Hamilton Approach to QCD in Coulomb Gauge

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J. Heffner, M. Leder, M. Pak, C. Popovici ,

J. Pawlowski, A. Szczepaniak, A. Weber

Hamilton approach to QCD

- Introduction
- Yang-Mills theory at T=0
- Yang-Mills theory at finite T
- QCD
- Conclusions

Canonical Quantization of Yang-Mills theory

Weyl gauge: $A_0^a(x) = 0$ cartesian coordinates $A_i^a(x)$

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gauge invariant wave functionals: $\Psi[A]$ K. Johnson etal........

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more convenient: gauge fixing $\partial A = 0$

Hamiltonian approach to YMT in Coulomb gauge $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi^{\perp} J \Pi^{\perp} + B^2) + H_C \qquad \Pi^{\perp} = \delta / i\delta A^{\perp}$$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial) \qquad D = \partial + gA$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}J\rho$$

Coulomb term

color charge density: $\rho = -A^{\perp}\Pi^{\perp} + \rho_m$

$$\langle \Phi | \ldots | \Psi \rangle = \int_{\Lambda} DAJ(A)\Phi^*(A)\ldots\Psi(A)$$

$$H\Psi[A] = E\Psi[A]$$

Perturbation theory

D. Campagnari, H. R. & A. Weber, Phys. Rev D(2009)

- Rayleigh-Schrödinger PT

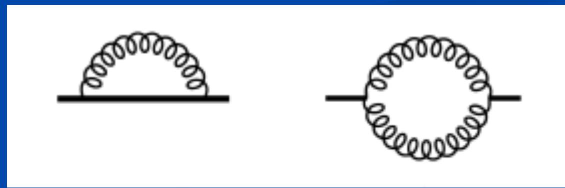
$$\widetilde{H} = H_0 + g\widetilde{H}_1 + g^2\widetilde{H}_2 + \mathcal{O}(g^3) \, ,$$

$$\left[|0\rangle + g\,|0\rangle^{(1)} + g^2\,|0\rangle^{(2)} + \mathcal{O}(g^3) \right] .$$

  - vacuum (QED)

$$\langle A|0\rangle = \mathcal{N} \exp\left\{ -\frac{1}{2} \int đk\, A^a_\sigma(\mathbf{k})\, |\mathbf{k}|\, A^a_\sigma(-\mathbf{k}) \right\} .$$

- ß-function



$$v(\mathbf{k}) = 1 + g^2 \frac{N_c}{(4\pi)^{2-\varepsilon}} \left\{ \frac{11}{3} \left[\frac{1}{\varepsilon} - \gamma - \ln\frac{\mathbf{k}^2}{\mu^2} \right] + \frac{31}{9} + \mathcal{O}(\varepsilon) \right\} .$$

$$\beta(g) = \frac{\partial g}{\partial \ln \mu} = \frac{1}{(4\pi)^2} \beta_0\, g^3 + \mathcal{O}(g^5) \, ,$$

$$\beta_0 = -\tfrac{11}{3} N_C$$

Variational approach

- Gaussian ansatz,

$$\Psi(A) = \exp\left[-\tfrac{1}{2}\int dx\, dy A(x)\omega(x,y)A(y)\right]$$

D. Schütte 1984

....

A. Szczepaniak & E. Swanson 2002

C. Feuchter & H. R. 2004
-ansatz
-FP determinant
-renormalization

- Greensite, Matevosyan,Olejnik,Quandt, Reinhardt, Szczepaniak,PRD83

Variational approach

- trial ansatz

C.Feuchter & H. R. PRD70(2004)

$$\Psi(A) = \frac{1}{\sqrt{\mathrm{Det}(-D\partial)}} \exp\left[-\tfrac{1}{2}\int dx dy A(x)\omega(x,y)A(y)\right]$$

- : gluon propagator $\langle A(x)A(y)\rangle = (2\omega(x,y))^{-1}$

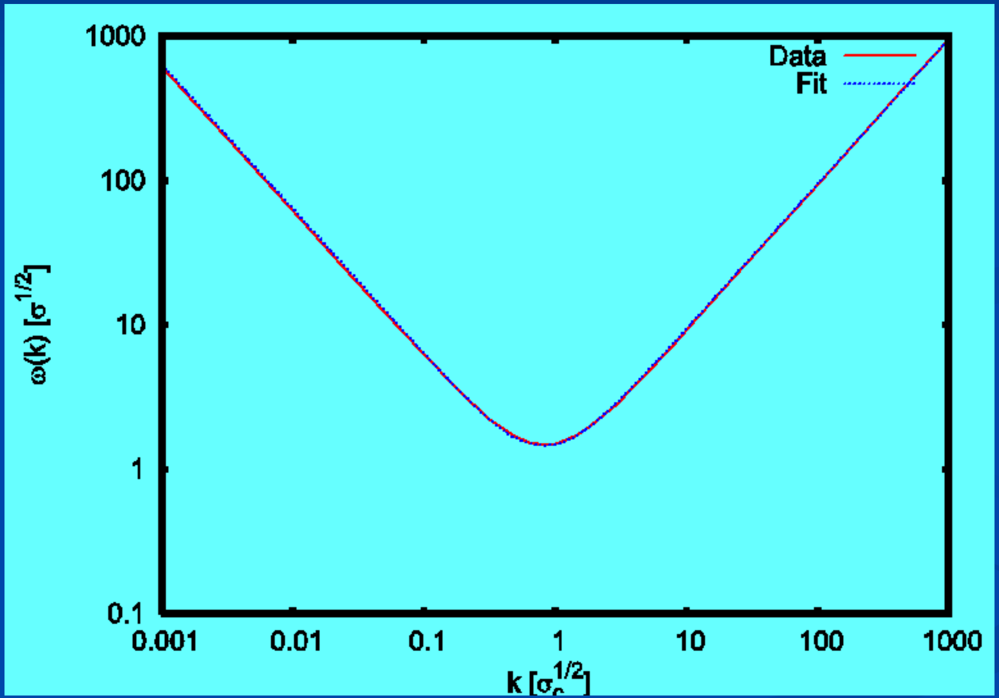
variational kernel $\omega(x,x')$ determined from

