Properties of hadronic resonances in a chiral model

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Motivation

• Development of a (chirally symmetric) linear sigma model for mesons and baryons

• Study of the model for $T = \rho = 0$ (spectroscopy in vacuum)

(decay, scattering lengths,…)

Interrelation between these two aspects!

• Second goal: properties at nonzero T and μ

(Condensates and masses in thermal/matter medium,…)

Fields of the model:

- Quark-antiquark mesons: scalar, pseudoscalar, vector and axial-vector quarkonia.
- Additional meson: The scalar glueball (and evt tetraquarks)
- Baryons: nucleon doublet and its partner

(in the so-called mirror assignment)

How to construct the model:

- (Global) Chiral symmetry: $SU_R(N_f) x SU_L(N_f) x U_V(1)$
- Retain operators of fourth order (dilatation invariance)

Mesonic sector (N_f = 2): 16 quark-antiquark fields

4 scalar and 4 pseudoscalar fields

$$\Phi = (\sigma + i\eta)t^0 + (\vec{a}_0 + i\vec{\pi})\cdot\vec{t}$$

chiral transformation :

$$\Phi \to U_R \Phi U_L^+ \qquad U_R, U_L \subset SU(2)$$

$$J^{PC} = 0^{-+}$$

$$\eta = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$$

$$\vec{\pi} \equiv \bar{u}d,\ \bar{d}u,\ \sqrt{\frac{1}{2}}(\bar{u}u - \bar{d}d)$$

$$J^{PC} = 0^{++}$$

$$\sigma = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$$

$$\vec{a}_0 \equiv \bar{u}d,\ \bar{d}u,\ \sqrt{\frac{1}{2}}(\bar{u}u - \bar{d}d)$$

⬇

First Problem: the scalar mesons!

4 vector and 4 axialvector fields

$$R^\mu = (\omega^\mu - f_1^\mu)t^0 + (\vec{\rho} - \vec{a}_1^{\,\mu})\cdot\vec{t}$$

$$L^\mu = (\omega^\mu + f_1^\mu)t^0 + (\vec{\rho} + \vec{a}_1^{\,\mu})\cdot\vec{t}$$

chiral transf. :

$$R^\mu \to U_R R^\mu U_R^+$$

$$L^\mu \to U_L L^\mu U_L^+$$

$$J^{PC} = 1^{--}$$

$$\omega = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \quad \vec{\rho} \equiv \bar{u}d,\ \bar{d}u,\ \sqrt{\frac{1}{2}}(\bar{u}u - \bar{d}d)$$

$$J^{PC} = 1^{++}$$

$$f_1 = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \quad \vec{a}_1 \equiv \bar{u}d,\ \bar{d}u,\ \sqrt{\frac{1}{2}}(\bar{u}u - \bar{d}d)$$

Lagrangian in the meson sector

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\left(\frac{G}{\Lambda}\right) - \frac{G^4}{4}\right) .$$

$$-\frac{11N_c}{48}\left\langle \frac{\alpha_s}{\pi}\left(\frac{1}{2}G^a_{\mu\nu}G^{a,\mu\nu}\right)\right\rangle = -\frac{11N_c}{48}C^4$$

$$\Lambda = \frac{\sqrt{11}}{2m_G}C^2 .$$

$$\begin{aligned}\mathcal{L} = {} & \mathcal{L}_{dil} + \mathrm{Tr}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi) - m_0^2\left(\frac{G}{G_0}\right)^2\Phi^\dagger\Phi - \lambda_2(\Phi^\dagger\Phi)^2\right] - \lambda_1(\mathrm{Tr}\left[\Phi^\dagger\Phi\right])^2 \\ & + c[\det(\Phi^\dagger) + \det(\Phi)] + \mathrm{Tr}\left[H\left(\Phi^\dagger + \Phi\right)\right] - \frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right] \\ & + \frac{m_1^2}{2}\left(\frac{G}{G_0}\right)^2\mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right] + \frac{h_1}{2}\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}[L_\mu L^\mu + R_\mu R^\mu] \\ & + h_2\mathrm{Tr}[\Phi^\dagger L_\mu L^\mu\Phi + \Phi R_\mu R^\mu\Phi^\dagger] + 2h_3\mathrm{Tr}[\Phi R_\mu\Phi^\dagger L^\mu] + \ldots ,\end{aligned}$$

In the chiral limit (H=0) two dimensional parameters: Λ (dilatation invariance) and c (anomaly). Both from the gauge sector.
Here the large Nc counting shows that all the state are quarkonia.
Only exception: the glueball. $(\Lambda \propto N_c, \lambda_2 \propto N_c^{-1}, \ldots)$

Details in: Denis Parganlija, F.G., Dirk H. Rischke, **Phys.Rev.D82:054024,2010**; **arXiv:1003.4934** [hep-ph].
S. Janwoski, D. Parganlija, F.G., D. Rischke, to appear in Phys. Rev. D, **arXiv:1103.3238** [hep-ph].

