Chiral Symmetry Restoration and Isospin Breaking

Ricardo Torres Andrés
Universidad Complutense, Madrid
rtandres@fis.ucm.es

EFFECTIVE FIELD THEORY GROUP
UCM, Madrid

Outline

A. Gómez Nicola, R. Torres Andrés:

Phys. Rev. D 83, 076005 (2011) and **arXiv:1009.2170v2[hep-ph]**

Quick review of Chiral Perturbation Theory and isospin breaking.

What kind of information can we extract? Bounds for the EM LEC and the role of the $\eta - \pi^0$ mixing in the flavour-breaking scalar susceptibilities, χ.

ChPT

Quick review of ChPT (I)

- EFT for low energy QCD based on the SSB of chiral symmetry

$$\mathrm{SU}_V(N_f) \text{ x } \mathrm{SU}_A(N_f) \to \mathrm{SU}_V(N_f), \quad N_f = 2,3$$

J. Gasser and H. Leutwyler, Annals Phys. **158**, (1984) 142.

- Describes NGB fields in the exponential representation $U = e^{i\phi/F}$, where the field matrices are

$$SU(2) : \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$SU(3) : \Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & \frac{-2}{\sqrt{3}}\eta \end{pmatrix}$$

transforming under chiral rotations in this way

$$U \to LUR^\dagger, \quad L \in SU_L(N_f), R \in SU_R(N_f)$$

Quick review of ChPT (II)

- $\mathcal{L}_{eff}$ consists of all the possible terms compatible with QCD symmetries: C,P,T, Lorentz and χSym
- Each term is accompanied by a LEC, which has to be determined experimentally
- Systematic expansion in even powers of $\frac{p}{\Lambda_\chi}$ (masses, external momentum), with $\Lambda_\chi \sim 4\pi F \sim 1$ GeV

$$\mathcal{L}_{eff}(U, \partial U, \partial^2 U, \dots) = \mathcal{L}_2 + \mathcal{L}_4 + \dots$$

- The contribution of a given diagram is controlled by Weinberg's power counting Theorem which depends on the number of loops, and the number of vertices coming from the different lagrangian terms.
- It has a finite number of counterterms to a given order, which means that ChPT is renormalizable order by order for a given proccess.

LO lagrangian NLO lagrangian

Quick review of ChPT (III)

Masses and charges

ChPT can incorporate both sources of IB through one vectorial and one scalar external fields

$$\boxed{\mathrm{v}_\mu = -QA_\mu, \quad s = M}$$

being

$$M = \mathrm{diag}(m_u, m_d) \oplus m_s, \qquad Q = \mathrm{diag}(\mathrm{e}_u, \mathrm{e}_d) \oplus \mathrm{e}_s$$

At LO, for example

$$\mathcal{L}_{p^2+\mathrm{e}^2} = \frac{F^2}{4}\,\mathbf{tr}\left[D_\mu U^\dagger D^\mu U + 2B_0\mathcal{M}\left(U + U^\dagger\right)\right] + C\,\mathbf{tr}\left[QUQU^\dagger\right]$$

with $D_\mu = \partial_\mu + iA_\mu[Q,\cdot]$, and C being the LEC that corrects the charged pion/kaon mass at tree level

$$M_\pm^2 - M_0^2 \sim 2\mathrm{e}^2\frac{C}{F^2} + \mathcal{O}(p^2, m_d - m_u)$$

NGB tree level masses

Isospin Breaking

Why do we study IB?

Motivation

Although $SU_V(N_f)$ is a good symmetry, IB is phenomenologically relevant at low energies

E.g: meson scattering with virtual photons, meson masses (corrections to Dashen's theorem), $a_0 - f_0$ mixing...

M. Knecht and R. Urech, Nucl. Phys. B **519**, 329 (1998).

U. G. Meiβner, G. Muller and S. Steininger, Phys. Lett. B **406**, 154 (1997)

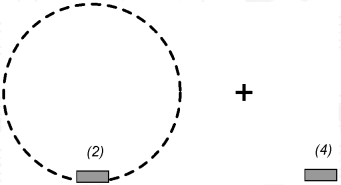
$\downarrow$

- Has IB anything to do with the T-scaling of the connected and disconnected susceptibilities at low temperature?
- The information about EM LEC $\{k_i, K_i\}$ is scarce$\rightarrow$ More info?