$$\langle\Psi|H|\Psi\rangle \to \min$$

Numerical results

D. Epple, H. R., W.Schleifenbaum,
PRD 75 (2007)

gluon energy



$$IR: \omega(k) \sim 1/k, \qquad UV: \omega(k) \sim k,$$

Static gluon propagator in D=3+1

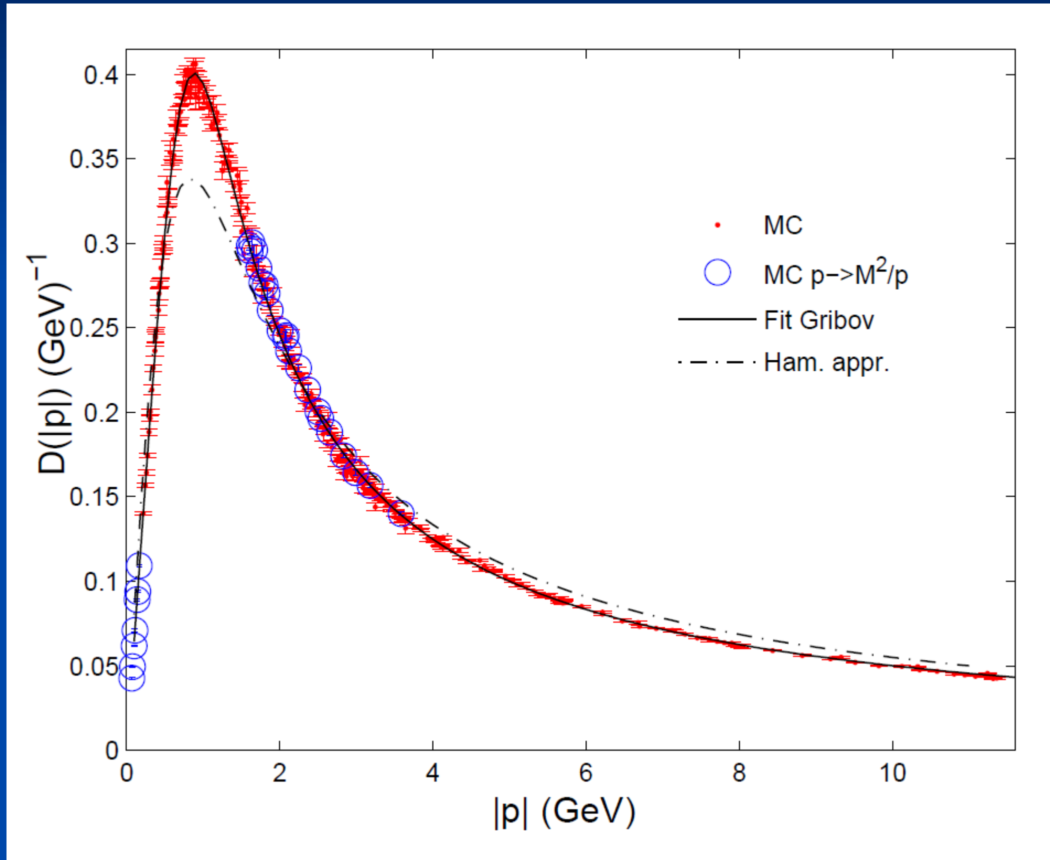
$$D(k) = (2\omega(k))^{-1}$$

Gribov's formula

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88GeV$$

missing strength in
mid momentum regime:
missing gluon loop



G. Burgio, M.Quandt , H.R., **PRL102(2009)**

Variational approach to YMT with non-Gaussian wave functional

D. Campagnari & H.R,
Phys.Rev.D82(2010)
See talk by D. C.

wave functional

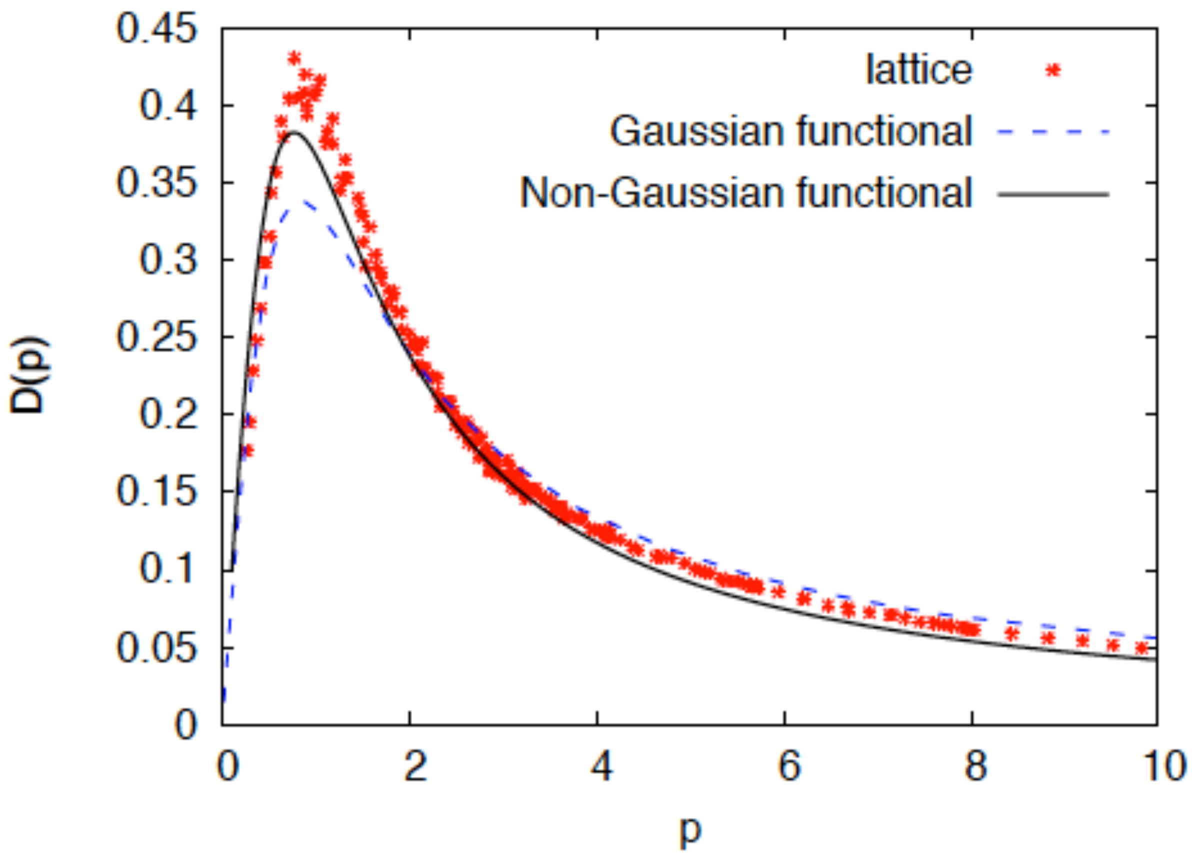
$$|\psi[A]|^2 = \exp(-S[A])$$

ansatz

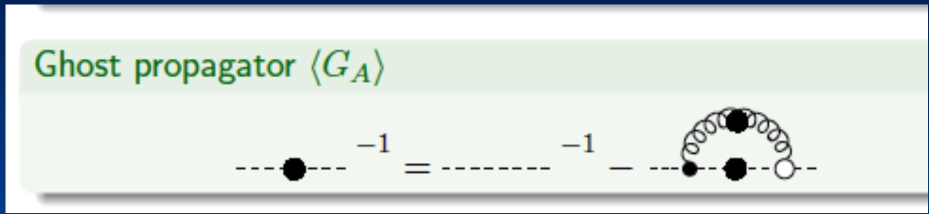
$$S[A] = \int \omega A^2 + \tfrac{1}{3!}\int \gamma^{(3)} A^3 + \tfrac{1}{4!}\int \gamma^{(4)} A^4$$

