The importance of pion and Extended Brueckner-Hartree-Fock theory

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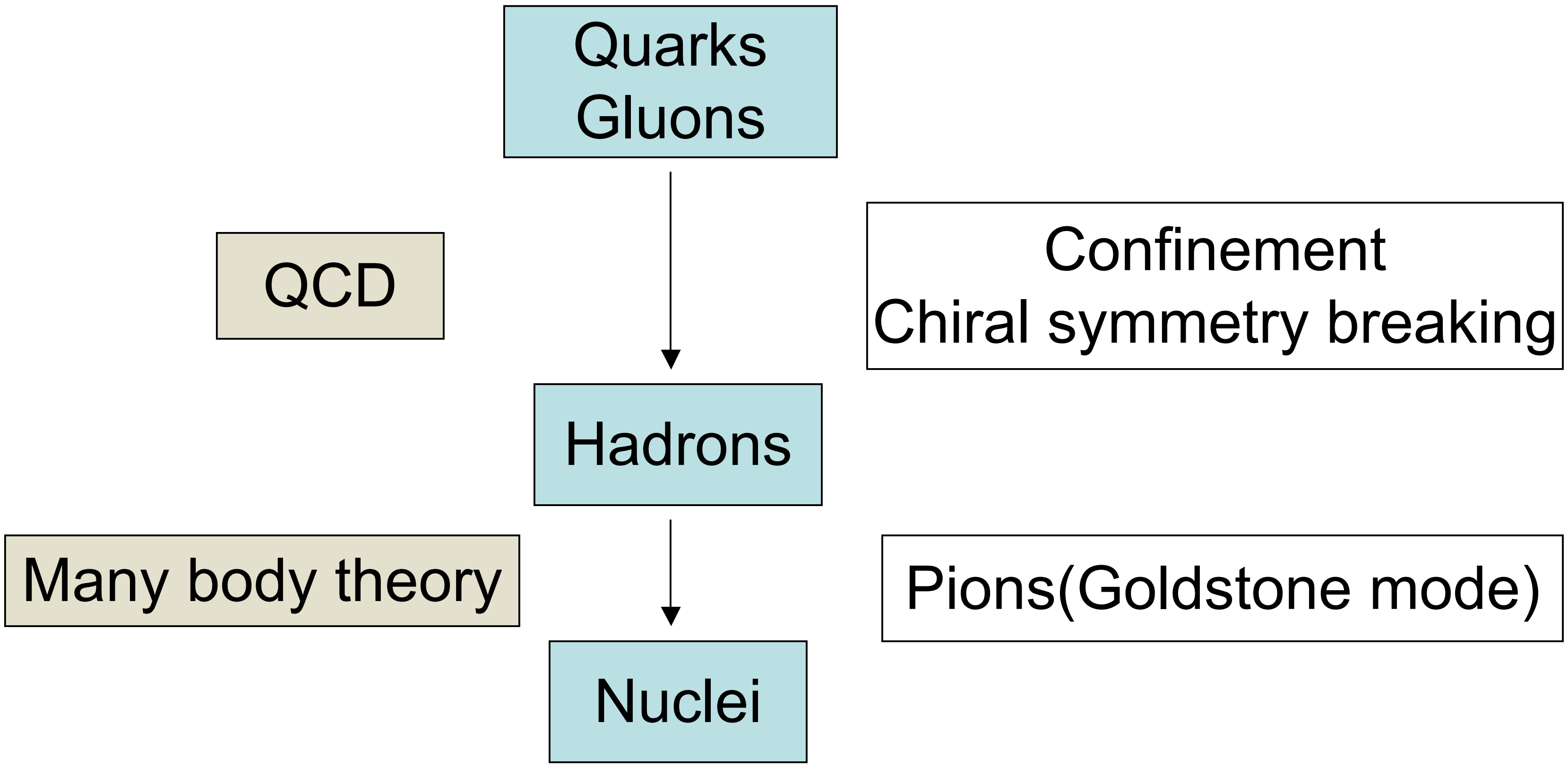
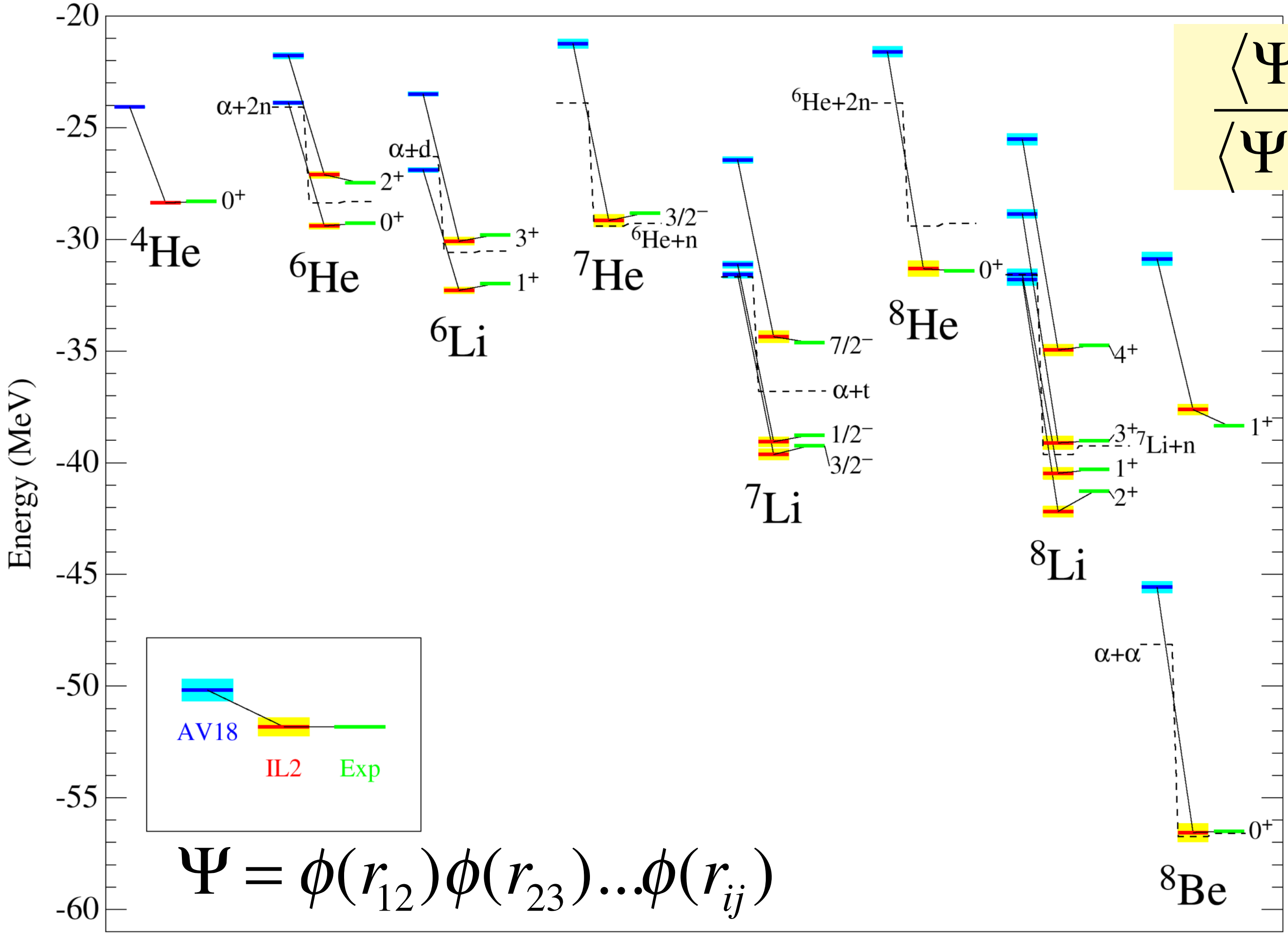
From quarks and gluons to hadrons and nuclei