Problem of scalars

$$\sigma \text{ is } f_0(600) \quad \text{or} \quad f_0(1370) \;\; ???$$

$$a_0 \text{ is } a_0(980) \quad \text{or} \quad a_0(1450) \;\; ???$$

This is an important issue. One shall do the correct assignment.

Many models use σ = $f_0(600)$ (Lσm, NJL). This has been the usual picture at nonzero temperature/density.

However, this assignment is found to be incorrect in many studies at zero temperature (Phenomenology, Large-N_c, Lattice) .

The quantitative effects of scalars both in the vacuum and in a medium are large!

Scenario I: $\sigma \cong f_0(600), a_0 \equiv a_0(980)$ and $G \approx f_0(1500)$

$M_\sigma \leq 550$ MeV from $\pi\pi$-scattering.

$$\Gamma\left[\sigma \equiv f_0(600) \to \pi\pi\right] \leq 200 \text{ MeV!} \text{ !!}$$

This is wrong! The experimental value is much larger (500 MeV).
Note, the role of axial-vector mesons is crucial for this result.

We conclude: the assignment is unfavored!
One should start from:

$$\sigma \approx f_0(1370) \quad \text{and} \quad a_0 \equiv a_0(1450)$$

Scenario II: $\sigma \approx f_0(1370), a_0 \equiv a_0(1450)$ and $G \approx f_0(1500)$

10 free parameters. 6 are fixed through $m_\pi, m_\rho, m_{\eta_N}, m_{a_1}, f_\pi, \Gamma_{a_1 \to \pi\gamma}$

For the remaining 4: fit to 5 exp quantities:

arXiv:1103.3238

Quantity	Our Value [MeV]	Experiment [MeV]
$M_{\sigma'}$	1191 ± 25	1200-1500
$M_{G'}$	1505 ± 5	1505 ± 6
$G' \to \pi\pi$	38 ± 5	38.04 ± 4.95
$G' \to \eta\eta$	5.3 ± 1.3	5.56 ± 1.34
$G' \to KK$	9.3 ± 1.7	9.37 ± 1.69

Fit in the scenario $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$. Note that the $f_0(1370)$ mass ranges between 1200 MeV and 1500 MeV [17] and therefore, as an estimate, we are using the value $m_{\sigma'} = (1350 \pm 150)$ MeV in the fit.

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \end{pmatrix} = \begin{pmatrix} \sqrt{0.75} & \sqrt{0.25} \\ -\sqrt{0.25} & \sqrt{0.75} \end{pmatrix} \begin{pmatrix} \sigma \equiv \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) \\ G = gg \end{pmatrix}$$

$$C^4 = \langle \tfrac{\alpha_s}{2\pi} G^a_{\mu\nu} G^{a,\mu\nu} \rangle = (698 \pm 39 \text{ MeV})^4$$

to be compared with

$$\langle \tfrac{\alpha_s}{2\pi} G^a_{\mu\nu} G^{a,\mu\nu} \rangle \approx (\underset{\text{sum rules}}{300} \text{ to } \underset{\text{lattice}}{600} \text{ MeV})^4$$

Ongoing studies in the meson sector

Nf=3: the Lagrangian is the same: only two more parameters (related to the mass of the s-quark).

Preliminary results in proceedings; long publication with details will come.

D. Parganlija et al, **Int.J.Mod.Phys.A26:607-609,2011**. **arXiv:1009.2250** [hep-ph]
AIP Conf.Proc.1343:328-330,2011. **arXiv:1011.6104** [hep-ph]

Substantial confirmation of the results of Nf = 2, but more results:
Good description of Φ, K*(892), f1(1525), K0(1450), …

Even for Nf = 3 the scalar mesons are above 1 GeV.

Still, there is the problems of the light scalar fields…

Tetraquark: outlook and short excursus at nonzero T

A possibility is to interpret the light scalar states below 1 GeV
[f_0(600), k(800), f_0(980) and a_0(980)]
as diquark-antidiquark objects: these are the Jaffe's tetraquarks.

