A lattice study of the strangeness content of the nucleon

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q and g → hadrons and nuclei

EMFCSC Erice, September 19, 2011

- Introduction
- How to compute Δq and $\langle N|\bar{q}q|N\rangle$
- Renormalization and results on $\sigma_{\pi N}$ and f_{T_s}
- Renormalization and results on Δu, Δd and Δs
- Summary

Based on LAT09-10: arXiv:0911.2407, arXiv:1011.2194

LAT11 proceedings and article in preparation.

Presentation of final results may differ.

Contribution of the quark spin to the spin of the nucleon.

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

Along with the quark angular momentum, L_q, the gluon spin ΔG and angular momentum L_g:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

Extraction of $\Delta\Sigma$ from experiment:

- measure the spin structure function $g_1(x, Q^2)$ for neutron, proton.
- extract $\Delta q(x, Q^2)$ from g_1's and integrate over x:

$$\Delta q(Q^2) = \int_0^1 dx\, \Delta q(x, Q^2)\,.$$

- Assumptions required to extrapolate data to $x < 10^{-3}$, e.g. fixing F/D from assuming $\mathrm{SU(3)_F}$ symmetry!

Hermes 2006 (COMPASS very similar): (at $Q^2 = 5\,\mathrm{GeV}^2$)

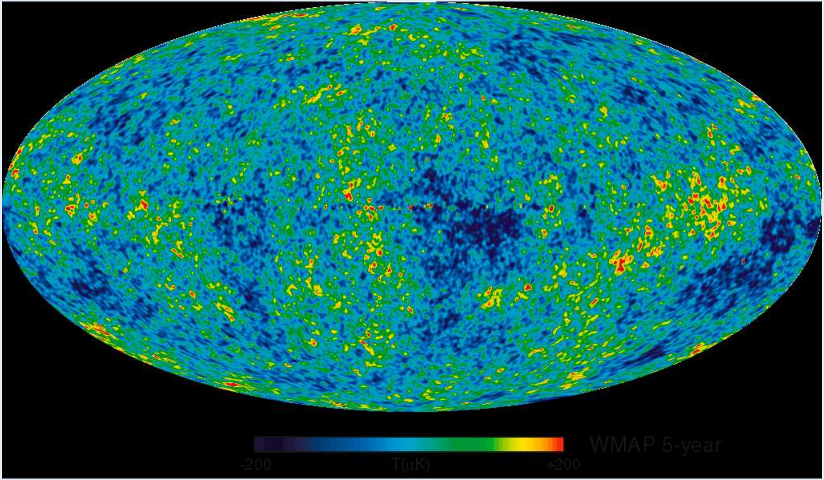
$$
\begin{aligned}
\Delta u &= 0.842(4)(8)(9) \\
\Delta d &= -0.427(4)(8)(9) \\
\Delta s &= -0.085(13)(8)(9)
\end{aligned}
$$

Errors are (theoretical)(experimental)(evolution).
However,

- data only for $x > 0.004$,
- restrict integration to range where $\exists$ data and new data $\Rightarrow \Delta s = 0.037(19)(27)$ at $Q^2 = 2.5\,\mathrm{GeV}^2$ (HERMES 2008).
- New COMPASS data at smaller x indeed indicate a less negative Δs.

$\longrightarrow$ F Bradamante, W Vogelsang, E-M Kabuß (Saturday), M Burkardt (Sunday), L Glozman (Tuesday), S Ohta (Thursday)

Rotational curves of galaxies, gravitational lensing, flat universe



$\Longrightarrow \exists$ dark matter R Young (Saturday), C Alexandrou

The (scheme- and scale-independent) scalar matrix elements $m_q\langle N|\bar{q}q|N\rangle$ determine the coupling strength of the nucleon to the Higgs at zero recoil:

$$\frac{f_N}{m_N} = \sum_{q\in\{u,d,s\}} f_{T_q}\frac{\alpha_q}{m_q} + \frac{2}{9(n_F-3)} f_{T_G} \sum_{q\in\{c,b,t,\ldots\}} \frac{\alpha_q}{m_q}\,,$$

where the couplings $\alpha_q \propto m_q/m_W$ and cross section $\propto |f_N|^2$.

