Transverse (Spin) Structure of Hadrons

Matthias Burkardt

New Mexico State University

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Outline

- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\boldsymbol{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\boldsymbol{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\boldsymbol{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs when the target is $\perp$ polarized

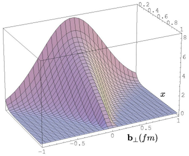
$\hookrightarrow$ Ji relation

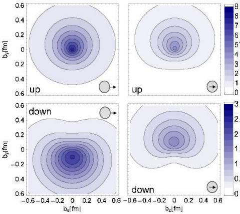
- Chromodynamik lensing and $\perp$ single-spin asymmetries (SSA)

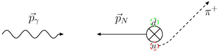
$$\left.\begin{array}{c}\text{transverse distortion of PDFs}\\ +\ \text{final state interactions}\end{array}\right\} \Rightarrow$$

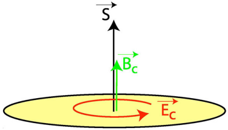
$\hookrightarrow$ SSA in $\gamma N \longrightarrow \pi + X$

- quark-gluon correlations $\rightarrow \perp$ force on q in DIS
- Summary







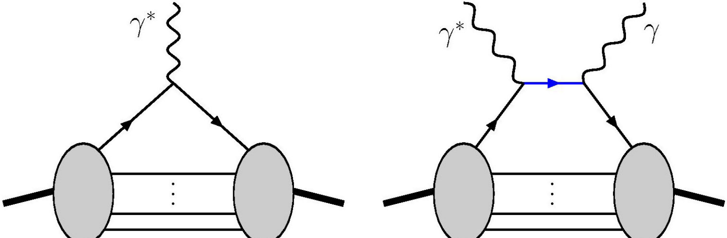


- virtual Compton scattering: $\gamma^* p \longrightarrow \gamma p$ (actually: $e^- p \longrightarrow e^- \gamma p$)
- 'deeply': $-q_\gamma^2 \gg M_p^2, |t| \longrightarrow$ Compton amplitude dominated by (coherent superposition of) Compton scattering off single quarks

$\hookrightarrow$ only difference between form factor (a) and DVCS amplitude (b) is replacement of photon vertex by two photon vertices connected by quark (energy denominator depends on quark momentum fraction x)

$\hookrightarrow$ DVCS amplitude provides access to momentum-decomposition of form factor = Generalized Parton Distribution (GPDs).

$$\int dx H_q(x, \xi, t) = F_1^q(t) \qquad \int dx E_q(x, \xi, t) = F_2^q(t)$$



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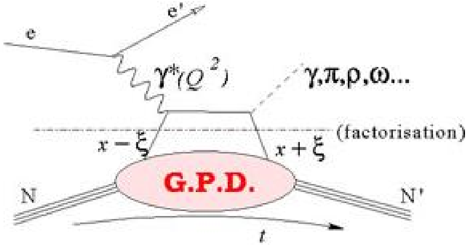


- form factors: $\stackrel{FT}{\longleftrightarrow} \rho(\vec{r})$
- $GPDs(x, \vec{\Delta})$: form factor for quarks with momentum fraction x

$\hookrightarrow$ suitable FT of $GPDs$ should provide spatial distribution of quarks with momentum fraction x

- careful: cannot measure longitudinal momentum (x) and longitudinal position simultaneously (Heisenberg)

$\hookrightarrow$ consider purely transverse momentum transfer

Impact Parameter Dependent Quark Distributions

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, \xi = 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

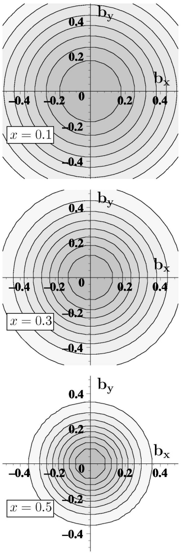
$q(x, \mathbf{b}_\perp)$ = parton distribution as a function of the separation $\mathbf{b}_\perp$ from the transverse center of momentum $\mathbf{R}_\perp \equiv \sum_{i\in q,g} \mathbf{r}_{\perp,i} x_i$
MB, Phys. Rev. D62, 071503 (2000)

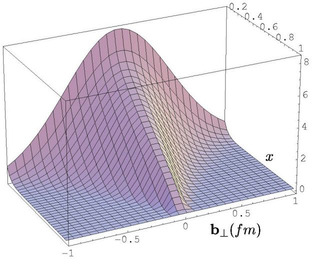
- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free of relativistic corrections

- probabilistic interpretation

$q(x, \mathbf{b}_\perp)$ for unpol. p



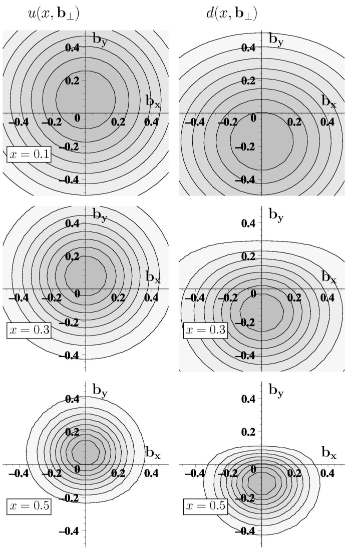


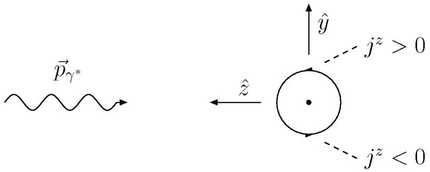
unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$
- x = momentum fraction of the quark
- $\vec{b} = \perp$ distance of quark from $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$

