The Confining String: Lattice Results versus Effective Field Theory

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From Quarks and Gluons to Hadrons and Nuclei, Erice, 23.9.2011

Collaboration:
Ferdinando Gliozzi (INFN Torino)
Michele Pepe (INFN Milano)

Outline

The Confining String in Yang-Mills Theory

Systematic Low-Energy Effective String Theory

Lüscher-Weisz Multi-Level Simulation Technique

String Width at Zero and at Finite Temperature

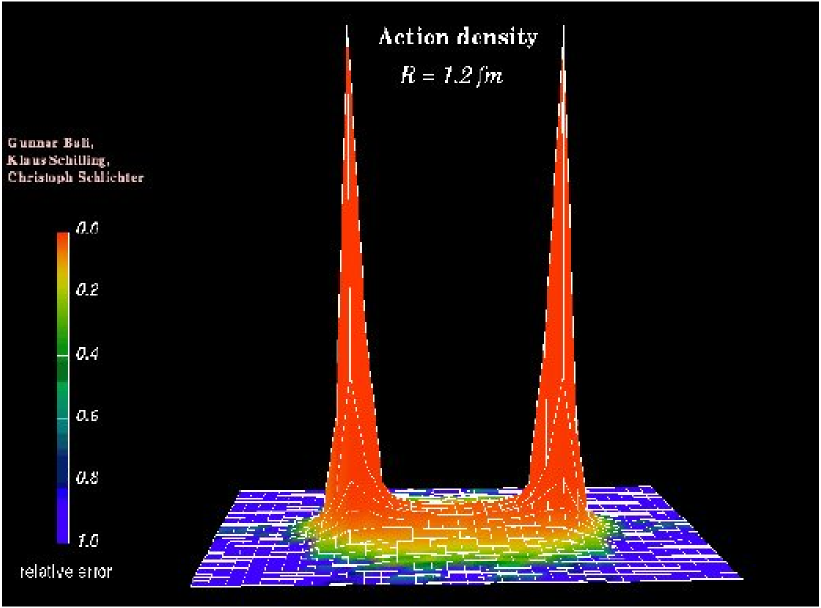
Anatomy of k-Strings in $SU(4)$ Yang-Mills Theory

String Breaking and String Decay

Conclusions

Outline

Action Density of Flux String in $SU(2)$ Yang-Mills Theory



G. S. Bali, K. Schilling, and C. Schlichter, Phys. Rev. D51 (1995) 5165.

String tension from Polyakov loop correlators

$$\Phi(\vec{x}) = \mathrm{Tr}\, \mathcal{P} \exp\left(\int_0^\beta dt \;\, G_0(\vec{x}, t) \right),$$

$$\langle \Phi(0)^* \Phi(r) \rangle \sim \exp\left(-\beta V(r)\right), \quad V(r) \sim \sigma r$$

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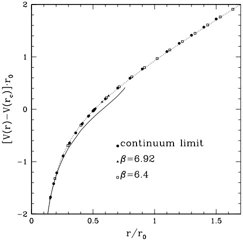

$\sigma \approx (0.4\mathrm{GeV})^2 \approx 10^5\mathrm{N}$

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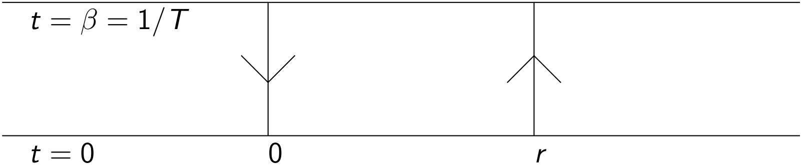