$$f_{T_q} = \frac{m_q\langle N|\bar{q}q|N\rangle}{m_N}$$

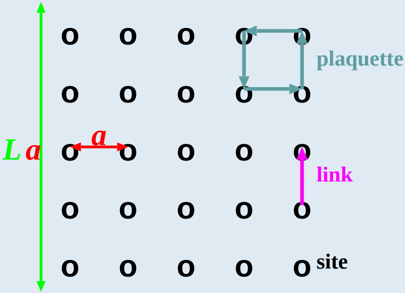
are the contributions of the light quark masses to the proton mass and,

$$f_{T_G} \approx 1 - \sum_{q\in\{u,d,s\}} f_{T_q}\,.$$

How large is $f_{T_u} + f_{T_d} = \sigma_{\pi N}/m_N$? How much do we know about f_{T_s} ???

Little is known experimentally but $\exists$ assumptions.

Lattice QCD



typical values:

$a^{-1} = 1.5–4$ GeV, $La = 1.5–6$ fm

continuum limit: $a \to 0$, La fixed

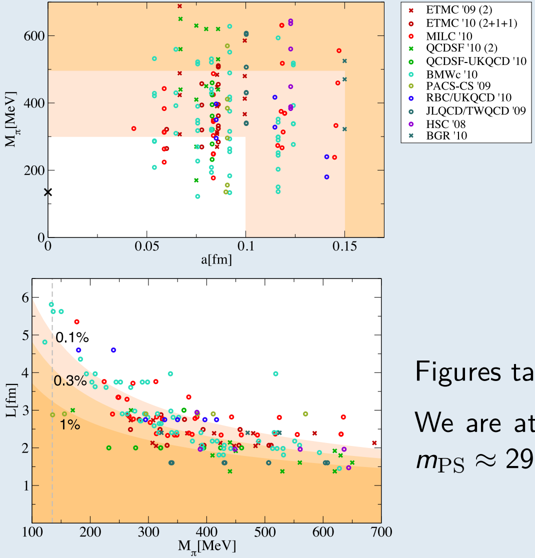
infinite volume: $La \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\,[d\psi][d\bar{\psi}]\; O[U] e^{-S[U,\psi,\bar{\psi}]}$$

"Measurement": average over a *representative* ensemble of gluon configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

Landscape of current lattice simulations



Figures taken from C Hölbling 11

We are at $a \approx 0.072$ fm, $La \approx 2.9$ fm, $m_{\rm PS} \approx 290$ MeV.

What is new?

Inclusion of flavour singlet contributions

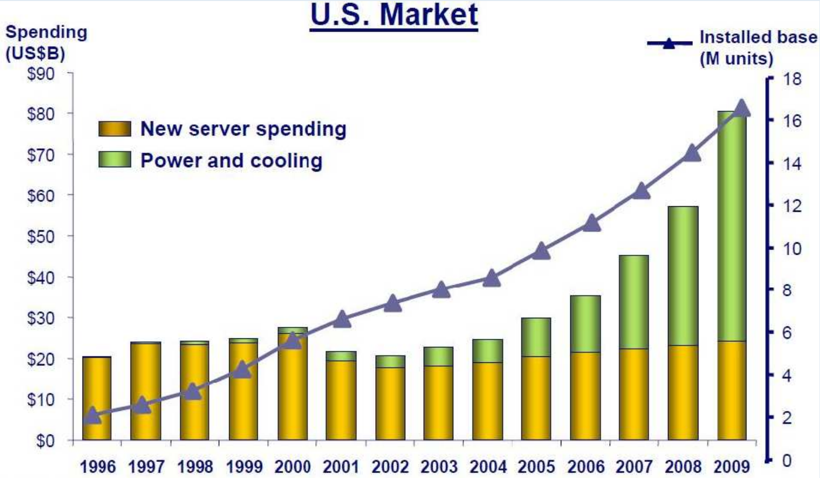
Full renormalization of the results

Improved creation operators, with good ground state overlap

Improved algorithms

New computer

The power wall



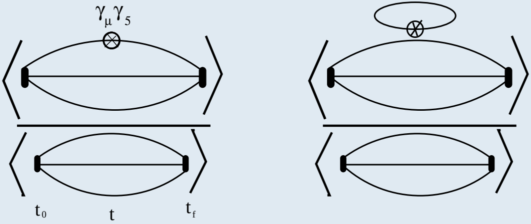
QPACE (QCD Parallel Computing on the Cell BE)

based on Enhanced Cell/BE € 10/GFlops, 5 W/GFlops (sustained)

Spin content:

$$\langle N, s|\bar{q}\gamma_\mu\gamma_5 q|N, s\rangle = 2m_N s_\mu \frac{\Delta q}{2}$$

On the lattice extracted using



(a) connected (b) disconnected

For Δs $\exists$ only the disconnected contribution.