Impact parameter dependent quark distributions





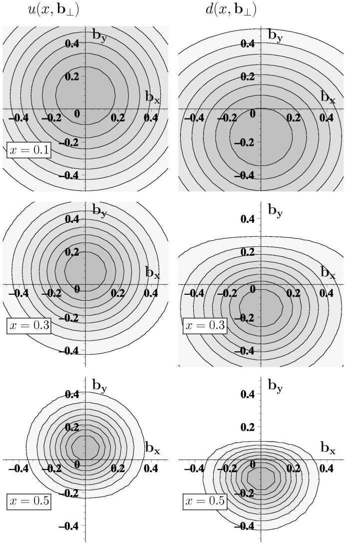
proton polarized in $+\hat{x}$ direction

no axial symmetry!

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$-\frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3



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$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$
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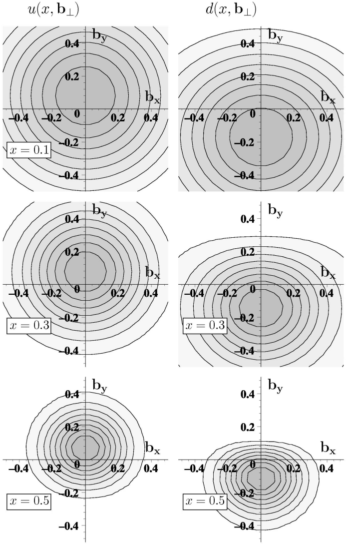
sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x,\mathbf{b}_\perp) b_y$$
$$= \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q}{2M}$$

Impact parameter dependent quark distributions



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$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + \ldots$

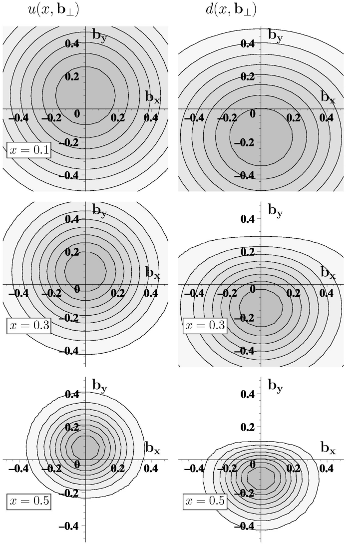
- u-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

$\hookrightarrow$ shift in $+\hat{y}$ direction

- d-quarks: $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$

$\hookrightarrow$ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}(\pm 0.2 fm)$!!!!



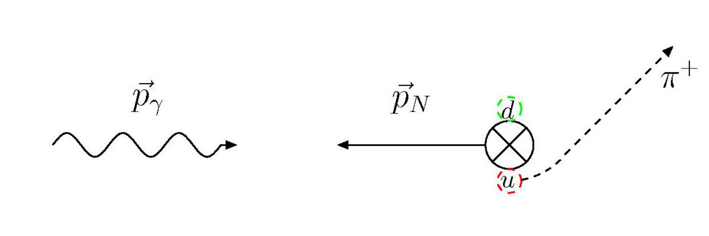
anomalous gravito-magnetic moment

- $B(0) \equiv \sum_{i \in q,g} B_i(0) = 0$ (→ S.J.Brodsky)
- $B_i(0)$ = shift of CoM for flavor i in $\perp$ pol. nucleon relative to nucleon CoM

↪ $B(0) = 0 \quad \leftrightarrow \quad$ CoM of quarks and gluons sum up to CoM of nucleon

- lattice (LHPC, QCDSF) u and d quarks seem to almost saturate $\sum_{i \in q,g} B_i(0) = 0$

example: semi-inclusive deep-inelastic scattering (SIDIS) $\gamma p \to \pi X$

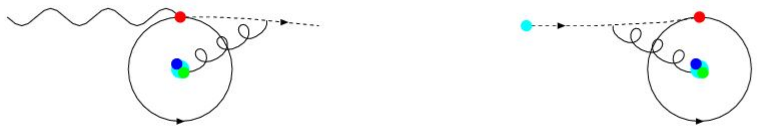


- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign 'determined' by κ_u & κ_d
- attractive FSI deflects active quark towards the CoM

↪ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction → 'chromodynamic lensing'

⇒ $\kappa_p\,,\kappa_n \quad \longleftrightarrow \quad$ sign of SSA!!!!!!!! (MB,2004)

- confirmed by HERMES (and recent COMPASS) p data; consistent with vanishing isoscalar Sivers (COMPASS) compare FSI for 'red' q that is being knocked out of nucleon with ISI for 'anti-red' $\bar{q}$ that is about to annihilate with a 'red' target q



a) b)

FSI in SIDIS

- knocked-out q 'red'

$\hookrightarrow$ spectators 'anti-red'

$\hookrightarrow$ interaction between knocked-out quark and spectators attractive

ISI in DY

- incoming $\bar{q}$ 'anti-red'

$\hookrightarrow$ struck target q 'red'

$\hookrightarrow$ spectators also 'anti-red'

$\hookrightarrow$ interaction between incoming $\bar{q}$ and spectators repulsive

test of $f_{1T}^{\perp}(x, \mathbf{k}-\perp)_{DY} = -f_{1T}^{\perp}(x, \mathbf{k}-\perp)_{SIDIS}$ critical test of TMD factorization approach

