

# QCD phenomenology from DSEs

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JLU Giessen

September 2011

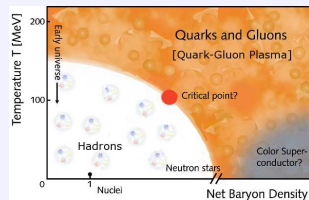
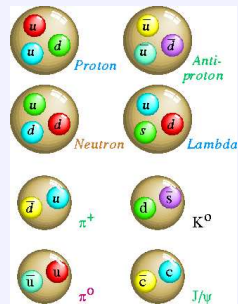
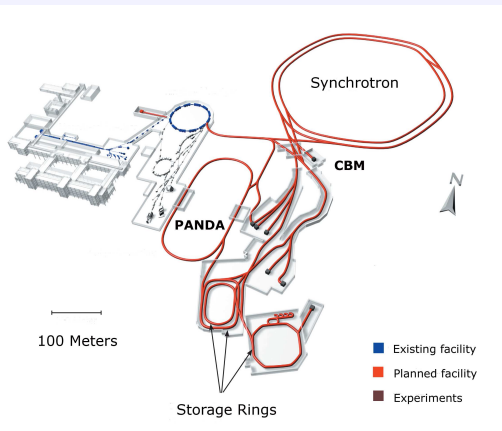
in collaboration with:

Jacqueline Bonnet, Gernot Eichmann, Tobias Goecke, Christian Kellermann, Jan Luecker,  
Axel Maas, Jens A. Mueller, Jan M. Pawlowski, Stefan Strauss, Lorenz von Smekal, Richard Williams

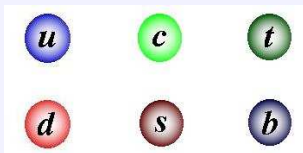
- 1 Introduction
- 2 Properties of SU(N) Yang-Mills theory
- 3 Dynamical chiral symmetry breaking: Quarks and mesons
- 4 Hadronic contributions to  $(g - 2)_\mu$
- 5 Chiral and deconfinement transitions in QCD

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# FAIR: CBM and PANDA



# Dynamical mass generation

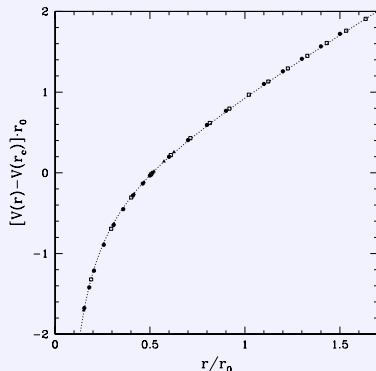


Quark mass generated by **weak** and **strong** interaction  
(QCD: explicit vs. dynamical breaking of chiral symmetry)

	u	d	s	c	b	t
$M_{\text{weak}}$ [MeV]	3	5	100	1300	4000	175000
$M_{\text{strong}}$ [MeV]	400	400	400	400	400	400
$M_{\text{tot}}$ [MeV]	400	400	500	1700	4400	175000

$M_{\text{strong}}$ : **Nonperturbative effect!**

# Confinement



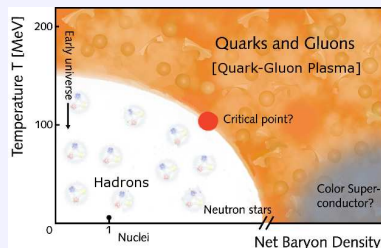
S. Necco and R. Sommer, Nucl. Phys. B **622** (2002) 328

- Linear rising potential:  
 $V(r) \sim r$
- Quark-Antiquark system cannot be split:  
Quark-Confinement
- Screening of gluons  
→ Glueballs

What are the driving mechanisms?  
Consequences for hadronic spectra ?

