# The $P\gamma$ transitions form factor in QCD

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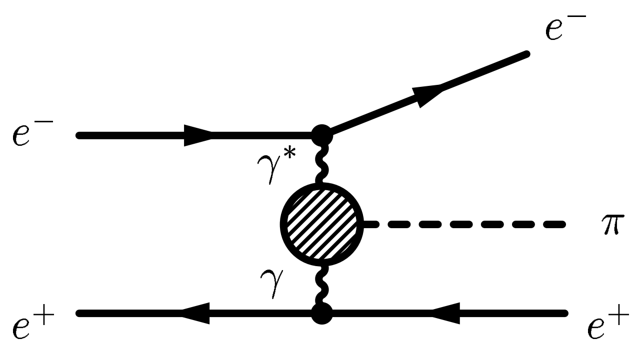
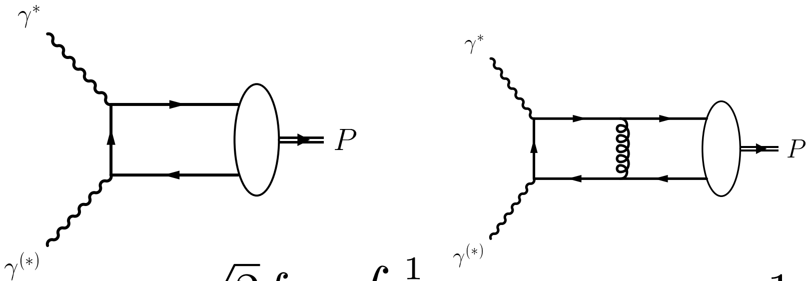
**Outline:**

- **The $\pi\gamma$ trans. form factor in coll. factorization**
- **The mod. pert. approach**
- **Generalization to $\eta, \eta', \eta_c$**
- **Summary**




based on arXiv: 1013.3542

# Theory: collinear factorization




at large $Q^2$

$$F_{\pi\gamma}(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 dx\, \Phi_\pi(x, \mu_F)\, \frac{1}{xQ^2} \left[1 + \frac{\alpha_s(\mu_R)}{\pi} \mathcal{K}(x, Q^2, \mu_R, \mu_F)\right]$$

$$\Phi_\pi(x, \mu_F) = 6x(1-x)\left[1 + \sum_{n=2,4,\ldots} a_n(\mu_0) \left(\frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)}\right)^{\gamma_n/\beta_0} C_n^{3/2}(2x-1)\right]$$

$f_\pi$ pion decay constant; $\mu_F, \mu_R, \mu_0$ factorization, renormalization, initial scale

$a_n$ embody soft physics convenient choice: $\mu_F = \mu_R = Q$ $\overline{MS}$ scheme

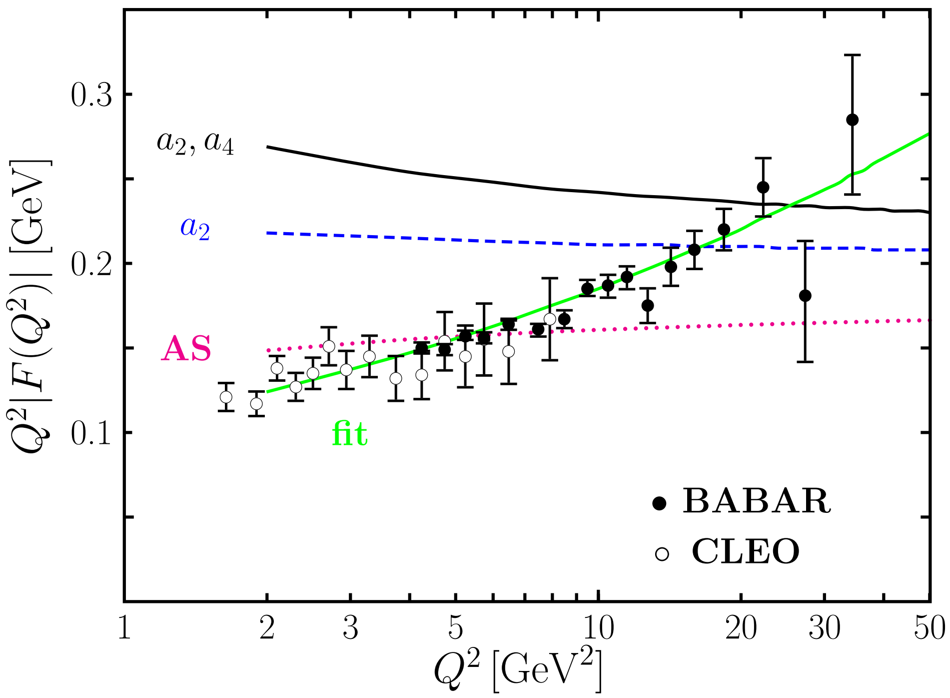
$\gamma_n$ anomalous dimensions (pos. fractional numbers, growing with $n$) LO evol.