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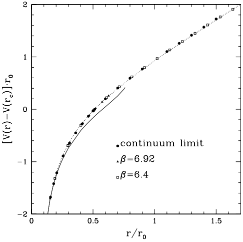
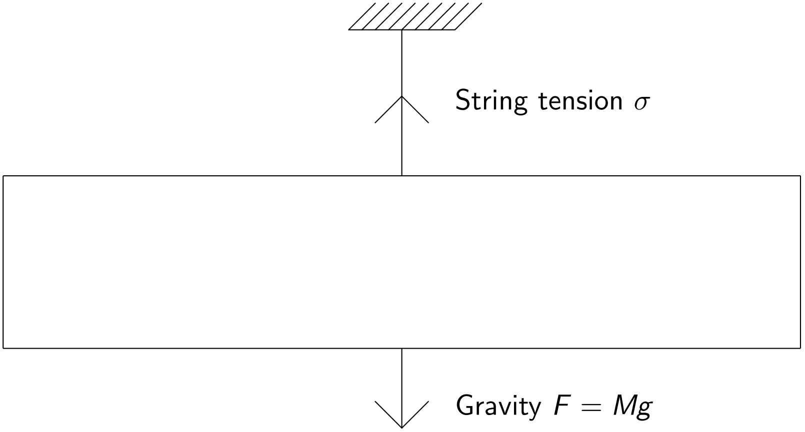


$\sigma \approx (0.4\mathrm{GeV})^2 \approx 10^5\mathrm{N}$

As strong as a cm-thick steel cable, but 13 orders of magnitude thinner.

How many people can be lifted by a Yang-Mills elevator?



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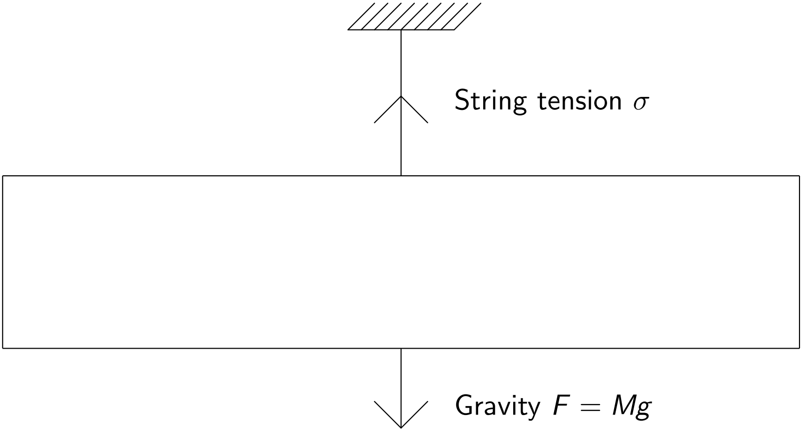

$$Mg = \sigma \approx 10^5\text{N}, \; g \approx 10\text{m/sec}^2 \;\Rightarrow\; M \approx 10^4\text{kg} \approx 100 \text{ People}$$

About 100 participants of the
33rd International School on Nuclear Physics, Erice 2011.

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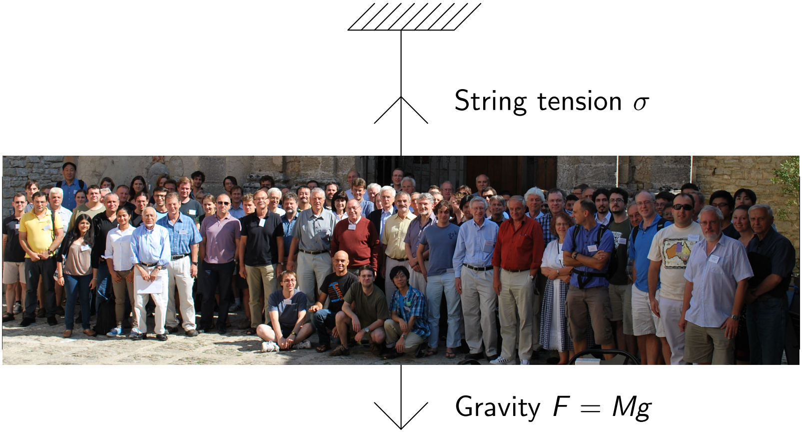

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Outline

Low-energy effective string theory

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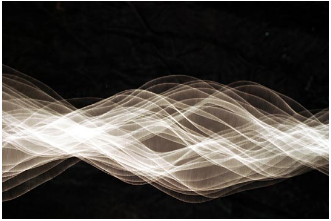

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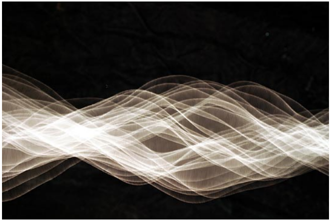

The height variable $\vec{h}(x,t)$ points to the fluctuating string world-sheet in the $(d-2)$ transverse dimensions ($\vec{h}(0,t) = \vec{h}(r,t) = 0$).