Extract matrix element from ratio (at zero momentum)

$$R^{con}(t,t_f) = \frac{\langle \Gamma^{\alpha\beta}_{\rm pol} C^{\beta\alpha}_{3pt}(t_0,t,t_f)\rangle}{\langle \Gamma^{\alpha\beta}_{\rm unpol} C^{\beta\alpha}_{2pt}(t_0,t_f)\rangle}$$

$$R^{dis}(t,t_f) = -\frac{\langle \Gamma^{\alpha\beta}_{\rm pol} C^{\beta\alpha}_{2pt}(t_0,t_f)\sum_{\mathbf{x}} \mathrm{Tr}(\gamma_j\gamma_5 M^{-1}(\mathbf{x},t;\mathbf{x},t))\rangle}{\langle \Gamma^{\alpha\beta}_{\rm unpol} C^{\beta\alpha}_{2pt}(t_0,t_f)\rangle}$$

- polarized in the j-direction.
- $\Gamma_{\rm pol} = i\gamma_j\gamma_5(1+\gamma_4)/2$, $\Gamma_{\rm unpol} = (1+\gamma_4)/2$.
- We smear C_{3pt} and C_{2pt} at source and sink.

For $t_f \gg t \gg t_0$:

$$R^{con}(t,t_f) + R^{dis}(t,t_f) \to 2\frac{\langle N,s|(\bar{q}\gamma_j\gamma_5 q)^{lat}|N,s\rangle}{2m_N} = \Delta q^{lat}$$

Renormalization

At finite a the Wilson action explicitly breaks chiral symmetry.
Scalar flavour singlets and non-singlets can renormalize differently.
We use $n_F = 2$ mass degenerate electrically neutral sea quarks.
Singlet mass from the vector Ward identity (VWI) quark mass:

$$\overline{m} = m^{\mathrm{sea}} = \frac{1}{2a}\left(\frac{1}{\kappa_{\mathrm{sea}}} - \frac{1}{\kappa_{\mathrm{c,sea}}}\right).$$

Renormalized quark mass,

$$m_q^{\mathrm{ren}}(\mu) = Z_m^s(\mu)\overline{m} + Z_m^{ns}(\mu)(m_q - \overline{m}).$$

$$m^{\mathrm{sea,ren}}(\mu) = Z_m^s(\mu)m^{\mathrm{sea}},$$

$$m^{\mathrm{sea,ren}}(\mu) - m^{\mathrm{val,ren}}(\mu) = Z_m^{ns}(\mu)(m^{\mathrm{sea}} - m^{\mathrm{val}}).$$

At $\kappa_{\rm val} = \kappa_{\rm c,val}$, $m^{\rm val,ren}$ vanishes and

$$\frac{Z_m^s}{Z_m^{ns}} = \left.\frac{m^{\rm sea} - m^{\rm val}}{m^{\rm sea}}\right|_{\kappa_{\rm val}=\kappa_{\rm c,val}}$$

$$= \frac{\kappa_{\rm sea}^{-1} - \kappa_{\rm c,val}^{-1}}{\kappa_{\rm sea}^{-1} - \kappa_{\rm c,sea}^{-1}} =: 1 + \alpha \,.$$