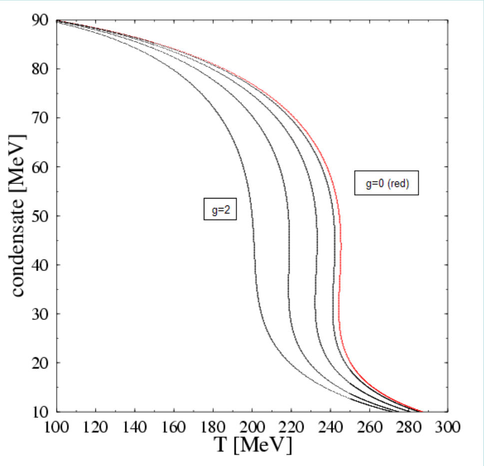
The N_f=3 case is an outlook. Mixing of these tetraquark-quarkonioa takes place.

Black et al, **Phys. Rev. D 64** (2001), F.G., **Phys.Rev.D 75**,(2007)

For N_f=2 only one tetraquark survives. In this case we studied a simplified system at nonzero T.

$$\text{The resonance } f_0(1370) \approx \sigma \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \text{ is the chiral partner of the pion } \vec{\pi}.$$

$$\text{The resonance } f_0(600) \approx \chi \equiv \frac{1}{2}[u,d][\bar{u},\bar{d}] \text{ is an extra-scalar state}$$



Increasing of mixing:

1) T_c decreases
2) First order softened
3) Cross-over obtained for tetraquark-quarkonium coupling large enough

Achim Heinz, Stefan Strube, F.G., Dirk H. Rischke
Phys.Rev.D79:037502,2009; **arXiv:0805.1134** [hep-ph]

Baryon sector

Strategy

Nucleon and its chiral partner : global chiral symmetry and operators of 4-th-order

(Axial-)vector mesons are included

Mirror assignment: (C. De Tar and T. Kunihiro, **PRD 39 (1989) 2805**)

$$\Psi_{1,R} \to U_R \Psi_{1,R} \qquad \Psi_{1,L} \to U_L \Psi_{1,L}$$

$$\Psi_{2,R} \to U_L \Psi_{2,R} \qquad \Psi_{2,L} \to U_R \Psi_{2,L}$$

A chirally invariant mass-term is possible!

$$m_0\left(\overline{\Psi}_{1,L}\Psi_{2,R} - \overline{\Psi}_{1,R}\Psi_{2,L} - \overline{\Psi}_{2,L}\Psi_{1,R} + \overline{\Psi}_{2,R}\Psi_{1,L}\right)$$

$$\begin{aligned}
L_{nucl} &= \overline{\Psi}_{1L} i\gamma_\mu \partial^\mu \Psi_{1L} + g\overline{\Psi}_{1L}\gamma_\mu L^\mu \Psi_{1L} + \overline{\Psi}_{1R} i\gamma_\mu \partial^\mu \Psi_{1R} + g\overline{\Psi}_{1R}\gamma_\mu R^\mu \Psi_{1R} \\
&+ \overline{\Psi}_{2L} i\gamma_\mu \partial^\mu \Psi_{2L} + g\overline{\Psi}_{2L}\gamma_\mu R^\mu \Psi_{2L} + \overline{\Psi}_{2R} i\gamma_\mu \partial^\mu \Psi_{2R} + g\overline{\Psi}_{2R}\gamma_\mu L^\mu \Psi_{2R} \\
&- \hat{g}_1(\overline{\Psi}_{1L}\Phi\Psi_{1R} + \overline{\Psi}_{1R}\Phi^+\Psi_{1L}) - \hat{g}_2(\overline{\Psi}_{2L}\Phi^+\Psi_{2R} + \overline{\Psi}_{2R}\Phi\Psi_{2L}) \\
&- m_0(\overline{\Psi}_{1L}\Psi_{2R} - \overline{\Psi}_{1R}\Psi_{2L} - \overline{\Psi}_{2R}\Psi_{1L} + \overline{\Psi}_{2L}\Psi_{1R})
\end{aligned}$$

Mass of the nucleon

$$\begin{pmatrix} N \\ N^* \end{pmatrix} = \hat{M} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \quad \text{with} \quad \hat{M} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix}$$