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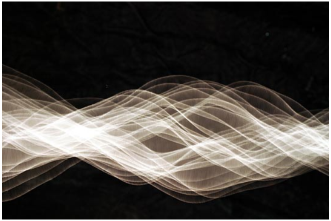

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$$S[\vec{h}] = \int_0^\beta dt \int_0^r dx \; \frac{\sigma}{2} \partial_\mu \vec{h} \cdot \partial_\mu \vec{h}, \quad \mu \in \{0,1\},$$

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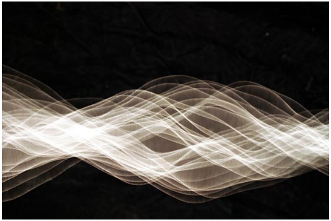

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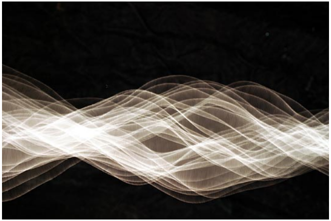

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$$w_{lo}^2(r/2) = \frac{d-2}{2\pi\sigma} \log(r/r_0)$$

M. Lüscher, K. Symanzik, P. Weisz, Nucl. Phys. B173 (1980) 365
M. Lüscher, Nucl. Phys. B180 (1981) 317
M. Lüscher, G. Münster, P. Weisz, Nucl. Phys. B180 (1981) 1

Effective string theory for $d = 3$ at the 2-loop level

$$S[h] = \int_0^\beta dt \int_0^r dx \; \frac{\sigma}{2} \left[\partial_\mu h \partial_\mu h - \frac{1}{8\sigma} \left(\partial_\mu h \partial_\mu h \right)^2 \right]$$

Even to the next order, the action agrees with the one of the Nambu-Goto string.

O. Aharony and E. Karzbrun, JHEP 0906 (2009) 012

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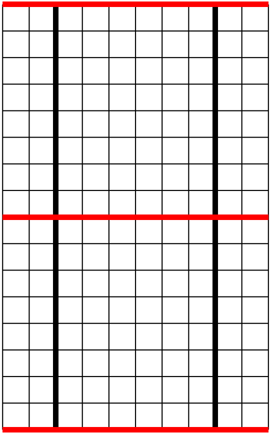
$$\begin{aligned}
w^2(r/2) &= \langle h(r/2,t)\, h(r/2+\epsilon, t+\epsilon') \rangle \\
&= \left(1 + \frac{4\pi f(\tau)}{\sigma r^2} \right) w_{lo}^2(r/2) - \frac{f(\tau)+g(\tau)}{\sigma^2 r^2}, \\
f(\tau) &= \frac{E_2(\tau) - 4E_2(2\tau)}{48}, \\
g(\tau) &= i\pi\tau \left(\frac{E_2(\tau)}{12} - q\frac{d}{dq} \right) \left(f(\tau) + \frac{E_2(\tau)}{16} \right) + \frac{E_2(\tau)}{96}, \\
E_2(\tau) &= 1 - 24 \sum_{n=1}^{\infty} \frac{n\, q^n}{1-q^n}, \quad q = \exp(2\pi i \tau), \quad \tau = i\beta/2r
\end{aligned}$$

F. Gliozzi, M. Pepe, UJW, Phys. Rev. Lett. 104 (2010) 232001
JHEP 1011 (2010) 053

Outline

Lüscher-Weisz Multi-Level Simulation Technique

M. Lüscher and P. Weisz, JHEP 0109 (2001) 010



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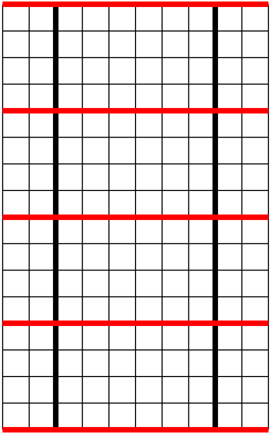