Total (Spin+Orbital) Quark Angular Momentum

$$J_q^x = L_q^x + S_q^x = \int d^3r \left[y T_q^{0z}(\vec{r}) - z T_q^{0y}(\vec{r}) \right]$$

- $T_q^{\mu\nu}(\vec{r})$ energy momentum tensor $(T_q^{\mu\nu}(\vec{r}) = T_q^{\nu\mu}(\vec{r}))$
- $T_q^{0i}(\vec{r})$ momentum density $[P_q^i = \int d^3r T_q^{0i}(\vec{r})\ ]$
- think: $(\vec{r} \times \vec{p})^x = yp^z - zp^y$

relate to impact parameter dependent quark distributions $q(x, \mathbf{r}_\perp)$:

Consider spherically symmetric wave packet with nucleon polarized in $+\hat{x}$ direction

- eigenstate under rotations about x-axis

$\hookrightarrow$ both terms in J_q^x equal:
$J_q^x = 2\int d^3r\, y T_q^{0z}(\vec{r}) = \int d^3r\, y \left[T_q^{0z}(\vec{r}) + T_q^{z0}(\vec{r}) \right]$

- $\int d^3r\, y T_q^{00}(\vec{r}) = 0 = \int d^3r\, y T_q^{zz}(\vec{r})$

$$\Rightarrow \quad J_q^x = \int d^3r\, y T_q^{++}(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

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- $\int dx\, xq(x,\mathbf{r}_\perp) = \frac{1}{2m_N}\int dz T^{++}(\vec{r})$
(note: here x is momentum fraction and not r^x)

$\hookrightarrow$ $J_q^x = m_N \int dx\, xr^y q(x,\mathbf{r}_\perp)$

- before applying this result to $\perp$ shifted PDFs, need to consider 'overall $\perp$ shift' of CoM for $\perp$ polarized target...

relate to impact parameter dependent quark distributions $q(x,\mathbf{b}_\perp)$:

- Thus $J_q^x = m_N \int dx\, x r^y q(x,\mathbf{r}_\perp)$ with $b^y = r^y - \frac{1}{2m_N}$, where $q(x,\mathbf{r}_\perp)$ is distribution relative to CoM of whole nucleon
- recall: $q(x,\mathbf{b}_\perp)$ for nucleon polarized in $+\hat{x}$ direction

$$\begin{aligned} q(x,\mathbf{b}_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} \\ &\quad - \frac{1}{2M_N}\frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} \end{aligned}$$

$$\begin{aligned} \Rightarrow J_q^x &= M_N \int dx\, x r^y q(x,\mathbf{r}_\perp) = \int dx\, x \left(m_N b^y + \frac{1}{2}\right) q(x,\mathbf{r}_\perp) \\ &= \frac{1}{2}\int dx\, x \left[H(x,0,0) + E(x,0,0)\right] \end{aligned}$$

relate to impact parameter dependent quark distributions $q(x, \mathbf{b}_\perp)$:

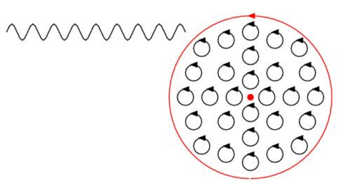
- Thus $J_q^x = m_N \int dx\, x r^y q(x, \mathbf{r}_\perp)$ with $b^y = r^y - \frac{1}{2m_N}$, where $q(x, \mathbf{r}_\perp)$ is distribution relative to CoM of whole nucleon
- recall: $q(x, \mathbf{b}_\perp)$ for nucleon polarized in $+\hat{x}$ direction

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} - \frac{1}{2M_N} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$\Rightarrow J_q^x = M_N \int dx\, x r^y q(x, \mathbf{r}_\perp) = \int dx\, x \left(m_N b^y + \frac{1}{2} \right) q(x, \mathbf{r}_\perp) = \frac{1}{2} \int dx\, x \left[H(x, 0, 0) + E(x, 0, 0) \right]$$

- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- partonic interpretation exists only for $\perp$ components!

q with polarization $\odot$

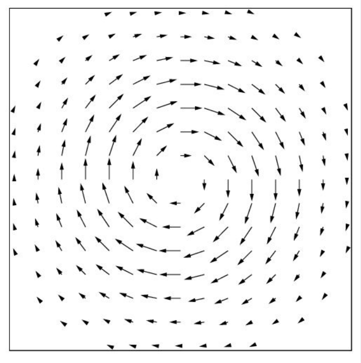


↪ counterclockwise current from lower component

↪ q distribution shifted to top

unpolarized target

- all q polns. equally likely



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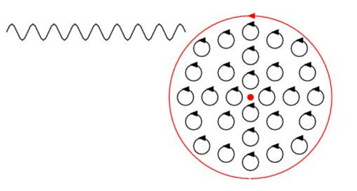
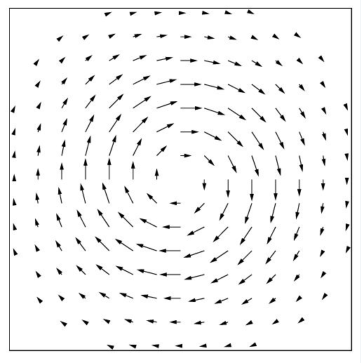