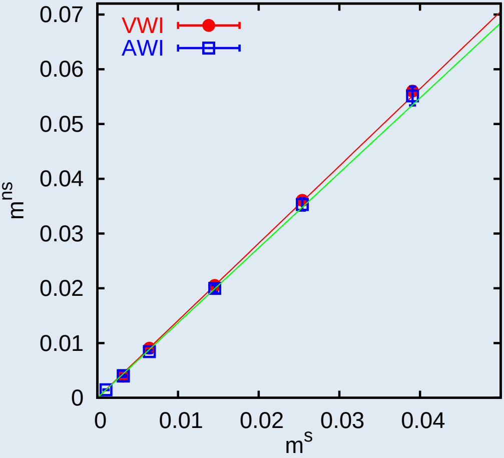
The non-singlet mass can also be obtained from the axial Ward identity (AWI),

$$\frac{Z_S^{ns}}{Z_P^{ns}} Z_A^{ns} m^{\rm AWI} = \frac{1}{2a}\left(\frac{1}{\kappa} - \frac{1}{\kappa_{\rm c,val}}\right) + O(a)\,,$$

where $Z_S Z_A / Z_P = 0.988(31)$ at $\beta = 5.29$ [M Göckeler et al 10]

We do not attempt $O(a)$ improvement. This would involve mixing with $a\langle N|{\rm Tr}\, F_{\mu\nu}F_{\mu\nu}|N\rangle$.

$$\alpha = \frac{Z_m^s}{Z_m^{ns}} - 1 = \text{slope} - 1 = \begin{cases} 0.411(13) & : \quad \text{VWI} \\ 0.369(22) & : \quad \text{AWI} \end{cases}$$



$$\begin{pmatrix} m_u(\mu) \\ m_d(\mu) \\ m_s(\mu) \end{pmatrix}^{\mathrm{ren}} = Z_m^{ns}(\mu, a) \begin{pmatrix} 1+\frac{\alpha(a)}{2} & \frac{\alpha(a)}{2} & 0 \\ \frac{\alpha(a)}{2} & 1+\frac{\alpha(a)}{2} & 0 \\ \frac{\alpha(a)}{2} & \frac{\alpha(a)}{2} & 1 \end{pmatrix} \begin{pmatrix} m_u(a) \\ m_d(a) \\ m_s(a) \end{pmatrix}^{\mathrm{lat}} .$$

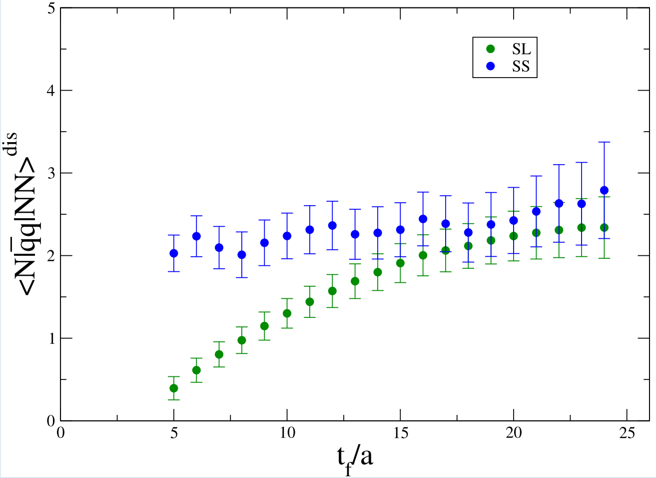
Now $m_q^{\mathrm{ren}}(\mu)\langle N|\bar{q}q|N\rangle^{\mathrm{ren}}(\mu)$ are renormalization group invariants, i.e. $\langle N|\bar{q}q|N\rangle^{\mathrm{lat}}(\mu)$ renormalize with the inverse of the above matrix:

$$\frac{m_u^{\mathrm{ren}}(\mu)+m_d^{\mathrm{ren}}(\mu)}{2}\langle N|\bar{u}u+\bar{d}d|N\rangle^{\mathrm{ren}}(\mu) = \frac{m_u^{\mathrm{lat}}+m_d^{\mathrm{lat}}}{2}\langle N|\bar{u}u+\bar{d}d|N\rangle^{\mathrm{lat}}$$

$$[m_s\langle N|\bar{s}s|N\rangle]^{\mathrm{ren}} = \left[m_s^{\mathrm{lat}} + \frac{\alpha}{2}\left(m_u^{\mathrm{lat}}+m_d^{\mathrm{lat}}\right)\right] \times \left(\langle N|\bar{s}s|N\rangle^{\mathrm{lat}} - \frac{\alpha}{2(1+\alpha)}\langle N|\bar{u}u+\bar{d}d|N\rangle^{\mathrm{lat}}\right)$$

f_{T_s} receives connected light quark contributions! C.f R Babich et al 10.