$N = N(940)$

$N^* = N^*(1535)$

$$\delta = ar\cosh\left[\frac{M_N + M_{N^*}}{2m_0}\right]$$

$$M_{N,N^*} = \frac{1}{2}\sqrt{4m_0^2 + (...)\phi^2} \pm (...)\phi$$

If $m_0 = 0 \rightarrow$ only the quark condensate generates the masses.

m_0 parametrizes the contribution which does not stem from the quark condensate

important at nonzero temperature and density

also in the so-called quarkyonik phase: L. McLerran, R. Pisarski **Nucl.Phys.A796:83-100,2007**

Axial couplings without vector d.o.f

Without vector and axial-vector mesons: $\left|g_A^N\right| \leq 1$ $\left|g_A^{N*}\right| \leq 1$ $g_A^{N*} = -g_A^N$

$g_A^N \approx 1$ $g_A^{N*} \approx -1$

$g_A^N = 1.26 \ \text{(exp)}, \quad g_A^{N*} \approx 0.2 \ \text{(latt)}$ cannot be described without vm.

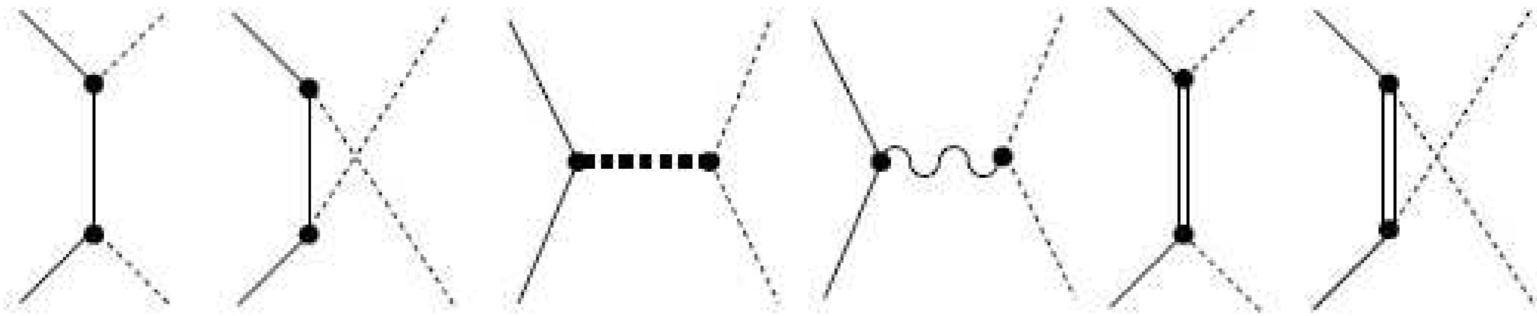
However, with (axial-)vector mesons are introduced the axial couplings are modified. A description of the axial couplings is then possible.

Result for m_0: $m_0 = 460 \pm 136 \ \text{MeV}$

Using $g_A^N = 1.26 \ \text{(exp)}, \quad g_A^{N*} \approx 0.2 \ \text{(latt)}$ and $\Gamma_{N^* \to N\pi} \approx 67 \ \text{MeV}$

S. Gallas, F.G, Dirk Rischke **Phys.Rev.D82:014004,2010**. e-Print: **arXiv:0907.5084** [hep-ph]

Pion-nulceon scattering lengths



Tree-level diagrams contributing to πN scattering. Dashed lines represent the pion, the bold dashed line the σ meso[n], the wavy line the ρ meson, full lines the nucleon, and double full lines the N^*, respectively.

$$a_0^- = (6.04 \pm 0.63) \cdot 10^{-4}\ \text{MeV}^{-1} \qquad a_0^{-(\exp)} = (6.4 \pm 0.1) \cdot 10^{-4}\ \text{MeV}^{-1}$$

$$a_0^+ \approx (\text{from} - 20\ \text{to} + 20 \cdot 10^{-4})\ \text{MeV}^{-1} \qquad a_0^{+(\exp)} = (-8.8 \pm 7.2) \cdot 10^{-4}\ \text{MeV}^{-1}$$

Large theoretical uncertainty due to the scalar-isosocalar

Importance of both vector mesons and mirror assignment in order to get these results

Where does m_0 comes from?

$$m_0\left(\overline{\Psi}_{1,L}\Psi_{2,R}-\overline{\Psi}_{1,R}\Psi_{2,L}-\overline{\Psi}_{2,L}\Psi_{1,R}+\overline{\Psi}_{2,R}\Psi_{1,L}\right)$$