# CBM: QCD phase transitions

- Existence and location of critical end point
- Propagation of quarks and gluons in plasma phase



- Chiral limit ( $M_{weak} \rightarrow 0$ ): order parameter chiral condensate

$$\langle \bar{\psi}\psi \rangle = Z_2 N_c \text{Tr}_D \int \frac{d^4 p}{(2\pi)^4} S(p)$$

- Static quarks ( $M_{weak} \rightarrow \infty$ ): order parameter Polyakov-loop

$$\Phi \sim e^{-F_q/T} \leftrightarrow \int_{\varphi} \langle \bar{\psi}\psi \rangle_{\varphi}$$

CBM/FAIR-Experiment

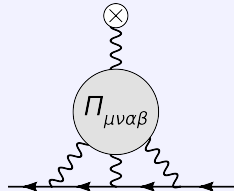
Experiment:  $11\,659\,2080(63) \times 10^{-11}$

G. W. Bennett *et al.* (Muon  $g-2$  Collaboration), PRD **73**, 072003 (2006)

Theory:  $11\,659\,1790(65) \times 10^{-11}$

F. Jegerlehner and A. Nyffeler, Phys. Rept. **477**, 1 (2009)

Problem: Hadronic 'LBL-contribution' with  $116(39) \times 10^{-11}$ :



- Cannot be determined from experimental input
- Multi-Scale problem!
- $\rightarrow$  NJL-model, VMD,... (Prades, Nyffeler, et al.)

**Goal: 'Ab initio' calculation of LBL contribution**



# QCD in covariant gauge

quarks, gluons and ghosts:

$$\mathcal{Z}_{\text{QCD}} = \int \mathcal{D}[\Psi, A, c] \exp \left\{ - \int d^4x \left( \bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

$$S_{\text{QCD}} = \int d^4x \left( \begin{array}{c} \text{---}^{-1} + \text{---} \bullet \text{---} + \text{---}^{-1} + \text{---} \bullet \text{---} + \\ \text{~~~~~}^{-1} + \text{~~~~~} \bullet \text{~~~~~} + \text{~~~~~} \bullet \text{~~~~~} \end{array} \right)$$

# Green's functions

## QCD Green's functions

- are connected to **confinement**:
  - Gribov-Zwanziger and Kugo-Ojima scenarios
  - Running Coupling
  - **Positivity**
  - **Polyakov Loop**
- encode  **$D\chi SB$**
- are ingredients for hadron phenomenology
  - **Bound state equations:**  
**Bethe–Salpeter equation / Faddeev equation**
  - Form factors, decays etc.

The Goal:

**Gauge invariant** information from **gauge fixed functional approach**

The Tool:

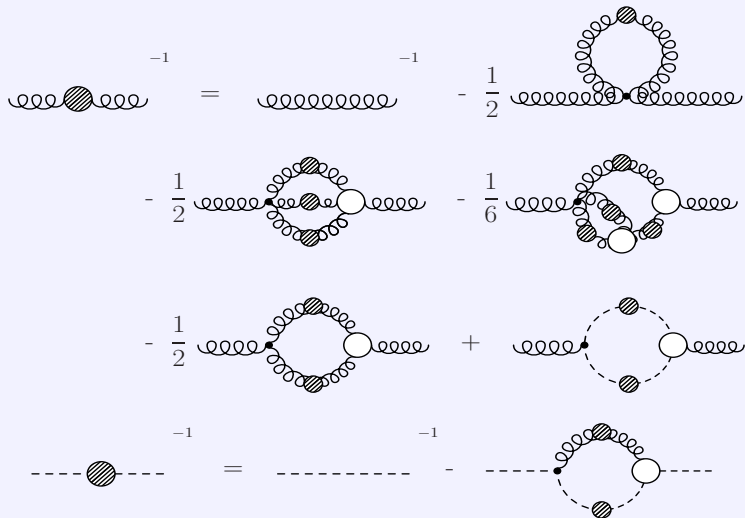
**Dyson-Schwinger** and **Bethe-Salpeter**-equations (DSE/BSE)