Variational calculation of few body system with NN interaction

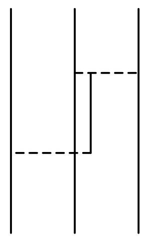
VMC+GFMC

$$\frac{\langle \Psi | V_{\pi} | \Psi \rangle}{\langle \Psi | V_{NN} | \Psi \rangle} \sim 80\%$$

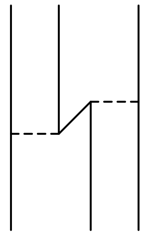


$$\Psi = \phi(r_{12})\phi(r_{23})\ldots\phi(r_{ij})$$

V_{NNN}



Fujita-Miyazawa



Relativistic

C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci.51(2001)

Heavy nuclei (Super model)

Pion is key

Pion is important in nucleus

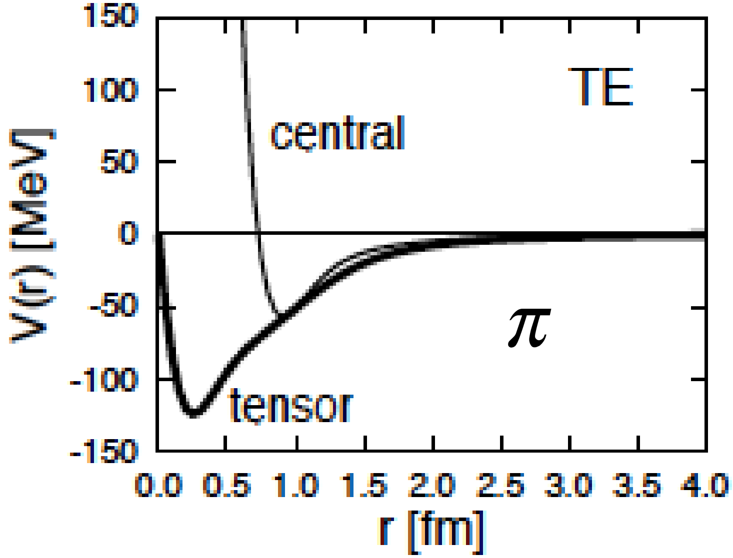
- 80% of attraction is due to pion
- Tensor interaction is particularly important

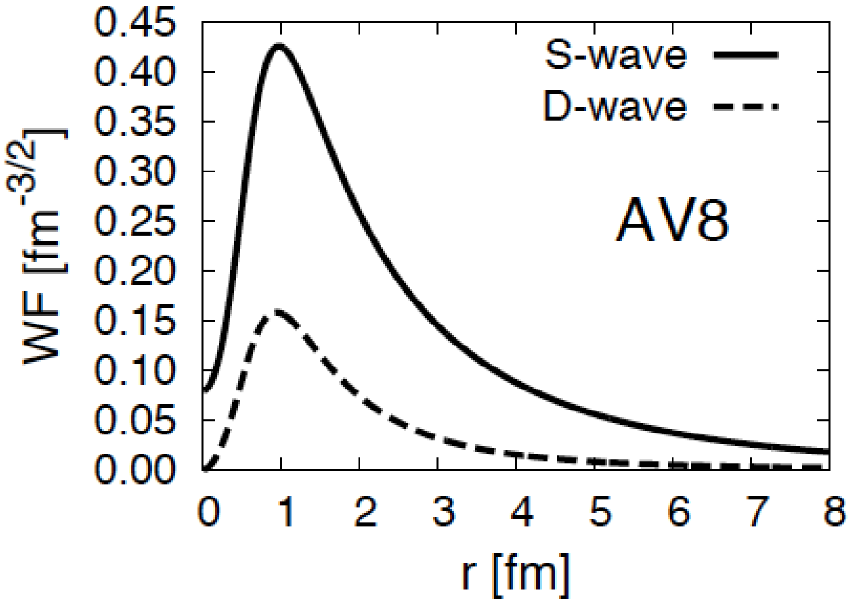
$$\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q} = \frac{1}{3} q^2 S_{12}(\hat{q}) + \frac{1}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 q^2 \qquad S_{12}(\hat{q}) = \sqrt{24\pi}\left[Y_2(\hat{q})[\sigma_1\sigma_2]_2\right]_0$$