T=0 Quark Condensates

$$\text{NLO ChPT} \rightarrow \mathcal{L}_{eff} = \mathcal{L}_{e^2+p^2} + \mathcal{L}_{e^4+p^4+p^2e^2}$$

- $\mathcal{L}_{(2)}$ generates the meson loops. LEC:$\{l_i, L_i\}$, $\{h_i, H_i\}$
- $\mathcal{L}_{(4)}$ gives the counter-terms to renormalize. LEC: $\{k_i, K_i\}$ (we will use natural values for the numerical calculations)



$$\langle \bar{q}_i q_i \rangle = -F^2 B \left(1 + \begin{array}{c}\text{Meson}\\ \text{loops}\end{array} + \begin{array}{c}\text{Counter}\\ \text{terms}\end{array} + \begin{array}{c}\text{Explicit EM}\\ \text{breaking}\end{array} \right)$$

LO lagrangian
NLO lagrangian

$T = 0$, SU(3) condensates comparation

We can compare our results, which consider properly all the EM LEC contributions in the ChPT framework, with those of G. Amoros, J. Bijnens and P. Talavera, Nucl. Phys. B **602**, 87 (2001), [ABT] where the EM corrections come from violations of Dashen's Theorem

J. Bijnens and J. Prades, Nucl. Phys. B **490**, 239 (1997)

	Our results	[ABT]	Isospin limit
$-\langle \bar{u}u \rangle_0/(B_0 F^2)$	1.278	1.271	1.29
$-\langle \bar{d}d \rangle_0/(B_0 F^2)$	1.297	1.284	1.29
$-\langle \bar{s}s \rangle_0/(B_0 F^2)$	1.899	1.964	1.90
$\frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle} - 1$	0.015	0.013	0

Considering properly all the EM LEC contributions results in corrections of about 2 % in the light condensate, 4 % in the strange one and between 15 % and 24 % for the vacuum asymmetry.

▸ Numerical values ▸ Explicit calculation of the condensates

Finding bounds for the EM LEC

Finding bounds for the EM LEC

Let us calculate the object

$$r := \frac{\langle \bar{q}q \rangle^{e \neq 0}}{\langle \bar{q}q \rangle^{e=0}}.$$

EM corrections induce an explicit break of χSym, so IF we ASSUME a ferromagnetic behaviour for the vacuum, then $\langle \bar{q}q \rangle$ increases with the inclusion of mass and charge terms. But this means that r is constrained!

Ferromagnetic behaviour... $\rightarrow r \geq 1 \rightarrow$ EM LEC are constrained!

Finding bounds for $SU(2)$ and $SU(3)$ LEC

SU(2) LEC bound to NLO

$$5\left[k_5^R(\mu) + k_6^R(\mu)\right] + k_7^R \geq \frac{9C}{F^4}\nu_{\pi^0}$$

SU(3) LEC bound to NLO

$$6\left(K_7 + K_8^r(\mu)\right) + 5\left(K_9^r(\mu) + K_{10}^r(\mu)\right) \geq \frac{9C}{2F^4}\left(2\nu_{\pi^\pm} + \nu_{K^\pm}\right)$$

being C the LEC that modifies the charged meson masses at LO, and with the usual definition for ν_i given by $\nu_i := \frac{1}{32\pi^2}\left[1 + \log\frac{M_i^2}{\mu^2}\right]$

J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 465 (1985)

Interest

- More restrictive than using *natural values* (coming only from dimensional analysis) for the EM LEC

Thermal condensates

Two order parameters

Let us rewrite them in terms of $\hat{m} = \frac{1}{2}(m_u + m_d)$, $m_\delta = \frac{1}{2}(m_u - m_d)$, then

$$\langle \bar{u}u + \bar{d}d \rangle = \frac{1}{\beta V}\frac{\partial}{\partial \hat{m}} \log Z \quad \rightarrow \chi\textbf{Sym Order Parameter}$$
$$\langle \bar{u}u - \bar{d}d \rangle = \frac{1}{\beta V}\frac{\partial}{\partial m_\delta} \log Z \quad \rightarrow \textbf{IB Order Parameter}$$

Results (I)

SU(3) condensates receive corrections through $\pi^0 - \eta$ mixing angle $\varepsilon \sim \frac{m_d - m_u}{\hat{m} - m_s}$:

- $\mathcal{O}(\varepsilon^2)$ and $\mathcal{O}(e^2\varepsilon)$ in $\langle \bar{u}u + \bar{d}d \rangle$
- $\mathcal{O}(\varepsilon)$ in $\langle \bar{u}u - \bar{d}d \rangle$ $\rightarrow$ will be important later...

