Kinematic effects in off-forward hard reactions

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based on

V.M. Braun, A.N. Manashov, arXiv:1108.2394 [hep-ph]

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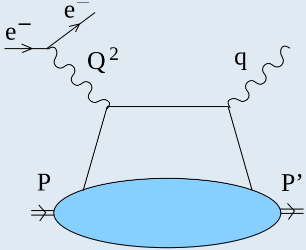
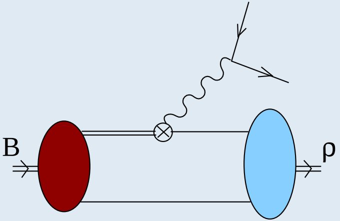
Hard exclusive processes involve off-forward matrix elements

DVCS: $\gamma^* P \to \gamma P'$



Form factors: $\gamma^* \pi \to \gamma$, $B \to \rho \ell \bar{\nu}_\ell$, ...



Operator Product Expansion

$$J(x)J(0) \sim \sum_N C_N(x^2,\mu^2)\,\mathcal{O}_N(\mu^2)$$

involves

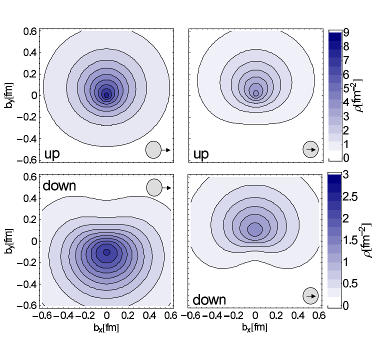
$$\langle P'|\mathcal{O}_N(\mu^2)|P\rangle \qquad \langle \rho(p)|\mathcal{O}_N(\mu^2)|0\rangle$$

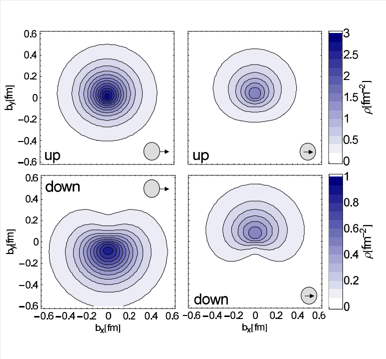
Kinematic variables: hadron mass m^2 momentum transfer $t = (P - P')^2$

How to calculate effects $\sim m^2/Q^2$ and t/Q^2?

Nucleon Tomography

access to three-dimensional picture of the nucleon (M. Burkardt)





$\hookrightarrow$ **first two moments of transverse spin parton density**

computer simulations: M. Göckeler *et al.*, Phys.Rev.Lett. 98 (2007) 222001

- **change of attitude: finite t a "nuisance" $\longrightarrow$ important tool**

Where is a problem?

- **Inclusive reactions $\leftrightarrow$ forward matrix elements**

O. Nachtmann, Nucl. Phys. **B63** (1973) 237:
all target mass corrections are due to subtraction of traces in leading twist operators $\mathcal{O}_{\mu\mu_1\ldots\mu_N}$

- **Exclusive reactions $\leftrightarrow$ off-forward matrix elements**

in addition, there are contributions of higher-twist operators that reduce to total derivatives of the operators of leading twist:

$$\partial^2\mathcal{O}_{\mu\mu_1\ldots\mu_N} \qquad \text{and} \qquad \partial^\mu\mathcal{O}_{\mu\mu_1\ldots\mu_N}$$

S. Ferrara, A. F. Grillo, G. Parisi and R. Gatto, Phys. Lett. **B38**, 333 (1972):
matrix elements of $\partial^\mu\mathcal{O}_{\mu\mu_1\ldots\mu_N}$ over free quarks vanish

The same problem in a different language

- **Using EOM $\partial^\mu \mathcal{O}_{\mu\mu_1\dots\mu_N}$ can be expressed in terms of quark-gluon operators:**