# Lattice QCD vs. DSE/FRG: Complementary!

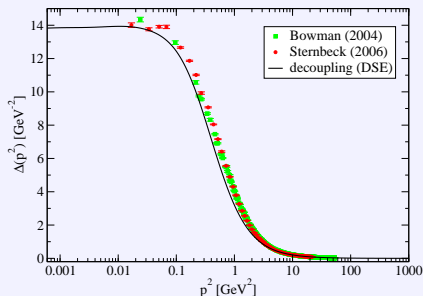
- Lattice simulations
  - ▶ Ab initio
  - ▶ Gauge invariant
- Functional approaches:
  - Dyson-Schwinger equations (DSE)
  - Functional renormalisation group (FRG)
  - ▶ Analytic solutions at small momenta
  - ▶ Space-Time-Continuum
  - ▶ Chiral symmetry: light quarks and mesons
  - ▶ Multi-scale problems feasible: e.g.  $(g-2)_\mu$
  - ▶ Chemical potential: no sign problem

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# Dyson-Schwinger equations (DSEs)



# Gluon propagator ( $T = 0$ )



C.F., Maas and Pawłowski, *Annals Phys.* **324** (2009) 2408.

Aguilar, Binosi, Papavassiliou, *PRD* **78**, 025010 (2008).

Cornwall, *PRD* **26** (1982) 1453.

$$\Delta(p^2) = \frac{Z(p^2)}{p^2}$$

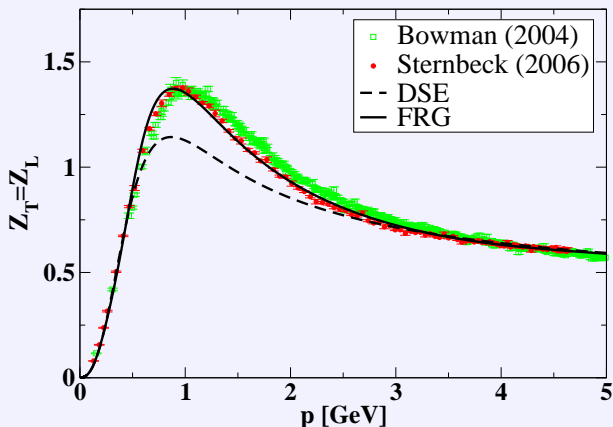
- Gluon 'mass' generation in agreement with lattice results

Cucchieri, Mendes, *PoS LAT2007* (2007) 297.

- Analytic structure of glue incl. positivity violations

→ **talk of Stefan Strauss**

# Gluon dressing function ( $T = 0$ )



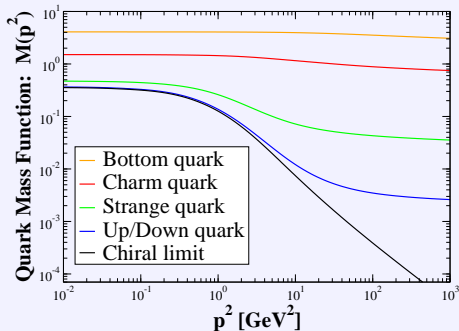
- DSE vs FRG: Effect of four-gluon-interaction
- Physics is in mid-momentum region !

C.F., A. Maas and J. M. Pawłowski, *Annals Phys.* **324** (2009) 2408-2437.

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# Explicit vs. dynamical chiral symmetry breaking



- $M(p^2)$ : momentum dependent!
- Dynamical masses  
 $M_{strong}(0) \approx 350 \text{ MeV}$
- Flavour dependence because of  $M_{weak}$

# From gluons and quarks to mesons

A Feynman diagram showing a gluon self-energy loop (a wavy line with a shaded circle) equal to the sum of a ghost loop (a dashed line with a shaded circle) and a quark loop (a solid line with a shaded circle).

A Feynman diagram showing a ghost self-energy loop (a dashed line with a shaded circle) equal to a quark loop (a solid line with a shaded circle).

A Feynman diagram showing a quark self-energy loop (a solid line with a shaded circle) equal to a gluon loop (a wavy line with a shaded circle).

A Feynman diagram showing a meson vertex (a circle with two external lines) equal to a quark-gluon loop (a circle with a quark line and a gluon line).