exploit DSE

Corrections to the gluon propagator

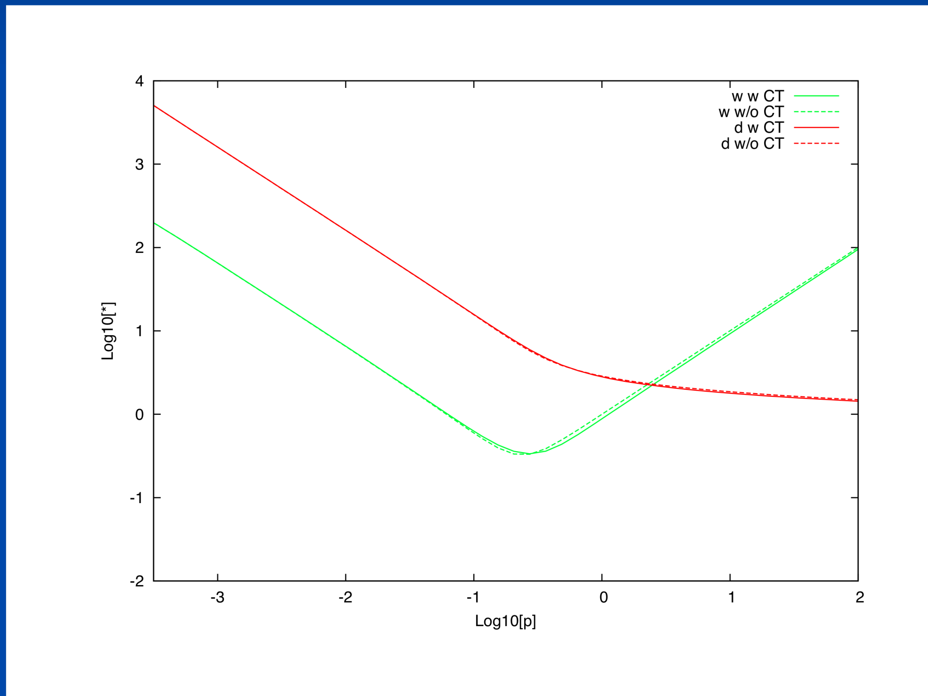


ghost propagator : $\left\langle (-D\partial)^{-1} \right\rangle = d(k)/k^2$



horizon condition $d^{-1}(0) = 0$

supported by the lattice

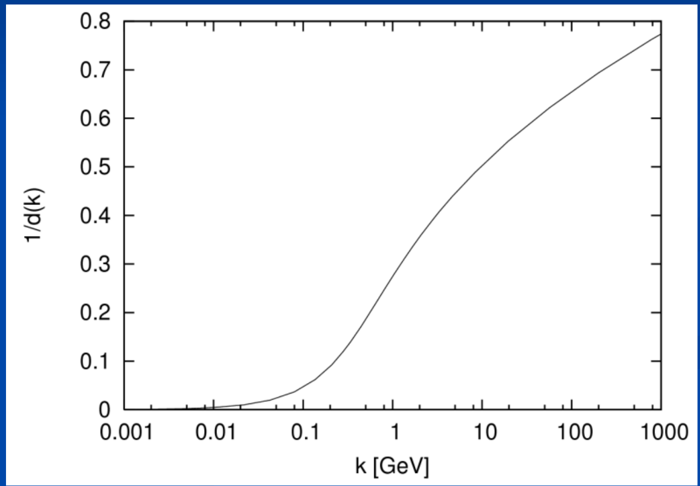


The color dielectric function of the QCD vacuum

- ghost propagator $\left\langle (-D\partial)^{-1} \right\rangle = d/(-\Delta)$
- dielectric „constant“

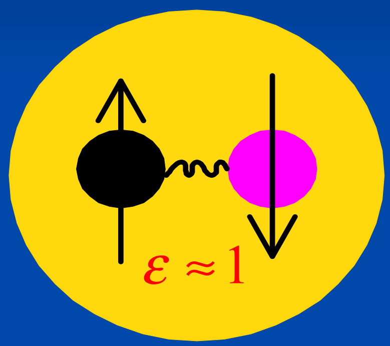
$$\varepsilon = d^{-1}$$

H.R. PRL101 (2008)

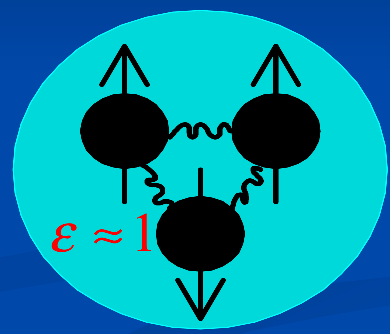


- horizon condition:
  - : $d^{-1}(k=0)=0$ $\varepsilon(k=0)=0$
- QCD vacuum: perfect color dia-electricum
  - dual superconductor
    - : $\varepsilon(k)<1$ anti-screening

$$D = \varepsilon E \qquad \partial D = \rho_{free}$$



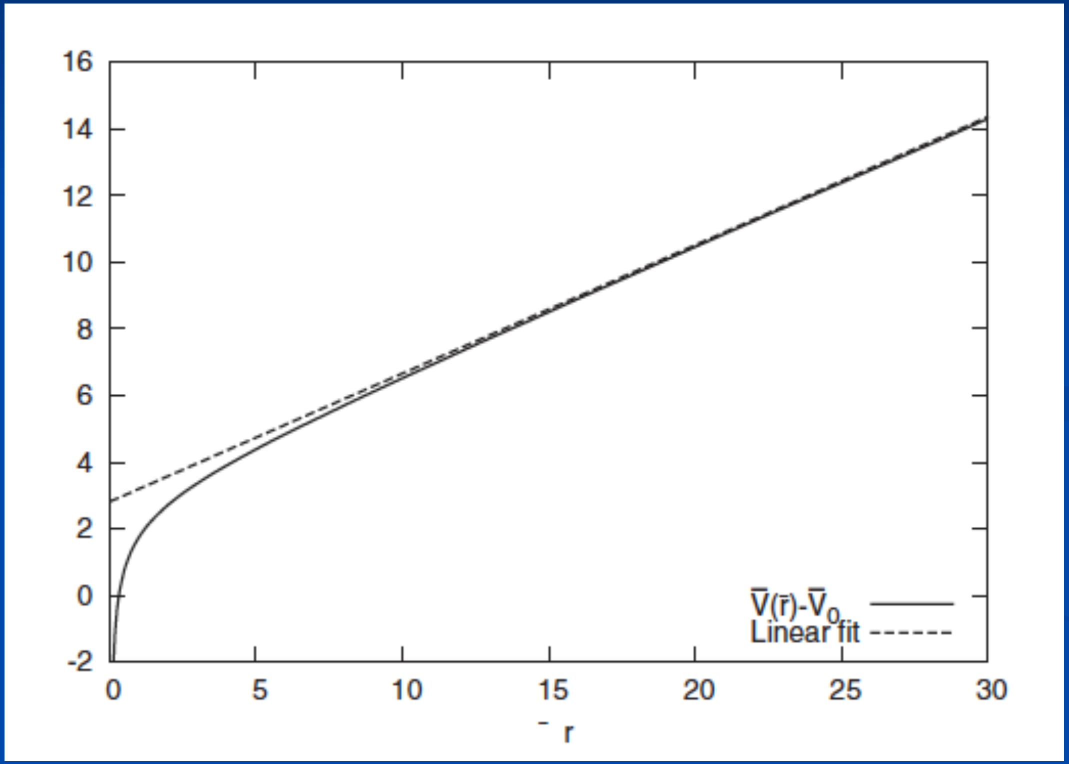
$$\varepsilon = 0$$



no free color charges in the vacuum: confinement

Static Coulomb potential

$$V(|x-y|) = g^2 \left\langle \langle x | (-D\partial)^{-1} (-\partial^2)(-D\partial)^{-1} | y \rangle \right\rangle$$



