Hadron Structure in Lattice QCD

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From Quarks and Gluons to Hadrons and Nuclei
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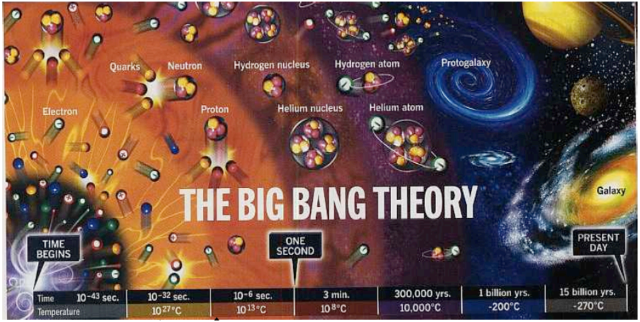
   - Δ electromagnetic form factors
   - Δ axial-vector form factors
   - Pseudo-scalar Δ form factors
5. **Conclusions**

Introduction

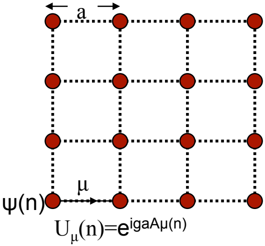
Numerical simulation of QCD already provides essential input for a wide class of strong interaction phenomena



- QCD phase diagram relevant for Quark-Gluon Plasma: $t \sim 10^{-32}$s and $T \sim 10^{27}$, studied in heavy ion collisions at RHIC and LHC
- Hadron structure: $t \sim 10^{-6}$ s, experimental program at JLab, Mainz.
  - Momentum distribution of quarks and gluons in the nucleon
  - Hadron form factors e.g. the nucleon axial charge g_A
- Nuclear forces: $t \sim 10^9$ years, affect the large scale structure of the Universe

Exa-scale machines are required to go beyond hadrons to nuclei

QCD on the lattice



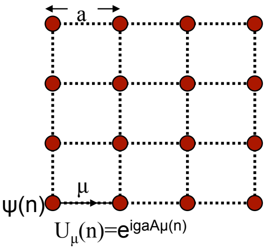
- Discretization of space-time in 4 Euclidean dimensions
  $\rightarrow$ Rotation into imaginary time is the most drastic modification
  Lattice acts as a non-perturbative regularization scheme with the lattice spacing a providing an ultraviolet cutoff at $\pi/a \rightarrow$ no infinities
- Gauge fields are links and fermions are anticommuting Grassmann variables defined at each site of the lattice. They belong to the fundamental representation of SU(3)
- Construction of an appropriate action such that when $a \rightarrow 0$ (and Volume$\rightarrow \infty$) it gives the continuum theory
- Construction of the appropriate operators with their renormalization to extract physical quantities
- Can be simulated on the computer using methods analogous to those used for Statistical Mechanics systems
  $\rightarrow$ Allows calculations of correlation functions of hadronic operators and matrix elements of any operator between hadronic states in terms of the fundamental quark and gluon degrees of freedom with only input parameters the coupling constant and the quarks masses.

$\Longrightarrow$ Lattice QCD provides a well-defined approach to calculate observables non-perturbative starting directly from the QCD Langragian.

Consider simplest isotropic hypercubic grid: $a = a_S = a_T$ and size $N_S \times N_S \times N_S \times N_T$, $N_T > N_S$.

- **Finite Volume:**
  1. Finite volume effects need to be studied $\rightarrow$ Take box sizes such that $L_S m_\pi \gtrsim 3.5$.
  2. Only discrete values of momentum in units of $2\pi/L_S$ are allowed.
- **Finite lattice spacing:** Need at least three values of the lattice spacing in order to extrapolate to the continuum limit.
- q^2**-values:** Fourier transform of lattice results in coordinate space taken numerically $\rightarrow$ for large values of momentum transfer results are too noisy $\Longrightarrow$ Limited to $Q^2 = -q^2 \sim 2\ \text{GeV}^2$.

QCD on the lattice



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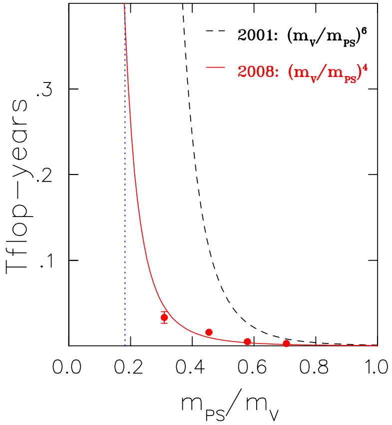
Computational cost

Simulation cost: $C_{\rm sim} \propto \left(\frac{300\mathrm{MeV}}{m_\pi}\right)^{c_m} \left(\frac{L}{2\mathrm{fm}}\right)^{c_L} \left(\frac{0.1\mathrm{fm}}{a}\right)^{c_a}$



L=2.1 fm, a=0.089 fm, K. Jansen and C. Urbach, arXiv:0905.3331

Coefficients c_m, c_L and c_a depend on the discretized action used for the fermions.
State-of-the-art simulations use improved algorithms:

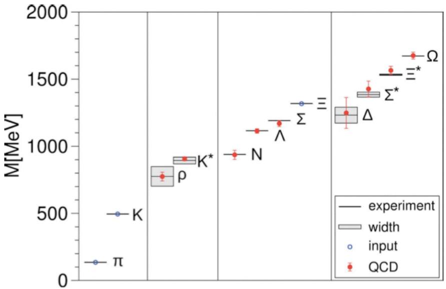
- Mass preconditioner, M. Hasenbusch, Phys. Lett. B519 (2001) 177
- Multiple time scales in the molecular dynamics updates

$\Longrightarrow$ for twisted mass fermions: $c_m \sim 4$, $c_L \sim 5$ and $c_a \sim 6$.

- Results at physical quark masses require $\mathcal{O}(1)$ Pflop.Years.
- After post-diction of well measured quantities the goal is to predict quantities that are difficult or impossible to measure experimentally.

Mass of low-lying hadrons

$N_F = 2+1$ smeared Clover fermions, BMW Collaboration, S. Dürr et al. Science 322 (2008)
$N_F = 2$ twisted mass fermions, ETM Collaboration, C. Alexandrou et al. PRD (2008)



- BMW with $N_F = 2+1$:
  - 3 lattice spacings: $a \sim 0.125, 0.085, 0.065$ fm set by m_Ξ
  - Pion masses: $m_\pi \gtrsim 190$ MeV
  - Volumes: $m_\pi^{\min} L \gtrsim 4$
- ETMC with $N_F = 2$:
  - 3 lattice spacings: $a = 0.089, 0.070, a = 0.056$ fm, set by m_N
  - $m_\pi \gtrsim 260$ MeV
  - Volumes: $m_\pi^{\min} L \gtrsim 3.3$

Good agreement between different discretization schemes $\Longrightarrow$ Significant progress in understanding the masses of low-lying mesons and baryons

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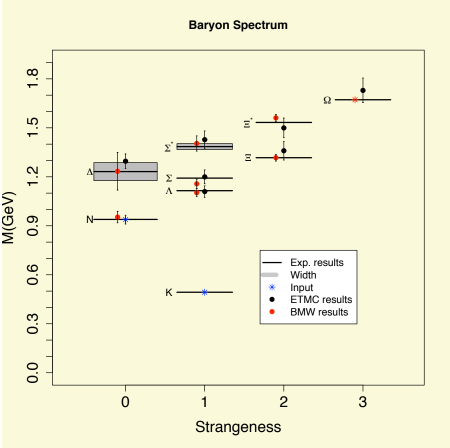