LO: Brodsky-Lepage (80) NLO: del Aguila-Chase (81); Braaten (83)

**LO:** $Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi \left[1 + \sum a_n(\mu_F)\right]$

for $\ln Q^2 \to \infty$ $\Phi_\pi \to 6x(1-x) = \Phi_{\rm AS}$ $Q^2 F_{\pi\gamma} \to \sqrt{2} f_\pi$

# The situation




CLEO: close to NLO result evaluated from asymp. DA (up to about $10\%$)

BaBar(09) strong increase with $Q^2$

green: $\sqrt{2} f_\pi \left(Q^2/10\,\mathrm{GeV}\right)^{0.25}$
(to guide the eyes)

AS; $a_2(1\,\mathrm{GeV}) = 0.39$
$a_2(1\,\mathrm{GeV}) = 0.39, \;\; a_4 = 0.24$

we have to worry:

a substantial increase of FF is difficult to accomodate in fixed order pQCD corr. due to $a_n$ $(> 0)$ only shift NLO pred. upwards, don't change shape

# The modified perturbative approach

many attempts to explain $F_{\pi\gamma}$: Here - MPA

LO pQCD + quark transv. momenta + Sudakov suppr. Sterman et al (89,92)

$\Longrightarrow$ coll. fact. a. for $Q^2 \to \infty$ ($k_\perp$ fact. based on work by Collins-Soper)

Sudakov factor: higher order pQCD in NLL, resummed to all orders

$$S \propto \ln \frac{\ln(\xi Q/\sqrt{2}\Lambda_{QCD})}{\ln(1/b\Lambda_{QCD})} + \text{NLL} + \text{RG}(\mu_F, \mu_R)$$

$$(\xi = x, 1-x) \qquad \Longrightarrow e^{-S}$$

exponentiation in $b$ space

($q - \bar{q}$ separation)

with $e^{-S} = 0$ for $b > 1/\Lambda_{QCD}$

interface - soft gluons in wave fct

(semi-)hard gluons in SF and $T_H$

fact. scale $\mu_F = 1/b$, $b$ plays role of IR cut-off:

$$\hat{\Psi}_\pi(x, b, \mu_F) = 2\pi \frac{f_\pi}{\sqrt{6}} \Phi_\pi(x, \mu_F) \exp\left[-\frac{x(1-x)b^2}{4\sigma_\pi^2}\right]$$

$$F_{\pi\gamma} = \int_0^1 dx \int_0^{1/\Lambda_{QCD}} db^2 \; \hat{\Psi}_\pi \left[\frac{2}{\sqrt{3}\pi} K_0(\sqrt{x}Qb)\right] e^{-S}$$

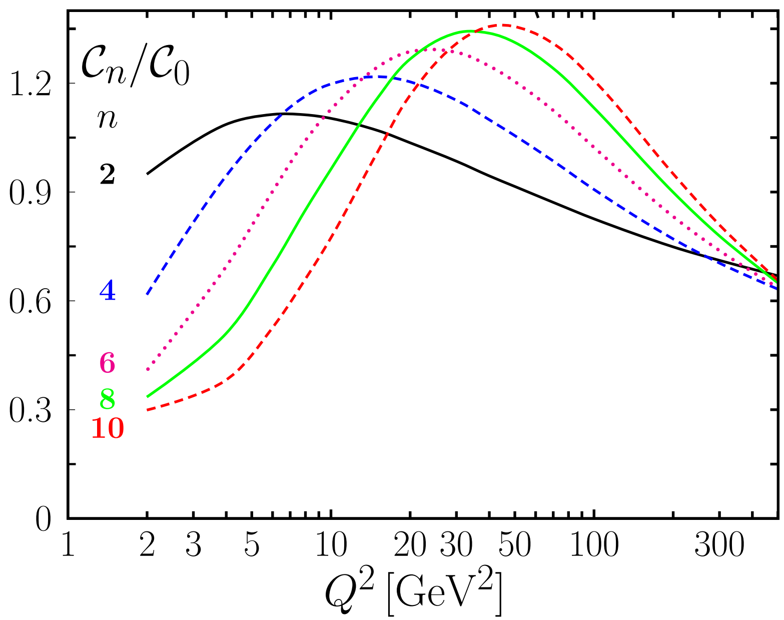
SF provides series of power suppressed terms accumulated at soft quark momenta ($x, 1-x \to 0$), grow with Gegenbauer index n (see also Agaev et al (11))