Varying the sink time t_f



$$V = 40^3 64,\ a^{-1} \approx 2.75\,\text{GeV}$$

Unrenormalized lattice results

$\kappa_{\rm sea} = 0.13632,\ m_{\rm PS} \approx 290\,\mathrm{MeV},\ a \approx 2.75\,\mathrm{GeV},\ Lam_{\rm PS} = 3.36, 4.20.$

$\kappa_{\rm val}$	$\kappa_{\rm cur}$	$m_{\rm PS,cur}$	V	$\langle N\|\bar{q}q\|N\rangle^{\rm lat}_{\rm dis}$	$\langle N\|\bar{q}q\|N\rangle^{\rm lat}_{\rm con}$
	0.13550	730 MeV	$32^3 64$	1.93(20)	
			$40^3 64$	2.17(25)	
0.13550	0.13609	460 MeV	$32^3 64$	2.18(22)	
			$40^3 64$	2.43(27)	
	0.13632	290 MeV	$32^3 64$	2.30(23)	
			$40^3 64$	2.55(29)	
	0.13550	730 MeV	$32^3 64$	2.18(22)	
			$40^3 64$	2.06(25)	
0.13609	0.13609	460 MeV	$32^3 64$	2.11(22)	
			$40^3 64$	2.28(26)	
	0.13632	290 MeV	$32^3 64$	2.16(23)	
			$40^3 64$	2.36(28)	
	0.13550	730 MeV	$32^3 64$	1.86(23)	
			$40^3 64$	1.93(27)	
0.13632	0.13609	460 MeV	$32^3 64$	1.93(24)	
			$40^3 64$	2.05(29)	
	0.13632	290 MeV	$32^3 64$	1.62(26)	8.43(73)
			$40^3 64$	1.86(31)	8.35(43)

The $\sigma_{\pi N}$ term

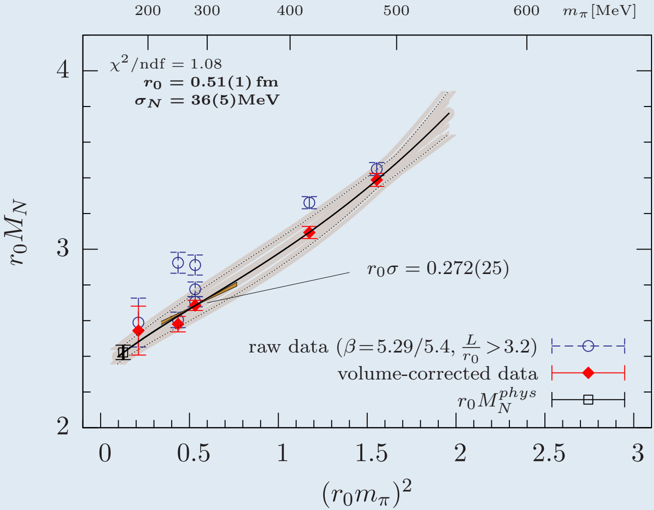
$$\sigma_{\mathrm{PS}\,N} = \begin{cases} 0.0376(39)a^{-1} = 0.263(26)(2)r_0^{-1} & (V = 32^3 64) \\ 0.0389(36)a^{-1} = 0.272(25)(2)r_0^{-1} & (V = 40^3 64) \end{cases},$$

where $r_0^{-1} = 386(20)$ MeV.
To LO χPT $dm_N/dm_{\mathrm{PS}}^2 = \mathrm{const}$ This means that $\sigma_{\mathrm{PS}\,N} \propto m_{\mathrm{PS}}^2$:

$$\sigma_{\mathrm{PS}\,N} = m_u \frac{\partial m_N}{\partial m_u} + m_d \frac{\partial m_N}{\partial m_d} \approx m_{\mathrm{PS}}^2 \frac{dm_N}{dm_{\mathrm{PS}}^2}.$$

So, naively, at the physical point, $\sigma_{\mathrm{PS}N} = 25(3)(1)$ MeV.
However ...