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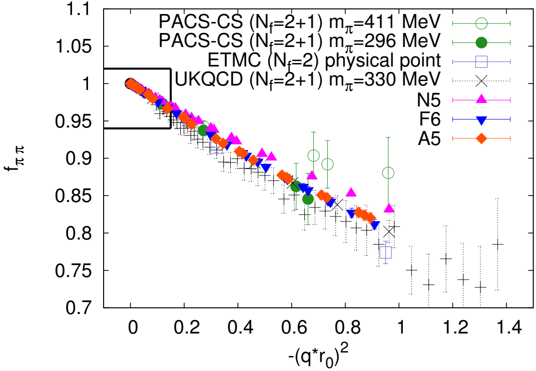
Pion form factor

Several Collaborations using dynamical quarks with pion masses down to about 300 MeV

ETMC, $N_F = 2$, R. Frezzotti, V. Lubicz and S. Simula, PRD 79, 074506 (2009)

- Examine volume and cut-off effects $\Rightarrow$ estimate continuum and infinite volume values
- Twisted boundary conditions to probe small $Q^2 = -q^2$, $\rightarrow$ Talk by A. Juttner
- All-to-all propagators and 'one-end trick' to obtain accurate results
- Chiral extrapolation using NNLO $\rightarrow$ $\langle r^2 \rangle$ and $F_\pi(Q^2) = \left(1 + \langle r^2 \rangle Q^2/6\right)^{-1}$



B.B. Brandt, A. Juttner, H. Wittig (CLS) arXiv:1109.0196

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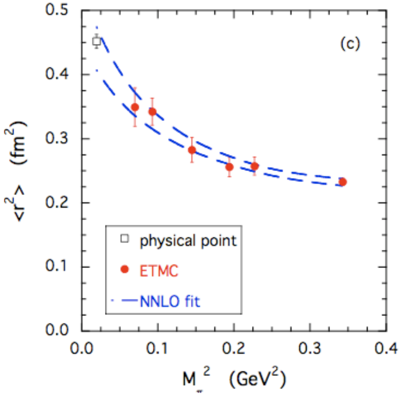

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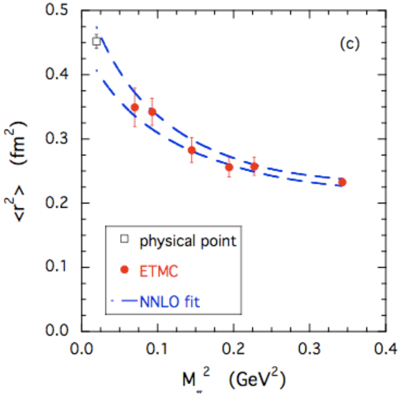
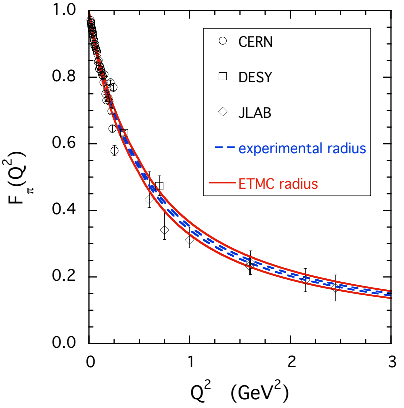
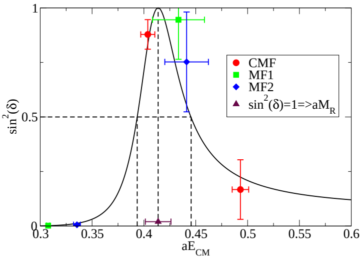




ρ-meson width

- Consider $\pi^+\pi^-$ in the $I = 1$-channel
- Estimate P-wave scattering phase shift $\delta_{11}(k)$ using finite size methods
- Use Lüscher's relation between energy in a finite box and the phase in infinite volume
- Use Center of Mass frame and Moving frame
- Use effective range formula: $tan\delta_{11}(k) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{E\left(m_R^2 - E^2\right)}$, $k = \sqrt{E^2/4 - m_\pi^2} \rightarrow$ determine m_R and $g_{\rho\pi\pi}$ and then extract $\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k_R^3}{m_R^2}$, $k_R = \sqrt{m_R^2/4 - m_\pi^2}$

$m_\pi = 309$ MeV, $L = 2.8$ fm



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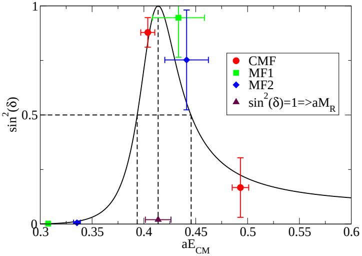

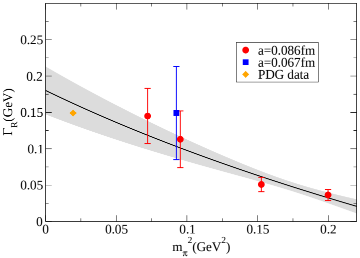

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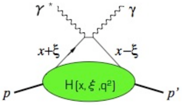
Definition of Generalized Form Factors

High energy scattering: Formulate in terms of light-cone correlation functions, M. Diehl, Phys. Rep. 388 (2003)
Consider one-particle states p' and $p \to$ Generalized Parton Distributions (GPDs), X. Ji, J. Phys. G24(1998)1181

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar\psi(-\lambda n/2)\Gamma \mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(n\alpha)} \psi(\lambda n/2)|p\rangle$$

"Handbag" diagram



where $q = p' - p$, $\bar P = (p'+p)/2$, n is a light-cone vector and $\bar P.n = 1$

$$\Gamma = \not{n} \quad \to \frac{1}{2}\bar u(p')\left[\not{n} H(x,\xi,q^2) + i\frac{n_\mu q_\nu \sigma^{\mu\nu}}{2m} E(x,\xi,q^2)\right] u(p)$$

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$$\Gamma = n_\mu \sigma^{\mu\nu} \quad \to \text{tensor GPDs}$$

Expansion of the light cone operator leads to a tower of local twist-2 operators $\mathcal{O}^{\mu_1\cdots\mu_n}$:

- Diagonal matrix element $\langle P|\mathcal{O}(x)|P\rangle$ (DIS) $\to$ parton distributions: $q(x)$, $\Delta q(x)$, $\delta q(x)$

$$\mathcal{O}_q^{\mu\mu_1\cdots\mu_n} = \bar q \gamma^{\{\mu} iD^{\mu_1}\ldots iD^{\mu_n\}} q \xrightarrow{unpolarized} \langle x^n\rangle_q = \int_0^1 dx\, x^n\left[q(x) - (-1)^n \bar q(x)\right]$$

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where $q = q_\downarrow + q_\uparrow$, $\Delta q = q_\downarrow - q_\uparrow$, $\delta q = q_T + q_\perp$

- Off-diagonal matrix elements (DVCS) $\to$ generalized form factors

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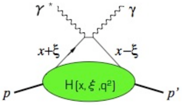

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Nucleon generalized form factors

Decomposition of matrix elements into generalized form factors:

$$\langle N(p',s')|\mathcal{O}_q^{\mu\mu_1\cdots\mu_n}|N(p,s)\rangle = \bar{u}(p',s')\Big[\sum_{\substack{i=0\\ \text{even}}}^{n}\Big(A_{n+1,i}(q^2)\gamma^{\{\mu} + B_{n+1,i}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha}{2m}\Big)q^{\mu_1}\ldots q^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_n\}}$$
$$+\text{mod}(n,2)C_{n+1,0}(q^2)\frac{1}{m}q^{\{\mu}q^{\mu_1}\ldots q^{\mu_n\}}\Big]u(p,s)$$