Pion	Tensor	spin-spin

The importance of pion is clear in deuteron

NN interaction





@ericepion

S=1 and L=0 or 2

$$\Psi_d = u(r)[Y_0(\hat{r}) \otimes \chi_1(\sigma_1\sigma_2)]_{1M} + w(r)[Y_2(\hat{r}) \otimes \chi_1(\sigma_1\sigma_2)]_{1M}$$

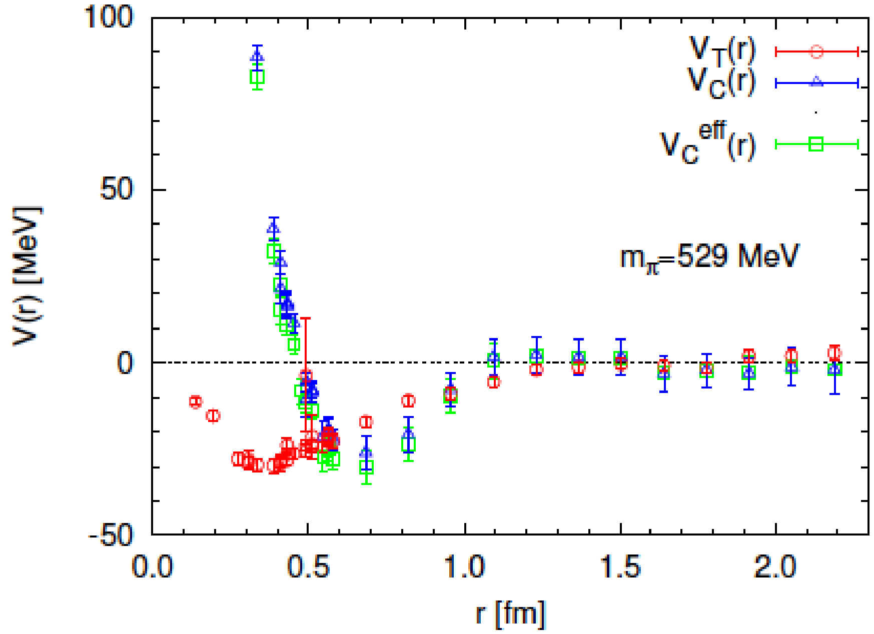
Deuteron (1^+)

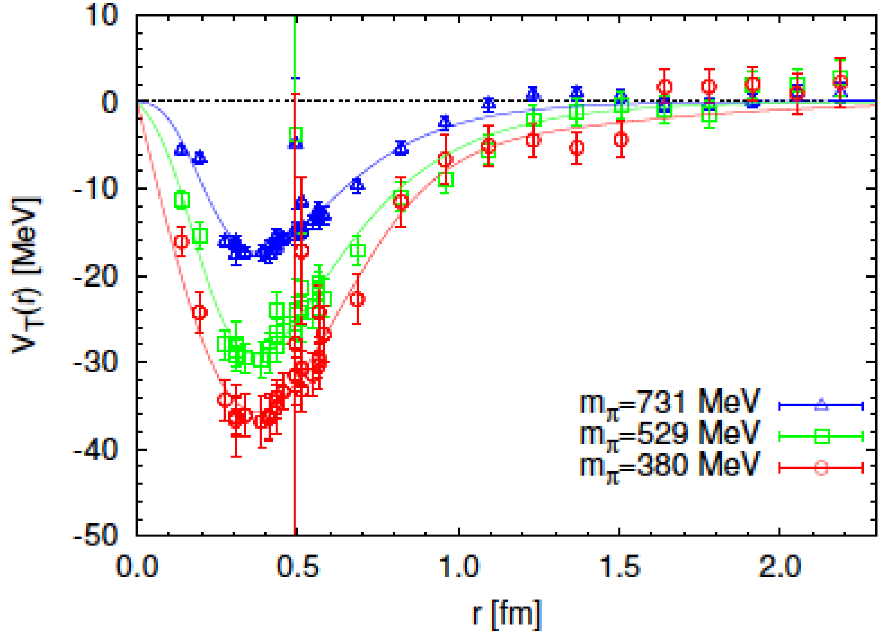
Energy	−2.24 [MeV]
Kinetic	19.88
(SS)	11.31
(DD)	8.57
Central	−4.46
(SS)	−3.96
(DD)	−0.50
Tensorc	−16.64
(SD)	−18.93
(DD)	2.29
LS	−1.02
P(*D*)	5.78 [%]
Radius	1.96 [fm]
(SS)	2.00 [fm]
(DD)	1.22 [fm]

Progress of Theoretical Physics, Vol. 123, No. 1, January 2010

Theoretical Foundation of the Nuclear Force in QCD and Its Applications to Central and Tensor Forces in Quenched Lattice QCD Simulations

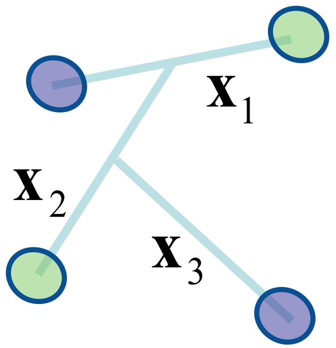
Sinya Aoki,[1] Tetsuo Hatsuda[2] and Noriyoshi Ishii[2]