↪ counterclockwise current from lower component

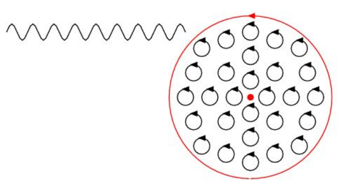
↪ q distribution shifted to top

unpolarized target

- q with pol. $\uparrow$ shifted to left



q with polarization $\odot$

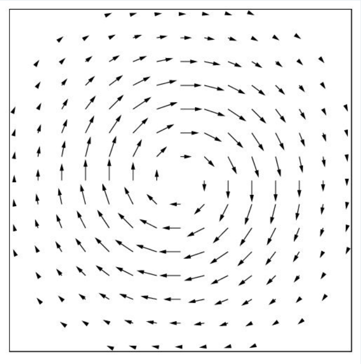


↪ counterclockwise current from lower component

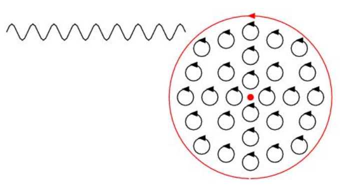
↪ q distribution shifted to top

unpolarized target

- q with pol. $\downarrow$ shifted to right



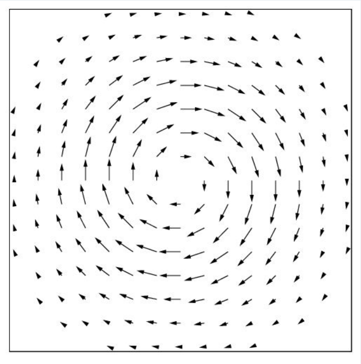
q with polarization $\odot$



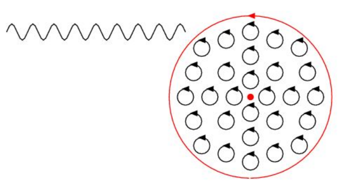
- ↪ counterclockwise current from lower component
- ↪ q distribution shifted to top

unpolarized target

- q with pol. $\rightarrow$ shifted to top



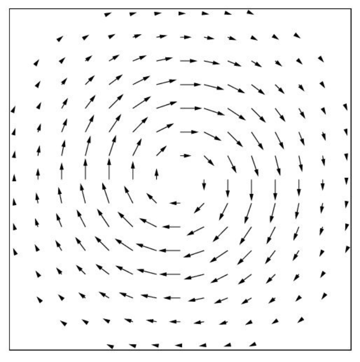
q with polarization $\odot$



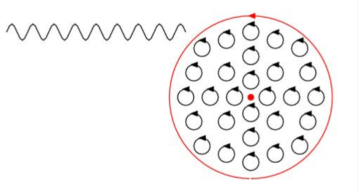
- ↪ counterclockwise current from lower component
- ↪ q distribution shifted to top

unpolarized target

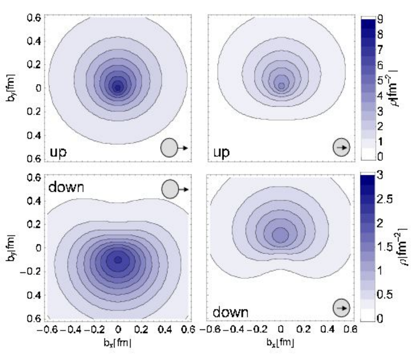
- q with pol. $\leftarrow$ shifted to bottom



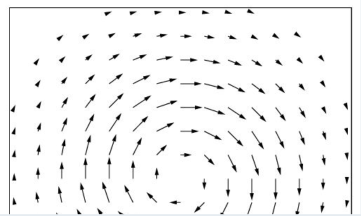
q with polarization $\odot$



lattice calculations (QCDSF)



unpolarized target



- transversity distribution in unpol. target described by chirally odd GPD $\bar{E}_T$
- $\bar{E}_T > 0$ for u & d (QCDSF)
- connection $h_1^\perp(x, \mathbf{k}_\perp) \leftrightarrow \bar{E}_T$ similar to $f_{1T}^\perp(x, \mathbf{k}_\perp) \leftrightarrow E$.

$\hookrightarrow$ $h_1^\perp(x, \mathbf{k}_\perp) < 0$ for u/p, d/p, u/π, $\bar{d}/\pi$, ..(MB+BH, 2008)

- different valence quarks add coherently $|h_1^\perp| > |f_1^\perp|$ (MB+BH; Musch)

higher twist in polarized DIS

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $g_1 = \frac{1}{2}\sum_q e_q^2 g_1^q$ with $g_1^q = q^\uparrow(x) + \bar{q}^\uparrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x)$
- g_2 involves quark-gluon correlations

↪ no parton interpret. as difference between number densities for g_2

- for ⊥ pol. target, g_1 & g_2 contribute equally

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

↪ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- What can one learn from g_2?
- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3\int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

$$d_2 \equiv 3\int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+q(0)\right|P,S\right\rangle$$

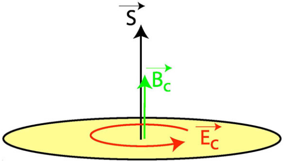
$$\sqrt{2}G^{+y} = G^{0y} + G^{zy} = -E^y + B^x$$

$\hookrightarrow$ write $d_2 = \frac{1}{4}\left(\chi_E + 2\chi_M\right)$ with

$$2M^2\vec{S}\chi_E = \left\langle P,S\left|\vec{j}_a \times \vec{E}_a\right|P,S\right\rangle$$

$$2M^2\vec{S}\chi_B = \left\langle P,S\left|j_a^0 \times \vec{B}_a\right|P,S\right\rangle$$

called color-electric and magnetic 'polarizabilities'



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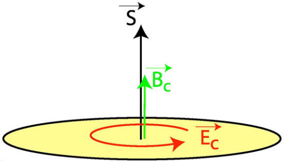
$$\sqrt{2} G^{+y} = G^{0y} + G^{zy} = -E^y + B^x$$

$\hookrightarrow$ write $d_2 = \frac{1}{4}\left(\chi_E + 2\chi_M\right)$ with

$$2M^2 \vec{S} \chi_E = \left\langle P, S \left| \vec{j}_a \times \vec{E}_a \right| P, S \right\rangle$$

$$2M^2 \vec{S} \chi_B = \left\langle P, S \left| j_a^0 \times \vec{B}_a \right| P, S \right\rangle$$

called color-electric and magnetic 'polarizabilities'



names misleading:

these are **not** the response of the nucleon due to an applied color field!