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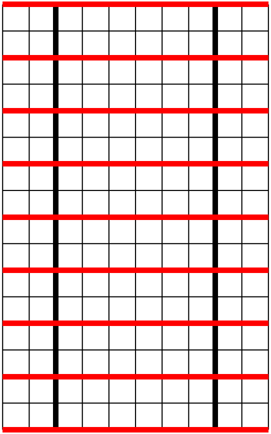

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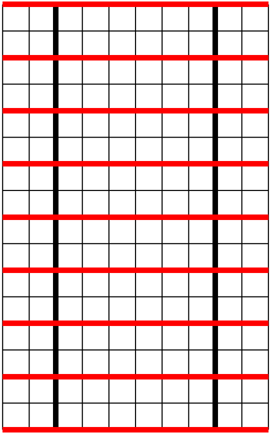

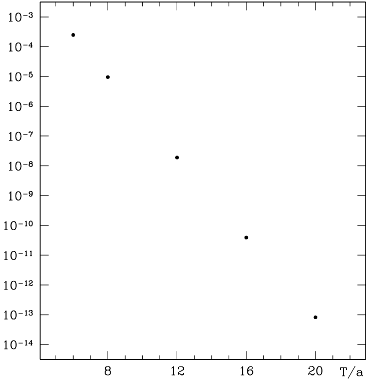

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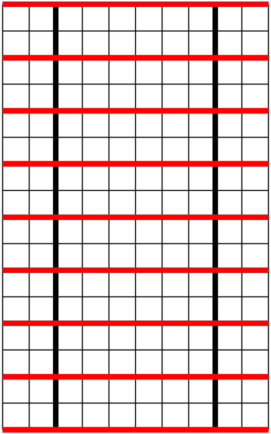

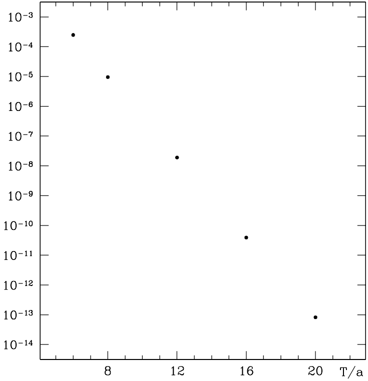

Extending this technique, we have reached $\langle \Phi(0)^* \Phi(r) \rangle \approx 10^{-135}$.

Outline

Simulations in $(2+1)$-d $SU(2)$ Yang-Mills theory

$$S[U] = -\frac{1}{g^2} \sum_{x,\mu,\nu} \mathrm{Tr}[U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu(x+\hat{\nu})^\dagger U_\nu(x)^\dagger],$$

$$\langle \Phi(0)\Phi(r) \rangle = \frac{1}{Z} \int \mathcal{D}U \; \Phi(0)\Phi(r) \exp(-S[U]) \sim \exp(-\beta V(r))$$

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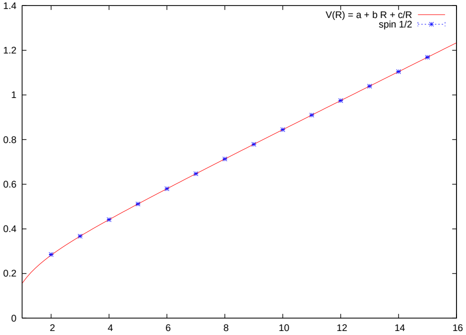