And similarly for $\mathcal{O}_{\Delta q}$ in terms of $\tilde{A}_{ni}(q^2)$, $\tilde{B}_{ni}(q^2)$ and $\mathcal{O}_{\delta q}$ in terms of $A^T_{ni},\ B^T_{ni},\ C^T_{ni}$ and D^T_{ni}

Special cases:

- $n=1$: ordinary nucleon form factors

$$A_{10}(q^2)=F_1(q^2)=\int_{-1}^{1}dxH(x,\xi,q^2),\quad B_{10}(q^2)=F_2(q^2)=\int_{-1}^{1}dxE(x,\xi,q^2)$$
$$\tilde{A}_{10}(q^2)=G_A(q^2)=\int_{-1}^{1}dx\tilde{H}(x,\xi,q^2),\quad \tilde{B}_{10}(q^2)=G_p(q^2)=\int_{-1}^{1}dx\tilde{E}(x,\xi,q^2)$$

  where

  - $j_\mu=\bar{\psi}\gamma_\mu\psi \Longrightarrow \gamma_\mu F_1(q^2)+\frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2)$
    The Dirac F_1 and Pauli F_2 are related to the electric and magnetic Sachs form factors:
    $G_E(q^2)=F_1(q^2)-\frac{q^2}{(2m)^2}F_2(q^2),\qquad G_M(q^2)=F_1(q^2)+F_2(q^2)$
  - $j_\mu=\bar{\psi}\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)\Longrightarrow i\left[\gamma_\mu\gamma_5 G_A(q^2)+\frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]\frac{\tau^a}{2}$

- $A_{n0}(0)$, $\tilde{A}_{n0}(0)$, $A^T_{n0}(0)$ are moments of parton distributions, e.g. $\langle x\rangle_q=A_{20}(0)$ and $\langle x\rangle_{\Delta q}=\tilde{A}_{20}(0)$ are the spin independent and helicity distributions
  $\rightarrow$ can evaluate quark spin, $J_q=\frac{1}{2}[A_{20}(0)+B_{20}(0)]=\frac{1}{2}\Delta\Sigma_q+L_q$
  $\rightarrow$ nucleon spin sum rule: $\frac{1}{2}=\frac{1}{2}\Delta\Sigma_q+L_q+J_g$, momentum sum rule: $\langle x\rangle_g=1-A_{20}(0)$

Nucleon generalized form factors

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    $G_E(q^2) = F_1(q^2) - \frac{q^2}{(2m)^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$
  - $j_\mu = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x) \Longrightarrow i\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]\frac{\tau^a}{2}$

- $A_{n0}(0)$, $\tilde{A}_{n0}(0)$, $A^T_{n0}(0)$ are moments of parton distributions, e.g. $\langle x\rangle_q = A_{20}(0)$ and $\langle x\rangle_{\Delta q} = \tilde{A}_{20}(0)$ are the spin independent and helicity distributions
  $\rightarrow$ can evaluate quark spin, $J_q = \frac{1}{2}[A_{20}(0) + B_{20}(0)] = \frac{1}{2}\Delta\Sigma_q + L_q$
  $\rightarrow$ nucleon spin sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma_q + L_q + J_g$, momentum sum rule: $\langle x\rangle_g = 1 - A_{20}(0)$

Nucleon generalized form factors

Decomposition of matrix elements into generalized form factors:

$$\langle N(p',s')|\mathcal{O}_q^{\mu\mu_1\cdots\mu_n}|N(p,s)\rangle = \bar{u}(p',s')\Big[\sum_{\substack{i=0\\ \text{even}}}^{n}\Big(A_{n+1,i}(q^2)\gamma^{\{\mu} + B_{n+1,i}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha}{2m}\Big)q^{\mu_1}\ldots q^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_n\}}$$
$$+\text{mod}(n,2)C_{n+1,0}(q^2)\frac{1}{m}q^{\{\mu}q^{\mu_1}\ldots q^{\mu_n\}}\Big]u(p,s)$$

And similarly for $\mathcal{O}_{\Delta q}$ in terms of $\tilde{A}_{ni}(q^2)$, $\tilde{B}_{ni}(q^2)$ and $\mathcal{O}_{\delta q}$ in terms of A^T_{ni}, B^T_{ni}, C^T_{ni} and D^T_{ni}

Special cases:

- $n=1$: ordinary nucleon form factors

$$A_{10}(q^2) = F_1(q^2) = \int_{-1}^{1} dx H(x,\xi,q^2), \quad B_{10}(q^2) = F_2(q^2) = \int_{-1}^{1} dx E(x,\xi,q^2)$$
$$\tilde{A}_{10}(q^2) = G_A(q^2) = \int_{-1}^{1} dx \tilde{H}(x,\xi,q^2), \quad \tilde{B}_{10}(q^2) = G_p(q^2) = \int_{-1}^{1} dx \tilde{E}(x,\xi,q^2)$$

  where

  - $j_\mu = \bar{\psi}\gamma_\mu\psi \Longrightarrow \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2)$
    The Dirac F_1 and Pauli F_2 are related to the electric and magnetic Sachs form factors:
    $G_E(q^2) = F_1(q^2) - \frac{q^2}{(2m)^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$
  - $j_\mu = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x) \Longrightarrow i\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]\frac{\tau^a}{2}$

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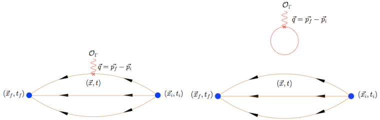
  $\to$ can evaluate quark spin, $J_q = \frac{1}{2}[A_{20}(0) + B_{20}(0)] = \frac{1}{2}\Delta\Sigma_q + L_q$
  $\to$ nucleon spin sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma_q + L_q + J_g$, momentum sum rule: $\langle x\rangle_g = 1 - A_{20}(0)$

Lattice evaluation

Evaluation of two-point and three-point functions

$$G(\vec{q},t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\,\Gamma^4_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\bar{J}_\beta(0)\rangle$$

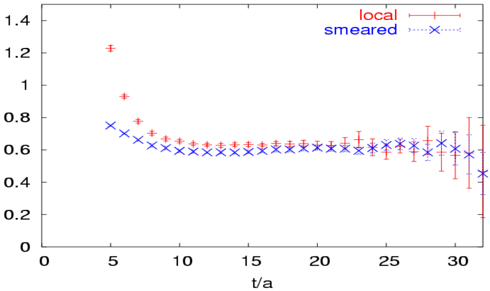
$$G^{\mu\nu}(\Gamma,\vec{q},t) = \sum_{\vec{x}_f,\vec{x}} e^{i\vec{x}\cdot\vec{q}}\,\Gamma_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\mathcal{O}^{\mu\nu}(\vec{x},t)\bar{J}_\beta(0)\rangle$$



Sequential inversion “through the sink” $\rightarrow$ fix sink-source separation $t_f - t_i$, final momentum $\vec{p}_f = 0$, Γ
Apply smearing techniques to improve ground state dominance in three-point correlators
Ratios: Leading time dependence cancels

$$aE_{\rm eff}(\vec{q},t) = \ln\left[G(\vec{q},t)/G(\vec{q},t+a)\right] \rightarrow aE(\vec{q})$$

$$R^{\mu\nu}(\Gamma,\vec{q},t) = \frac{G^{\mu\nu}(\Gamma,\vec{q},t)}{G(\vec{0},t_f)}\sqrt{\frac{G(\vec{p}_i,t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(\vec{p}_i,t)G(\vec{p}_i,t_f)}} \rightarrow \Pi^{\mu\nu}(\vec{q},\Gamma)$$