- Central quantity:  
quark-gluon vertex

Alkofer, C.F., Llanes-Estrada, Schwenzer,  
Annals Phys.324:106-172,2009.

- Meson structure beyond  
rainbow-ladder

C.F. and Williams, PRD **78**, 074006 (2008).  
C.F. and Williams, PRL **103** (2009) 122001.

- Baryon structure  
→ talk of Gernot Eichmann

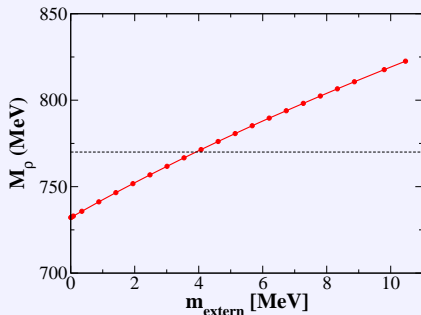
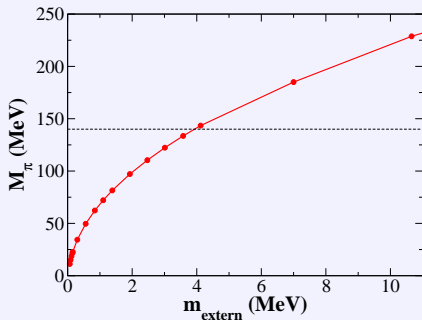
- $U_A(1)$ -problem

Alkofer, C.F. and Williams, Eur. Phys. J. A **38**, 53 (2008)

- $\pi\gamma\gamma$  and  $g-2$

Goecke, C.F., Williams, PRD **83** (2011) 094006.

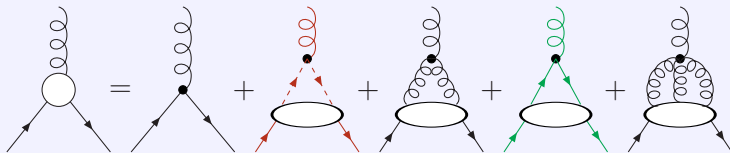
# Pions and Rho-Mesons



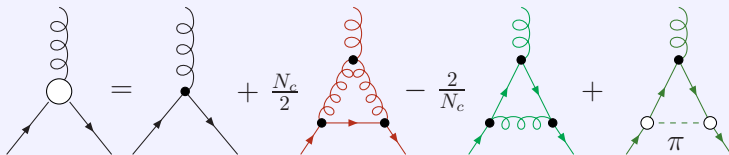
- $m_{u,d} \rightarrow 0$ :  $m_\pi^2 f_\pi^2 = (m_u + m_d) \langle \bar{\psi}\psi \rangle$   
Pion is massless Goldstone-boson with massive constituents
- Simple model: 'rainbow-ladder approximation'  
only vector coupling between quark and gluon included

P. Maris and C. D. Roberts, Phys. Rev. C **56** (1997) 3369

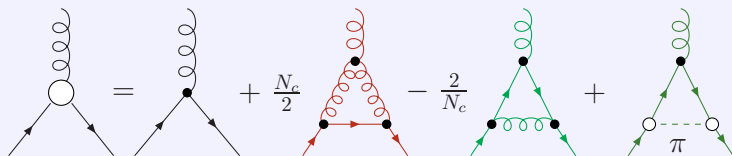
# Quark-gluon vertex I



- Truncation via skeleton expansion leads to (all propagators and vertices dressed!):



# Quark-gluon vertex II



- Abelian diagram extensively explored

Roberts, Tandy, Thomas, Watson *et al.*

- Gluon self-interaction leading in large  $N_c$
- Gluon self-interaction also leading at  $N_c = 3$

Alkofer, C.F., Llanes-Estrada, Schwenzer, *Annals Phys.*324:106-172,2009.