Tensor Optimized Few-body Model (TOFM)

$$\langle D | S_{12} | S \rangle \neq 0$$



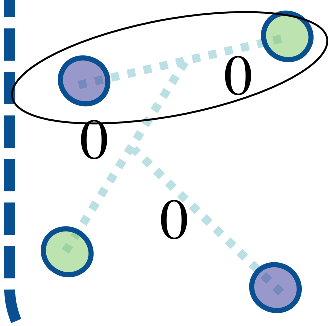
Jacobi coordinate x_1, x_2, x_3

$$|\Psi\rangle = |\Psi\rangle_S + |\Psi\rangle_D$$

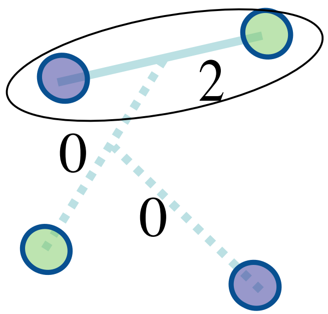
$$|\Psi\rangle_{S,D} = \sum_{i=1}^{k} c_i \psi_i = \sum_{i=1}^{k} c_i \mathcal{A} \left[\psi_L^{space} \chi_S^{spin} \right]_J \chi_T^{isospin}$$

For ^{4}He Total J=0, S-wave (L=0,S=0)
D-wave(L=2,S=2)

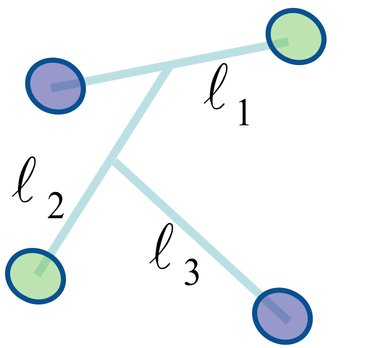
S-wave (L=0)



D-wave (L=2)