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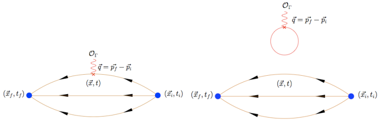

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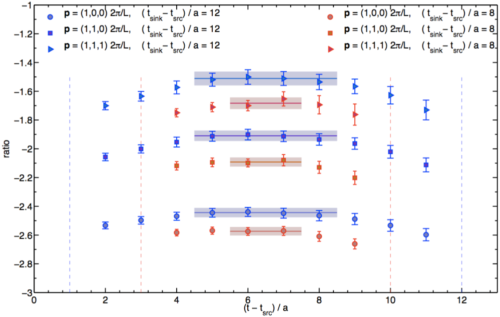
Variational approach in the case of mesons leads to improvement on plateaux: B. Blossier *et at.*, (Alpha Collaboration), JHEP 0904 (2009)

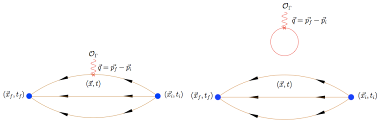


Lattice evaluation

Evaluation of two-point and three-point functions

$$G(\vec{q},t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\, \Gamma^4_{\beta\alpha}\, \langle J_\alpha(\vec{x}_f,t_f)\bar{J}_\beta(0)\rangle$$

$$G^{\mu\nu}(\Gamma,\vec{q},t) = \sum_{\vec{x}_f,\vec{x}} e^{i\vec{x}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_f,t_f)\mathcal{O}^{\mu\nu}(\vec{x},t)\bar{J}_\beta(0)\rangle$$



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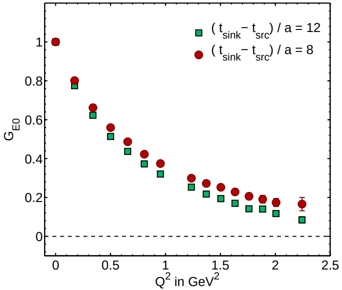


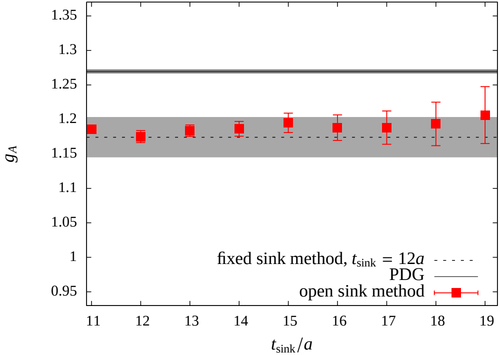
Electric form factor: $t_f - t_i > 1$ fm

However, this might be operator dependent

Study of excited state contributions

$N_F = 2 + 1 + 1$ with $m_\pi \sim 380$ MeV and $a = 0.08$ fm

Vary source- sink separation:



S. Dinter, C.A. M. Constantinou, V. Drach, K. Jansen and D. Renner, arXiv: 1108.1076

Study of excited state contributions

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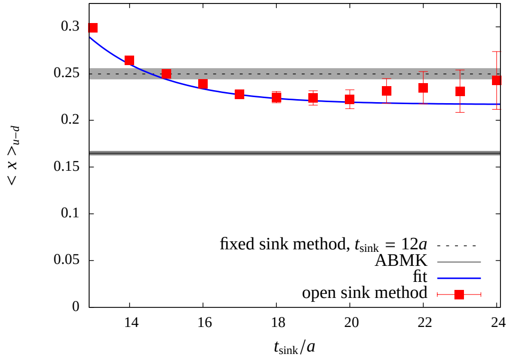


$\Longrightarrow$ Excited contributions are operator dependent

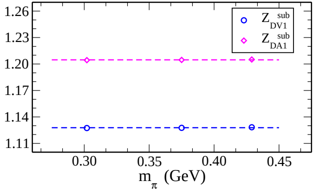
g_A unaffected, $\langle x \rangle_{u-d}$ 10% lower

S. Dinter, C.A. M. Constantinou, V. Drach, K. Jansen and D. Renner, arXiv: 1108.1076

Non-perturbative renormalization

Most collaborations use non-perturbative renormalization.

ETMC: RI'-MOM renormalization scheme as in e.g. M. Göckeler *et al.*, Nucl. Phys. B544,699



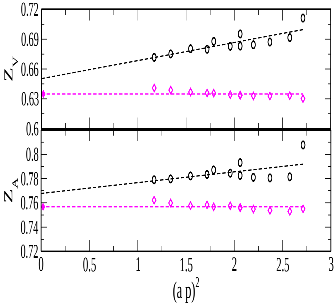
- Fix to Landau gauge and compute:

  $S^u(p) = \frac{a^8}{V} \sum_{x,y} e^{-ip(x-y)} \langle u(x)\bar{u}(y)\rangle$

  $G(p) = \frac{a^{12}}{V} \sum_{x,y,z,z'} e^{-ip(x-y)} \langle u(x)\bar{u}(z)\mathcal{J}(z,z')d(z')\bar{d}(y)\rangle$

  $\rightarrow$ Amputated vertex functions:

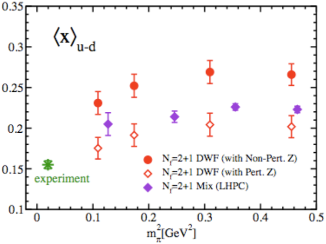
  $\Gamma(p) = (S^u(p))^{-1}\, G(p)\, (S^d(p))^{-1}$
- Renormalization functions: Z_q and $Z_\mathcal{O}$
- Mass independent renormalization scheme $\rightarrow$ need chiral extrapolations
- Subtract $\mathcal{O}(a^2)$ perturbatively.



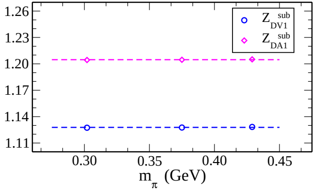
Non-perturbative renormalization

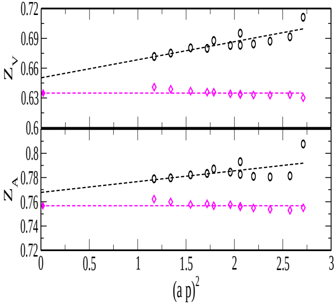
Most collaborations use non-perturbative renormalization.

- RBC: Also uses a RI'-MOM renormalization scheme but with momentum independent source, Y. Aoki *al.* arXiv:1003.3387



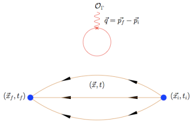
Similarly for $< x >_{\Delta u - \Delta d}$ $\rightarrow$ non-perturbative renormalization may explain the lower values observed by LHPC



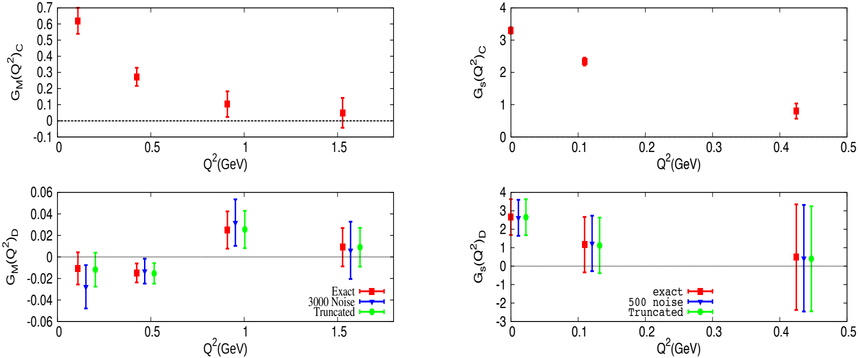


Disconnected contributions

- Approximate using stochastic techniques
- A study was carried out comparing stochastic methods to the exact evaluation enabled using GPUs; $N_f = 2$ Wilson fermions (SESAM Collaboration)