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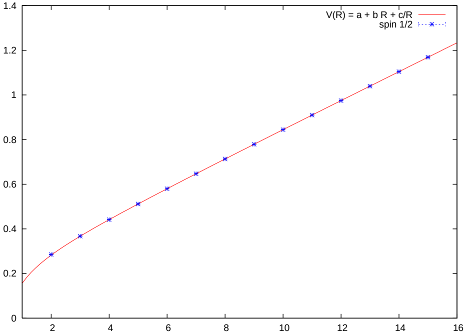
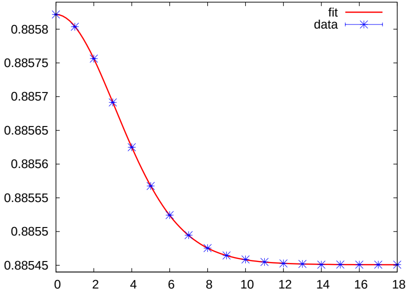


$$V(r) = \sigma r - \frac{\pi}{24r} + \dots$$

There is very good agreement with the effective string theory.
The value of the string tension obtained on a $54^2 \times 48$ lattice at $4/g^2 = 9$ is $\sigma = 0.025897(15)/a^2$.

Computation of the string width

$$P(x) = \text{Tr}[U_1(x)U_0(x+\hat{1})U_1(x+\hat{0})^\dagger U_0(x)^\dagger],$$
$$C(x_2) = \frac{\langle \Phi(0)\Phi(r)P(x)\rangle}{\langle \Phi(0)\Phi(r)\rangle} - \langle P(x)\rangle$$



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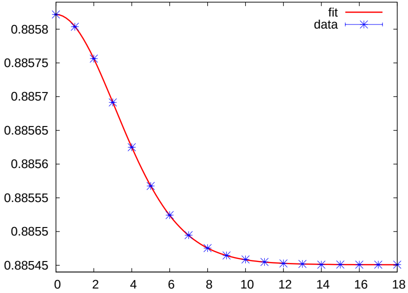

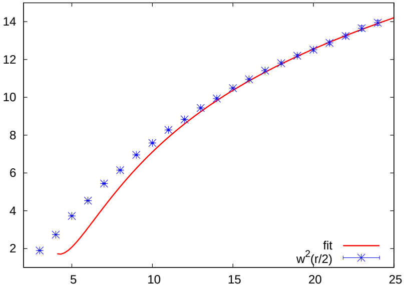

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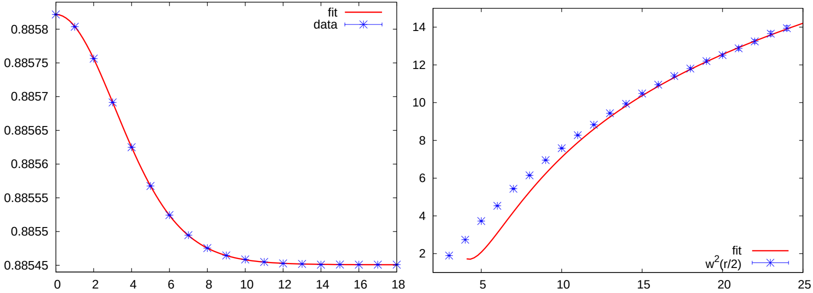

There is very good agreement with the effective string theory at the two-loop level. The values of the low-energy parameters are $\sigma = 0.025897(15)/a^2$, $r_0 = 2.26(2)a = 0.364(4)/\sqrt{\sigma} \approx 0.2$ fm.

F. Gliozzi, M. Pepe, UJW, Phys. Rev. Lett. 104 (2010) 232001
H. Meyer, Phys. Rev. D82 (2010) 106001

String width at finite temperature

$$w^2(r/2) = \frac{1}{2\pi\sigma}\log\left(\frac{\beta}{4r_0}\right) + \frac{r}{2\beta} + \dots$$

At finite temperature, the effective string theory predicts a linear increase of the width as one separates the static sources.