SU(2) IB order parameter $\langle \bar{u}u - \bar{d}d \rangle$ has no T contributions (pion loop corrections of u and d condensates cancel), while in SU(3)-ChPT it has.

Results (II)

In $M_\pi \ll T \ll M_K$, kaons and eta are exp. suppressed so pion contribution dominates SU(3) $\langle \bar{u}u - \bar{d}d \rangle$ and makes it grow with T

$$\langle \bar{u}u - \bar{d}d \rangle_T - \langle \bar{u}u - \bar{d}d \rangle_0 \sim B_0 \frac{2\varepsilon}{\sqrt{3}} g_1(M_{\pi^0}, T)$$

where $g_1(M,T) = \frac{1}{2\pi^2} \int_0^\infty dp \frac{p^2}{E_p} \frac{1}{e^{\beta E_p} - 1}$, $\quad E_p^2 = p^2 + M^2$, grows with T

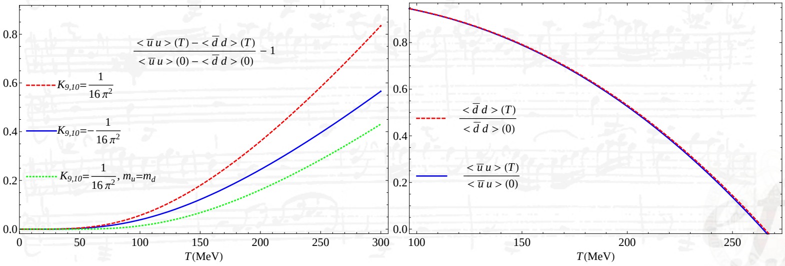


FIGURA: Left: $u-d$ condensate difference at finite temperature in $SU(3)$, relative to its $T=0$ value. Right: The two condensates separately.

Isospin breaking in χ_{con} and χ_{dis}

Connected, disconnected and total suscept.

It is possible to relate the connected and disconnected parts of χ in terms of the flavour breaking susceptibilities

$$\chi_{ij} = -\frac{\partial}{\partial m_i}\langle \bar{q}_j q_j \rangle_T = \frac{1}{\beta V}\frac{\partial^2}{\partial m_i \partial m_j}\log Z$$

A. V. Smilga and J. J. M. Verbaarschot, Phys. Rev. D 54 (1996) 1087

$$\chi = \chi_{uu} + \chi_{dd} + 2\chi_{ud}$$
$$\chi_{con} = \frac{1}{2}(\chi_{uu} + \chi_{dd}) - \chi_{ud}$$
$$\chi_{dis} = \chi_{ud}$$

▸ Con. and Dis. definitions

We can put them as derivatives of condensates with respect to masses

Remarks

$$\chi = -\frac{\partial}{\partial \hat{m}}\langle \bar{q}q \rangle_T, \qquad \chi_{con} = -\frac{1}{2}\partial_{m_\delta}\langle \bar{u}u - \bar{d}d \rangle$$
$$\chi_{dis} = -\frac{1}{4}\left[\partial_{\hat{m}}\langle \bar{u}u + \bar{d}d \rangle - 2_{m_\delta}\langle \bar{u}u - \bar{d}d \rangle\right]$$

When approaching chiral phase transition $(m_q, T) \rightarrow (0^+, T_c)$ (IR regime), it is important to know the scaling with m and T.

Disconnected

- Sensitive to χSym restoration
- Scales as $\chi_{dis}^{IR}(0) \sim \log M_\pi^2$ and $\chi_{dis}(T) \sim T/M_\pi$
  A. V. Smilga and J. J. M. Verbaarschot, Phys. Rev. D **54** (1996)

Connected

- Directly related to IB
- The behaviour is not so clear
  - its IR divergent piece is $\propto N_f^2 - 4$ (N_f = identical light flavors)
  - in staggered lattice formalism, χ_{con} receives "false"GB-like corrections coming from taste violation

  S. Ejiri *et al.*, Phys. Rev. D **80**, 094505 (2009).
  W. Unger [RBC-Bielefeld Collaboration], PoS **LAT2009**, 180 (2009).