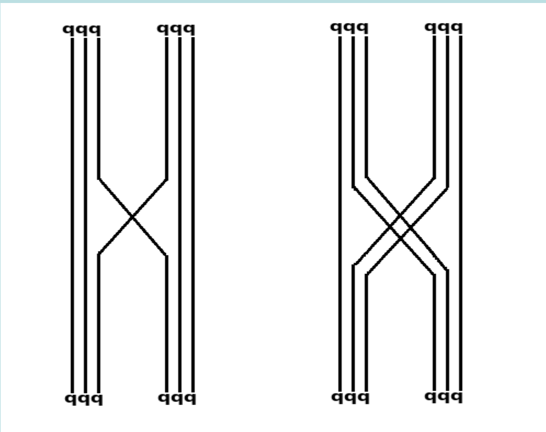
By requiring dilatation invariance one should modify the mass - terms as :

$$(a\chi+bG)\left(\overline{\Psi}_{1,L}\Psi_{2,R}-\overline{\Psi}_{1,R}\Psi_{2,L}-\overline{\Psi}_{2,L}\Psi_{1,R}+\overline{\Psi}_{2,R}\Psi_{1,L}\right)$$

tetraquark dilaton

By shifting : $\chi\rightarrow\chi_0+\chi,\quad G\rightarrow G_0+G$ one has : $m_0=a\chi_0+bG_0$

m_0 originates form the tetraquark and the gluon condensates.
Note, also, a tetraquark exchange naturally arises in nucleon-nucleon interactions



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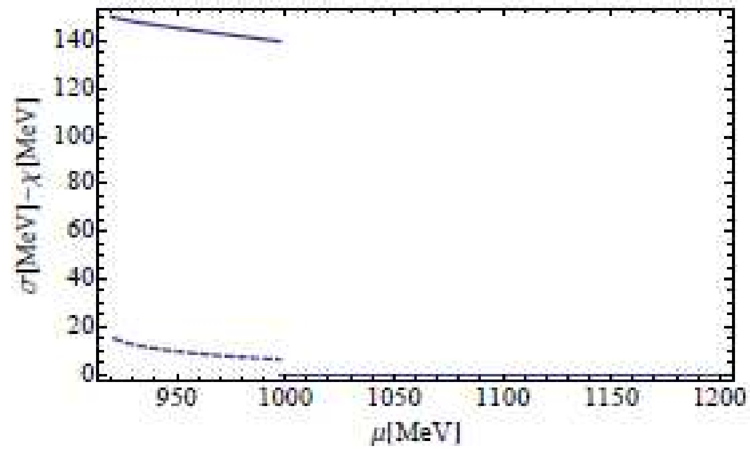


FIG. 3: Mean σ (solid line) and χ (dashed line) fields. A strong first order phase transition is present at $\mu \sim 1$ GeV. The transition point is the same for the two fields. Note, the minimal value for the chemical potential corresponds to nuclear matter.

The model with the light tetraquark has been studied at finite density in
S. Gallas, F.G, G. Pagliara, to appear in **Nucl. Phys. A**, **arXiv:1105.5003**

Saturation: ok. Compressibility: K is about 200 MeV (in agreement with experiment)

Related question: does nuclear matter binds at large Nc?

As soon as the lightest scalar f0(600) is not a quarkonium, nuclear matter ceases to exist already for Nc=4.

Luca Bonanno and F.G., **Nucl.Phys.A859:49-62,2011**. **arXiv:1102.3367** [hep-ph]

Summary and outllok

Chiral model for hadrons based on dilatation invariance and global symmetry

Important role of (axial)vector mesons in all phenomenology

Scalar quarkonium and glueball above 1 GeV (effects in the medium)

Contribution to the nucleon mass which does not stem fro the chiral condensate (but from the tetraquark and glueball condensates)