D. Epple, H. Reinhardt
W.Schleifenbaum,
PRD 75 (2007)

$$V(R) = \xrightarrow[R\to\infty]{} \sigma_C R,$$

lattice $\sigma_C = 2...3\sigma_W$

$$V(R) = \xrightarrow[R\to 0]{} \sim 1/R$$

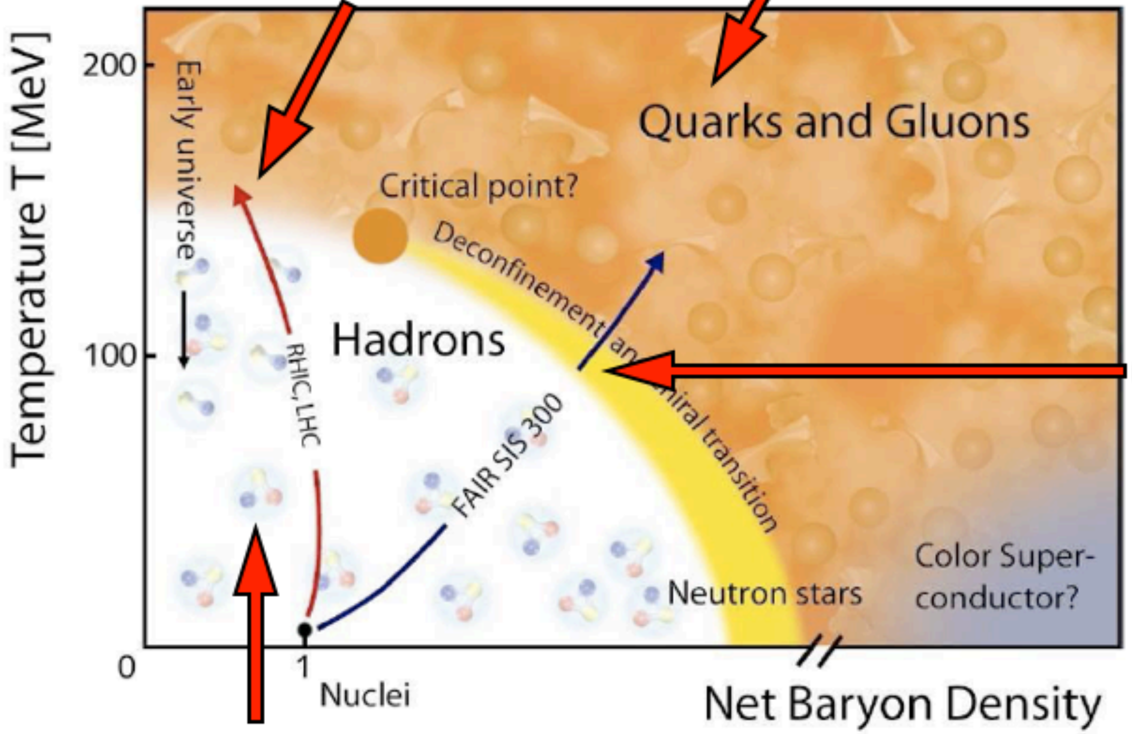
Phase diagram of QCD

Strongly correlated quark-gluon-plasma

'RHIC serves the perfect fluid'

massless quarks (chiral symmetry)

deconfinement



FAIR, www.gsi.de

quarkyonic:

confinement & chiral symmetry?

hadronic phase

confinement & chiral symmetry breaking

Hamiltonian approach to YMT at finite T

Reinhardt, Campagnari, Szczepaniak, PRD,2011
Heffner, Reinhardt, Campagnari, to be published

- Grand canonical ensemble with $\mu = 0$
  - density matrix

$$D = \exp(-\beta H) \qquad \beta = 1/k_B T$$

  - thermal averages

$$\langle \ldots \rangle_T = \frac{Tr(D\ldots)}{TrD}, \qquad Tr(\ldots) = \sum_k \langle \tilde{k} | \ldots | \tilde{k} \rangle$$

  - complete basis $|\tilde{k}\rangle$

Grand canonical ensemble

- choice of basis

$$\left|\tilde{k}\right\rangle = \frac{1}{\sqrt{Det(-D\partial)}}\left|k\right\rangle$$

gauge field

$$A(k) = \frac{1}{\sqrt{2\omega(k)}}(a(k) + a^{\dagger}(-k)), \qquad \omega(k) - arbitrary$$

vacuum

$$a(k)\left|0\right\rangle = 0, \qquad \left\langle A\middle|0\right\rangle = \exp(-\tfrac{1}{2}\int A\omega A)$$

complete basis

$$\left|0\right\rangle, \quad a^{\dagger}(k)\left|0\right\rangle, \quad a^{\dagger}(k)a^{\dagger}(k)\left|0\right\rangle, \ldots.$$

Grand canonical ensemble

- density operator $D = \exp(-\beta H)$

$$D = \exp(-\beta \tilde{H}), \qquad \tilde{H} = \int dk \Omega(k) a^\dagger(k) a(k)$$

- occupation numbers

$$\langle a^\dagger a \rangle \equiv n(k) = [\exp(\beta\Omega(k)) - 1]^{-1}$$

  - gluon propagator $\langle AA \rangle = (1+2n)/(2\omega)$

- entropy $S = -k_B Tr(D \ln D)$
- free energy $F[\Omega,\omega] = \langle H \rangle_T - TS$

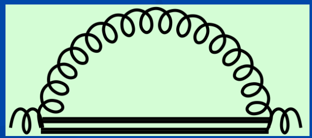
Finite T variational principle

$$n(k) = [\exp(\beta\Omega(k) - 1]^{-1} \qquad \langle A|0\rangle = \exp(-\tfrac{1}{2}\int A\omega A)$$

- free energy $F[\Omega,\omega] = E[\Omega,\omega] - TS[\Omega] \to \min$
- variation w.r.t. the density operator

$$\delta F[n,\omega]/\delta n(k) = 0 \qquad \Rightarrow \quad \Omega(k) = \omega(k)(1 + I_C(k))$$

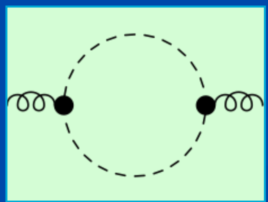
quasi-gluon energy

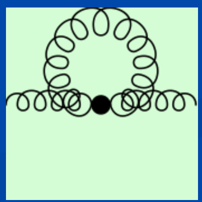


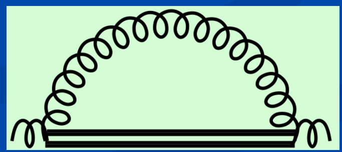
- variation w.r.t. the basis $\delta F[\Omega,\omega]/\delta\omega(k) = 0$

$$\omega^2(k) = k^2 + \chi^2(k) + I_{tad}[n] + I_C[n](k)$$

gap equation







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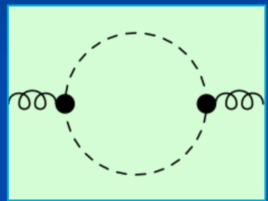
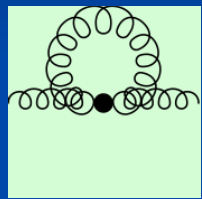
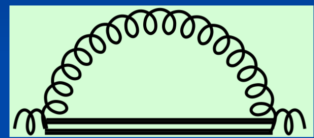
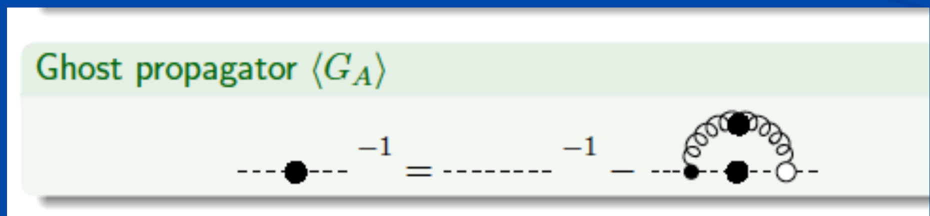