only lowest $a_n$ affect FF

# A remarkable property

with the Gegenbauer expansion

$$Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi \mathcal{C}_0(Q^2, \mu_0, \sigma_\pi) \Big[ 1 + \sum_{n=2,4,\cdots} a_n(\mu_0)\, \mathcal{C}_n/\mathcal{C}_0 \Big]$$




$Q^2 \to \infty$: $\mathcal{C}_0 \to 1$ and $\mathcal{C}_n \to 0$ due to evolution

low $Q^2$: strong suppr. of higher terms

increasing $Q^2$: higher $n$ terms become gradually more important

# Fit to BaBar data

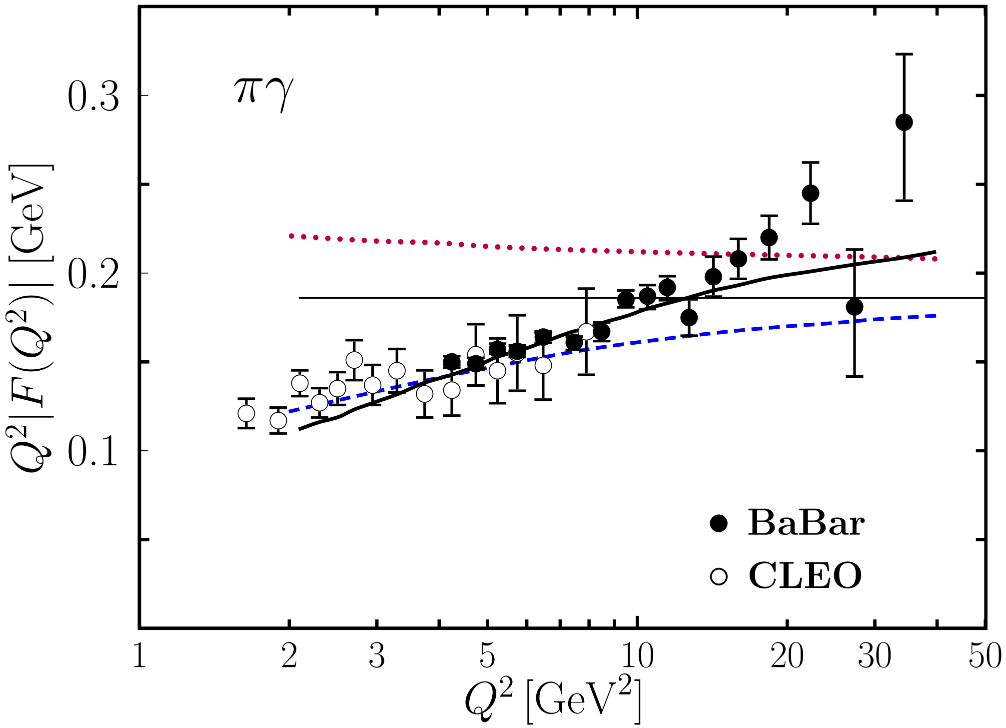
present data allow to fix only one Gegenbauer coefficient

fit to CLEO and Babar data (initial scale $\mu_0 = 1\,\mathrm{GeV}$):