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

$$\sqrt{2} G^{+y} = G^{0y} + G^{zy} = -E^y + B^x$$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

$$\sqrt{2} G^{+y} = G^{0y} + G^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

↪ $d_2 \leftrightarrow$ average **color Lorentz force** acting on quark moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by γ^*

$$\langle F^y \rangle = -2M^2 d_2 = -\frac{M}{P^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

cf. Qiu-Sterman matrix element $\langle k_\perp^y \rangle \equiv \int_0^1 dx \int d^2 k_\perp\, k_\perp^2 f_{1T}^\perp(x, k_\perp^2)$

$$\langle k_\perp^y \rangle = -\frac{1}{2p^+} \left\langle P,S \left| \bar{q}(0) \int_0^\infty dx^- g G^{+y}(x^-) \gamma^+ q(0) \right| P,S \right\rangle$$

semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

matrix element defining d_2 $\leftrightarrow$ 1^{st} integration point in QS-integral

color Lorentz force

$d_2 \leftrightarrow$ average **color Lorentz force** acting on quark moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by γ^*

$$\langle F^y \rangle = -2M^2 d_2 = -\frac{M}{P^{+2} S^x} \left\langle P, S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P, S \right\rangle$$

sign of d_2

- $\kappa_q / p \longrightarrow$ sign of deformation

↪ direction of average force

↪ $d_2^u > 0$, $d_2^d < 0$

- *cf.* $f_{1T}^{\perp u} < 0$, $f_{1T}^{\perp u} < 0$

magnitude of d_2

- $\langle F^y \rangle = -2M^2 d_2 = -10 \frac{GeV}{fm} d_2$
- expect partial cancellation of forces in SSA

↪ $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm}$

↪ $d_2 = \mathcal{O}(0.01)$

lattice (Göckeler et al., 2005)

$d_2^u \approx 0.010$, $d_2^d \approx -0.0056$

please: up to date lattice calcs.

color Lorentz force

$e_2 \leftrightarrow$ average **color Lorentz force** (in $\hat{y}$-direction) acting on quark (with transversity $\hat{x}$) moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by γ^*

$$\langle F^y \rangle = M^2 e_2 \equiv M^2 \int_0^1 dx \bar{e}_2(x) = \frac{M}{4P^{+2}} \sum_{i=1,2} \left\langle P \left| \bar{q}(0) g G^{+i}(0) \sigma^{+i} q(0) \right| P \right\rangle$$

chirally even

- GPD $E_q \Rightarrow \mathbf{b}_\perp$ deformation of unpol. q distr. in $\perp$ pol. target

$\hookrightarrow$ $f_{1T}^\perp$

$\hookrightarrow$ $d_2 \equiv \int dx x^2 \bar{g}_2$ force

chirally odd

- GPD $\bar{E}_T \Rightarrow \mathbf{b}_\perp$ deformation of quarks with transversity in unpol. target

$\hookrightarrow$ $h_1^\perp$

$\hookrightarrow$ $e_2 \equiv \int dx x^2 \bar{e}_2$ force

lattice (Göckeler et al., 2005)

$d_2^u \approx 0.010$, $d_2^d \approx -0.0056$

please: up to date lattice calcs.

lattice

B.Musch ...

- Deeply Virtual Compton Scattering (DVCS) $\longrightarrow$ GPDs

$\hookrightarrow$ impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$

- $E^q(x, 0, -\boldsymbol{\Delta}_\perp^2) \leftrightarrow \kappa_{q/p}$ (contribution from quark flavor q to anomalous magnetic moment)
- $E^q(x, 0, -\boldsymbol{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- $\perp$ deformation $\leftrightarrow$ (sign of) SSA (Sivers; Boer-Mulders)
- parton interpretation for Ji-relation
- $L_q \neq \mathcal{L}_q$
- higher-twist ($\int dx\, x^2 \bar{g}_2(x)$, $\int dx\, x^2 \bar{e}(x)$) $\leftrightarrow \perp$ force in DIS
- $\perp$ deformation $\leftrightarrow$ (sign of) quark-gluon correlations ($\int dx\, x^2 \bar{g}_2(x)$, $\int dx\, x^2 \bar{e}(x)$)

first: QED without electrons

- apply $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{b}(\vec{a} \cdot \vec{c})$ to $\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)$

$$\begin{aligned} \vec{J} &= \int d^3r\, \vec{x} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{x} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right] \\ &= \int d^3r \left[E^j \left(\vec{x} \times \vec{\nabla}\right) A^j - \vec{x} \times (\vec{E} \cdot \vec{\nabla})\vec{A}\right] \end{aligned}$$