C.F, R. Williams, *PRL* **103** (2009) 122001

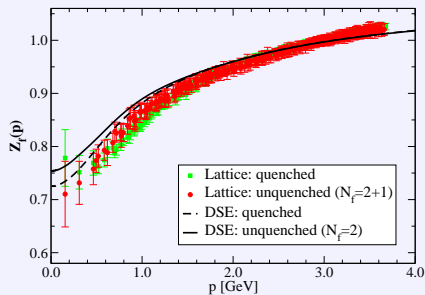
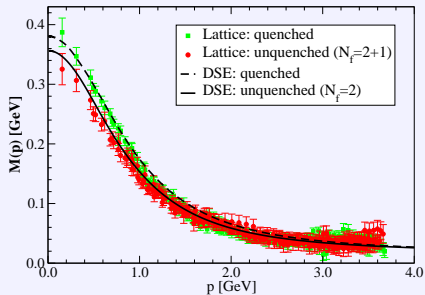
- Pion backreaction  $\rightarrow$  pion cloud effects

C.F, D. Nickel and J. Wambach, *PRD* **76** (2007) 094009

C.F., D. Nickel and R. Williams, *EPJC* **60**, 1434 (2008)

C.F. and R. Williams, *PRD* **78**, 074006 (2008).

# Pion cloud effects in the quark propagator



CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

## ● Unquenching effects of similar size as lattice

P. O. Bowman, et al. Phys. Rev. D **71** (2005) 054507

# Pion cloud effects in light mesons

	RL	3g	3g+ $\pi$	Experiment
$M_\pi$	138	138	138	138
$f_\pi$	94	111	105	93
$M_\rho$	758	881	805	776
$f_\rho$	154	176	168	162
$M_\sigma$	645	884	820	450
$M_{a_1}$	926	1055	1040	1230
$M_{b_1}$	912	972	940	1229

- Attractive effects of 'pion cloud'
- Corrections from decay channels missing

CF and R. Williams, PRL **103** 122001 (2009).

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# Hadronic contributions to $g-2$ I

Experiment:  $11\,659\,2080(63) \times 10^{-11}$

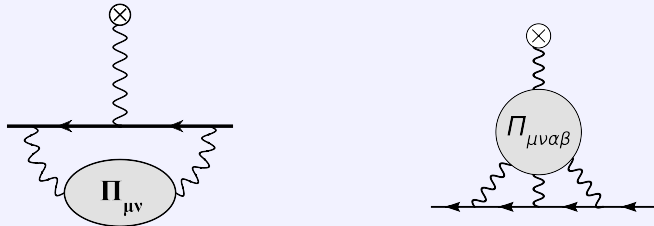
G. W. Bennett *et al.* (Muon  $g-2$  Collaboration), PRD **73**, 072003 (2006)

Theory:  $11\,659\,1790(65) \times 10^{-11}$

F. Jegerlehner and A. Nyffeler, Phys. Rept. **477**, 1 (2009)

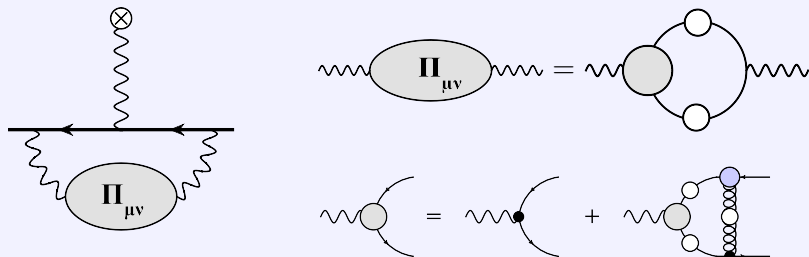
Hadronic contributions:

vacuum polarization and light-by-light scattering



**Goal: 'Ab initio' calculation of LBL contribution**

# Results: hadronic vacuum polarisation

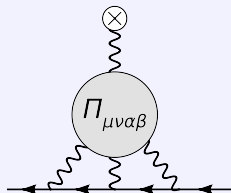


- Building blocks: quark propagator and quark-photon vertex
- $\rightarrow$  our value ( $N_f = 5$ ):  $(a_\mu^{HVP})_{DSE} = 6760 \times 10^{-11}$   
 $(a_\mu^{HVP})_{Exp.} = 6949.1(37.2)(21.0) \times 10^{-11}$