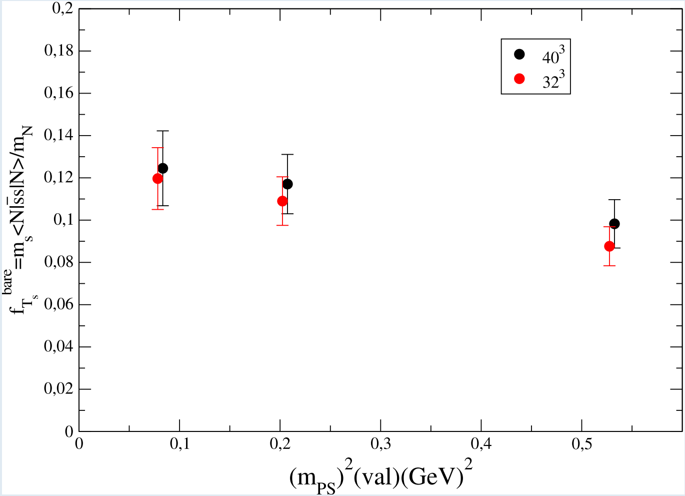
$O(p^4)$ χ perturbation theory fit. A Sternbeck



$$\sigma_{\pi N}^{\rm phys} = 36(5)\,\mathrm{MeV}$$

$$f_{T_u} + f_{T_d} = 0.038(5)$$

The strangeness contribution f_{T_s}



However $\cdots$

$$f_{T_s} = \frac{[m_s \langle N|\bar{s}s|N\rangle]^{\mathrm{ren}}}{m_N} = 0.013(15)(10)$$

This means that $\sigma_s = m_N f_{T_s} \approx (12 \pm 17)$ MeV and

$$y = \frac{2\langle N|\bar{s}s|N\rangle^{\mathrm{ren}}}{\langle N|\bar{u}u + \bar{d}d|N\rangle^{\mathrm{ren}}} = \begin{cases} 0.039(37)(29) & (L = 32) \\ 0.041(37)(29) & (L = 40) \end{cases},$$

χ extrapolation increases the denominator by 1.4 – 1.5.
The numerator will increase less (if at all).

$\Rightarrow y < 0.13$ with 95 % confidence.

Comparison with $n_F = 2 + 1$

Feynman-Hellmann method:

QCDSF: R Horsley et al LAT 11

$$\sigma_{\pi N} = 31(5)(??)\ \mathrm{MeV}\,,$$
$$\sigma_s = 91(34)(??)\ \mathrm{MeV}\,.$$

BMW-c: A Ramos et al LAT 11

$$\sigma_{\pi N} = 39(4)\left(\begin{array}{c}+18\\-7\end{array}\right)\ \mathrm{MeV}\,,$$
$$\sigma_s = 67(27)\left(\begin{array}{c}+55\\-47\end{array}\right)\ \mathrm{MeV}\,.$$

The direct method seems to be more efficient for σ_s.

It can also constrain the chiral extrapolation of $\sigma_{\pi N}$.

$n_F = 2+1$:

$$(\Delta u + \Delta d + \Delta s)^{\overline{MS}}(\mu) = Z_A^s(\mu, a)(\Delta u + \Delta d + \Delta s)^{\mathrm{lat}}(a)\,,$$
$$(\Delta u + \Delta d - 2\Delta s)^{\overline{MS}} = Z_A^{ns}(a)(\Delta u + \Delta d - 2\Delta s)^{\mathrm{lat}}(a)\,,$$
$$(\Delta u - \Delta d)^{\overline{MS}} = Z_A^{ns}(a)(\Delta u - \Delta d)^{\mathrm{lat}}(a)\,,$$

$$\begin{pmatrix} \Delta u(\mu) \\ \Delta d(\mu) \\ \Delta s(\mu) \end{pmatrix}^{\overline{MS}} = \begin{pmatrix} Z_A^{ns} + \frac{z(\mu)}{3} & \frac{z(\mu)}{3} & \frac{z(\mu)}{3} \\ \frac{z(\mu)}{3} & Z_A^{ns} + \frac{z(\mu)}{3} & \frac{z(\mu)}{3} \\ \frac{z(\mu)}{3} & \frac{z(\mu)}{3} & Z_A^{ns} + \frac{z(\mu)}{3} \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta d \\ \Delta s \end{pmatrix}^{\mathrm{lat}}\,,$$

where $z(\mu) = Z_A^s(\mu) - Z_A^{ns}$.

