truncations break the theory
- quantum mechanics is *not* a limit of quantum field theory

✢ ÆRIC MEC HEHT GEWYRCAN

OTHER ISSUES

- when is the instantaneous approx applicable?
- odd quantum numbers/ hybrids
- spurious states
- multiplicative renormalisability
- gauge invariance
- the QM limit of scalar QED
- the transition to QM is not complete or trivial!