Nf=3 (ongoing) and Nf=4, additonal tetraquark states, weak decays

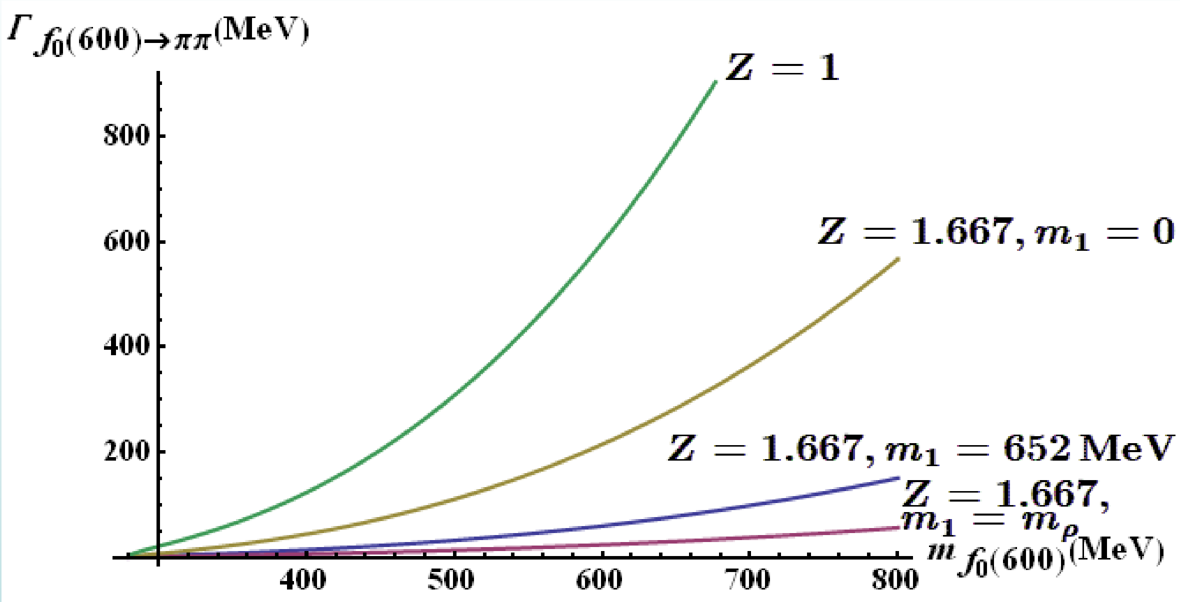
Systematic studies of the phase diagrams of QCD

Thank You

for the

attention

arXiv:1003.4934 [hep-ph].



Z = 1 corresponds to the (unphysical) decoupling of axial-vector mesons.
The reason for the big change is technical: the a1-π mixing.

arXiv:1103.3238

Quantity	Our Value [MeV]	Experiment [MeV]
$G' \to \rho\rho \to 4\pi$	30	53.96 ± 7.06
$G' \to \eta\eta'$	0.8	2.07 ± 1.01
$\sigma' \to \pi\pi$	284 ± 95	-
$\sigma' \to \eta\eta$	72 ± 14	-
$\sigma' \to K\bar{K}$	4.6 ± 3.3	-
$\sigma' \to \rho\rho \to 4\pi$	0.1	-

Further results regarding the $\sigma' \equiv f_0(1370)$ and $G' \equiv f_0(1500)$ decays.

$f_0(1370)$ decays mostly in the pion-pion channel. (Exp. still ddebated)

Consistent phenomenological picture also above 1 GeV.

Phenomenology of ρ, a1,… is satisfatory

Global chiral symmetry and VMD

Here we follow a different way: we consider –as in QCD- only global chiral symmetry and we restrict to terms up to order four.

M. Urban, M. Buballa and J. Wambach**, *Nucl. Phys. A* 697, 338 (2002)**

D. Parganlija, F. G. and D. Rischke**, AIP Conf. Proc. 1030, 160 (2008)**

Denis Parganlija, F.G., Dirk H. Rischke, **Phys.Rev.D82:054024,2010**; **arXiv:1003.4934** [hep-ph].

We use another realization of VMD: the photon is first introduced via minimal substitution. The form factors of hadrons receive corrections from the (gauge-invariant) mixing term $\propto \rho_{\mu\nu} F^{\mu\nu}$

In this realization of VMD no need of universality. Details in:

[H. O' Connell, B. Pearce, A. Thomas and A. Williams, Prog.Part.Nucl.Phys.39:201-252,1997

Why only up to 4-th order

$$Tr\left[\left(\Phi^{+}\Phi\right)\right], \quad Tr\left[\left(\Phi^{+}\Phi\right)^{2}\right], \quad Tr\left[\left(\Phi^{+}\Phi\right)^{4}\right], \quad Tr\left[\left(\partial_{\mu}\Phi^{+}\partial^{\mu}\Phi\right)^{2}\right]$$

One could think that the reason is renormalizability! However, a low-energy theory of QCD does not need to be renormalizable.