T. Goecke, C.F., R. Williams, PLB in press, arXiv:1107.2588

K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, T. Teubner, J. Phys. G **G38** (2011) 085003

# Results: anomalous magnetic moment



Group	Approach	$\pi^0, \eta, \eta'$	quark-loop
BPP	ENJL	85(13)	21(3)
KN	LMD+V	83(12)	—
MV	LMD+V	114(10)	short distance
	<b>DSE</b>	<b>84(13)</b>	<b>107(48)</b>

J. Bijnens, E. Pallante and J. Prades Phys. Rev. Lett. **75**, 1447 (1995)

M. Knecht and A. Nyffeler, Phys. Rev. D **65** 073034 (2002)

K. Melnikov and A. Vainshtein, Phys. Rev. D **70** 113006 (2004)

T. Goecke, C.F., R. Williams, PRD 83 (2011) 094006.

C.F., T. Goecke and R. Williams, in preparation

Total result:

Experiment: 11 659 **2080(63)**  $\times 10^{-11}$

G. W. Bennett *et al.* (Muon g-2 Collaboration), PRD **73**, 072003 (2006)

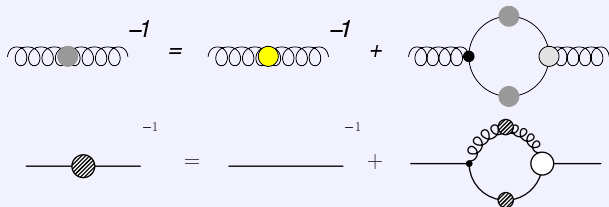
Our value: 11 659 **1865(97)**  $\times 10^{-11}$

C.F., T. Goecke and R. Williams, in preparation

→ **talk of Tobias Goecke**

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# The ordinary chiral condensate



- Order parameter for **chiral symmetry breaking: condensate**

$$\langle \bar{\psi}\psi \rangle = Z_2 N_c T \sum_{n_p} \int \frac{d^3 p}{(2\pi)^3} \text{Tr}_D S(\vec{p}, \omega_p)$$

- Order parameter for **deconfinement: dressed Polyakov loop**

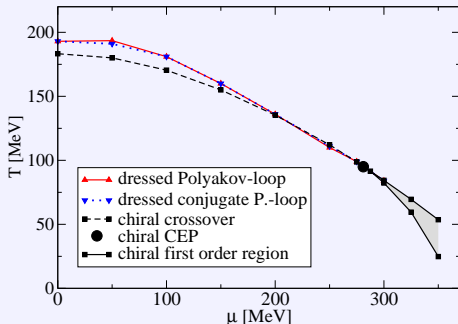
$$\Sigma_1 = - \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \langle \bar{\psi}\psi \rangle_\varphi$$

C.F., PRL **103** (2009) 052003

Synatschke, Wipf and Wozar, PRD **75**, 114003 (2007).

Bilgici, Bruckmann, Gattringer and Hagen, PRD **77** 094007 (2008).

# $N_f = 2$ : QCD phase diagram



→ talk of Jan Luecker

C.F., J. Luecker, J. A. Mueller, PLB 702 (2011) 438-441.

- CEP at  $(T, \mu) \simeq (95, 280)$  MeV
- in agreement with expectations from lattice

de Forcrand, Philipsen, JHEP **0811** (2008) 012; Nucl. Phys. **B642** (2002) 290-306.

G. Endrodi, Z. Fodor, S. D. Katz, K. K. Szabo, JHEP **1104** (2011) 001.

- qualitative agreement with PQM model

Herbst, Pawłowski, Schaefer, PLB **696** (2011)

- imaginary chemical potential (FRG):

Braun, Haas, Marhauser, Pawłowski, PRL 106 (2011)

# Thank you for your attention!

## Helmholtz Young Investigator Group "Nonperturbative Phenomena in QCD"



 **LOEWE** – Landes-Offensive zur Entwicklung  
Wissenschaftlich-ökonomischer Exzellenz