$N = 1$:

$$\partial^\mu O_{\mu\nu} = 2i\bar{q} g G_{\nu\mu}\gamma^\mu q\,, \qquad O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overset{\leftrightarrow}{D}_\nu q + (\mu \leftrightarrow \nu)]$$

$N = 2$:

$$\frac{4}{5}\partial^\mu \mathcal{O}_{\mu\alpha\beta} = -12i\bar{q}\gamma^\rho \left\{ gG_{\rho\beta}\vec{D}_\alpha - \overset{\leftarrow}{D}_\alpha gG_{\rho\beta} + (\alpha\leftrightarrow\beta)\right\} q - 4\partial^\rho \bar{q}(\gamma_\beta g\widetilde{G}_{\alpha\rho} + \gamma_\alpha g\widetilde{G}_{\beta\rho})\gamma_5 q$$
$$- \frac{8}{3}\partial_\beta \bar{q}\gamma^\sigma \widetilde{G}_{\sigma\alpha}\gamma_5 q - \frac{8}{3}\partial_\alpha \bar{q}\gamma^\sigma \widetilde{G}_{\sigma\beta}\gamma_5 q + \frac{28}{3} g_{\alpha\beta}\partial_\rho \bar{q}\gamma^\sigma \widetilde{G}_{\sigma\rho} q\,,$$

where

$$\mathcal{O}_{\mu\alpha\beta} = \mathrm{Sym}_{\mu\alpha\beta}\left[\frac{15}{2}\bar{q}\gamma_\mu \overset{\leftrightarrow}{D}_\alpha \overset{\leftrightarrow}{D}_\beta\, q - \frac{3}{2}\partial_\alpha\partial_\beta \bar{q}\gamma_\mu q\right] - \text{traces}$$

etc.

- **How to distinguish between “kinematic” and genuine “dynamic” degrees of freedom?**
- **Are matrix elements of twist-four $\bar{q}Gq$ operators $\sim \Lambda^2_{QCD}$ or are they $\sim t \gg \Lambda^2_{QCD}$?**

Let G_{Nk} be the complete basis of twist-four operators

general structure of such relations:

$$(\partial\mathcal{O})_N = \sum_k a_k^{(N)}\, G_{Nk}$$

and simultaneously

$$T\{j(x)j(0)\}^{t=4} = \sum_{N,k} c_{N,k}(x)\, G_{Nk}$$

A separation of "kinematic" and "dynamic" contributions implies rewriting expansion of T-product in such a way that the particular combination appearing in $(\partial\mathcal{O})_N$ is separated from the "remainder".

The "kinematic" approximation would correspond to taking into account this term only, and neglecting contributions of "genuine" quark-gluon operators.

Guidung principle:

- **"Kinematic" and "Dynamic" contributions must have autonomous scale-dependence**

Let $\mathcal{G}_{N,k}$ be the set of *multiplicatively renormalizable* twist-four operators

$$\mathcal{G}_{N,k} = \sum_{k'} \psi^{(N)}_{k,k'}\, G_{N,k'}$$

- **One solution of the RG equations in known without calculation !**

$$(\partial\mathcal{O})_N = \sum_{k} a^{(N)}_{k}\, G_{Nk}$$

assume $(\partial O)_N$ corresponds to $k=0$, so $\mathcal{G}_{N,k=0} \equiv (\partial\mathcal{O})_N$ and $\psi^{(N)}_{k=0,k'} = a_{k'}$

Inverting the matrix of coefficients $\psi^{(N)}_{k,k'}$

$$G_{N,k} = \phi^{(N)}_{k,0}(\partial\mathcal{O})_N + \sum_{k'\neq 0} \phi^{(N)}_{k,k'}\, \mathcal{G}_{N,k'}$$

leading to

$$T\{j(x)j(0)\}^{\mathrm{tw}-4} = \sum_{N,k} c_{N,k}(x)\phi^{(N)}_{k,0}\,(\partial\mathcal{O})_N + \ldots$$

the ellipses stand for the contributions of “genuine” twist-four operators

The problem is that finding $\phi^{(N)}_{k,0}$ requires the knowledge of the full matrix $\psi^{(N)}_{k,k'}$, alias explicit solution of the twist-four RG equations.