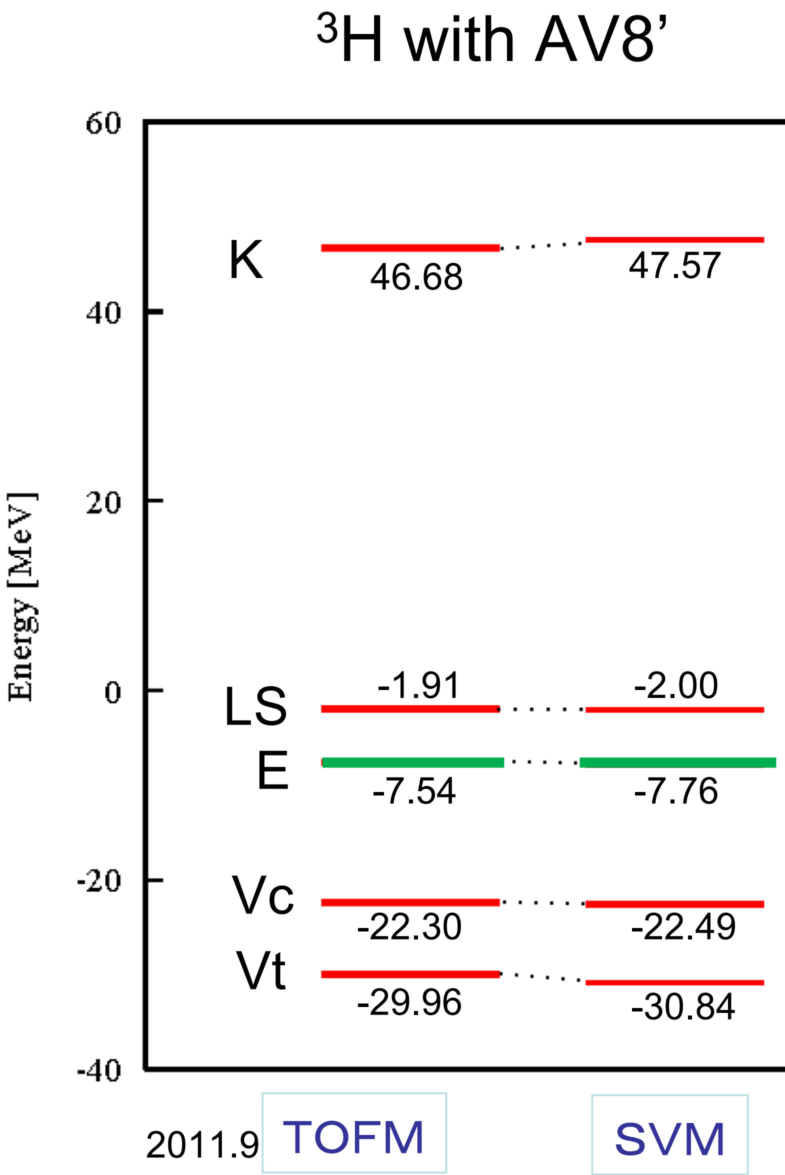


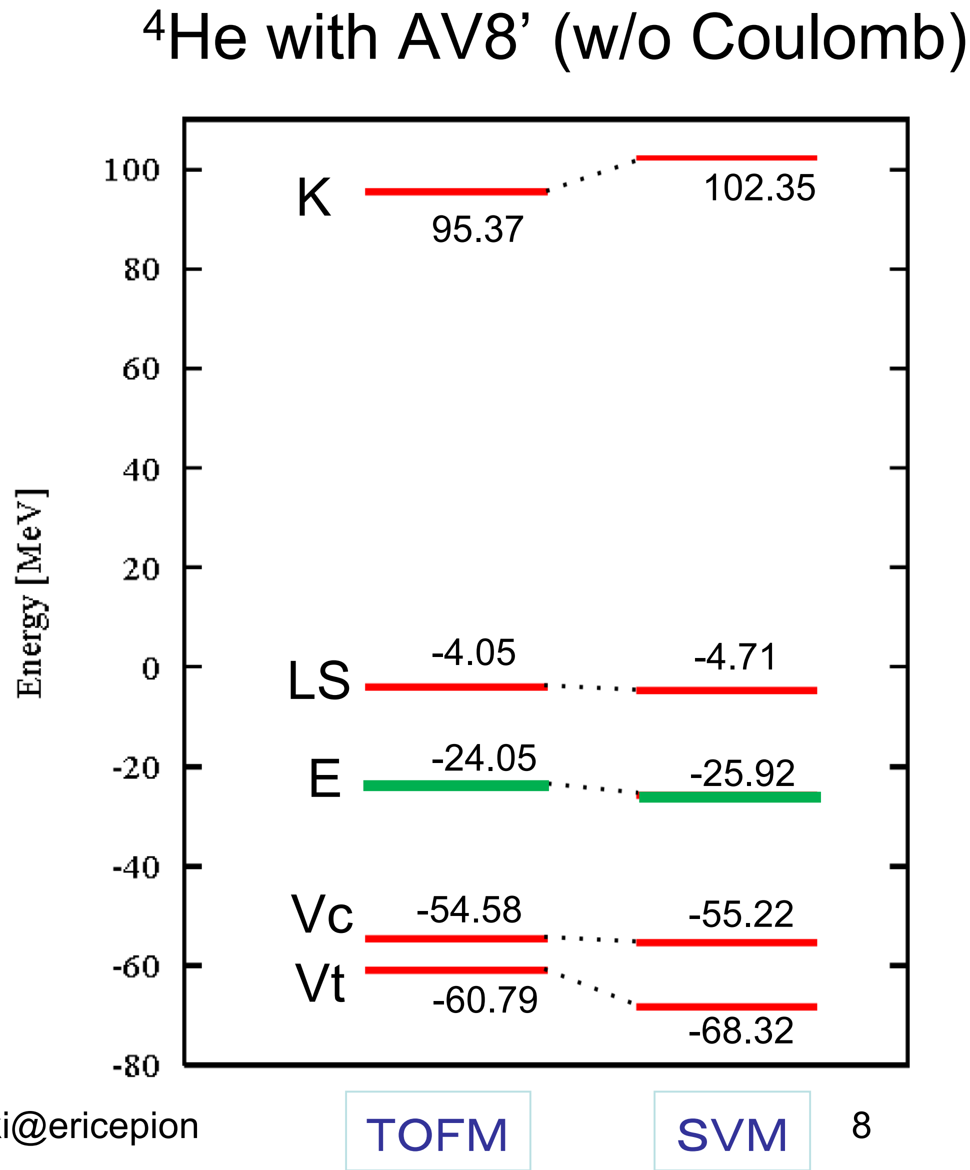
$$l_1 \neq l_2 \neq l_3 = any$$



deuteron like state

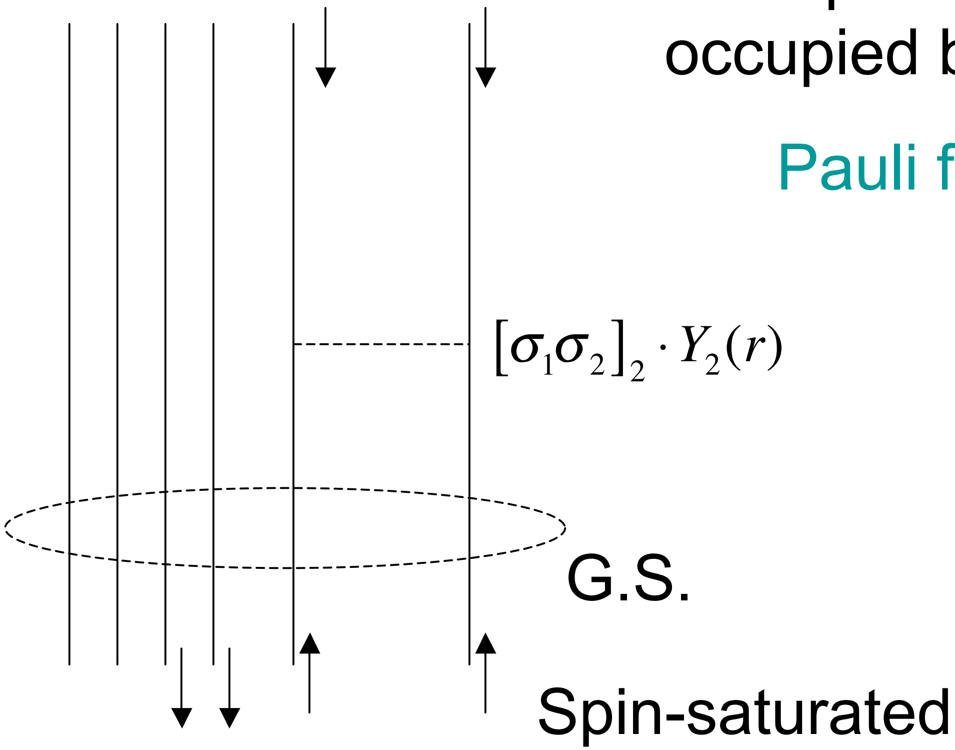
Comparison of TOFM with rigorous calculation (SVM)





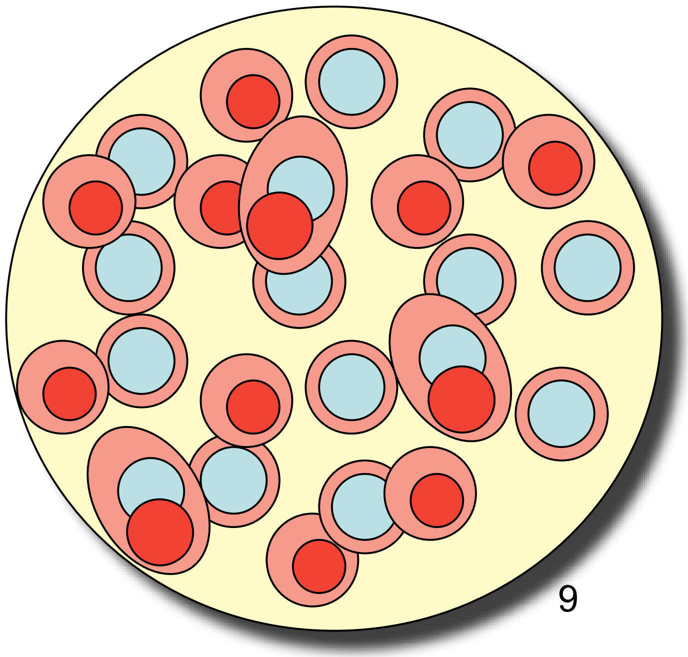
TOFM (TOSM) should be used for nuclear many body problem

2p-2h excitation is essential for treatment of pion



The spin flipped states are already occupied by other nucleons.