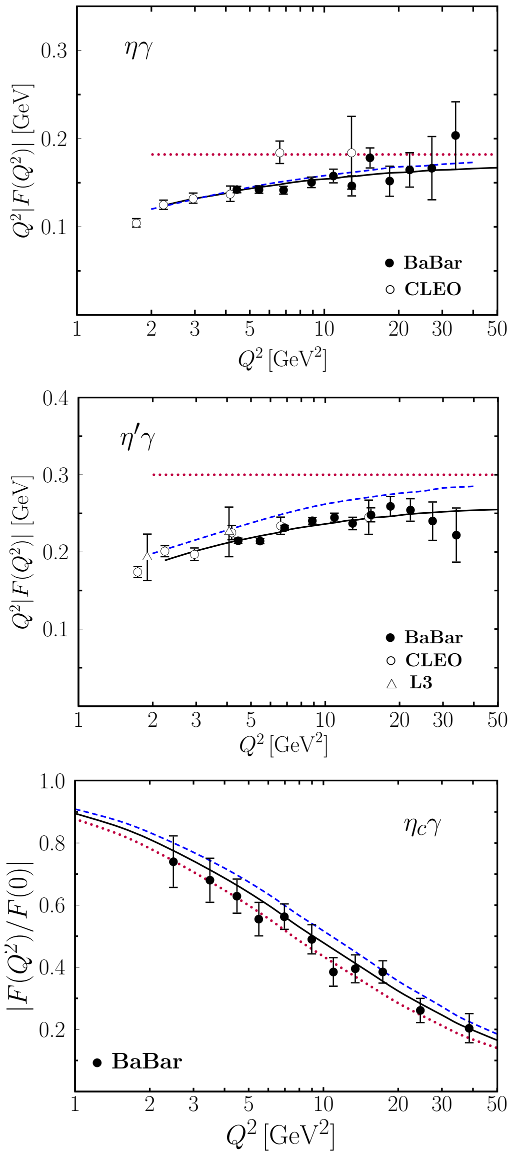
$a_2 = 0.25$ (fixed from lattice Braun(06))

$a_4 = 0.01 \pm 0.06$ $\sigma_\pi = 0.40 \pm 0.06\,\mathrm{GeV}^{-1}$ (trans. size parameter)

dashed line: $\Phi_{AS}$ K.-Raulfs(95); dotted line: coll. NLO result



# Generalization to $\eta, \eta', \eta_c$




$F_{P\gamma} = F^8_{P\gamma} + F^1_{P\gamma}$

dashed: $\Phi_{\rm AS}$ Feldmann-K.(97)

dotted: asymp. behavior

solid: at $\mu_0 = 2\,\mathrm{GeV}$

$\sigma_8 = 0.84 \pm 0.14\,\mathrm{GeV}^{-1};\ a_2^8 = -0.06 \pm 0.06$

$\sigma_1 = 0.74 \pm 0.05\,\mathrm{GeV}^{-1};\ a_2^1 = -0.07 \pm 0.04$

$\eta_c$ data BaBar(10) 2nd large scale

$$T_H = \frac{2\sqrt{6}\,e_c^2}{xQ^2 + (1 + 4x(1-x))m_c^2 + k_\perp^2}$$

$$\Phi_{\eta_c} = Nx(1-x)\exp\left[-\sigma_{\eta_c}^2 M_{\eta_c}^2 \frac{(x-1/2)^2}{x(1-x)}\right]$$

Wirbel-Stech-Bauer(85) ($\sigma_{\eta_c} = 0.44\,\mathrm{GeV}^{-1}$)

Sudakov unimportant

lines: $m_c = 1.35(1.49, 1.21)\,\mathrm{GeV}$

behavior predicted Feldmann-K(97)

# Summary

Surprising behavior of BaBar data:
strong increase of scaled $\pi\gamma$ FF but flat behavior of other FF
implies strong violation of flavor symmetry $(\pi^0 - \eta)$ at large $Q^2$