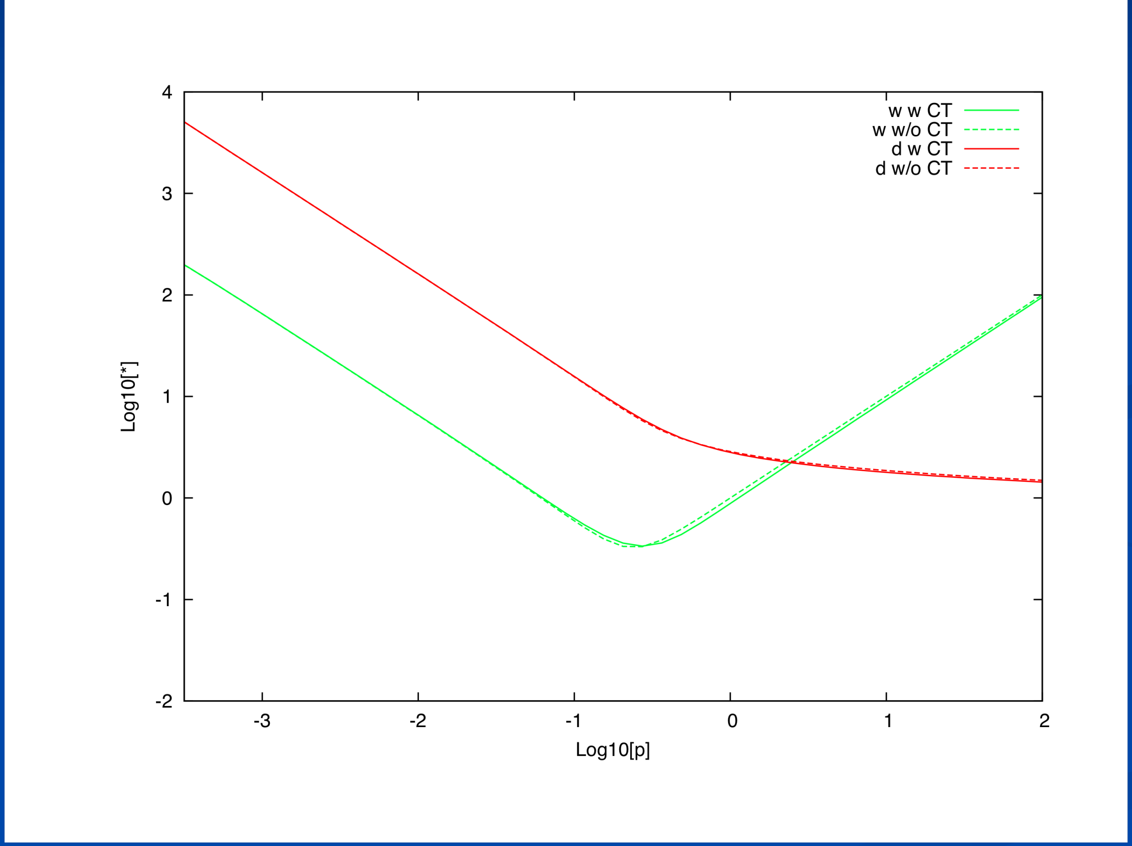
renormalization as at T=0



The influence of the Coulomb term

$$H_C = \frac{1}{2}\int J^{-1}\Pi J^{\|}\Pi^{\|} = \frac{1}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}J\rho$$

color charge density: $\rho = -A^{\perp}\Pi^{\perp} + \rho_m$



–irrelevant in the YM sector

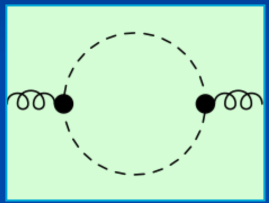
–utterly important for quarks:

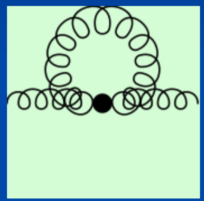
confining potential

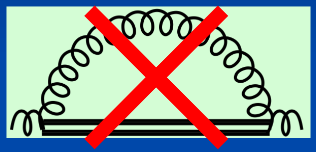
Without Coulomb term

Heffner, Reinhardt & Campagnari

$$\omega^2(k) = k^2 + \chi^2(k) + I_{tad}[n] + I_C[n](k)$$







quasi-gluon energy

$$\Omega(k) = \omega(k)$$

vacuum $\langle A|0\rangle = \exp(-\frac{1}{2}\int A\omega A)$

occupation number $n(k) = \left[\exp(\beta\Omega(k)) - 1\right]^{-1}$

Infrared analysis

gluon energy $\omega(p) = A / p^{\alpha}$ ghost form factor $d(p) = B / p^{\beta}$

$T = 0$ *sum rule* $\alpha = 2\beta + 2 - d$

$d = 3$ $\beta = 1.0(0.99)$ $\beta = 0.796(0.85)$

$d = 2$ $\beta = 0.5(0.45)$

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$T \to \infty$ $n(k) \simeq 1 / \beta\omega(k)$

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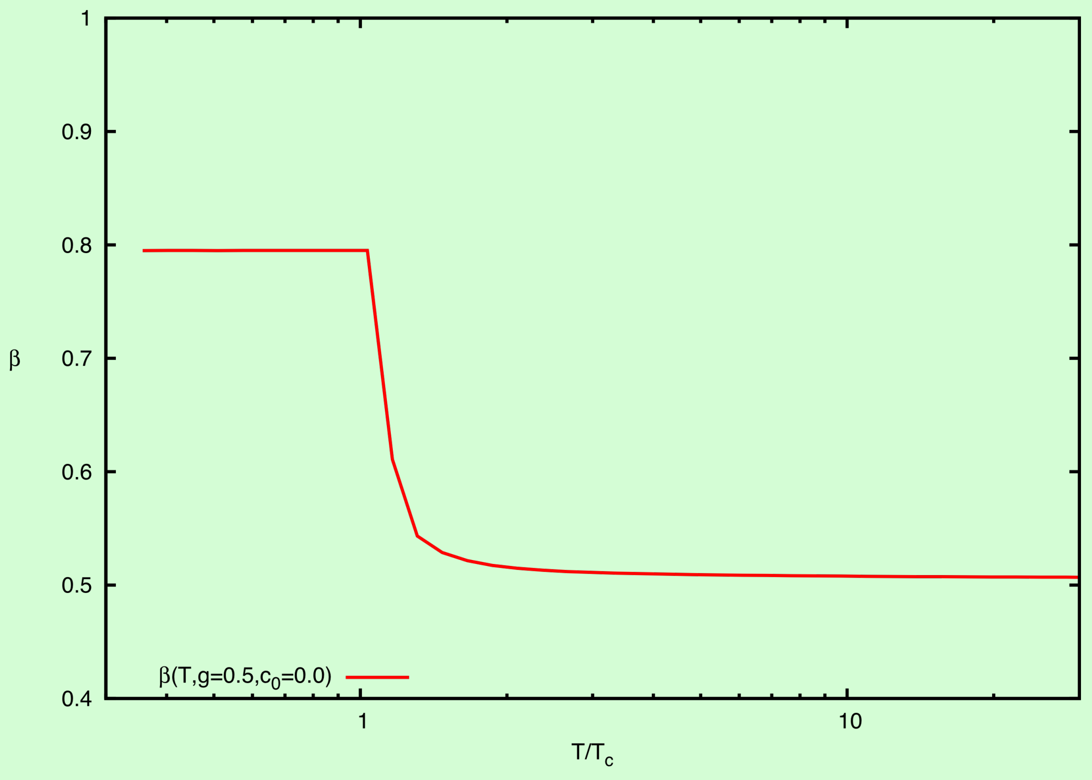
sum rule $\alpha = 2\beta + 2 - d$