- integrate by parts (drop surface term)

$$\vec{J} = \int d^3r \left[E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \left(\vec{x} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]$$

- drop 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = 0$), yielding $\vec{J} = \vec{L} + \vec{S}$ with

$$\vec{L} = \int d^3r\, E^j \left(\vec{x} \times \vec{\nabla}\right) A^j \qquad \vec{S} = \int d^3r\, \vec{E} \times \vec{A}$$

- note: $\vec{L}$ and $\vec{S}$ not separately gauge invariant

- treat FSI to lowest order in g

$\hookrightarrow$
$$\left\langle k_q^i \right\rangle = -\frac{g}{4p^+}\int \frac{d^2\mathbf{b}_\perp}{2\pi}\frac{b^i}{\left|\mathbf{b}_\perp\right|^2}\left\langle p,s\left|\bar{q}(0)\gamma^+\frac{\lambda_a}{2}q(0)\rho_a(\mathbf{b}_\perp)\right|p,s\right\rangle$$

with $\rho_a(\mathbf{b}_\perp) = \int dr^- \rho_a(r^-,\mathbf{b}_\perp)$ summed over all quarks and gluons

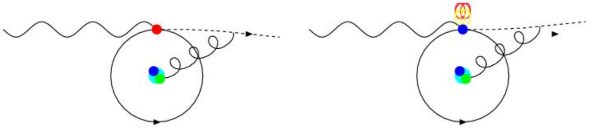
$\hookrightarrow$ SSA related to dipole moment of parton density-density correlations

- similar density-density correlations as appear in multiparton correlations ($\rightarrow$ M.Diehl)
- GPDs (N polarized in $+\hat{x}$ direction): $u \longrightarrow +\hat{y}$ and $d \longrightarrow -\hat{y}$

$\hookrightarrow$ expect density density correlation to show same asymmetry $\langle b^y \bar{u}(0)\gamma^+ \frac{\lambda_a}{2} u(0)\rho_a(\mathbf{b}_\perp)\rangle > 0$

$\hookrightarrow$ sign of SSA opposite to sign of distortion in position space

'Chromodynamic lensing' mechanism for $\perp$ SSA requires long range coherence of color field!



before 'dressing' active quark 'dressed' with glue

QCD-evolution: long-range color decoherence:

- after 'dressing' itself with a gluon, previously red quark more likely to be blue or green

$\hookrightarrow$ attraction to far-away spectators mostly gone

- only attracted to close-by (high Q^2) g from dressing
- high Q^2: q at low x likely to have dressed itself with perturbative gluon!

$\hookrightarrow$ 'Chromodynamic lensing' mechanism suppressed for high Q^2 & small x?

sea quarks

- consider pert. $\bar{q}$ from splitting of *glue*
- $f_{1T,g}^{\perp}$ expected to be small due to sum rule

$$\sum_{i\in q,g}\int dx \int d^2\mathbf{k}_\perp \mathbf{k}_\perp^2 f_{1T}^{\perp,i}(x,\mathbf{k}_\perp) = 0$$

- expect $f_{1T,\bar{q}}^{\perp}$ also to be small for pert. $\bar{q}$

spherically symmetric wave packet has center of momentum off-center:

- relativistic effect $\longrightarrow$ use Dirac wave packet for nucleon

$$\psi = \begin{pmatrix} f(r) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M_N} f(r) \end{pmatrix} \chi \qquad \text{with} \qquad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\int d^3r f^2(r) = 1$, take limit of large 'radius' for wave packet

- evaluate $T_q^{0z} = \frac{i}{2}\bar{q}\left(\gamma^0\partial^z + \gamma^z\partial^0\right) q$ in this state
- $\psi^\dagger \partial_z \psi$ even under $y \to -y$, i.e. no contribution to $\langle y T_q^{0z} \rangle$
- use $i\psi^\dagger\gamma^0\gamma^z\partial^0\psi = E\psi^\dagger\gamma^0\gamma^z\psi$

$$\begin{aligned}
\langle T^{0z} y \rangle &= E\int d^3r \psi^\dagger \gamma^0\gamma^z \psi y = E\int d^3r \psi^\dagger \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix} \psi y \\
&= \frac{2E}{E+M_N}\int d^3r \chi^\dagger \sigma^z \sigma^y \chi f(r)(-i)\partial^y f(r) y \\
&= \frac{E}{E+M_N}\int d^3r f^2(r) \stackrel{R\to\infty}{\longrightarrow} \frac{1}{2}
\end{aligned}$$

$\hookrightarrow$ p pol. in $+\hat{x}$ direction has CoM shifted by $\frac{1}{2M_N}$ in $+\hat{y}$ direction!

spherically symmetric wave packet has center of momentum off-center:

- relativistic effect $\longrightarrow$ use Dirac wave packet for nucleon

$$\psi = \begin{pmatrix} f(r) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M_N} f(r) \end{pmatrix} \chi \qquad \text{with} \qquad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

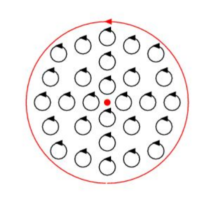
$$\langle T^{0z} y \rangle \stackrel{R\to\infty}{\longrightarrow} \frac{1}{2}$$

$\hookrightarrow$ p pol. in $+\hat{x}$ direction has CoM shifted by $\frac{1}{2M_N}$ in $+\hat{y}$ direction!