Solution:

Bukhvostov, Frolov, Lipatov, Kuraev, Nucl. Phys. **B258** (1985) 601

- **Four-particle twist-4 operators have autonomous scale-dependence $\longrightarrow$ irrelevant**

Braun, Manashov, Rohrwild, Nucl. Phys. **B807** (2009) 89; Nucl. Phys. **B826** (2010) 235.

- **RG equations for three-particle (non-quasipartonic) operators are hermitian w.r.t. a certain scalar product**

Hence different solutions are mutually orthogonal w.r.t. a certain weight function:

$$\sum_k \mu_k^{(N)} \psi_{l,k}^{(N)} \psi_{m,k}^{(N)} \sim \delta_{l,m}$$

so that

$$\phi_{k,0}^{(N)} = a_k^{(N)} ||a^{(N)}||^{-2}, \qquad ||a^{(N)}||^2 = \sum_k \mu_k^{(N)} (a_k^{(N)})^2$$

and finally

$$T\{j(x)j(0)\}^{\text{tw}-4} = \sum_N \left(\sum_k \frac{c_{N,k}(x) a_k^{(N)}}{||a^{(N)}||^2} \right) (\partial\mathcal{O})_N + \text{dynamic higher twist}$$

Results: Time-ordered product of electromagnetic currents

$$S_{\mu\alpha\nu\beta} = g_{\mu\alpha} g_{\nu\beta} + g_{\nu\alpha} g_{\mu\beta} - g_{\mu\nu} g_{\alpha\beta}$$

$$\begin{aligned} i\,T\Big\{ j_\mu^{em}(x) j_\nu^{em}(0) \Big\} \quad = \quad & -\frac{1}{\pi^2 x^4} \Big\{ x^\alpha \Big[S_{\mu\alpha\nu\beta} \mathbb{V}^\beta + i\epsilon_{\mu\nu\alpha\beta} \mathbb{A}^\beta \Big] \\ & + x^2 \Big[(x_\mu \partial_\nu + x_\nu \partial_\mu) \mathbb{X} + (x_\mu \partial_\nu - x_\nu \partial_\mu) \mathbb{Y} \Big] \Big\} \end{aligned}$$

twist expansion:

$$\begin{aligned} \mathbb{V}_\beta &= \mathbb{V}_\beta^{t=2} + \mathbb{V}_\beta^{t=3} + \mathbb{V}_\beta^{t=4} + \ldots \\ \mathbb{A}_\beta &= \mathbb{A}_\beta^{t=2} + \mathbb{A}_\beta^{t=3} + \mathbb{A}_\beta^{t=4} + \ldots \\ \mathbb{X} &= \mathbb{X}^{t=4} + \ldots \\ \mathbb{Y} &= \mathbb{Y}^{t=4} + \ldots \end{aligned}$$

final answer:

$$\mathbb{V}_\mu^{t=4} = \frac{1}{2}\sum_{N,\text{odd}} \varkappa_N \frac{1}{(N+2)^2}\int_0^1 du\,(u\bar u)^{N+1}\Big\{x_\mu\,[(\partial\mathcal{O})_N^V(ux)]_{l.t.} + \frac{1}{2}N(N+3)\int_0^1 dv\,v^{N-1}\,x^2\partial_\mu\,[(\partial\mathcal{O})_N^V(uvx)]_{l.t.}\Big\}$$

$$\mathbb{A}_\mu^{t=4} = \frac{1}{4}x^2\partial_\mu \sum_{N,\text{even}} \varkappa_N \frac{N(N+3)}{(N+2)^2}\int_0^1 du\,(u\bar u)^{N+1}\int_0^1 dv\,v^{N-1}\,[(\partial\mathcal{O})_N^A(uvx)]_{l.t.}\,,$$