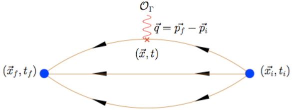
- Disconnected loops contributing to nucleon form factors show slow convergence
  The truncated solver method is best suited, G. Bali, S. Collins, A. Schafer Comput.Phys.Commun. 181 (2010) 1570
- Loops with a scalar inversion are much easier to compute



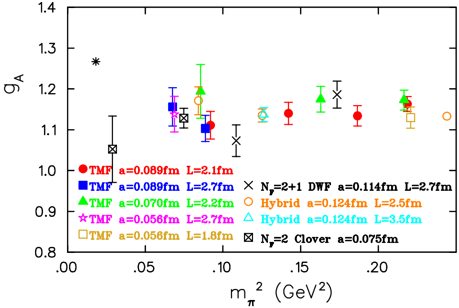
C.A., K. Hadjiyiannakou, G. Koutsou, A. 'O Cais, A. Strelchenko, arXiv:1108.2473

Nucleon form factors

- Many lattice studies down to lowest pion mass of $m_\pi \sim 300$ MeV
  $\Longrightarrow$ Lattice data in general agreement
- Axial-vector FFs: $A^a_\mu = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
  $\Longrightarrow \frac{1}{2}\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]$



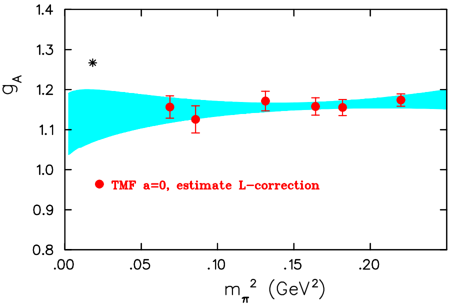
Axial charge is well known experimentally, straight forward to compute in lattice QCD



- Agreement among recent lattice results - all use non-perturbative Z_A
- Weak light quark mass dependence
- What can we say about the physical value of g_A?

Similar discrepancy also for the momentum fraction

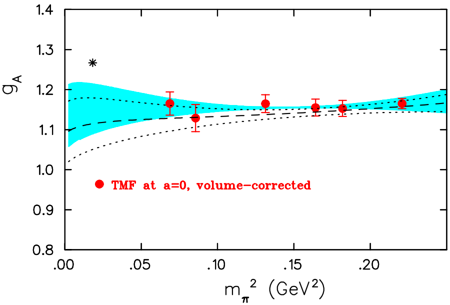
Physical results on g_A



- What can we say about the physical value of g_A?
- Use results obtained with twisted mass fermions C. A. *et al.* (ETMC), Phys. Rev. D83 (2011) 045010
- Take continuum limit and estimate volume corrections, A. Ali Khan, *et al.*, PRD **74**, 094508 (2006)
- Use one-loop chiral perturbation theory in the small scale expansion (SSE), T. R. Hemmert, M. Procura and W. Weise, PRD **68**, 075009 (2003).
- 3 fit parameters, $g_A^0 = 1.10(8)$, $g_{\Delta\Delta}$=2.1(1.3), $C^{SSE}(1\ GeV) = -0.7(1.7)$, axial N$\Delta$ coupling fixed to 1.5: $\Rightarrow g_A = 1.14(6)$
- Fitting lattice results directly leads to $g_A = 1.12(7)$

Lattice determination of the axial charges of other baryons can provide input for χPT, H.- W. Lin and K. Orginos, PRD 79, 034507 (2009); M. Gockeler *et l.*, arXiv:1102.3407
Volume and discretization errors small compared to the uncertainty in the chiral expansion

Physical results on g_A



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Nucleon form factors

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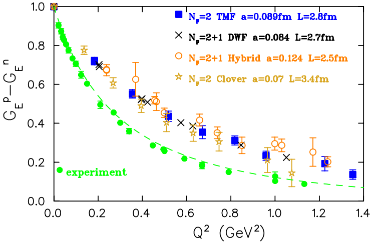
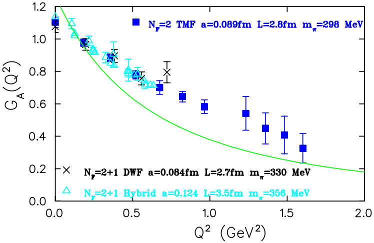
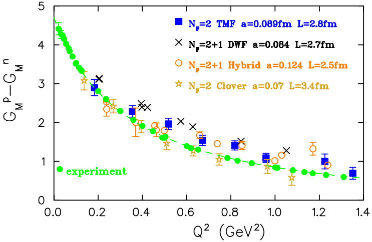
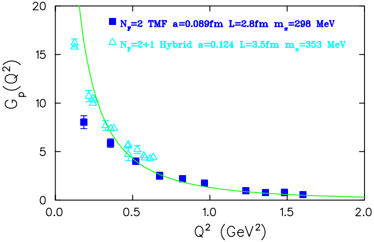








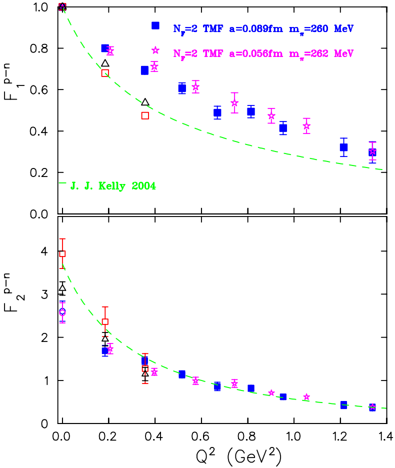
Results from ETMC (arXiv:0910.3309), LHPC using DWF (S. N. Syritsyn, PRD 81, 034507 (2010)) and a hybrid action (J. D. Bratt *et al.*, arXiv:1001:3620), and from CLS using Clover, (H. Wittig)

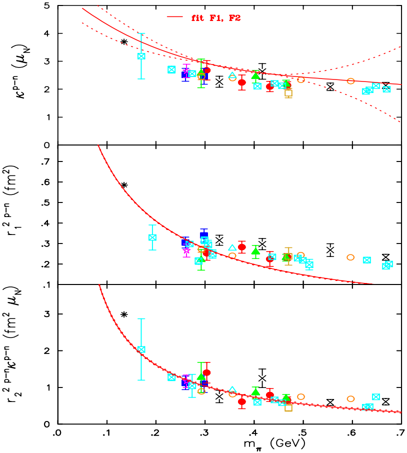
Can we get results at physical point?

Chiral extrapolation of electromagnetic form factors

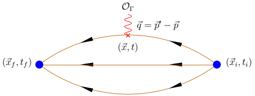
Baryon chiral perturbation theory to one-loop, with Δ d.o.f.(SSE) and iso-vector $N\Delta$ coupling included in LO, T. R. Hemmert and W. Weise, Eur. Phys. J. A **15**,487 (2002); M. Gockeler *et al.*, PRD **71**, 034508 (2005).