Large-Nc arguments allow to neglect some, but not all higher order diagrams

The reason is: dilatation invariance of the interaction terms and finiteness of potential

$$V_{dilaton}(G) \propto G^{4}\left(\log\frac{G}{\Lambda}+\frac{1}{4}\right) \qquad G \to G_{0}+\Lambda .$$

Λ only dimensionful param.-which breas dilatation inv.- in the chiral limit!

$$V(G,\Phi) \propto G^{2}Tr\left[\Phi^{+}\Phi\right]+\ldots+G^{-4}Tr\left[\left(\partial_{\mu}\Phi^{+}\partial^{\mu}\Phi\right)\right]+\ldots.$$

Local vs global chiral symmetry

In many works: Local Chiral symmetry: $SU_R(2)xSU_L(2)$.

[S. Gasiorowicz and D. A. Geffen (1969), U. G. Meissner (1988), P. Ko and S. Rudaz (1994)]

In agreement with the VMD-Sakurai: $\propto \rho_\mu A^\mu$

Coupling universality. Small number of parameters

However: ρ decay and other observables are not correct.

To keep ‘local chiral symmetry’ further terms of higher orders must be included

Contributions to the rho mass

$$M_\rho^2 = \underbrace{\phi^2}_{\text{quark condensate}} (\ldots) + m_1^2 \qquad M_{a_1}^2 = M_\rho^2 + \phi^2 (\ldots)$$

The quark condensate decreases with T; the behavior of the rho mass strongly depends on the composition

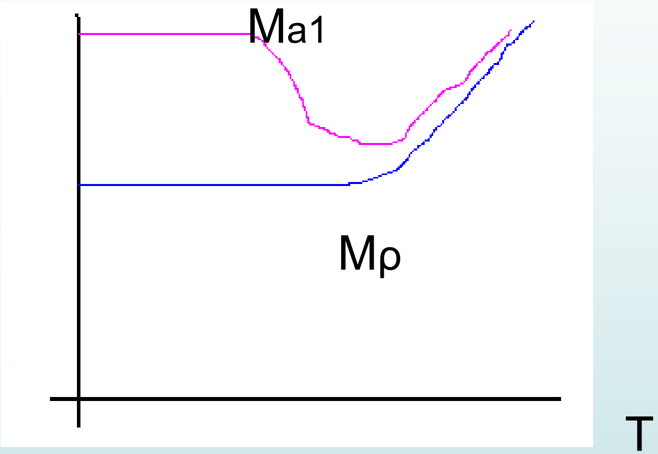
$$\text{In the local case}: \; M_\rho^2 = m_1^2$$

(no contribution of the quark condensate)

In general we recognize 3 scenarios

Digression: 3 scenarios for the ρ-meson at nonzero T

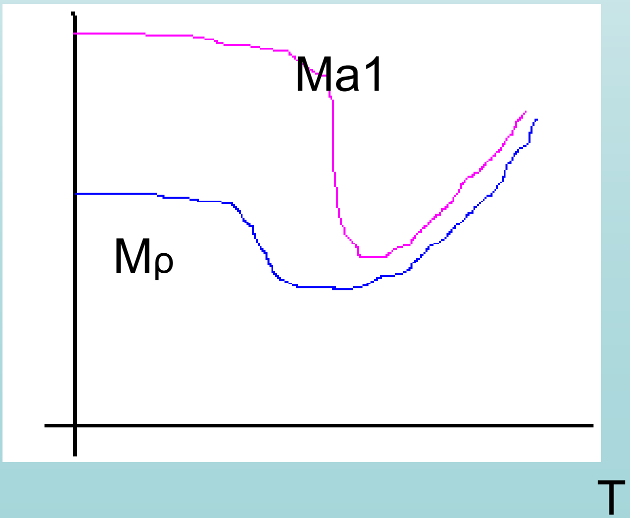
$$M_\rho^2 = \underbrace{\phi^2}_{\text{quark condensate}} (\ldots) + \underbrace{G_0^2}_{\text{gluon condensate}} (\ldots)$$