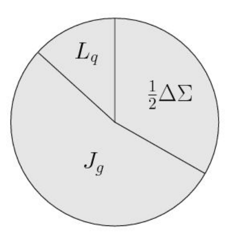
origin of 'shift' of CoM

- nucleon polarization: $\odot$
- counterclockwise momentum density from lower component
- $p \sim \frac{1}{R}$, but $y \sim R$

$\hookrightarrow$ $\langle T^{++} y \rangle = \mathcal{O}(1)$



Ji decomposition



'pizza tre stagioni'

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S| \, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \, |P,S\rangle$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Ji decomposition

- Δq from polarized DIS
- $J_q \equiv \frac{1}{2}\Delta q + L_q = \frac{1}{2}\int_0^1 dx \, [H_q(x,0,0) + E_q(x,0,0)]$ from DVCS
- J_g in principle from gluon-GPDs; in practice $J_g = \frac{1}{2} - J_q$
- spin 1 $\longrightarrow$ Goldstein, Liuti, ..

$$\begin{aligned}
\vec{J}_\gamma &= \int d^3r\, \vec{r} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{r} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right] \\
&= \int d^3r\, \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla})\vec{A}\right] \\
&= \int d^3r\, \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j + \left(\vec{r} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]
\end{aligned}$$

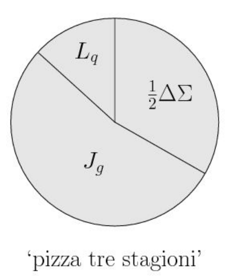
- replace 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \vec{E} \times \vec{A}\right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

↪ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

Ji decomposition



$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

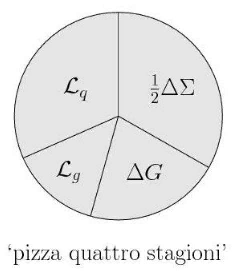
$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S|\, q^\dagger(\vec{x})\Sigma^3 q(\vec{x})\, |P,S\rangle$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x})\left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x})|P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe decomposition



light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S|\, \bar{q}(\vec{r})\gamma^+\left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r})|P,S\rangle$$

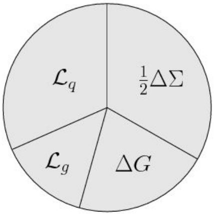
$$\Delta G = \varepsilon^{+-ij}\int d^3r \, \langle P,S|\, \mathrm{Tr} F^{+i} A^j \,|P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S| \mathrm{Tr} F^{+j}\left(\vec{x} \times i\vec{\partial}\right)^z A^j |P,S\rangle$$

Jaffe decomposition

- Δq from polarized DIS
- ΔG from $\Delta g(x)$ $(\vec{p}\overleftarrow{p}$ & $\frac{d}{d\ln Q^2}\Delta q(x))$
- ΔG gauge invariant! Nonlocal for $A^+ \neq 0$
- no exp./lattice access to $\mathcal{L}_q$, $\mathcal{L}_g$
- only $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q$, by subtraction $\mathcal{L} = \frac{1}{2} - \Delta G - \sum_q \frac{1}{2}\Delta q$

Jaffe decomposition



'pizza quattro stagioni'

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S|\, \bar{q}(\vec{r})\gamma^+ \left(\vec{r}\times i\vec{\partial}\right)^z q(\vec{r})|P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij}\int d^3r\, \langle P,S|\, \mathrm{Tr} F^{+i} A^j\, |P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S|\mathrm{Tr} F^{+j}\left(\vec{x}\times i\vec{\partial}\right)^z A^j|P,S\rangle$$

- L_q matrix element of

$$q^\dagger \left[\vec{r}\times\left(i\vec{\partial}-g\vec{A}\right)\right]^z q = \bar{q}\gamma^0\left[\vec{r}\times\left(i\vec{\partial}-g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0+\gamma^z)$

$$\bar{q}\gamma^+\left[\vec{r}\times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- (for $\vec{p}=0$) matrix element of $\bar{q}\gamma^z\left[\vec{r}\times\left(i\vec{\partial}-g\vec{A}\right)\right]^z q$ vanishes (parity!)

↪ L_q identical to matrix element of $\bar{q}\gamma^+\left[\vec{r}\times\left(i\vec{\partial}-g\vec{A}\right)\right]^z q$ (nucleon at rest)

↪ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger\left(\vec{r}\times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger\left(r^x gA^y - r^y gA^x\right)q\Big|_{A^+=0}$

scalar diquark model

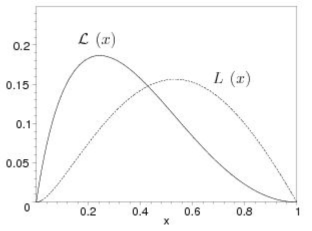
- 'mother functions' $\psi_s^S(x, \mathbf{k}_\perp)$

$\hookrightarrow$ $\mathcal{L}_q$ from $|\psi_s^S(x, \mathbf{k}_\perp)|^2$

- GPDs from overlap integrals of $\psi^\dagger \psi$

$\hookrightarrow$ L_q from Ji

- $L_q = \mathcal{L}_q$.
  No surprise since $L_q - \mathcal{L}_q \sim \langle q^\dagger \vec{r} \times \vec{A} q \rangle$ and no $\vec{A}$ in scalar diquark model
- $L_q(x) \neq \mathcal{L}_q(x)$



M.B. + Hikmat BC,
PRD **79**, 071501 (2009)

QED for dressed e^- in QED

- 'mother functions' $\psi_{sh}^S(x, \mathbf{k}_\perp)$

$\hookrightarrow$ $\mathcal{L}_q$ from $|\psi_{sh}^S(x, \mathbf{k}_\perp)|^2$

- GPDs from overlap integrals of $\psi^\dagger \psi$

$\hookrightarrow$ L_q from Ji

- $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$
- Wakamatsu-pizza?
- Goldstein-pizza?