Pauli forbidden



Extended Brueckner–Hartree–Fock theory with pionic correlation in finite nuclei

Yoko Ogawa *, Hiroshi Toki

Annals of Physics (2011)

$$\langle 0|S_{12}|0\rangle = 0, \qquad S_{12} = \sqrt{\frac{24\pi}{5}}[Y_2(\hat{r}) \times [\sigma_1 \times \sigma_2]_2]^{(0)}.$$

Hartree-Fock theory cannot handle tensor interaction

$$|\Psi\rangle = C_0|0\rangle + \sum_{\alpha} C_\alpha |2p-2h:\alpha\rangle$$

$$\delta\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} = 0 \qquad \langle\Psi|\Psi\rangle = |C_0|^2 + \sum_{\alpha}|C_\alpha|^2 = 1$$

Total energy

Equation for finite nuclei

$$\langle \Psi | H | \Psi \rangle = |C_0|^2 \langle 0 | H | 0 \rangle + C_0^* \sum_\alpha C_\alpha \langle 0 | H | 2p - 2h : \alpha \rangle$$
$$+ C_0 \sum_\alpha C_\alpha^* \langle 2p - 2h : \alpha | H | 0 \rangle + \sum_{\alpha\beta} C_\alpha^* C_\beta \langle \alpha | H | \beta \rangle$$

$$|2p - 2h : \alpha \rangle \equiv | \alpha \rangle$$

Variational principle

$$\frac{\partial}{\partial C_\alpha^*} \langle \Psi | H - E | \Psi \rangle = 0$$

$$C_0 \langle \alpha | H | 0 \rangle + \sum_\beta C_\beta \langle \alpha | H | \beta \rangle = E C_\alpha$$

$$\frac{\partial}{\partial \psi_a^*(x)} \left[\langle \Psi | H | \Psi \rangle - \sum_b e_b \psi_b^*(x) \psi_b(x) \right] = 0$$

$$|0\rangle = \prod_a \psi_a(x)$$

$$|C_0|^2 \frac{\partial}{\partial \psi_a^*} \langle 0 | H | 0 \rangle + C_0^* \sum_\alpha C_\alpha \frac{\partial}{\partial \psi_a^*} \langle 0 | H | \alpha \rangle + \sum_{\alpha\beta} C_\alpha^* C_\beta \frac{\partial}{\partial \psi_a^*} \langle \alpha | H | \beta \rangle = e_a \psi_a(x)$$

EBHF equation

$$C_\alpha = -C_0 \sum_\beta \frac{1}{\langle\alpha|H|\beta\rangle - E\delta_{\alpha\beta}} \langle\beta|H|0\rangle = -C_0 \sum_\beta \langle\alpha| \frac{1}{H-E} |\beta\rangle\langle\beta|H|0\rangle$$

$$|C_0|^2 \frac{\partial}{\partial\psi_a^*}\langle 0|H|0\rangle - |C_0|^2 \sum_{\alpha\beta} \frac{\partial}{\partial\psi_a^*}\langle 0|H|\alpha\rangle\langle\alpha|\frac{1}{H-E}|\beta\rangle\langle\beta|H|0\rangle$$

$$+|C_0|^2 \sum_{\alpha\alpha'\beta\beta'} \langle 0|H|\alpha'\rangle\langle\alpha'|\frac{1}{H-E}|\alpha\rangle \frac{\partial}{\partial\psi_a^*}\langle\alpha|H|\beta\rangle\langle\beta|\frac{1}{H-E}|\beta'\rangle\langle\beta'|H|0\rangle = e_a\psi_a(x)$$

Effective Hamiltonian in HF method

$$H_{eff} = |C_0|^2 H - |C_0|^2 \sum_{\alpha\beta} H|\alpha\rangle\langle\alpha|\frac{1}{H-E}|\beta\rangle\langle\beta|H$$

$$\frac{\partial}{\partial\psi_a^*(x)}\langle 0|H_{eff} - \sum_b e_b \psi_b^* \psi_b|0\rangle = 0$$

Extended BHF equation

Feshbach projection method

$$P + Q = 1$$

$$H(P+Q)\Psi = E(P+Q)\Psi.$$

$$\begin{cases} PHP\Psi + PHQ\Psi = EP\Psi, \\ QHP\Psi + QHQ\Psi = EQ\Psi. \end{cases}$$

$$Q\Psi = \frac{1}{E - QHQ} QHP\Psi$$

$$PHP\Psi + PHQ\frac{1}{E - QHQ}QHP\Psi = EP\Psi.$$

$$H_{eff} = PHP - PHQ\frac{1}{QHQ - E}QHP$$

$$P = |0\rangle\langle 0|$$

$$Q = \sum_{\alpha} |2p2h:\alpha\rangle\langle 2p2h:\alpha|$$

$$\langle 0|H_{eff}|0\rangle = |C_0|^2\langle 0|H|0\rangle - |C_0|^2\sum_{\alpha\beta}\langle 0|H|\alpha\rangle\langle\alpha|\frac{1}{H-E}|\beta\rangle\langle\beta|H|0\rangle$$

Variational method and Feshbach theory completely agree.