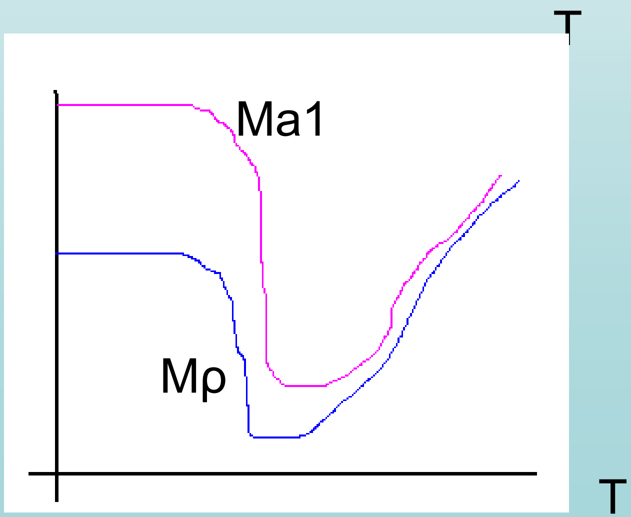
Case A: G0-term dominates

In our case: $\underbrace{G_0^2}_{\text{gluon condensate}} (\ldots) \approx (600\ \text{MeV})^2$

We expect case A to hold;
small drop of the masses in the medium



Case B: both terms are similar



Case C: the condensate dominates

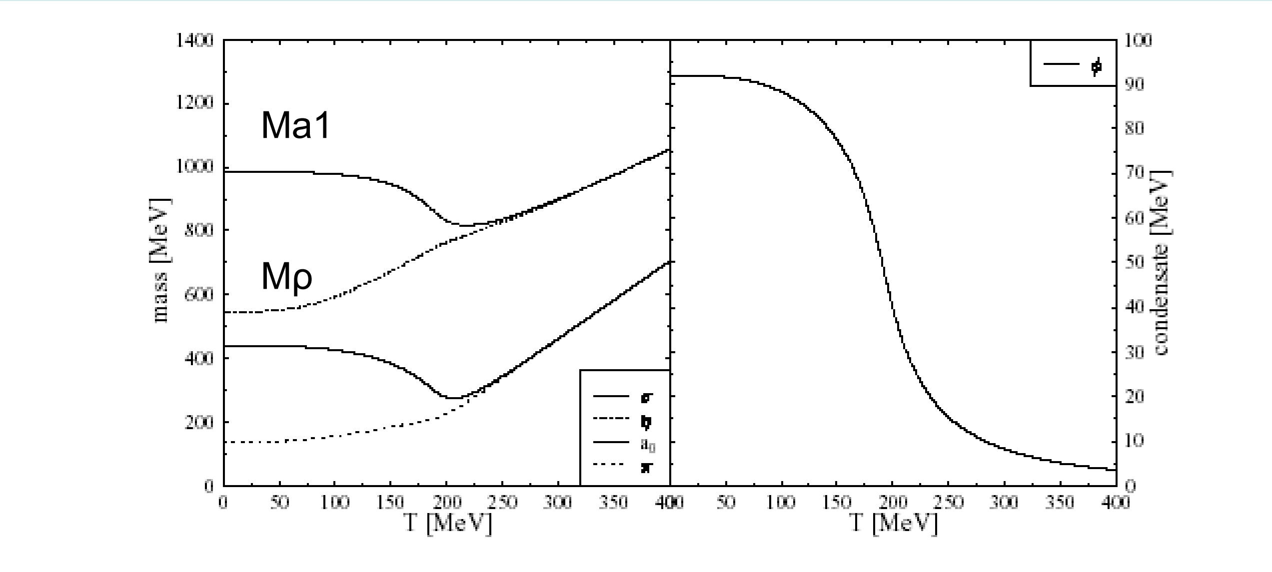
Explicit calculation in the local case

- Calculation in the chirally local case by: S. Strüber and D. Rischke, **Phys.Rev.D77:085004,2008**

- 2Pi (CJT) formalism: system of coupled Dyson-Schwinger eqs at 2 Pi

- In this work –although with the simplified local case- the first calculation at nonzero T has been performed.

- Role of vector mesons in the chiral phase transition important



$$M_\rho^2 = m_1^2$$

As expected –case A. We would like to do it in the global model. Outlook for the future,

Fixing the free parameters in Scenario I

In order to fix the parameters we have first to specify the assignment of the scalar mesons

$$\varphi \equiv f_0(600) \quad \text{and} \quad a_0 \equiv a_0(980)$$

This is the problematic assignment. Nevertheless, as also done in many studies we test it.

For the remaining 5 parameters use:

$$\rho \to \pi\pi \qquad a_1 \to \pi\gamma \qquad f_1 \to a_0\pi \qquad a_0^0 \qquad a_0^2$$

One can fix the parameters…BUT then the f0(600) is too narrow.