$n_F = 2$: ($\Delta u + \Delta d$ is now the singlet.)

$$\begin{pmatrix} \Delta u(\mu) \\ \Delta d(\mu) \\ \Delta s(\mu) \end{pmatrix}^{\overline{MS}} = \begin{pmatrix} Z_A^{ns} + \frac{z(\mu)}{2} & \frac{z(\mu)}{2} & 0 \\ \frac{z(\mu)}{2} & Z_A^{ns} + \frac{z(\mu)}{2} & 0 \\ \frac{z(\mu)}{2} & \frac{z(\mu)}{2} & Z_A^{ns} \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta d \\ \Delta s \end{pmatrix}^{\mathrm{lat}}\,.$$

$$Z_A^{ns} = 0.76485(64)(73)$$

has been determined non-perturbatively M Göckeler et al 10.
Due to the axial anomaly, Z_A^s acquires and anomalous dimension,

$$\gamma_A^s(\alpha_s) = -6C_F n_{\rm f} \left(\frac{\alpha_s}{4\pi}\right)^2 + \cdots .$$

A Skouroupathis, H Panagopoulos 09 have determined,

$$z(\mu) := Z_A^s(\mu) - Z_A^{ns} = C_F n_{\rm f} \left[15.8380(8) - 6\ln(a^2\mu^2)\right] \left(\frac{\alpha_s}{4\pi}\right)^2 + O(\alpha_s^3) ,$$

for the conversion to the $\overline{MS}$ scheme, where we use,

$$\alpha_s = \frac{g^2}{4\pi} = -\frac{3}{4\pi}\ln\langle U_\Box\rangle = 0.14278(5) .$$

$O(a)$ improvement:

$$Z_A^{ns} \mapsto Z_A^{ns}(1 + b_A am)\,,$$
$$z(\mu) \mapsto z(\mu)[1 + (b_A^s - b_A)am]\,,$$

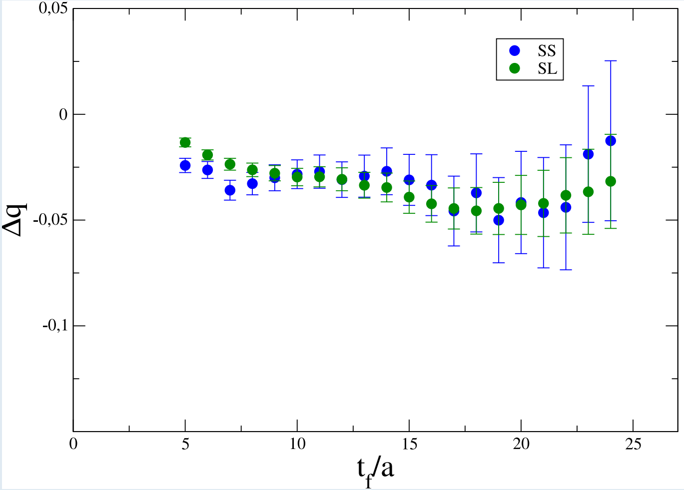
$$b_A = b_A^s + O(\alpha_s^2) \approx 1 + C_F \times 18.02539\frac{\alpha_s}{4\pi}\,.$$

The total correction reads,

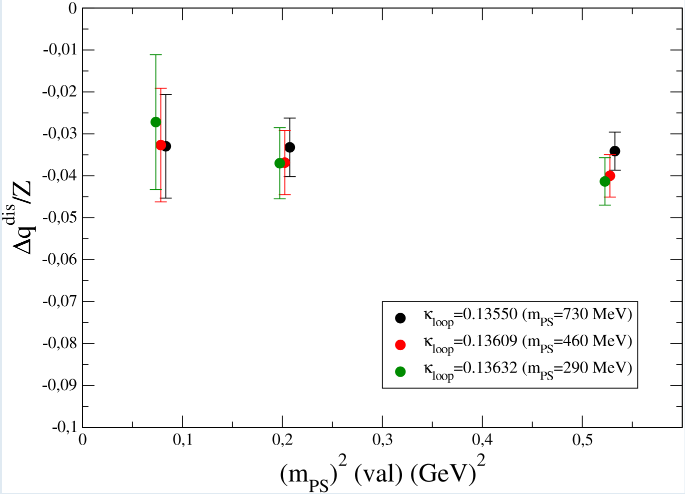
$$1 + b_A am = \begin{cases} 1.0324(3)(47) & (m_s\,, \kappa = 0.13550) \\ 1.0120(3)(17) & (\kappa = 0.13609) \\ 1.0041(3)(5) & (m_u = m_d\,, \kappa = 0.13632) \end{cases}\,.$$