Fit $F_1(m_\pi, Q^2)$ and $F_2(m_\pi, Q^2)$ with 5 parameters: (κ_v), the isovector (c_v) and axial N to Δ (c_A) couplings and two counterterms

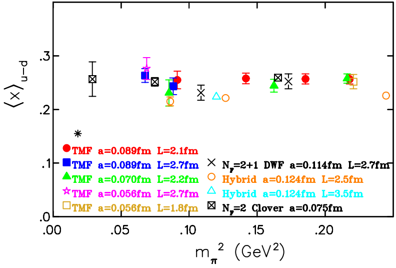


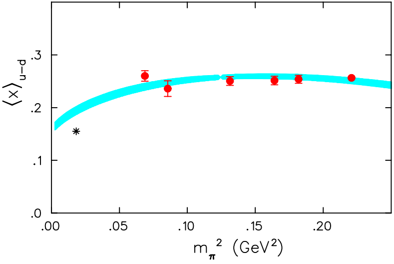


Nucleon momentum fraction



Momentum fraction $\langle x \rangle_{u-d} = A_{20}^{\text{isovector}}$ is calculated similarly to g_A





Physical point: $\langle x \rangle_{u-d}$ from S. Alekhin *et al.* arXiv:0908.2766

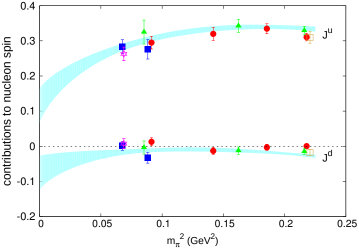
HBχPT for $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u-\Delta d}$, D. Arndt, M. Savage, NPA 697, 429 (2002); W. Detmold, W Melnitchouk, A. Thomas, PRD 66, 054501 (2002)

Fit ETMC results with $\lambda^2 = 1$ GeV^2

$$\langle x \rangle_{u-d} = C\left[1 - \frac{3g_A^2+1}{(4\pi f_\pi)^2} m_\pi^2 \ln\frac{m_\pi^2}{\lambda^2}\right] + \frac{c_8(\lambda^2)m_\pi^2}{(4\pi f_\pi)^2} \qquad \langle x \rangle_{\Delta u-\Delta d} = \tilde{C}\left[1 - \frac{2g_A^2+1}{(4\pi f_\pi)^2} m_\pi^2 \ln\frac{m_\pi^2}{\lambda^2}\right] + \frac{\tilde{c}_8(\lambda^2)m_\pi^2}{(4\pi f_\pi)^2}$$

Nucleon spin

Results using $N_F = 2$ TMF for 270 MeV $< m_\pi <$ 500 MeV, C. Alexandrou *et al.* (ETMC), arXiv:1104.1600
In qualitative agreement with J. D. Bratt *et al.* (LHPC), PRD82 (2010) 094502



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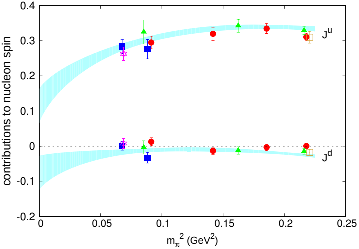
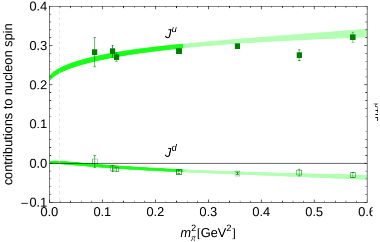
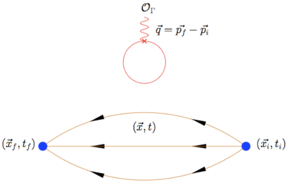
In qualitative agreement with J. D. Bratt *et al.* (LHPC), PRD82 (2010) 094502





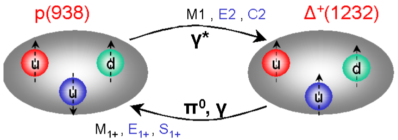
$\Longrightarrow$ Total spin for u-quarks $J^u \sim 0.25$ and for d-quark $J^d \sim 0$

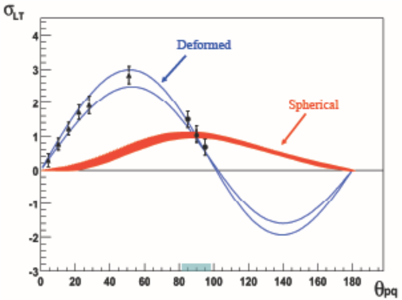


Disconnected contributions neglected

$N\gamma^* \to \Delta$ form factors

- A dominant magnetic dipole, M1
- An electric quadrupole, E2 and a Coulomb, C2 signal a deformation in the nucleon/Δ
- 1/2-spin particles have vanishing quadrupole moment in the lab-frame
- Probe nucleon shape by studying transitions to its excited Δ-state
- Difficult to measure/calculate since quadrupole amplitudes are sub-dominant





- $R_{EM}(\mathrm{EMR}) = -\dfrac{G_{E2}(Q^2)}{G_{M1}(Q^2)}$,

  $R_{SM}(\mathrm{CMR}) = -\dfrac{|\vec{q}|}{2m_\Delta}\dfrac{G_{C2}(Q^2)}{G_{M1}(Q^2)}$,

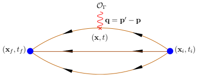
  in lab frame of the Δ.
- Precise data strongly “suggesting” deformation in the Nucleon/Δ
  At $Q^2 = 0.126$ GeV^2:
  EMR=$(-2.00 \pm 0.40_{\mathrm{stat+sys}} \pm 0.27_{\mathrm{mod}})$%,
  CMR=$(-6.27 \pm 0.32_{\mathrm{stat+sys}} \pm 0.10_{\mathrm{mod}})$%

C. N. Papanicolas, Eur. Phys. J. A18 (2003); N. Sparveris *et al.*, PRL **94**, 022003 (2005)

Lattice evaluation

$$\langle \Delta(p',s')|j_\mu|N(p,s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta\; m_N}{E_\Delta(\mathbf{p}')\; E_N(\mathbf{p})}\right)^{1/2}\bar{u}_\sigma(p',s')\left[G^*_{M1}(q^2)K^{M1}_{\sigma\mu} + G^*_{E2}(q^2)K^{E2}_{\sigma\mu} + G^*_{C2}K^{C2}_{\sigma\mu}\right]u(p,s)$$

- Evaluation of two-point and three-point functions



$$G(\vec{q},t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\,\Gamma^4_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\bar{J}_\beta(0)\rangle$$

$$G^{\mu\nu}(\Gamma,\vec{q},t) = \sum_{\vec{x}_f,\vec{x}} e^{i\vec{x}\cdot\vec{q}}\,\Gamma_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\mathcal{O}^\mu(\vec{x},t)\bar{J}_\beta(0)\rangle$$

$$R^J_\sigma(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma_\tau;\mu) = \frac{\langle G^{\Delta J_\mu N}_\sigma(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma_\tau)\rangle}{\langle G^{\Delta\Delta}_{ii}(t_2,\mathbf{p}';\Gamma_4)\rangle}\left[\frac{\langle G^{\Delta\Delta}_{ii}(t_2,\mathbf{p}';\Gamma_4)\rangle}{\langle G^{NN}(t_2,\mathbf{p};\Gamma_4)\rangle}\;\frac{\langle G^{NN}(t_2-t_1,\mathbf{p};\Gamma_4)\rangle\;\langle G^{\Delta\Delta}_{ii}(t_1,\mathbf{p}';\Gamma_4)\rangle}{\langle G^{\Delta\Delta}_{ii}(t_2-t_1,\mathbf{p}';\Gamma_4)\rangle\;\langle G^{NN}(t_1,\mathbf{p};\Gamma_4)\rangle}\right]^{1/2}$$