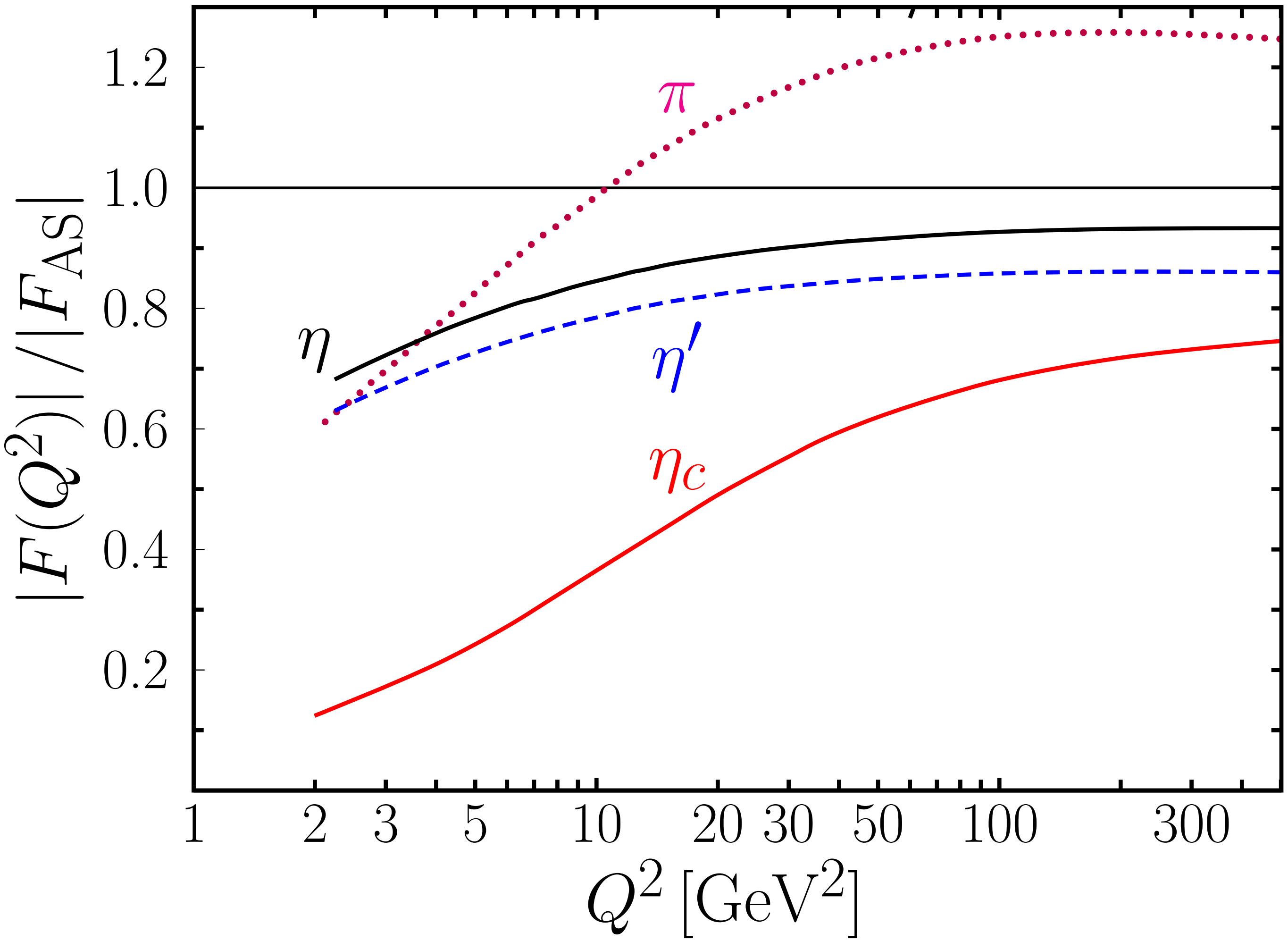
Theoretical description of $\pi\gamma$ FF requires strong power corrections
but mild power corrections for the others (or logarithmic corr.)

Quark-transverse momenta and Sudakov suppressions is one way to estimate power corrections; existing data on $P\gamma$ trans. form factor $(P = \pi, \eta, \eta', \eta_c)$ can be described well within that approach.
One Gegenbauer coeff. of each DA can be determined from data

BaBar data on $\pi\gamma$ FF cast severe doubts on any attempt to explain other excl. observables within coll. factorization frame work (e.g. pion or proton FF)

$\pi\gamma$ form factor should be remeasured by BELLE

# Comparison



1.2
1.0
0.8
0.6
0.4
0.2
$|F(Q^2)|\,/\,|F_{\rm AS}|$
$\pi$
$\eta$
$\eta'$
$\eta_c$
1
2
3
5
10
20
30
50
100
300
$Q^2\,[\mathrm{GeV}^2]$