Our results: IR behavior

Setting $m_u = m_d = \hat{m}$ and $e = 0$ after the calculation with IB ...

Disconnected part

$$\frac{\left[\chi_{dis}(T) - \chi_{dis}(0)\right]^{IR}}{B_0^2} = \frac{3T}{16\pi M_\pi}, \quad M_\pi \ll T \ll M_K$$

$$\frac{\chi_{dis}^{IR}(T=0)}{B_0^2} = -\frac{3}{32\pi^2}\log\frac{M_\pi^2}{\mu^2} + 32L_6^r(\mu) + \frac{1}{288\pi^2}\left(-28 + 5\log\frac{M_\eta^2}{\mu^2}\right)$$

- Coincides with A. V. Smilga and J. J. M. Verbaarschot, Phys. Rev. D **54** (1996)

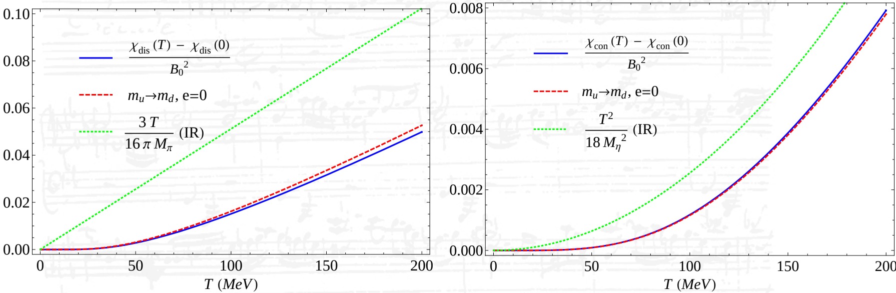
Connected part

$$\frac{\left[\chi_{con}(T) - \chi_{con}(0)\right]^{IR}}{B_0^2} = \frac{T^2}{18M_\eta^2}, \quad M_\pi \ll T \ll M_K$$

$$\frac{\chi_{con}^{IR}(T=0)}{B_0^2} = 8\left[H_2^r(\mu) + 2L_8^r(\mu)\right] - \frac{1}{16\pi^2}\left(1 + \log\frac{M_K^2}{\mu^2} + \frac{2}{3}\log\frac{M_\eta^2}{\mu^2}\right)$$

- Controlled by the mass scale $M_\eta^2 \leftarrow$ Directly related to IB ($\pi^0\eta$ mixing)

Temperature dependence.



- Connected behaves much softer than disconnected near the transition, growing with T^2 for low and moderate temperatures
- Isospin corrections are more important in the disc. part (differences between red and blue curves)
- IR limit gives qualitatively behavior near restoration, but numerically is not a good approx. for physical pion masses

Summary

Conclusions

EM LEC BOUNDS

- Scale independent bounds for certain combinations of EM LEC.

QUARK CONDENSATES AT FINITE T

- $\langle \bar{q}q \rangle_T$ (order param. of χSym) receives small IB corrections for phys. masses and electric charges (T_c is affected less than 1 %).
- $\langle \bar{u}u - \bar{d}d \rangle_T$ (order param. of IB) is T-independent in SU(2), but in SU(3) grows with T (in the chiral limit is given by $(m_d - m_u)T^2/M_\eta^2$,i.e controlled by the eta mass and not by the usual energy scale F^2). Anyway, this growth does not reflect in T_c of $\langle \bar{u}u \rangle_T$ and $\langle \bar{d}d \rangle_T$.

LIGHT SCALAR SUSCEPTIBILITIES AND THE ROLE OF THE $\pi^0 - \eta$ MIXING

- The terms in $\langle \bar{u}u - \bar{d}d \rangle_T$ linear in ε give contributions to the connected part of the suscept. which don't vanish in the IL and, therefore, can only be calculated setting IB from the beginning.
- In accordance with the behavior of the order parameters, χ_{dis} is IR divergent as T/M_π whereas χ_{con} is IR regular surviving the chiral limit as T^2/M_η^2.
- Although our small-mass analysis does not show the peaks expected in the transition, it allows to infer that the disc.part should have a more pronounced peak than the conn. one.

Thanks for your attention!