$d = 3$ $\beta = 0.5$ $\alpha = 0$

Infrared exponent of ghost



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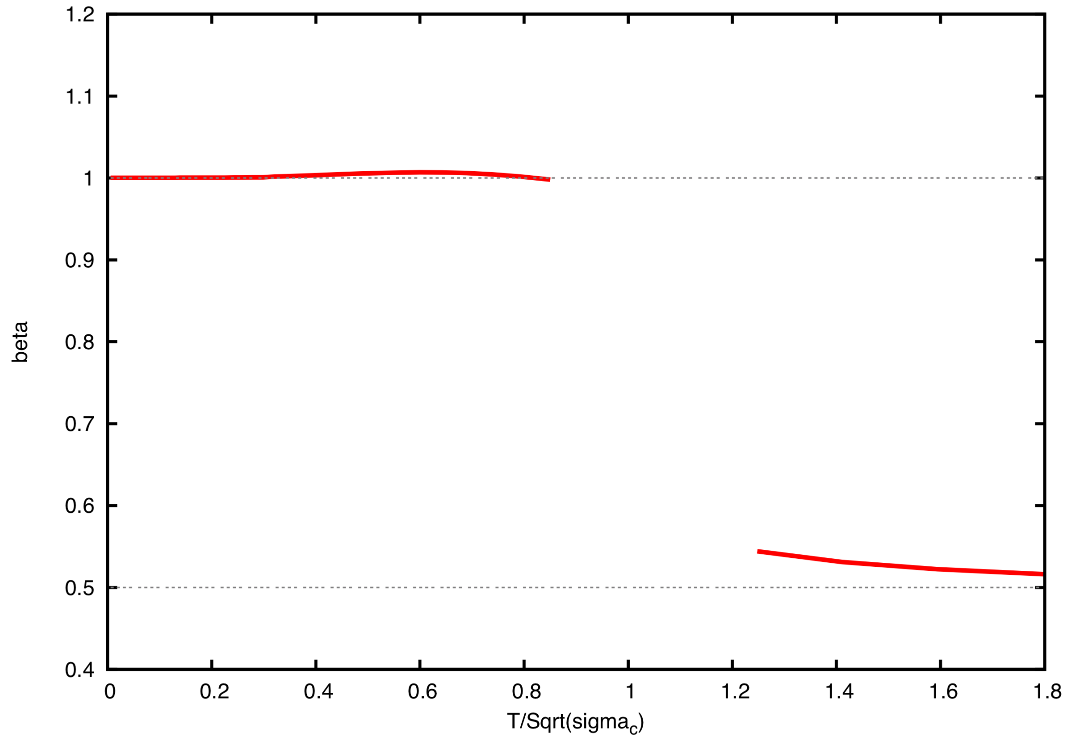
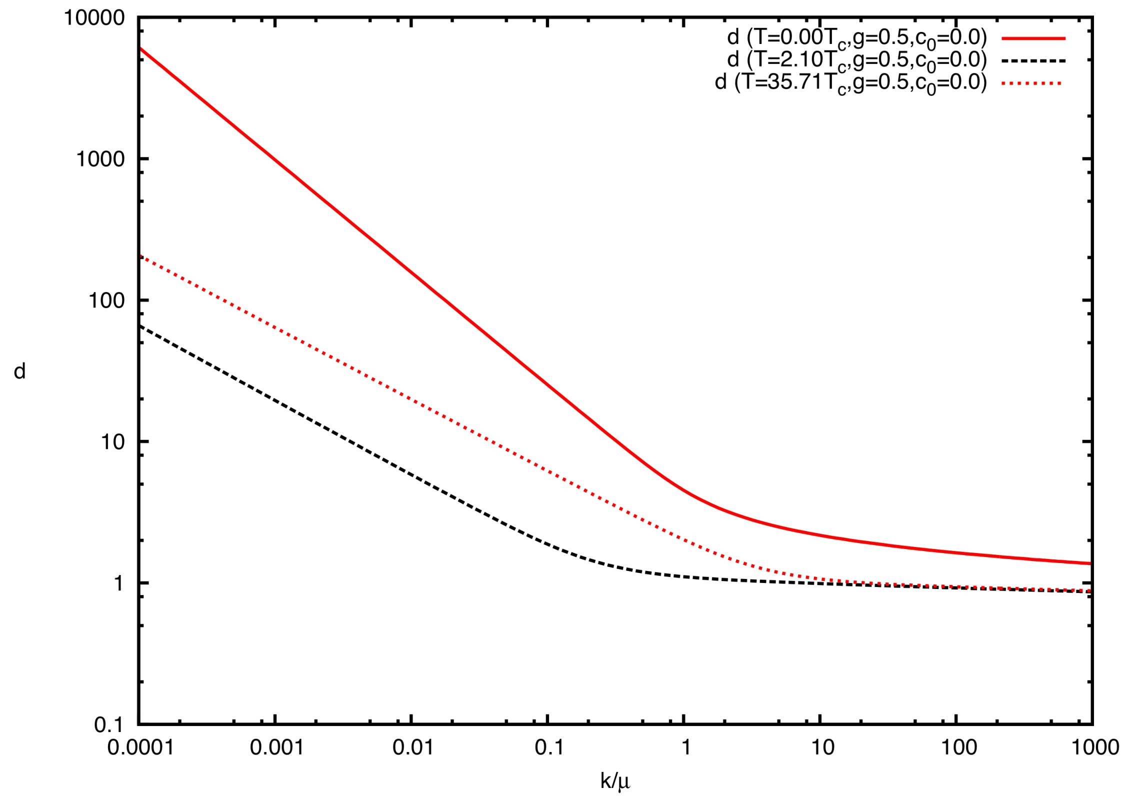
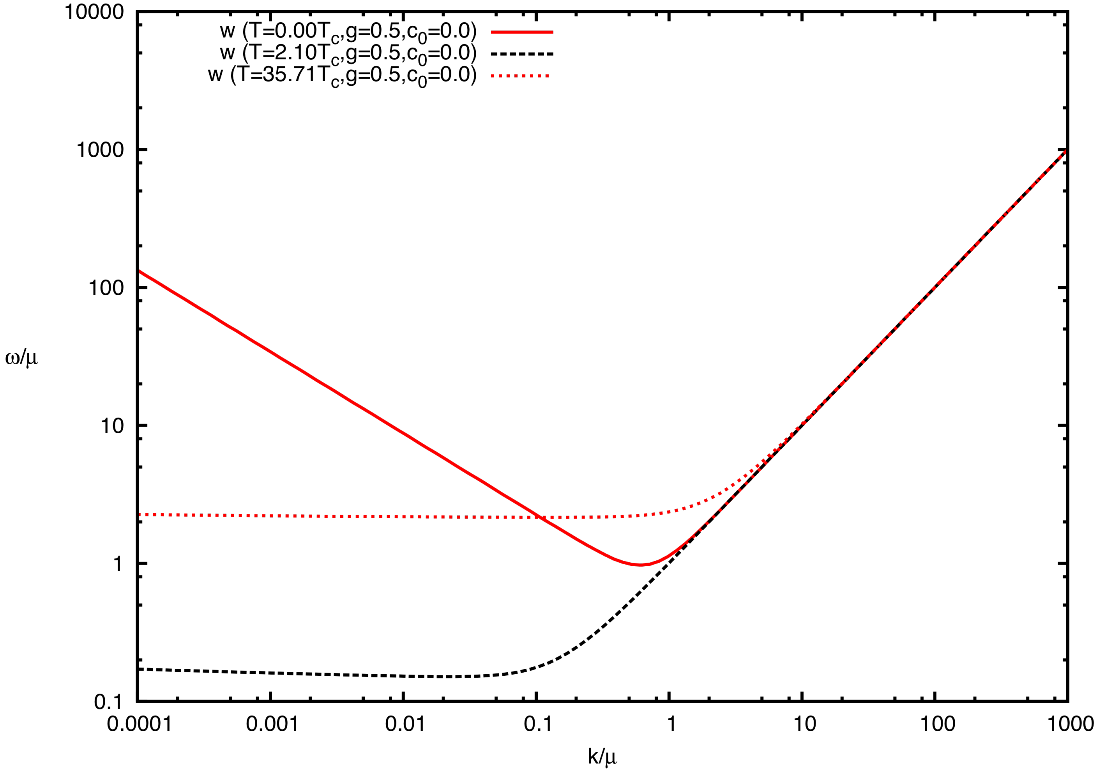