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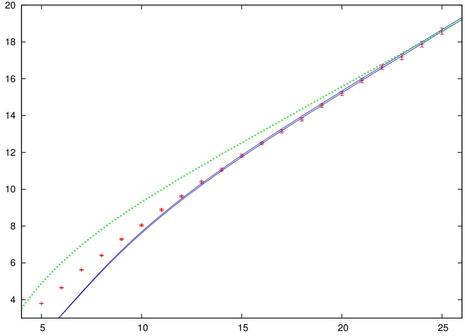

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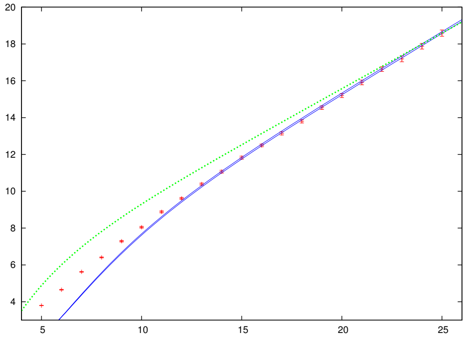
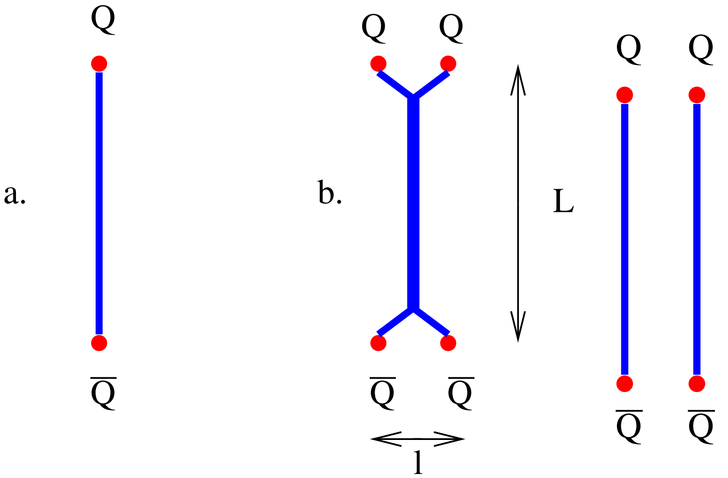


Again, there is excellent agreement with the effective string theory at the two-loop level, now without any adjustable parameters.

F. Gliozzi, M. Pepe, UJW, JHEP 1101 (2011) 057

Outline

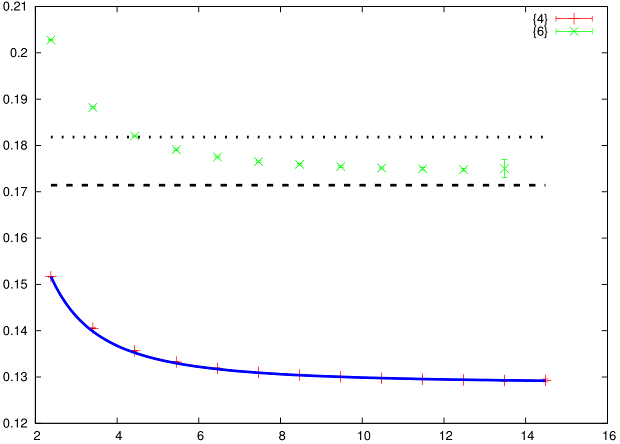
In $SU(4)$ Yang-Mills theory the $\{4\}$- and $\{6\}$-string are stable



Does the $\{6\}$-string consist of two strands of $\{4\}$-string?

Forces exerted by the {4}- and {6}-strings:

$$-F(r) = \frac{dV(r)}{dr} \rightarrow \sigma + \frac{\pi}{24r^2}$$

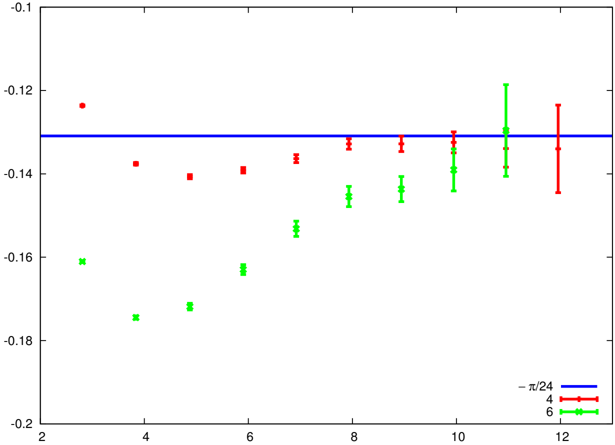


There are small deviations from both Casimir scaling and sine-law.