$$\mathbb{X}^{t=4} = \frac{1}{4}\sum_{N,\text{odd}} \varkappa_N \frac{N+1}{(N+2)^2}\int_0^1 du\,(u\bar u)^{N}(u-\bar u)\int_0^1 dv\,v^{N-1}\,[\partial\mathcal{O}_N^V(uvx)]_{l.t.}\,,$$

$$\mathbb{Y}^{t=4} = -\frac{1}{4}\sum_{N,\text{odd}} \varkappa_N \frac{N+1}{(N+2)^2}\int_0^1 du\,(u\bar u)^{N}(u^2+\bar u^2)\int_0^1 dv\,v^{N-1}\,[(\partial\mathcal{O})_N^V(uvx)]_{l.t.}\,.$$

$\varkappa_N = 2(2N+3)/(N+1)!$

$[\ldots]_{l.t.}$: leading-twist projection

Application to GPDs

$$P=(p+p')/2,\quad \Delta=p'-p$$

$$\langle p'|\mathcal{O}_N|p\rangle = \Big[\bar u(p')\not{n} u(p)\sum_{k=even}^{N} A_{N,k}(t)\Delta_+^k P_+^{N-k} + \frac{\bar u(p')u(p)}{m}\sum_{k=even}^{N+1} B_{N,k}(t)\Delta_+^k P_+^{N+1-k}\Big]_{l.t.}$$

$A_{N,k}(t)$, $B_{N,k}(t)$: generalized form factors; conformal moments of GPDs

$$\langle P'|(\partial\mathcal{O})_N|P\rangle = i\Big[\bar u(p')\not{n} u(p)\sum_{k=even}^{N+1} \widehat{A}_{N,k}(t)\Delta_+^{k-1} P_+^{N-k} + \frac{\bar u(p')u(p)}{m}\sum_{k=even}^{N+2} \widehat{B}_{N,k}(t)\Delta_+^{k-1} P_+^{N+1-k}\Big]_{l.t.}$$

$$\begin{aligned}
\widehat{A}_{N,k}(t) &= t\,A_{N,k}(t)\frac{k(2N+3-k)}{2(N+1)^2} - \left(m^2-\frac{t}{4}\right)A_{N,k-2}\frac{(N-k+2)(N-k+1)}{2(N+1)^2}\\
\widehat{B}_{N,k}(t) &= t\,B_{N,k}(t)\frac{k(2N+3-k)}{2(N+1)^2} - \left(m^2-\frac{t}{4}\right)B_{N,k-2}\frac{(N-k+3)(N-k+2)}{2(N+1)^2}\\
&\quad - \frac{m^2}{(N+1)^2}(N-k+2)A_{N,k-2}(t)
\end{aligned}$$

Conformal symmetry and $SU(1,1)$ scalar product

collinear conformal transformations $\quad SL(2,\mathbb{R}) \Leftrightarrow SU(1,1)$

$$x_\mu = z n_\mu\,, \qquad z \in \mathbb{R} \to z' = \frac{az+b}{cz+d}\,, \qquad \Leftrightarrow \qquad z \in \mathbb{C} \to z' = \frac{az+b}{\bar{b}z+\bar{a}}$$

representations are labeled by conformal spin

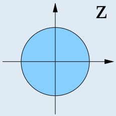
$$\varphi(z) \to T^j\varphi(z) = \frac{1}{(\bar{b}z+\bar{a})^{2j}}\varphi\left(\frac{az+b}{\bar{b}z+\bar{a}}\right)$$