Bakker Leader Trueman

Bakker Leader Trueman

$\perp$ pol. nucleon at $p \to \infty$

$$J_q^x = L_q^x + S_q^x = \int d^3r \left[yT_q^{0z}(\vec{r}) - zT_q^{0y}(\vec{r}) \right]$$

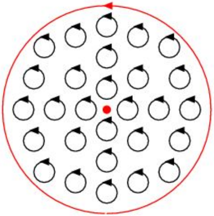
BIG small

$\hookrightarrow$ p pol. in $+\hat{x}$ direction has CoM shifted by $\frac{1}{2M_N}$ in $+\hat{y}$ direction!

- for wave packet centered around origin: $J^x \sim \frac{1}{2M_N} P_N \longrightarrow \infty$

$\hookrightarrow$ subtract that infinity

- quark flavor q has CoM shifted by $\frac{1}{2M_N} \int dx x E(x_q, 0, 0)$
- $J_q^x \sim \frac{1}{2M_N} \int dx x E(x_q, 0, 0) P_N \longrightarrow \infty$



now: QED with electrons

- start from what X.Ji would call $\vec{J}_\gamma$:

$$\vec{J}_\gamma = \int d^3r\, \vec{x} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{x} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right]$$

- integrate by parts

$$\vec{J} = \int d^3r \left[E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \left(\vec{x} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]$$

- replace 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \vec{E} \times \vec{A}\right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

↪ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

Ji decomposition

$$\vec{J} = \int d^3x \left[\psi^\dagger \vec{\Sigma}\psi + \psi^\dagger \vec{x} \times \left(i\vec{\partial} - g\vec{A}\right)\psi + \vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]$$

with $\Sigma^i = \frac{i}{2}\varepsilon^{ijk}\gamma^j\gamma^k$

- Ji does not integrate gluon term by parts, nor identify gluon spin/OAM separately
- Ji-decomposition valid for all three components of $\vec{J}$, but usually only applied to $\hat{z}$ component, where the quark spin term has a partonic interpretation

(+) all three terms manifestly gauge invariant

(+) DVCS can be used to probe $\vec{J}_q = \vec{S}_q + \vec{L}_q$

(-) quark OAM contains interactions

(-) only quark spin has partonic interpretation as a single particle density

Jaffe/Manohar decomposition

- in light-cone framework & light-cone gauge $A^+ = 0$ one finds for $J^z = \int dx^- d^2\mathbf{r}_\perp M^{+xy}$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

where $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\begin{aligned}
\mathcal{L}_q &= \int d^3r \, \langle P,S| \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) \, |P,S\rangle \\
\Delta G &= \varepsilon^{+-ij} \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle \\
\mathcal{L}_g &= 2 \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j \, |P,S\rangle
\end{aligned}$$

Jaffe/Manohar decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

- $\Delta\Sigma = \sum_q \Delta q$ from polarized DIS (or lattice)
- ΔG from $\vec{p}\overleftarrow{p}$ or polarized DIS (evolution)

↪ ΔG gauge invariant, but local operator only in light-cone gauge

- $\int dx x^n \Delta G(x)$ for $n \geq 1$ *can* be described by manifestly gauge inv. local op. ($\longrightarrow$ lattice)
- $\mathcal{L}_q$, $\mathcal{L}_g$ independently defined, but
  - no exp. identified to access them
  - not accessible on lattice, since nonlocal except when $A^+ = 0$
- parton net OAM $\mathcal{L} = \mathcal{L}_g + \sum_q \mathcal{L}_q$ by subtr. $\mathcal{L} = \frac{1}{2} - \frac{1}{2}\Delta\Sigma - \Delta G$
- in general, $\mathcal{L}_q \neq L_q$ $\qquad$ $\mathcal{L}_g + \Delta G \neq J_g$
- makes no sense to 'mix' Ji and JM decompositions, e.g. $J_g - \Delta G$ has no fundamental connection to OAM

What is Orbital Angular Momentum

- light-cone wave function in $e\gamma$ Fock component

$$\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\frac{k^1 - ik^2}{x(1-x)}\phi \qquad \Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp) = -\sqrt{2}\frac{k^1 + ik}{1-x}$$

$$\Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\left(\frac{m}{x} - m\right)\phi \qquad \Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = 0$$

- OAM of e^- according to Jaffe/Manohar
  $\mathcal{L}_e = \int_0^1 dx \int d^2\mathbf{k}_\perp \left[(1-x)\left|\Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp)\right|^2 - \left|\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp)\right|^2\right]$
- e^- OAM according to Ji $L_e = \frac{1}{2}\int_0^1 dx\, x\left[q(x) + E(x,0,0)\right] - \frac{1}{2}\Delta q$

$\rightsquigarrow$ $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$

- Likewise, computing J_γ from photon GPD, and $\Delta\gamma$ and $\mathcal{L}_\gamma$ from light-cone wave functions and <u>defining</u> $\hat{L}_\gamma \equiv J_\gamma - \Delta\gamma$ yields
  $\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma$
- $\frac{\alpha}{4\pi}$ appears to be small, but here $\mathcal{L}_e$, L_e are all of $\mathcal{O}(\frac{\alpha}{\pi})$