M. Pepe, PoS LATTICE 2010 (2010) 017.

Does the {6}-string consist of two strands of {4}-string?

$$c(r) = -\frac{1}{2} r^3 \frac{dF(r)}{dr} \rightarrow -\frac{\pi}{24}$$

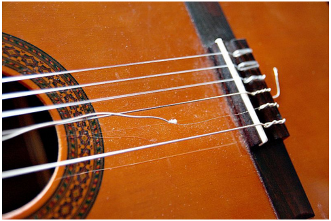


The Lüscher term of the 6-string is the same as the one of the 4-string. Hence there is only one strand.

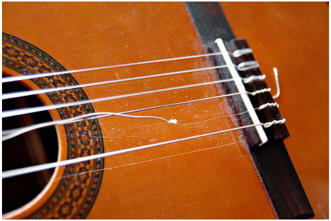
B. Bringoltz and M. Teper, Phys. Lett. B663 (2008) 429.
M. Pepe, PoS LATTICE 2010 (2010) 017.

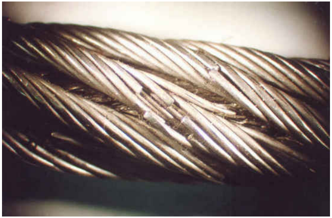
Outline

String breaking



String breaking and string decay (strand rupture?)





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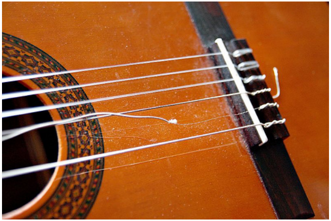
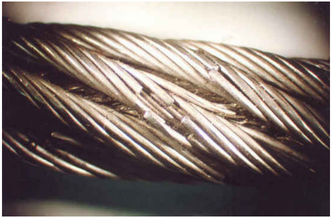
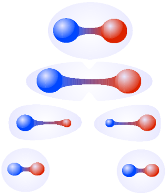




Static potential between triplet ($Q = 1$) charges in (2+1)-d $SU(2)$ Yang-Mills theory



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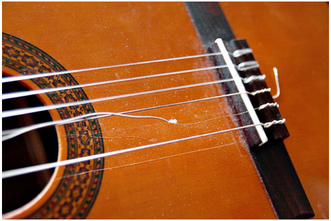

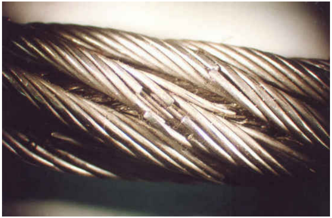

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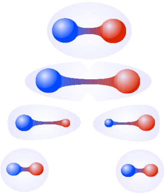
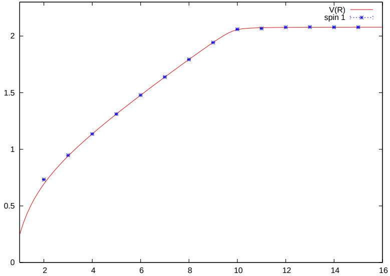
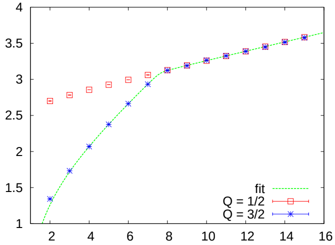
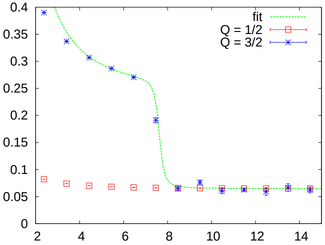




M. Pepe and UJW, Phys. Rev. Lett. 102 (2009) 191601

String decay





Potential and force between quadruplet ($Q = 3/2$) charges

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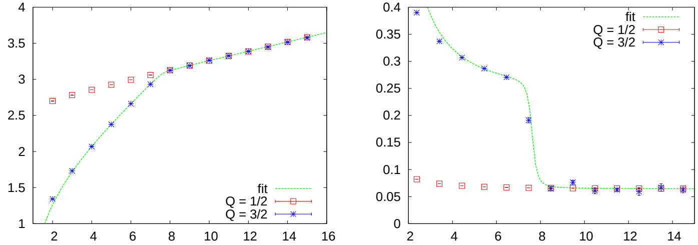