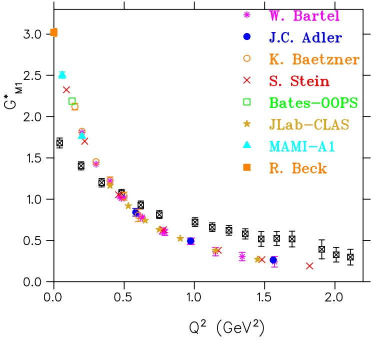
- Construct optimized sources to isolate quadrupoles $\rightarrow$ three-sequential inversions needed

$$S^J_1(\mathbf{q};J) = \sum_{\sigma=1}^{3}\Pi^J_\sigma(\mathbf{0},-\mathbf{q};\Gamma_4;J)\quad,\qquad S^J_2(\mathbf{q};J) = \sum_{\sigma\neq k=1}^{3}\Pi^J_\sigma(\mathbf{0},-\mathbf{q};\Gamma_k;J)$$

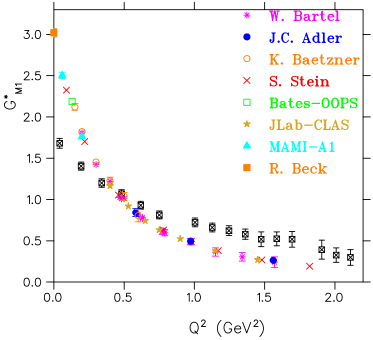
$$S^J_3(\mathbf{q};J) = \Pi^J_3(\mathbf{0},-\mathbf{q};\Gamma_3;J) - \frac{1}{2}\left[\Pi^J_1(\mathbf{0},-\mathbf{q};\Gamma_1;J) + \Pi^J_2(\mathbf{0},-\mathbf{q};\Gamma_2;J)\right]$$

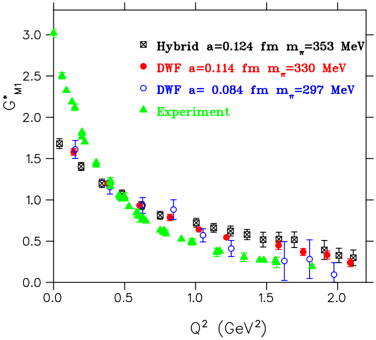
- Use the coherent sink technique: create four sets of forward propagators for each configuration at source positions separated in time by one-quarter of the total temporal size, Syritsyn et al. (LHPC), Phys. Rev. D81 (2009) 034507.

Results on magnetic dipole



Results on magnetic dipole





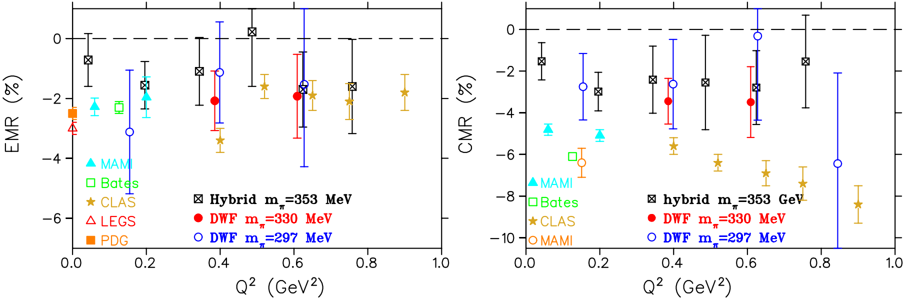
Slope smaller than experiment, underestimate G^*_{M1} at low Q^2 $\rightarrow$ pion cloud effects?

New results using $N_f = 2 + 1$ dynamical Domain Wall Fermions, simulated by RBC-UKQCD Collaborations $\Longrightarrow$ No visible improvement.
C. A., G.Koutsou, J.W. Negele, Y. Proestos, A. Tsapalis, Phys. Rev. D83 (2011)

Situation like for nucleon form factors, independent of lattice discretization

Results on EMR and CMR

Systematic errors may cancel in rations: G_{E2} and G_{C2} are suppressed at low Q^2 like G^*_{M1}
$\Longrightarrow$ look at EMR and CMR



New results using $N_f = 2 + 1$ dynamical domain wall fermions by RBC-UKQCD Collaborations
Need large statistics to reduce the errors $\Longrightarrow$ as $m_\pi \rightarrow 140$ MeV $\mathcal{O}(10^3)$ need to be analyzed.

N to Δ axial-vector and pseudoscalar form factors

$$\langle \Delta(p', s')|A^3_\mu|N(p,s)\rangle = \mathcal{A}\bar{u}^\lambda(p',s')\left[\left(\frac{C^A_3(q^2)}{m_N}\gamma^\nu + \frac{C^A_4(q^2)}{m_N^2}p'^\nu\right)\left(g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu}\right)q^\rho + C^A_5(q^2)g_{\lambda\mu} + \frac{C^A_6(q^2)}{m_N^2}q_\lambda q_\mu\right]u(p,s)$$

$$\mathcal{A} = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta m_N}{E_\Delta(\mathbf{p}')E_N(\mathbf{p})}\right)^{1/2}$$

- $C^A_5(q^2)$ analogous to the nucleon $G_A(q^2)$
- $C^A_6(q^2)$, analogous to the nucleon $G_p(q^2)$ $\longrightarrow$ pion pole behaviour
- $C^A_3(q^2)$ and $C^A_4(q^2)$ are suppressed (transverse part of the axial-vector)

- Study also the pseudo-scalar transition form factor $G_{\pi N\Delta}(q^2)$
$\Longrightarrow$ Non-diagonal Goldberger-Treiman relation:

$$C^A_5(q^2) + \frac{q^2}{m_N^2}C^A_6(q^2) = \frac{1}{2m_N}\frac{G_{\pi N\Delta}(q^2)f_\pi m_\pi^2}{m_\pi^2 - q^2} \quad .$$

Pion pole dominance relates C^A_6 to $G_{\pi N\Delta}$ through:

$$\frac{1}{m_N}C^A_6(q^2) \sim \frac{1}{2}\frac{G_{\pi N\Delta}(q^2)f_\pi}{m_\pi^2 - q^2}$$

Goldberger-Treiman relation becomes

$$G_{\pi N\Delta}(q^2)\, f_\pi = 2m_N C^A_5(q^2)$$

N to Δ axial-vector and pseudoscalar form factors

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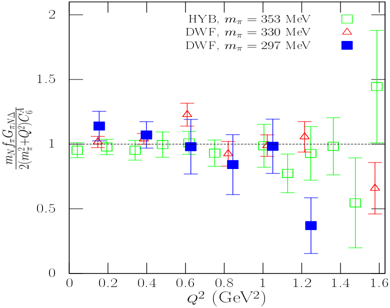
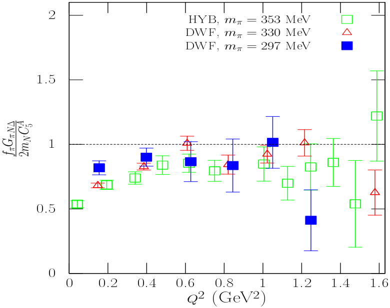
N to Δ Goldberger-Treiman relation





Pion-pole dominance: $\frac{1}{m_N} C_6^A(Q^2) \sim \frac{1}{2} \frac{G_{\pi N\Delta}(Q^2) f_\pi}{m_\pi^2 + Q^2}$ Goldberger-Treiman rel.: $G_{\pi N\Delta}(Q^2) f_\pi = 2 m_N C_5^A(Q^2)$

Similar behavior as in the nucleon system, i.e. between $G_{\pi NN}$ and G_p, and $G_{\pi NN}$ and G_A.