This is a unitary transformation with respect to the following scalar product:

$$\langle\phi,\psi\rangle_j = \frac{2j-1}{\pi}\int_{|z|<1} d^2z\,(1-|z|^2)^{2j-2}\bar{\phi}(z)\psi(z) \equiv \int_{|z|<1}\mathcal{D}_j z\,\bar{\phi}(z)\psi(z)\,, \qquad ||\phi||^2 = \langle\phi,\phi\rangle$$

similar for several variables

$$\langle\phi,\psi\rangle_{j_1,j_2} = \int_{|z_1|<1}\mathcal{D}_{j_1}z_1\int_{|z_2|<1}\mathcal{D}_{j_2}z_2\,\bar{\phi}(z_1,z_2)\psi(z_1,z_2)$$



- different powers are orthogonal

$$\langle z^n, z^{n'} \rangle = \delta_{nn'} ||z^n||^2 , \qquad \langle (z_1 - z_2)^n, (z_1 - z_2)^{n'} \rangle = \delta_{nn'} ||z_{12}^n||^2$$

- reproducing operator (δ-function on a disc)

$$\phi(z) \quad = \quad \int_{|w|<1} \mathcal{D}_j w \, \mathcal{K}_j(z, \bar{w}) \phi(w) \, , \qquad \mathcal{K}_j(z, \bar{w}) = \frac{1}{(1 - z\bar{w})^{2j}}$$

- Fourier

$$\rho_N \iint_{|z_i|<1} \mathcal{D}_1 z_1 \mathcal{D}_1 z_2 \, (\bar{z}_1 - \bar{z}_2)^N e^{ip_1 z_1 + ip_2 z_2} \quad = \quad i^N (p_1 + p_2)^N C_N^{3/2} \left(\frac{p_1 - p_2}{p_1 + p_2} \right)$$

- conformal operator $O_+(z_1,z_2) = \bar\psi_+(z_1 n)\psi_+(z_2 n)$

$$\begin{aligned}\mathcal{O}_N &= (-\partial_+)^N \bar\psi_+ C_N^{3/2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right)\psi_+ = \rho_N \Big\langle (z_1-z_2)^N, O_+(z_1,z_2)\Big\rangle \\ \partial_+^k \mathcal{O}_N &= \rho_N \Big\langle (S_{12}^+)^k (z_1-z_2)^N, O_+(z_1,z_2)\Big\rangle \equiv \rho_N \Big\langle \Phi_{N,k}(z_1,z_2), O_+(z_1,z_2)\Big\rangle\end{aligned}$$

$S_{12}^+ = z_1^2\partial_{z_1} + z_2^2\partial_{z_2} + 2z_1 + 2z_2$

- The functions $\Phi_{Nk}(z_1,z_2)$ form an orthogonal basis

$$\Big\langle \Phi_{Nk}, \Phi_{N'k'}\Big\rangle = \delta_{NN'}\delta_{kk'}\,||\Phi_{Nk}||^2$$

so that the light-ray operator can be expanded as

$$O_{++}(z_1,z_2) = \sum_{N=0}^{\infty}\sum_{k=0}^{\infty} \rho_N^{-1}\frac{1}{||\Phi_{Nk}||^2}\Phi_{Nk}(z_1,z_2)\partial_+^k\mathcal{O}_N$$

Conformal basis for twist-four non-quasipartonic operators

Braun, Manashov, Rohrwild, Nucl. Phys. **B826** (2010) 235.

$$Q_1(z_1,z_2,z_3) = \bar\psi_+(z_1)f_{+-}(z_2)\psi_+(z_3)\,, \qquad T^{j=1}\otimes T^{j=1}\otimes T^{j=1}$$
$$Q_2(z_1,z_2,z_3) = \bar\psi_+(z_1)f_{++}(z_2)\psi_-(z_3)\,, \qquad T^{j=1}\otimes T^{j=3/2}\otimes T^{j=1/2}$$
$$Q_3(z_1,z_2,z_3) = \frac{1}{2}[D_{-+}\bar\psi_+](z_1)f_{++}(z_2)\psi_+(z_3)\,, \qquad T^{j=3/2}\otimes T^{j=3/2}\otimes T^{j=1}$$