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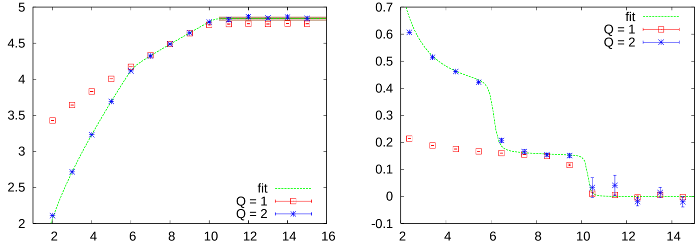

Potential and force between quintet ($Q = 2$) charges

Constituent gluon model: $E_{Q,n}(r) = \sigma_Q r - \frac{c_Q}{r} + 2M_{Q,n}$

$$H_1(r) = \begin{pmatrix} E_{1,0}(r) & A \\ A & E_{0,1}(r) \end{pmatrix},$$

$$H_{3/2}(r) = \begin{pmatrix} E_{3/2,0}(r) & B \\ B & E_{1/2,1}(r) \end{pmatrix},$$

$$H_2(r) = \begin{pmatrix} E_{2,0}(r) & C & 0 \\ C & E_{1,1}(r) & A \\ 0 & A & E_{0,2}(r) \end{pmatrix},$$

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Q	$\sigma_Q a^2$	σ_Q/σ	$4Q(Q+1)/3$
1/2	0.06397(3)	1	1
1	0.144(1)	2.25(2)	8/3
3/2	0.241(5)	3.77(8)	5
2	0.385(5)	6.02(8)	8

We observe deviations from Casimir scaling.

Masses and mass differences: $\Delta_{Q,n} = M_{Q-1,n+1} - M_{Q,n}$

Q	$M_{Q,0}a$	$M_{Q-1,1}a$	$M_{Q-2,2}a$	$\Delta_{Q,0}a$	$\Delta_{Q-1,1}a$
1/2	0.109(1)	—	—	—	—
1	0.37(3)	1.038(1)	—	0.67(3)	—
3/2	0.72(5)	1.32(5)	—	0.60(5)	—
2	1.04(3)	1.71(3)	2.42(1)	0.67(3)	0.71(3)

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H. Meyer and M. Teper, Nucl. Phys. B668 (2003) 111

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A simple constituent gluon model describes the data rather well.

Outline

Conclusions

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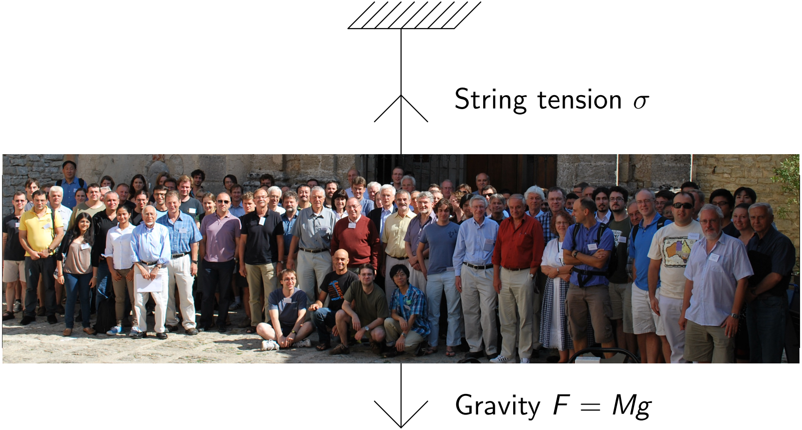
• A constitutent gluon model accounts for the "brown muck" surrounding a screened color charge.

Yang-Mills elevator as a metaphor for our meeting:



Thanks to Amand Faessler and Jochen Wambach for installing a "Knowledge Elevator" in Erice that elevates us to a higher level of understanding of the strong interaction.