Comparison of BHF and EBHF theories

$$(T+V)\psi = E\psi$$

$$G = V - V\frac{Q}{T-(E_1+E_2)}G$$

$$G = V - V\frac{Q}{H_{HF}-E_{HF}^{h}}G = V - V\frac{Q}{H_{HF}-E_{HF}^{h}}V + V\frac{Q}{H_{HF}-E_{HF}^{h}}V\frac{Q}{H_{HF}-E_{HF}^{h}}V\ldots$$

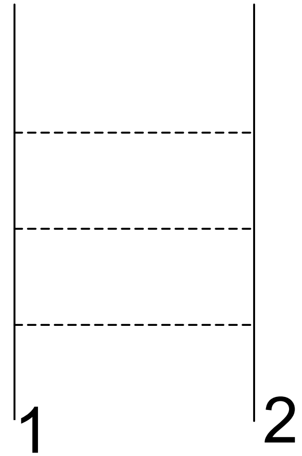
$$G = V - V\frac{Q}{H_{HF}-E_{HF}^{h}+V}V \qquad \left(H_{HF}-E_{HF}^{h}\right)\left|2p-2h:\alpha\right\rangle = E_\alpha\left|2p-2h:\alpha\right\rangle$$

$$\langle 0|T+G|0\rangle = \langle 0|T+V|0\rangle - \sum_{\alpha\beta}\langle 0|V|\alpha\rangle\langle\alpha|\frac{1}{E_\alpha\delta_{\alpha\beta}+V}|\beta\rangle\langle\beta|V|0\rangle$$

EBHF

$$\langle 0|H_{eff}|0\rangle = \underline{|C_0|^2}\langle 0|T+V|0\rangle - \underline{|C_0|^2}\sum_{\alpha\beta}\langle 0|V|\alpha\rangle\langle\alpha|\frac{1}{H-E}|\beta\rangle\langle\beta|V|0\rangle$$

$$\langle\alpha|H-E|\beta\rangle = E_\alpha\delta_{\alpha\beta} + \langle\beta|V|\alpha\rangle + \underline{\langle 0|H|0\rangle\delta_{\alpha\beta} - E\delta_{\alpha\beta}}$$