and

$$\bar Q_1(z_1,z_2,z_3) = \bar\psi_+(z_1)\bar f_{+-}(z_2)\psi_+(z_3)\,, \qquad T^{j=1}\otimes T^{j=1}\otimes T^{j=1}$$
$$\bar Q_2(z_1,z_2,z_3) = \bar\psi_-(z_1)\bar f_{++}(z_2)\psi_+(z_3)\,, \qquad T^{j=1/2}\otimes T^{j=3/2}\otimes T^{j=1}$$
$$\bar Q_3(z_1,z_2,z_3) = \frac{1}{2}\bar\psi_+(z_1)\bar f_{++}(z_2)[D_{+-}\psi_+](z_3)\,, \qquad T^{j=1}\otimes T^{j=3/2}\otimes T^{j=3/2}\,.$$

C.f. in usual notation

$$\bar q_L(z_1)\left[F_{+\mu}(z_2)+i\widetilde{F}_{+\mu}(z_2)\right]\gamma^\mu q_L(z_3) = Q_2(z_1,z_2,z_3) - Q_1(z_1,z_2,z_3)$$
$$\bar q_L(z_1)\left[F_{+\mu}(z_2)-i\widetilde{F}_{+\mu}(z_2)\right]\gamma^\mu q_L(z_3) = \bar Q_2(z_1,z_2,z_3) - \bar Q_1(z_1,z_2,z_3)$$

$$\vec{Q}(z_1,z_2,z_3) = \begin{pmatrix} Q_1(z_1,z_2,z_3)\\ Q_2(z_1,z_2,z_3)\\ Q_3(z_1,z_2,z_3)\end{pmatrix} \qquad (1)$$

Kinematic projection operators

$$
\begin{aligned}
2(\partial\mathcal{O})_N &= \frac{ig\rho_N}{(N+1)^2}\Big[\langle\!\langle \overrightarrow{\Psi}_N, \overrightarrow{Q}\rangle\!\rangle - \langle\!\langle \overrightarrow{\bar{\Psi}}_N, \overrightarrow{\bar{Q}}\rangle\!\rangle\Big] + \dots \\
\langle\!\langle \overrightarrow{\Phi}, \overrightarrow{\Psi}\rangle\!\rangle &= 2\langle\Phi_1, \Psi_1\rangle_{111} + \langle\Phi_2, \Psi_2\rangle_{1\frac{3}{2}\frac{1}{2}} + \frac{1}{2}\langle\Phi_3, \Psi_3\rangle_{\frac{3}{2}\frac{3}{2}1}
\end{aligned}
$$

$$
\begin{aligned}
ig\overrightarrow{Q}(z_1, z_2, z_3) &= \sum_{N=0}^{\infty}\sum_{k=0}^{\infty}\frac{p_{Nk}(N+1)^2}{\rho_N\|\Psi_N\|^2}\,\overrightarrow{\Psi}_{Nk}(z_1, z_2, z_3)\,\partial_+^k(\partial\mathcal{O})_N + \dots \\
ig\overrightarrow{\bar{Q}}(z_1, z_2, z_3) &= -\sum_{N=0}^{\infty}\sum_{k=0}^{\infty}\frac{p_{Nk}(N+1)^2}{\rho_N\|\Psi_N\|^2}\,\overrightarrow{\bar{\Psi}}_{Nk}(z_3, z_2, z_1)\,\partial_+^k(\partial\mathcal{O})_N + \dots
\end{aligned}
$$

- All entries known explicitly
- The ellipses stand for “dynamic” operators

Outlook

- Done:

A theoretical framework for the calculation of finite t and target mass corrections in hard off-forward processes

- To do:
  - **Factorization of kinematic contributions to DVCS to twist-4 accuracy**
  - **Concrete predictions and applications to data analysis in DVCS, $\gamma^* \to \eta\gamma$**
  - **Flavor-singlet contributions, gluon transversity**
  - **An alternative derivation**