Ghost form factor

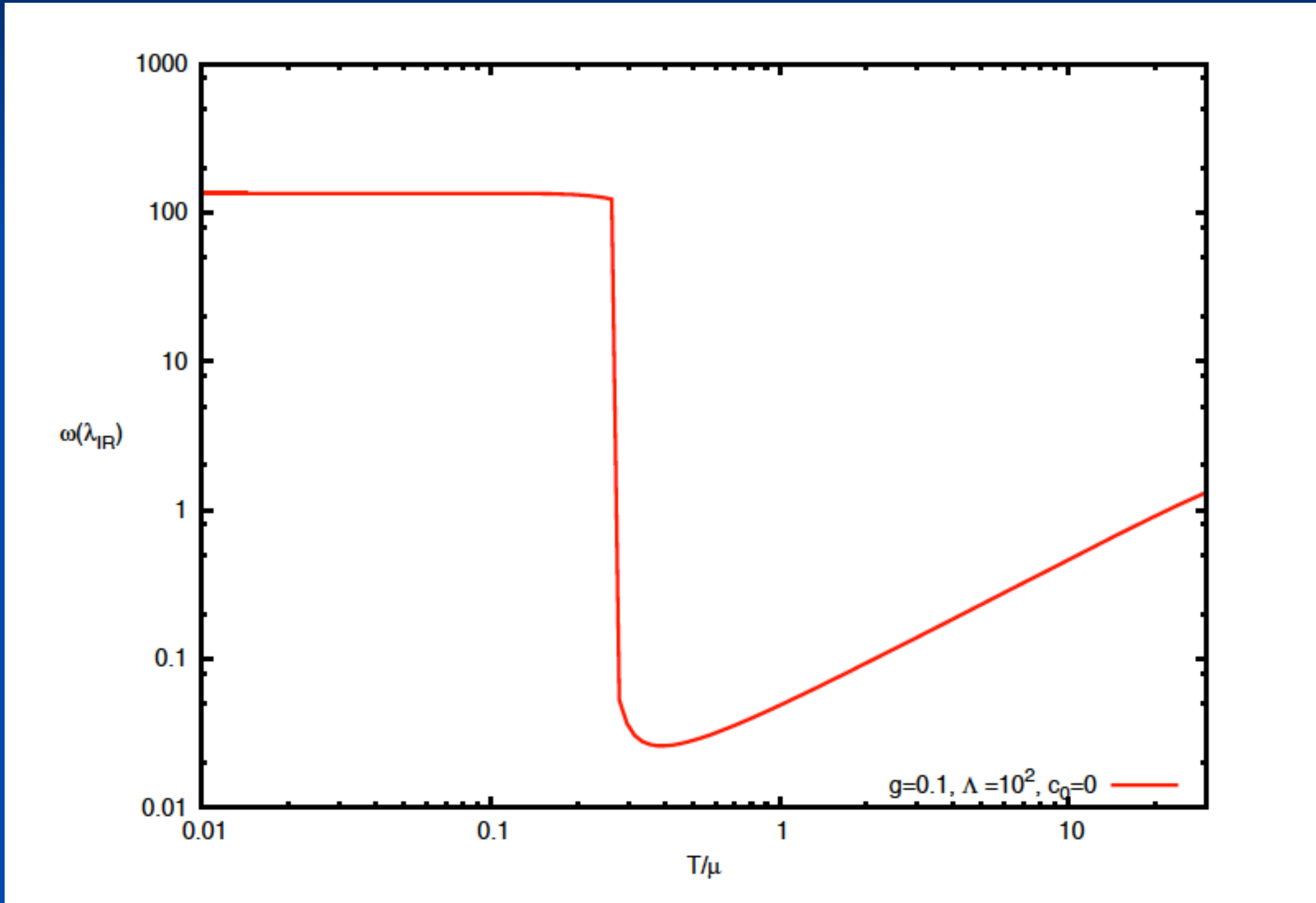


Gluon energy



Effective gluon mas (IR-value of gluon energy)

$$m(T) = \omega(k = \lambda_{IR}, T)$$



Critical temperature

$T_C = 162\,MeV\sqrt{\sigma_C / \sigma_W}$ $\sigma_C - Coulomb\ string\ tension$

$\sigma_W - Wilson\ string\ tension$

$SU(2) - lattice : \sigma_C = 2.....3\sigma_W$

$\Rightarrow\ T_C = 230......280\,MeV$

$SU(2) - lattice : T_C = 295\,MeV$

Effective gluon mas (IR-value of gluon energy)

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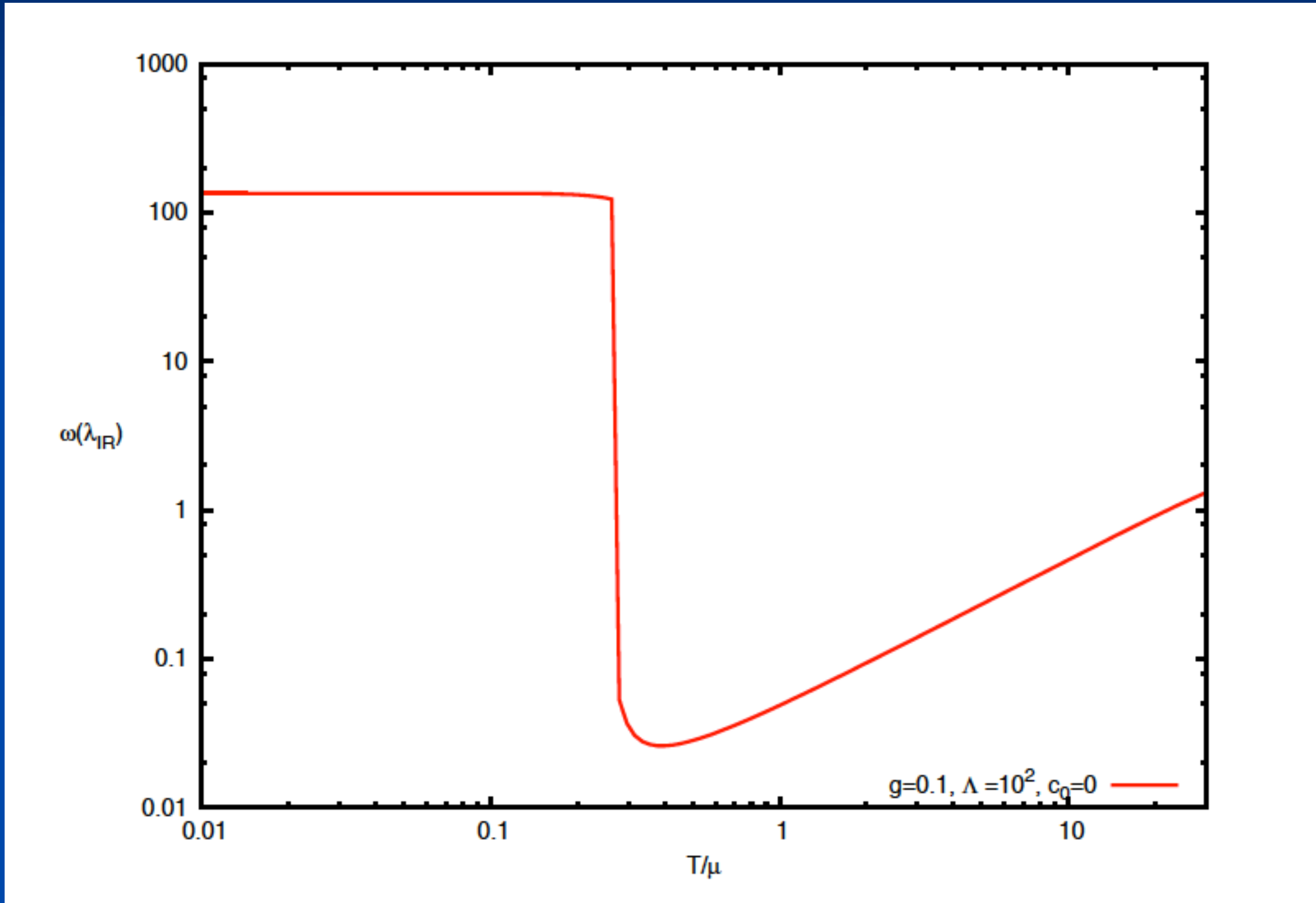