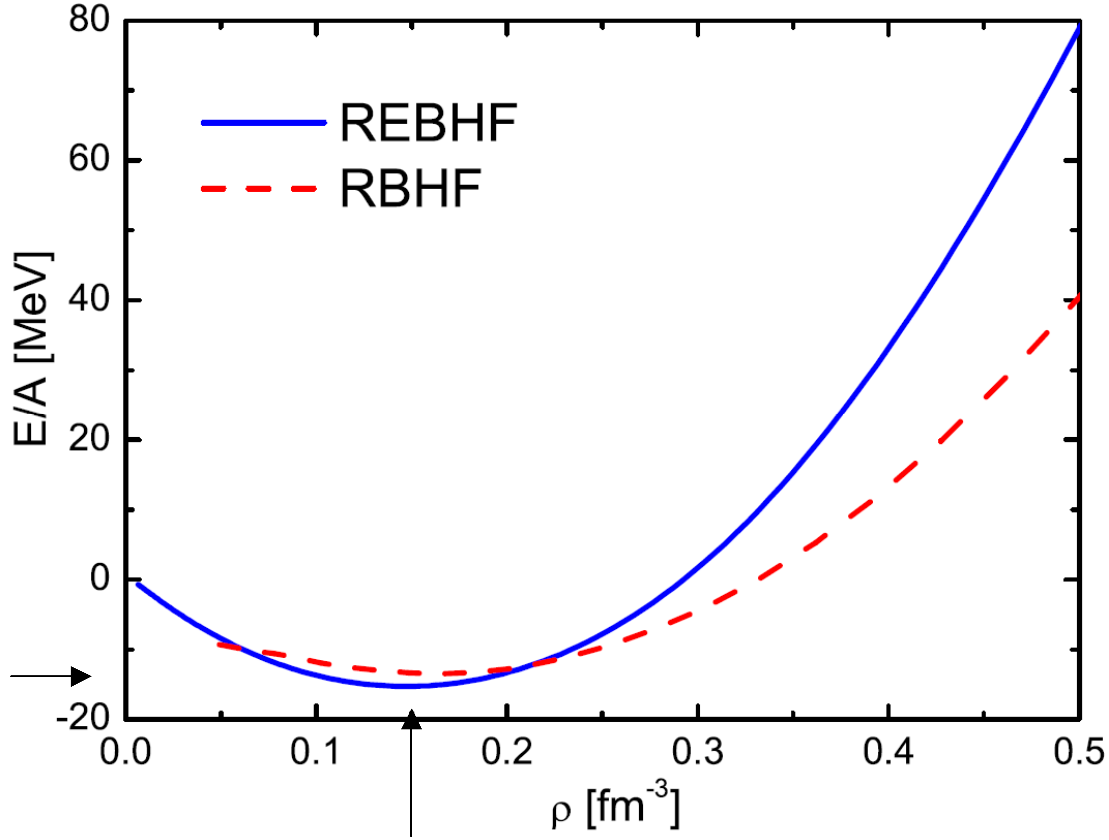


Nuclear matter

J. Hu, H. Toki and Y. Ogawa

Calculating finite nuclei

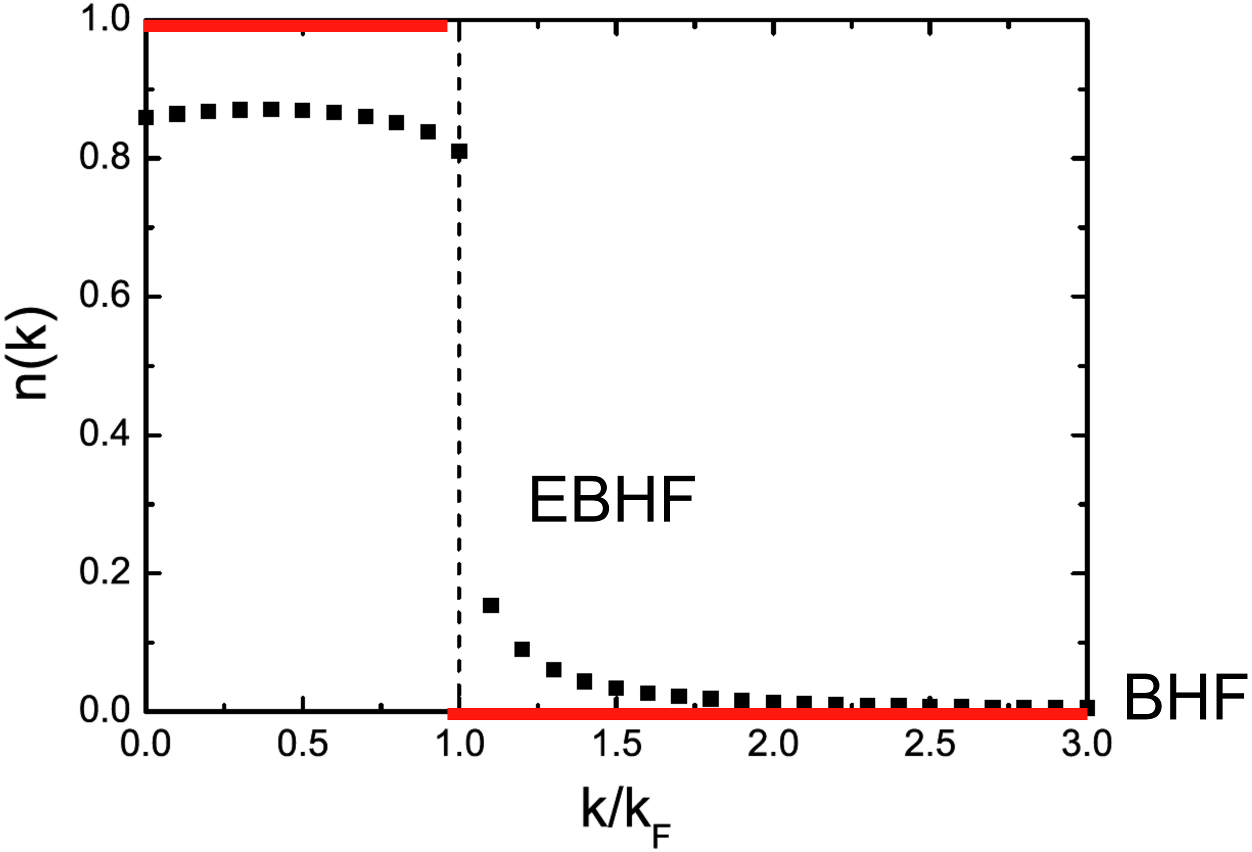
Bonn B



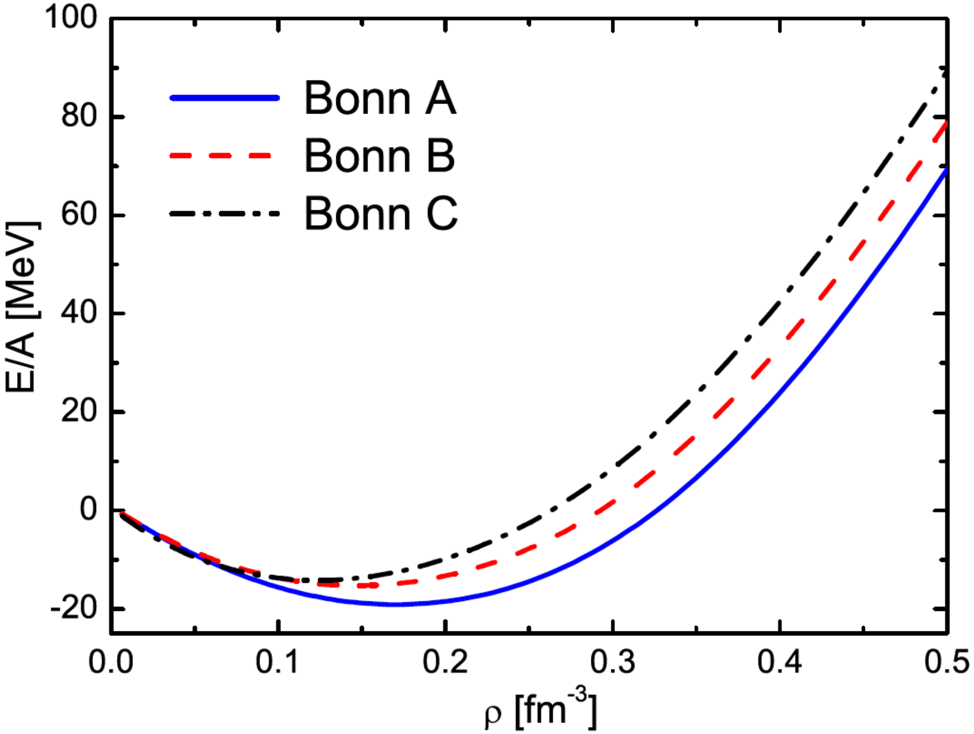
$$E/A = 0.16 \pm 0.01$$

$$\rho_0 = 0.15 \pm 0.01$$

Momentum distribution in nuclear matter



$$n(k) = \langle \Psi | a_k^+ a_k | \Psi \rangle$$



Tensor interaction

A	B	C
small	medium	large

$$E/A = -0.16 \pm 0.01$$
$$\rho_0 = 0.15 \pm 0.01$$
$$K = 250 \sim 280$$

Methods	Potential	ρ [fm^{-3}]	E/A [MeV]	K [MeV]	M_N^*/M_N	$\lvert C_0\rvert^2$
RBHF	Bonn A	0.1814	-15.38	302.9	0.598	-
	Bonn B	0.1625	-13.44	240.3	0.621	-
	Bonn C	0.1484	-12.12	181.6	0.640	-
REBHF	Bonn A	0.1699	-19.13	374.1	0.632	0.712
	Bonn B	0.1484	-15.28	294.2	0.664	0.710
	Bonn C	0.1227	-14.20	222.1	0.703	0.689

Conclusion

- We have developed Extended Brueckner-Hartree-Fock (EBHF) theory for tensor interaction.
- We are able to calculate many body system by using bare nucleon-nucleon interaction.
- We calculate nuclear matter with Bonn potential.
- Bonn B interaction reproduces the saturation property.