We allow for a 50 % error on all perturbative results.
We obtain $z(\sqrt{7.5}\,\text{GeV}) = 0.0055(1)(27)$.
(Note that $z(\sqrt{10}\,\text{GeV}) = 0.0049(25)$: very slow running with the scale.)

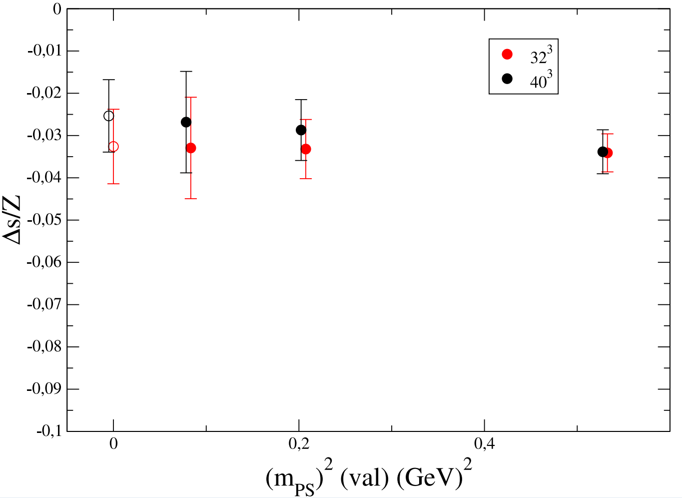
Varying t_f



Dependence on current and valence quark masses



Finite volume and valence quark mass effects



Bare lattice results ($\kappa_{\rm val} = \kappa_{\rm sea}$)

q	V	$\Delta q^{\rm lat}_{\rm con}$	$\Delta q^{\rm lat}_{\rm dis}$
u		1.065(22)	-0.027(16)
d	$32^3 64$	-0.344(14)	-0.027(16)
s		0	-0.033(12)
u		1.071(15)	-0.049(17)
d	$40^3 64$	-0.369(9)	-0.049(17)
s		0	-0.027(12)

Numbers have to be multiplied by ≈ 0.77.

The (renormalized) sea quark contributions are all small (-0.05 to -0.01).

Renormalized results

Everything is in the $\overline{MS}$ scheme at $\mu^2 = 7.5\,\mathrm{GeV}^2$.

q	Δq^{ren}, 32^3	Δq^{ren}, 40^3
u	0.799(21)(2)	0.787(18)(2)
d	-0.283(16)(2)	-0.319(15)(2)
s	-0.024(12)(2)	-0.020(10)(2)

Δu and Δd are reduced by ≈ 0.03 by the sea quark contributions.
Δs in increased by ≈ 0.002 from mixing with light quark flavours.

Note that $g_A = \Delta u - \Delta d \approx 1.1 < 1.27$.

Main systematics: $m_{\mathrm{PS}} \approx 2m_\pi^{\mathrm{phys}}$. No chiral extrapolation!

This would probably increase Δu and $|\Delta d|$.

Summary & Outlook

We have directly calculated quark line disconnected contributions to the nucleon structure for $n_F = 2$ improved Wilson sea quarks. Main results:

- $\sigma_{\pi N} = (36 \pm 5)$ MeV, complementing the indirect method.
- $\sigma_s = (12 \pm 17)$ MeV, however no chiral extrapolation.
- $\Delta s^{\overline{MS}}(\sqrt{7.5}\,\mathrm{GeV}) = -0.022(11)$.
- Both, the Δs and possibly the $\sigma_{\pi N}$ values indicate substantial $\mathrm{SU(3)_F}$ flavour symmetry violations.

Accurate calculations of isosinglet contributions to hadronic structure observables are possible and feasible. Work in progress:

- $n_F = 2 + 1$, lighter quark masses.
- Other disconnected contributions, e.g. to $\langle x \rangle$, δx, G_M, G_E.