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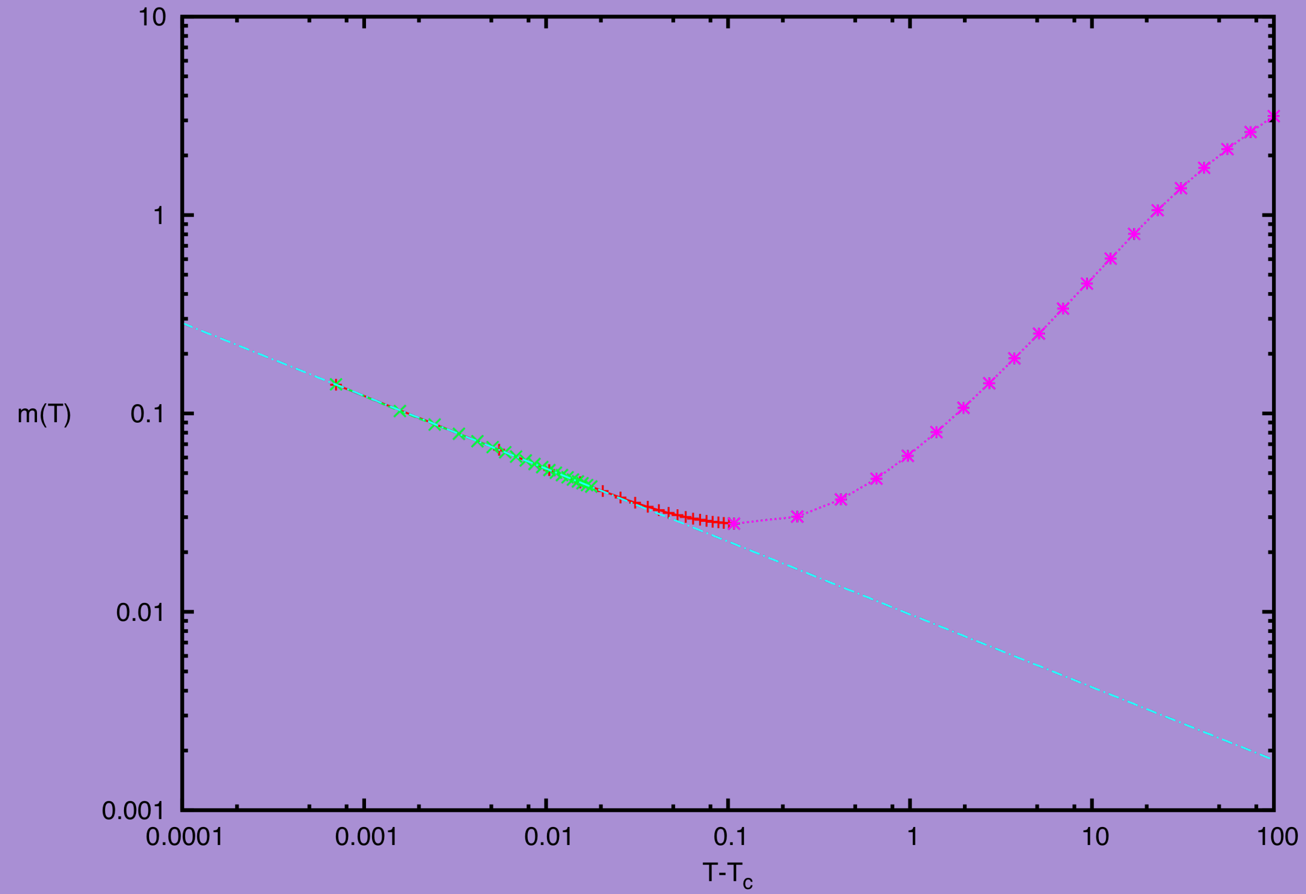

Critical exponent

critical behaviour of effective gluon mass near T_C :

$m_{crit}(T) \sim (t-1)^{-\kappa} \qquad t = T/T_C \qquad\qquad \kappa \simeq 0.37$

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$m_{crit}(T) \sim (t-1)^{-\kappa} \qquad t = T/T_C \qquad\qquad \kappa \simeq 0.37$

quasi-gluon picture (Castorina, Miller, Satz 2011):

$\kappa = (1+\eta-\nu)$

ν – critical exponent of the correlation length $\xi(T) \sim (t-1)^{-\nu}$

η – anomalous dimension exponent

SU(2) $\triangleq$ d=3 Ising model: $\nu \simeq 0.63,\ \eta \simeq 0.04, \qquad \kappa \simeq 0.41$

Hamiltonian approach to QCD in Coulomb gauge $\partial A = 0$

$$H = H_{YM} + H_C + H_q$$

Christ and Lee

$$H_{YM} = \tfrac{1}{2}\int (J^{-1}\Pi^{\perp} J \Pi^{\perp} + B^2)$$

$$J(A^{\perp}) = Det(-D\partial) \qquad D = \partial + gA$$

$$\mathrm{H_C} = \tfrac{1}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}J\rho$$

color charge density: $\rho^a = (-\hat{A}^{\perp}\Pi^{\perp})^a + \Psi^{\dagger}(x)t^a\Psi(x)$

$$\mathrm{H_q} = \int \Psi^{\dagger}(x)[\alpha(p+gA)+\beta m_0]\Psi(x)$$

Quark wave functional

M. Pak & H. R. 2011

$$quark\ w.f.\ (Thoules\ theorem)$$

$$\langle A | \Phi \rangle_q = \exp\left[\int \Psi^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A}) \Psi \right] | 0 \rangle$$

$v = 0:\quad BCS - wave\ function\ (Adler\ \&\ Davis)$

$v \neq 0:\ quark - gluon\ coupling$

Quark condensate

$without\ quark-gluon\ coupling(\nu=0)$

$$\langle\bar{q}q\rangle=(-113MeV\sqrt{\sigma_C/\sigma_W})^3$$

$with\ quark-gluon\ coupling$

$$\langle\bar{q}q\rangle=(-135MeV\sqrt{\sigma_C/\sigma_W})^3 \quad 20\%-increase$$

$lattice: \quad \sigma_C/\sigma_W=2...3$

$$\langle\bar{q}q\rangle=(-191....234MeV)^3$$

$\exp: \quad \langle\bar{q}q\rangle=(-230MeV)^3$

Constituent quark mass

$without\ \ quark-gluon\ \ coupling(\mathrm{v}=0)$

$$\mathrm{m}=84\,MeV\sqrt{\sigma_C/\sigma_W}$$

$with\ \ quark-gluon\ \ coupling$

$$\mathrm{m}=132\,MeV\sqrt{\sigma_C/\sigma_W}\qquad 57\%-increase$$

$lattice:\qquad \sigma_C/\sigma_W=2...3$

$$\mathrm{m}=187....230\,MeV$$

Hamiltonian approach to QCD in Coulomb gauge

- variational approach toYMT at T=0
  - decent description of IR sector

    $\omega(k) \sim k^{-\alpha}$ $\quad d(k) \sim k^{-\beta}$ $\quad \alpha = 2\beta - 1$ $\quad \beta = 1$ $\quad \beta = 0.8$
  - dual superconductor
- FRG flow equation
  - similar results as in variational approach
- YMT at finite T
  - deconfinement phase transition $\quad \beta = 1/2$
  - critical temperature $\quad T_C = 230 \ldots 280\, MeV$
- QCD
  - spontaneous breaking of chiral symmetry
  - quark condensate: $\quad \langle \bar{q}q \rangle = (-191 \ldots 234\, MeV)^3$

Thanks for your attention