Ratio: $G_{\pi N\Delta}/G_{\pi NN} \sim 1.6$

Δ electromagnetic form factors

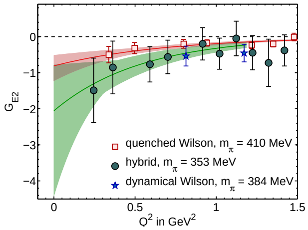
$$\langle \Delta(p',s')|j^\mu(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\left\{\left[F_1^*(Q^2)g^{\alpha\beta} + F_3^*(Q^2)\frac{q^\alpha q^\beta}{(2M_\Delta)^2}\right]\gamma^\mu + \left[F_2^*(Q^2)g^{\alpha\beta} + F_4^*(Q^2)\frac{q^\alpha q^\beta}{(2M_\Delta)^2}\right]\frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}\right\}u_\beta(p,s)$$

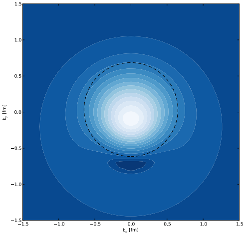
with e.g. the quadrupole form factor given by: $G_{E2} = \left(F_1^* - \tau F_2^*\right) - \frac{1}{2}(1+\tau)\left(F_3^* - \tau F_4^*\right)$, where $\tau \equiv Q^2/(4M_\Delta^2)$

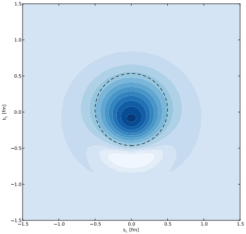
Construct an optimized source to isolate G_{E2} $\rightarrow$ additional sequential propagators needed.
Neglect disconnected contributions in this evaluation.

Transverse charge density of a Δ polarized along the x-axis can be defined in the infinite momentum frame $\rightarrow$ $\rho^\Delta_{T\,\frac{3}{2}}(\vec{b})$ and $\rho^\Delta_{T\,\frac{1}{2}}(\vec{b})$.

Using G_{E2} we can predict 'shape' of Δ.







Δ with spin 3/2 projection elongated along spin axis compared to the Ω^-

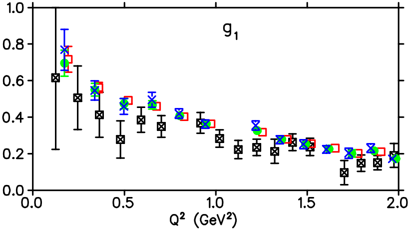
C. A., T. Korzec, G. Koutsou, C. Lorcé, J. W. Negele, V. Pascalutsa, A. Tsapalis, M. Vanderhaeghen, NPA825 ,115 (2009).

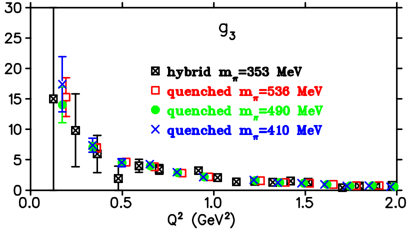
Δ axial-vector form factors

Axial-vector current: $A^a_\mu(x) = \overline{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$

$$\langle \Delta(p', s')|A^3_\mu(0)|\Delta(p, s)\rangle = -\bar{u}_\alpha(p', s')\frac{1}{2}\left[-g^{\alpha\beta}\left(g_1(q^2)\gamma^\mu\gamma^5 + g_3(q^2)\frac{q^\mu}{2M_\Delta}\gamma^5\right) + \frac{q^\alpha q^\beta}{4M^2_\Delta}\left(h_1(q^2)\gamma^\mu\gamma^5 + h_3(q^2)\frac{q^\mu}{2M_\Delta}\gamma^5\right)\right]u_\beta(p, s)$$

i.e. 4 axial form-factors, g_1, g_3, h_1 and h_3 $\longrightarrow$ at $q^2 = 0$ we can extract the Δ axial charge





C. A., E. Gregory, T. Korzec, G. Koutsou, J. W. Negele, T. Sato, A. Tsapalis, arXiv:1011.0411

Δ pseudoscalar couplings

Pseudoscalar current: $P^a(x) = \overline{\psi}(x)\gamma_5 \frac{\tau^a}{2}\psi(x)$

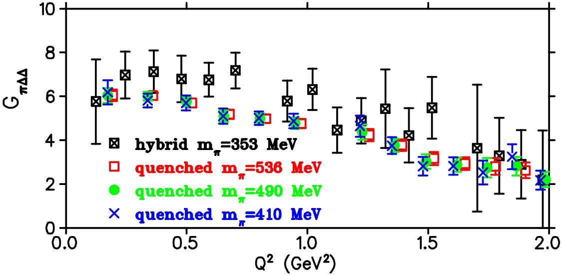
- $\Delta - \Delta$ matrix element:

$$\langle \Delta(p',s')|P^3(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\frac{1}{2}\left[-g^{\alpha\beta}\tilde{g}(q^2)\gamma^5 + \frac{q^\alpha q^\beta}{4M_\Delta^2}\tilde{h}(q^2)\gamma^5\right]u_\beta(p,s)$$

  i.e. two $\pi\Delta\Delta$ couplings $\Longrightarrow$ two Goldberger-Treiman relations.

- $G_{\pi\Delta\Delta}$ is given by: $m_q\tilde{g}(Q^2) \equiv \frac{f_\pi m_\pi^2 G_{\pi\Delta\Delta}(Q^2)}{(m_\pi^2+Q^2)}$ and $H_{\pi\Delta\Delta}$ is given by: $m_q\tilde{h}(Q^2) \equiv \frac{f_\pi m_\pi^2 H_{\pi\Delta\Delta}(Q^2)}{(m_\pi^2+Q^2)}$

- Goldberger-Treiman relations: $f_\pi G_{\pi\Delta\Delta}(Q^2) = m_\Delta g_1(Q^2)$, $\quad f_\pi H_{\pi\Delta\Delta}(Q^2) = m_\Delta h_1(Q^2)$

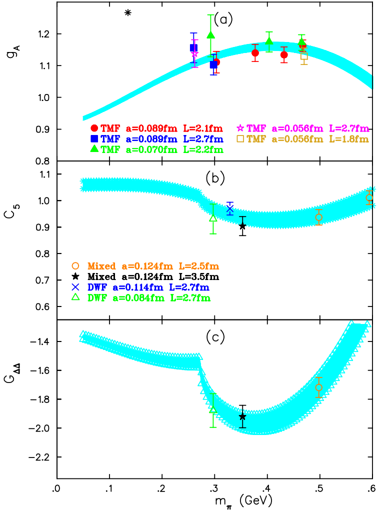


C. A., E. Gregory, T. Korzec, G. Koutsou, J. W. Negele, T. Sato, A. Tsapalis, arXiv:1011.0411

Global chiral fit to the axial couplings

Use heavy baryon χPT to describe the pion mass dependence of the nucleon and Δ axial charge g_A and g_Δ as well as the N to Δ axial transition C_5^A.

T. R. Hemmert, M. Procura, W. Weise, PRD68, 075009 (2003); F. J. Jiang and B. C. Tiburzi, PRD 78, 017504 (2008); M. Procura, PRD 78, 094021 (2008).



Conclusions

- Large scale simulations using the underlying theory of the Strong Interactions have made spectacular progress
  $\Longrightarrow$ we now have simulations of the full theory at near physical parameters
- The low-lying hadron spectrum is reproduced
- Nucleon form factors are being computed by a number of collaborations aiming at reproducing the experimental values
- N to Δ transition form factors can be extracted in a similar way to the nucleon ones
  $\rightarrow$ EMR and CMR allow comparison to experiment
- Δ form factors are predicted
- Resonance width can be computed within Euclidean Lattice QCD as illustrated for the ρ-meson $\rightarrow$ similar techniques can be applied to Δ

$\Longrightarrow$ We expect many physical results using dynamical simulations at physical pion mass in the next few years.

Thank you for your attention